THE BEST TYPE OF INDEX NUMBER

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The formulae presented in this study are the ratio of the two value aggregates measured in real terms. Such a formula is called a quantity or volume index. One of its main properties is that when multiplied by an appropriate price index, the product represents a value index.

One can derive many formulae to arrive at such ratios by giving different averages and different weights. Because of the different possible types of averages and weights there is a problem of statistical technique to be solved in the choice of formulae.[1]

In practice one can not achieve direct measurement of the concepts involved, and it is necessary to rely on indirect means.[2]

There are two ways in which indexes can be approached. The atomistic approach characterizes the usual kind of indexes in which the different variables (e.g. prices and quantities) are considered independent. In the functional approach certain characteristic relations are assumed to exist between prices and quantities.[3] These two approaches have been called by Samuelson (1947) "The Statistical Theory of Index Numbers" and "The Economic Theory of Index Numbers" respectively.[4]

The economic theory of index numbers is largely confined to "cost of living" comparisons, while the statistical theory of index numbers is more concerned with general purpose index numbers (e.g. wholesale price indexes, export import price and volume indexes and production indexes). For limitation of time, the scope of this paper has been confined to the statistical theory of index numbers is more concerned with general purpose index numbers(e.g. wholesale price indexes, export-import price and volume indexes and production indexes).For limitation of time,the scope of this paper has been confined to the statistical theory of index numbers.As has been discussed above, the present study is confined to the classical statistical theory only and not the new statistical theory, i.e. minimizing the sum of squares of the residual enor.[5]

In this study the attempt has been made to choose from different types of formulae the best average and the correct weighting system for forming the ideal index.These formulae are then treated by Fisher's classical test. For empirical analysis of choosing the best index,the present writer chooses the production index from general economic indicators.

The arrangement of this paper is as follows:In part 1, different types of formulae are presented.In part 2,the indexes will be tested.Part 3,analyses the empirical example. In part 4 a brief conclusion on the subject willbe discussed.

Types of Formulae

There are two main types of average or aggregate.These are the arithmetic and geometric means. Modes, median and harmonic means are less commonly used for index numbers.

The arithmetic and geometric means for volume (quantity) and price indexes take the form:

(1) $$Q^a = \sum_{i=1}^{n} w_q^i \frac{q_\ell^i}{p_o^i}$$

(2) $$P^a = \sum_{i=1}^{n} w_p^i \frac{p_\ell^i}{p_o^i}$$

(3) $$Q^g = \prod_{i=1}^{n} \left(\frac{q_\ell^i}{q_o^i}\right)^{w^i}$$

(4) $$P^g = \prod_{i=1}^{n} \left(\frac{p_\ell^i}{p_o^i}\right)^{w^i}$$

where (w^i)s are weights. Righthand superscript of the indexes a and g denote arithmetic and geometric means respectively. Q and P denote quantity and price indexes. $\sum$ and Π denote summation and multiplication. Righthand superscripts (i) denote commodities (i=1....n). Righthand subscripts o and ℓ denote base and given years respectively;

(ℓ=o+1, ... m). q and p denote the quantities and prices of individual commodities.

Weights are usually expressed as relatives and may be written by one of the following forms:

$$(5) \qquad w^i = \frac{pq}{\sum pq}$$

$$(6) \qquad w^i_q = \frac{pqo}{\sum pqo}$$

$$(7) \qquad w^i_p = \frac{poq}{\sum poq}$$

where $w^i \geq o$ and $\sum_{i=1}^{n} w^i = 1$

To arrive at the best index (P)s in equation (6), (q)s in equation (7) or (pq)s in equation (5) should be constant.[6] (6) and (7) are weights for quantity and price indexes - arithmetic means. (5) is weight for geometric means.

1 (a) Arithmetic Means:

i. *Arithmetic means with fixed weight:* Application of equation (6) in (1) and (7) in (2) gives the reduced arithmetic means formula:

$$(8) \qquad Q^a = \frac{\sum_{i=1}^{n} p^i q^i_\ell}{\sum_{i=1}^{n} p^i q^i_o}$$

$$(9) \qquad P^a = \frac{\sum_{i=1}^{n} p^i q^i_\ell}{\sum_{i=1}^{n} p^i q^i_o}$$

If p refers to situation o, then equation (8) becomes the Laspeyers[7] quantity index:

$$(10) \qquad {}_LQ^a = \frac{\sum_{i=1}^{n} p_o^i q_\ell}{\sum_{i=1}^{n} p_o^i q_o^i}$$

Equation (10) is almost used by all countries for quantity indexes.

If q refers to situation o, then equation (9) becomes the Laspeyers price index:

$$(11) \qquad {}_LP^a = \frac{\sum_{i=1}^{n} p_\ell^i q_o^i}{\sum_{i=1}^{n} p_o^i q_o^i}$$

If p refers to situation ℓ, then equation (8) becomes the Paasche[8] quantity index:

$$(12) \qquad {}_PQ^a = \frac{\sum_{i=1}^{n} p_\ell^i q_\ell^i}{\sum_{i=1}^{n} p_\ell^i q_o^i}$$

and if q refers to situation (ℓ), equation (9) becomes the Paasche price index:

$$(13) \qquad {}_PP^a = \frac{\sum_{i=1}^{n} p_\ell^i q_\ell^i}{\sum_{i=1}^{n} p_o^i q_\ell^i}$$

Equation (13) is almost used by all countries for price indexes.

$\sum_{i=1}^{n} p_o^i q_o^i$ and $\sum_{i=1}^{n} p_\ell^i q_\ell^i$ are the actual money aggregates in situations o and ℓ respectively. Cross valuation gives two hypothetical money aggregates

$\sum_{i=1}^{n} p_\ell^i q_o^i$ and $\sum_{i=1}^{n} p_o^i q_\ell^i$

ii. *Arithmetic means with variable weight:*

$$(14) \qquad {}_M Q^{va} = \frac{\sum w_\ell^i q_\ell^i}{\sum w_o^i q_o^i}$$

$$(15) \qquad {}_M P^{va} = \frac{\sum w_\ell^i p_\ell^i}{\sum w_o^i p_o^i}$$

Equations (14) and (15) have been developed by the present writer. The geometric means of these equations have been proposed by Mizutani (1967).[10]

1. (b) Geometric means:

If pq in equation (5) refers to situation o, then (3) and (4) will be:

$$(16) \qquad Q^{go} = \prod_{i=1}^{n} \left(\frac{q_\ell^i}{q_o^i} \right)^{w_o^i}$$

$$(17) \qquad P^{go} = \prod_{i=1}^{n} \left(\frac{p_\ell}{p_o} \right)^{w_o^i}$$

where $w_o^i = \frac{p_o \, q_o}{\sum p_o \, q_o}$

Equations (16) and(17) have been applied by the Harvard Committee on Economic Research.[11]

If pq in equation (5) refers to situation ℓ then (3) and (4) will be:

(18) $$Q^{g\ell} = \prod_{i=1}^{n} \left(\frac{q_\ell^i}{q_o^i}\right)^{w_\ell^i}$$

(19) $$P^{g\ell} = \prod_{i=1}^{n} \left(\frac{p_\ell^i}{p_o^i}\right)^{w_\ell^i}$$

Equations (18) and (19) have applied by Fisher himself.[12]

Equations (16) - (19) are geometric means of quantity and price indexes in direct comparison.

1. (c) <u>Cross-weighted means</u>:

Many compromise formulae can be constructed from equations (8)-(19) since none of these equations can be trueindexes as their weight systems are not constant. In other words, the constant weight in both means (i.e. arithmetic & geometric) will be somewhere between o and ℓ.

i. *Cross-weighted arithmetic means:* Assuming that the constant parameter in equation (8) is equal to $\frac{1}{2}(p_o + p_\ell)$, which is a compromise between o and ℓ, one can obtain the following formula:

(20) $$E^{Q^{ca}} = \frac{\sum_{i=1}^{n} \frac{1}{2}(p^i + p_\ell^i)\, q_\ell^i}{\sum_{i=1}^{n} \frac{1}{2}(p_o^i + p_\ell^i)\, q_o^i} = \frac{\sum_{i=1}^{n} (p_o^i + p_\ell^i)\, q_\ell^i}{\sum_{i=1}^{n} (p_o^i + p_\ell^i)\, q_o^i}$$

and given that q in equation (9) is equal to $\frac{1}{2}(q_o+q_\ell)$, one can obtain the following formula:

(21) $$E^{P^{ca}} = \frac{\sum_{i=1}^{n} p_\ell^i \frac{1}{2}(q_o^i + q_\ell^i)}{\sum_{i=1}^{n} p_o^i \frac{1}{2}(q_o^i + q_\ell^i)} = \frac{\sum_{i=1}^{n} p_\ell^i (q_o^i + q_\ell^i)}{\sum_{i=1}^{n} p_o^i (q_o^i + q_\ell^i)}$$

Equations (20) and (21) are cross-weighted arithmetic means of quantity and price indexes respectively. These formulae were first introduced by Edgeworth (1925).[14]

ii. *Cross-weighted geometric means:* This formula can be obtained by multiplying the base-weight by current-weight of the geometric means, then taking square roots:

$$Q^{cg} = \sqrt{\prod_{i=1}^{n} \left(\frac{q_\ell^i}{q_o^i}\right)^{w_o^i} \cdot \prod_{i=1}^{n} \left(\frac{q_\ell^i}{q_o^i}\right)^{w_\ell^i}}$$

$$(22) \qquad = \prod_{i=1}^{n} \left(\frac{q_\ell^i}{q_o^i}\right)^{\frac{1}{2}(w_o^i + w_\ell^i)}$$

$$(23) \qquad P^{cg} = \prod_{i=1}^{n} \left(\frac{p_\ell^i}{p_o^i}\right)^{\frac{1}{2}(w_o^i + w_\ell^i)}$$

Equations (22) and (23) are quantity and price indexes of cross-weighted geometric means.[15]

It is easier to do the calculation by using logarithms of (22) and (23) than to calculate the actual numbers.

The two formulae will be:

$$(24) \qquad \log Q^{cg} = \tfrac{1}{2}(w_o^i + w_\ell^i) \log q_\ell^i / q_o^i$$

$$(25) \qquad \log P^{cg} = \tfrac{1}{2}(w_o^i + w_\ell^i) \log p_\ell^i / p_o^i$$

1. (d) Mizutani variable weighted geometric means:

Mizutani (1967) has developed the following quantity and price indexes:

$$(26) \qquad \log M^{Q^{vg}} = \sum_{i=1}^{n} w_\ell^i \log p_\ell^i - \sum_{i=1}^{n} w_o^i \log q_o^i$$

(27) $$\log M^{P^{vg}} = \sum_{i=1}^{n} w_{\ell}^{i} \log p_{\ell}^{i} - \sum_{i=1}^{n} w_{o}^{i} \log p_{o}^{i}$$

1. (e) Cross-weighted arithmetic means by geometric means:

(28) $$_{F}Q^{cag} = \sqrt{\frac{\sum_{i=1}^{n} p_{o}^{i} q_{\ell}^{i}}{\sum_{i=1}^{n} p_{o}^{i} q_{o}^{i}} \cdot \frac{\sum_{i=1}^{n} p_{\ell}^{i} q_{\ell}^{i}}{\sum_{i=1}^{n} p_{\ell}^{i} q_{o}^{i}}} = \sqrt{_{L}Q^{a} \cdot {_{P}Q^{a}}}$$

(29) $$_{F}P^{cag} = \sqrt{\frac{\sum_{i=1}^{n} p_{\ell}^{i} q_{o}^{i}}{\sum_{i=1}^{n} p_{o}^{i} q_{o}^{i}} \cdot \frac{\sum_{i=1}^{n} p_{\ell}^{i} q_{\ell}^{i}}{\sum_{i=1}^{n} p_{o}^{i} q_{\ell}^{i}}} = \sqrt{_{L}P^{a} \cdot {_{P}P^{a}}}$$

Equations (28) and (29) are a mixture of linearity (Laspeyers and Paasche indexes being ratios of linear forms) and log-linearity (the geometric mean).

These two formulae are called the "Ideal" quantity and price indexes by Irving Fisher. Fisher, I. (1923)chose formula (29) from 143 algebraic ratios (indexes) as the "best" index.

Fisher's identification number of equation (29) is (353).[17]

2. Testing the indexes:

Fisher (1923) formulated several formula tests in order to decide which formula of his 143 equations was the best (ideal).

The most important tests are:[18]

(i) The Circular Test
(ii) The Time Reversal Test
(iii) The Factor Reversal Test

2 (a) Circular test:

P_{o1} should be independent of the choice of a third ime period. The argument can be shown as follows:

(30) $$P_{o1} \cdot P_{12} = P_{o2}$$

Circular test consists of three tests:[19]

(a) Time Reversal test: $P_{o1} \cdot P_{1o} = 1$

(b) Identity test : $P_{oo} = 1$

(c) Base test : $\frac{\partial}{\partial \ell} \left(\frac{P_{o\ell}}{P'_{o\ell}} \right) = 0$

If the Identity test and Base test are fulfilled in an index, the Circular test will be equivalent to the Time Reversal test.

This test has been fully analysed by Fisher himself. In the end he abandoned it with the words:

> "I am to show that the Circular test is theoretically a mistaken one, that a necessary irreducible minimum of divergency from such fulfilment is entirely right and proper, and, therefore, that a perfect fulfilment of this so-called Circular test should really be taken as proof that the formula which fulfils it is erroneous."[20]

The present writer therefore did not consider this test on the formulae discussed.

2. (b) The Time Reversal test:

(31) $$P_{o1} \cdot P_{o1} = 1$$

The price index of one direction should be the inversion of the price index in the other direction no matter which of the two is taken as the base.[21] In other words, P_{o1} represents any index number for time "1" relatively to time "o". The other way round is P_{1o}, and this, turned upside down, is $\frac{1}{P_{1o}}$, which, therefore, is the general expression for the time antithesis of P_{o1}.

Thus, P^{Pa} is the time antithesis of L^{Pa},

and P^{g} is the time antithesis of P^{go}.

5 formulae fulfil the Time Reversal test (TRT). (See: table (1)

It is a necessary condition for a best index to satisfy (TRT), but in order the formula (or formulae) to be "ideal" a sufficient condition is needed. Factor Reversal test will be discussed in the following pages and may meet this condition.

2. (c) <u>The Factor Reversal test</u>:

(32) $P . Q = V$

where $V = \sum_{i=1}^{n} p_\ell^i q_\ell^i / \sum_{i=1}^{n} p_o^i q_o^i$

This test is whether the index permits the interchanging of the prices and quantities of any formula, i.e. the two results multiplied together should give the true value ratio.[22]

Fisher (1923) justified this test in the following way:

> "1. No reason can be given for employing a given formula for one of the two factors which does not apply to the other, and 2. Such reversibility already applies to each pair of individual price and quantity ratios and should, in all logic, apply to the index numbers which aim to represent them in the mass."[23]

The only formulae which satisfy the Factor Reversal test are "Ideal" Fisher formulae (equations (28) and 29). In other words, "Ideal" Fisher formulae were arrived at mainly from this test.

Table 1 shows the nine price indexes, which have been shown previously, with their Fisher's identification numbers and the two tests.i.e. the Time Reversal test and Factor Reversal test.

Table 1. Different formulae of price indexes with their classical tests

Items	Equation Number	Symbols	Types of Formulae	Fisher's Numbers	Tests*	
					1	2
1	11	L^{Pa}	Laspeyers	53		
2	13	P^{Pa}	Paasche	54		
3	17	$P^{g^{o}}$	Weighted geometric means (base)	23		
4	19	$P^{g^{\ell}}$	Weighted geometric means (current)	29		
5	15	$M^{P^{va}}$	Variable weight (a.m.)		X	
6	27	M^{Pvg}	Variable weight (g.m.)		X	
7	21	E^{Pca}	Edgeworth	2153	X	
8	23	P^{cg}	Cross-weighted (g.m.)	123	X	
9	29	F^{Pcag}	"Ideal" Fisher	353	X	X

*Cross signals indicate the formula fulfils the test.
Test 1 is the Time Reversal Test and Test 2 is the Factor Reversal Test.

Many economists reject the Factor Reversal test emphasized by Fisher as it has no economic significance.[24] The reason is that the component of the "Ideal" Fisher formula consists of hypothetical money aggregates, i.e. $\sum_{i=1}^{n} p_o q_\ell$ and $\sum_{i=1}^{n} p_\ell q_o$; therefore the geometric mean of both indexes,i.e. Paasche and Laspeyers forms, is based on hypothetical cross-values.

Most of the index number makers and users suggest that the "Ideal" Fisher index is useful and recommend that it should be compiled. As pointed out by Stone (1956), "The form proposed by Fisher.... may be useful for the technical purpose of removing small discrepancies between (LQ^a) and

($_PQ^a$) or between ($_LP^a$) and ($_PP^a$), but where both the base- and current-weighted weighted index numbers can be calculated it is desirable that they should be given separately".[25]

Geary and Pratschke (1968) whilst remaking on index numbers constructed from Irish data recommend that:

> "our own empirical rule is to compute both Laspeyers and Paasche indexes whenever this is possible and to regard the "true" index as somewhere between the two figures, on elementary indifference curve considerations: the wider the difference between the figures, the less reliable the index from any formula. The Fisher "Ideal" (the geometric mean of the two) is taken as the estimate of the "true" index".[26]

After illustrating the practical usefulness of the Fisher "Ideal" formula, one would like to know whether all the different indexes discussed above fulfil the Factor Reversal test, i.e. the product of quantity and price indexes should be equal to the ratio of the total values.

Price and quantity indexes introduced so far do not meet formula (32). For example,

(33) $$_LP^a \,.\, _LQ^a \neq V^a$$

Therefore, a correction coefficient (δ) will be introduced to fulfil formula (32). As P and Q each require a correction coefficient, thus:

(34) $$V = \delta^2 P \,.\, Q$$

(35) $$\delta = V \,/\, P \,.\, Q$$

(36) $$\delta Q = V \,.\, Q \,/\, P$$

(37) $$\delta P = V \,.\, P \,/\, Q$$

By definition and algebraically "Ideal" Fisher formula's coefficient (δ) is equal to one. That is to say,

(38) $$F\delta = V^{cag} / {}_{F}Q^{cag} \cdot {}_{F}P^{cag}$$
$$= 1$$

The corrected Laspeyers' quantity index can be obtained from formula (36) as follows:

(39) $$\delta L^{Q^a} = \sqrt{V^a_{\cdot} {}_{L}Q^a / {}_{L}P^a}$$
$$= \sqrt{\frac{\sum p^i_\ell q^i_\ell}{\sum p^i_o q^i_o} \cdot \frac{\sum p^i_o q^i_\ell}{\sum p^i_o q^i_o} \cdot \frac{\sum p^i_o q^i_o}{\sum p_\ell^i q^i_o}}$$
$$= \sqrt{\frac{\sum p^i_\ell q^i_\ell}{\sum p^i_\ell q^i_o} \cdot \frac{\sum p^i_o q^i_\ell}{\sum p^i_o q^i_o}}$$
$$= \sqrt{{}_{P}Qa \cdot {}_{L}Qa}$$

Formula (39) is the "Ideal" Fisher index, i.e. formula (28).

From formula (37) the corrected Laspeyers' price index will then be as follows:

(40) $$\delta_{L}Pa = \sqrt{{}_{P}Pa \cdot {}_{L}Pa}$$

Formula (40) is the "Ideal" Fisher price index, i.e. formula (29).

One can get the same result as shown in formulae (39) and (40) by applying formulae (36) and (37) to Paasche's quantity and price indexes respectively.

The remaining formulae i.e. items (3-8) in Table 1, can be corrected similar to equation (36) in order to satisfy Factor Reversal test (FRT).

The main advantage of correcting an index according to (34) is that the controversary part that surrounds Fisher's "ideal" index ceases to exist.

Empirical analyses

Iraqi Monthly Industrial Survey (MIS) for the years 1960-63 was the source of the data used in the compilation of annual indexes (quantity and unit value) of the knitting industry.

As value and quantity of each material consumed are covered by the survey while only quantity produced of each product with values of total output have been covered by the survey, so the former has been chosen as an empirical example for our study.

Price and quantity indexes for years 1961-63 with 1960 as a base year for the 9 formulae have been calculated and have been shown in the tables 2 and 3 respectively.

Table 2: Different formulae for price indexes of trico industry: 1960=100 Materials

Item	Equations	Formulae	1961	1962	1963
1	11	L^{pa}	105.26	98.21	95.12
2	13	P^{pa}	106.39	96.90	77.84
3	17	$P^{g^{o}}$	103.65	97.90	84.47
4	19	$P^{g^{\ell}}$	106.82	98.38	81.03
5	15	$M^{P^{va}}$	111.70	100.32	90.59
6	27	$M^{P^{vg}}$	106.50	97.73	88.02
7	21	$E^{P^{ca}}$	105.84	99.10	86.70
8	23	P^{cg}	105.60	98.27	86.05
9	29	$F^{P^{cag}}$	105.82	98.06	86.05

Source: Table 1.

Table 3: Different formulae for quantity indexes of trico industry: 1960=100 materials

Item	Equations	Formulae	1961	1962	1963
1	10	$_LQ^a$	89.51	151.20	314.82
2	12	$_PQ^a$	90.46	150.71	257.64
3	16	Q^{go}	88.21	152.42	279.55
4	18	$Q^{g\ell}$	90.83	151.40	268.20
5	14	$_MQ^{va}$	94.99	154.39	299.85
6	26	$_MQ^{vg}$	90.56	150.40	291.33
7	20	$_EQ^{ca}$	89.99	150.96	286.95
8	22	Q^{cg}	89.80	151.23	288.70
9	28	$_FQ^{cag}$	89.98	150.90	284.80

Source: Table 1.

The total value ratios for years 1961-63 have been calculated as well and are as follows: 95.22, 148.02 and 245.08 respectively.

As has been shown in Table 1, the first 8 formulae do not meet P.Q=V. A correction coefficient (δ), therefore,has been calculated for all formulae in order to fulfil the second condition i.e. sufficient condition. (see: Table 4).

In judging which formula is the best, four criteria are selected and discussed in the following pages.

(a) Testing V. correction coefficient:

By multiplying (δ) to all formulae in Tables 2 and 3 i.e. Original indexes, the corrected formulae, then, will satisfy Factor Reversal Test (FRT).

Table (4) shows that all (δ)s are not far from unity for the years 1961-62. The discrepancies are $\pm$ 2 per cent or less (except item 5-$_M\delta^{va}$- which shows a divergence of (-5) per cent for 1961). In 1963, (δ)s have shown a wide discrepancy among them i.e. $\pm$10 per cent or less.

Table 4. Correction coefficient (δ)

Item		1961	1962	1963
1	$L\delta^{a}$	1.0054	.9984	0.9047
2	$P\delta^{a}$	.9946	1.0016	1.1055
3	δ^{go}	1.0200	.9900	1.0187
4	δ^{ge}	.9906	.9967	1.0619
5	$M\delta^{va}$	.9473	.9774	0.9498
6	$M\delta^{vg}$	.9936	1.0033	.9776
7	$E\delta^{ca}$	.9998	.9996	.9925
8	δ^{cg}	1.0020	.9978	.9865
9	$F\delta^{cag}$	1.0000	1.0000	1.0000

Source: Tables 2 and 3.

One concludes, then, that the closest (δ) to unity (in our case: items 1, 2, 7 and 8 besides item 9- "ideal" Fisher) the index will be nearly the best. (see; Table 5, column 4).

(b) Speed:

Table 5 gives the ranks of time studies for calculating the 9 indexes. This calculation has been done with a desk electric calculating machine (Facit) and logarithmic tables.

The first prize for speed goes to formula (14). The booby prize is captured by formula (26). All the other formulae occupy the 7 intermediate ranks.

Fisher "Ideal" formula ranks 6 and its two components i.e. Laspeyers and Paasche rank 2 and 3 respectively.

It will be noted from Table 5, column 5 that, the geometric formulae took more time than arithmetic formulae.

Table 5: Order of rank of formulae

Item (1)	Equations (2)	Formulae (3)	(4) δ^*	Speed (5)
1	10	$_LQ^a$	2	2
2	12	$_PQ^a$	3	3
3	16	Q^{go}	8	5
4	18	$Q^{g\ell}$	7	7
5	14	$_MQ^{va}$	9	1
6	26	$_MQ^{vg}$	6	9
7	20	$_EQ^{ca}$	4	4
8	22	Q^{cg}	5	8
9	28	$_FQ^{cag}$	1	6

*average of 1961&1962.

The most important result of the time studies is that all general economic indicatiors are calculated by governmental departments in most countries, using electronic computors. So the most complex formula can be calculated in few minutes.

(c) Accuracy (errors in data):

It is vital that the original data be as accurate as possible.

In price indexes, the larger number of commodities are used the net effect on the index will be smaller even though the original data are inaccurate. But as has been said before, production indexes have been chosen in our study, therefore, the problem of the accuracy of the original data will be different from that price indexes.

The errors in quantity indexes are mainly due to the quality of the original data. In this matter, one can recall what Royal Meeker (1929) wrote. He pointed out that "when the quantities and the prices are thoroughly satisfactory I think it will make but little difference what mathematical formula is used in calculating the index numbers."[27]

Table (4) shows that the (δ)s have shown a wide discrepancy among the 9 formulae for the year 1963, while in the case of 1961 and 1962 the discrepancy between them are very low. (see: Table 5, column 4). This could mean that the 1963 prices and quantities are inaccurate. This inaccuracy could be due to the political changes of 1963, more than 40 employees of the Central Bureau of Statistics were dismissed and replaced by new staff.[28]

As one knows, the experience of the staff is as important as their number in promoting the efficiency and reliability of statistical work. Thus, such changes in staff could but effect adversely the (MIS).

From the discussion above, one can conclude that Trico industry's data for the year 1963 are inaccurate.

(d) Understandability:

It is important to see that an index number, besides being accurate and quickly calculated, can be easily understood. In this respect, it is obvious that Laspeyers from (10) is a more understandable index than others.

Conclusion

The main reason for choosing the 9 formulae in our study out of hundreds of ratios is that they are often applied by the index users (except formula (15) which is newly developed by the present writer).

P^{P^a} is the time antithesis of L^{P^a} as well as $P^{g\ell}$ is the time antithesis of P^{go}. The remaining formulae satisfy (TRT).

Correction coefficient can replace (FRT) by multiplying (δ) to its original index.

When the data on the quantities and the prices are accurate, Laspeyers form is very close to Paasche form, and Laspeyers form is then recommended for measuring general economic indicators.

When Laspeyers and Paasche forms are far from each other this means the data could be inaccurate; therefore, "Ideal" Fisher form should be used.

The more the gap between the base year and the current year, the more the divergence between Laspeyers and Paasche forms will be significant. Thus, it is desirable to calculate "Ideal" Fisher form besides compiling both Laspeyers and Paasche forms.

Notes

1. This problem of index number arises "whenever we want a quantitative expression for a complex that is made up of individual measurements for which no common physical unit exists", Frisch, R. (1936). "The Problem of Index Number: Annual Survey of General Economic Theory", *Econometrica*. Vol.4, no.1, page 1.
2. See the classical definiton of an index number which is put forward by Edgeworth who states that "it shows by its variations the changes in a magnitude which is not susceptible either of accurate measurement in itself or of direct valuation in practice", F.Y. Edgeworth (1925). "The Plurality of Index Numbers", *Economic Journal*, 1925; also, R.G. D. Allen (1949) in defining the index number by saying that "an index number is an indirect measure of something of a statistical concept". He adds that "it is like an average or a measure of dispersion and, once again, we must expect to get several alternative measures, and not just one, to serve our purpose", R.G.D. Allen (1949), *Statistics for Economists*, chapter VI, Index Numbers, P.98.
3. Frisch, R. (1936), page 3.
4. Samuelson, P.A. (1947), *Foundations of Economic Analysis* , Cambridge, Mass., page 146.
5. For the new statistical theory of index numbers see:

1. Theil, H. (1960),"Best Linear Index Numbers of prices and Quantities", *Econometrica,* Vol. 1, 28, 1960. 2. Kloek and van Rees (1961), "On the Method of 'deflated' Best Linear Index Numbers", *Bulletin de l'Institut International de Statistique,* Vol.39 (1961). 3. Kloek and De Wit (1961), "Best Linear and Best Linear Unbiased Index Numbers", *Econometrica,* vol.29. 4. The application and significancy of the statistical theory of Index Numbers (the above three methods is discussed in Al-Jazairi (N.T.), (1967), *"Movements in the Foreign Trade of Five Arab Countries",* L.S.E. (London) thesis.

6. In deriving constant weights for geometric and arithmetic means in Divisia's Integral index, see:Roy, Rene, (1927) "Les index economiques", *Revue d'economie Politique,* PP. 1264-1277, and Tornqvist, L. (1937) "Fin-

1a lands Banks Konsumptionsprisindex". *Nordisk Tidsskrift for Teknisk Okonoci,* Kobenhavn, P.81.

7. Named after the German economist E. Laspeyers (1834-1913).

8. Named after the German economist H. Paasche(1851-1925).

9. In the study of bias of Paasche and Laspeyers indexes see; Bortkiewicz, L. von (1922), "Zweck und Struktur einer Preisindexzahl", *Nordisk Statistisk Tidskrift,* II, PP.334-379; Bortkiewicz, L. Von (1924). "Zwech und Struktur ein er Preisindexzahl", *Nordisk Statistisk Tidskrift,* III, PP. 218-222; Siegel, I.H. (1941a),"The difference between the Paasche and Laspeyers Index-Number Formulas", *J.A.S.A.,* Sept. 1941, PP. 343-350; Siegel, I.H. (1941b), "Further Note on the Difference between Index Formulas", *J.A.S.A.,* Dec. 1941, PP. 519-24; Marris, R.L. (1958), *Economic Arithmetic* Macmillan & Co. Ltd., London, Chapt. 8; Zarnowitz, V. (1961), "Index Numbers and the Seasonality of Quantities and Prices", in *The Price Statistics of the Federal Government* N.B.E.R., 1961, PP. 233-304.

10. Mizutani, K. (1967), "New Formulas for Making Price and Quantity Index Numbers", in *Essay on Mathematical Economics,* edited by Shubick, M., PP. 467-475.

11. See Fisher (1923) *The Making of Index Numbers* second Ed. Boston. Houghton Mifflin Co. P.53.

12. Fisher (1923) P.49.

13. Fisher, I. (1923) page 468.

14. See: Edgeworth, F.Y. (1887-): Several articles

collected in papers relating to Political Economy:Vol. 1, London 1925. This formula appeared as number 2153 in Fisher's textbook, P. 484, and has been developed by Bowley, A.L. (1928), "Notes on Index Numbers", *Economic Journal*, Vol. 38, P.216; see also Fabricant, S. (1940), *The Output of Manufacturing Industries*, 1899-1937, NBER, Publication no. 39. Fabricant was one of the first to apply Edgeworth's formula for U.S. industries, see page 34.

15. Formula (23) appears as number (123) in Fisher's textbook on page 473. There are fourteen index numbers which he regards as superior to this one. First application of (22) and (23) has been made by Tornqvist, L. C. (1957), "Tariffs and Economic Development of the Post and Telegraph Office During the Years 1931-1955", *Appendix of the Annual Report of the Administration of posts and Telegraphs for year 1955*. Helsinki; then applied by Theil, H. (1965), "The Information Approach to Demand Analysis", *Econometrica*, Vol.33, PP. 67-87; and Theil, H. *Economic and Information Theory*, chapt. 5-8, Vol.7, in the series Studies in Mathematical and Managerial Economics, Amsterdam. This formula has been developed and applied by Klock, T. (1967), *On Quadratic Approximations of Cost-of-living and Real-Income Index Numbers* Netherlands of Economics, Econometric Institute, August 4, 1967.

16. See: These formulae have been applied by Mizutani, K. (1967), and can satisfy the following tests:
    1. Circular test
    2. Factor reversal test in special case
    3. Continuity test, i.e. quality change
    4. Avoiding the fallacy of "The Beauty Contest"
    5. Test of grouping

17. The "Ideal" index has been recommended by C.M. Walsh (1901), *The Measurement of General Exchange Value*, P.429; Pigou, A.C. (1932), *The Economics of Welfare* (4th ed. 1932), P.69; Young A.A. (1921), "The Measurement of Changes of General Price Level", *Quarterly Journal of Economics*, Vol.35 (1921), P.572; and Pearson, W.M. (1928), *The Construction of Index Numbers*, P.85.

18. Fisher introduced as well another group of tests of minor importance which are as follows:
    T1. *Proportionality test:* If all prices increase in

the same proportion from time o to time 1,the P_{01} must show exactly this proportionality.
T2. *Determinateness test:* P_{01} shall not become zero, infinite, or interminate, if an individual price or quantity becomes zero.
T3. *Commensurability test:* P_{01} should be uneffected by any unit of measurement of prices or quantities.
T4. *Withdrawal of Entry Test:* P_{01} should be uneffected by the withdrawal or entry of a price relative agreeing with the index number. Fisher (1923) examined T1, T2 and T4 to all (143) formulae and concludes that "all these tests relate to the behaviour of the formula under some special circumstances, such as when all the relatives are equal, when one is zero, or when one coincides with the index number, and have little value as a general guide", page 421. Frisch (1930) tried to show that T2 and T3 and base test were inconsistent, see: Frisch (1930), "Necessary and Sufficient Conditions Regarding the form of an Index Number, which shall Meet Certain of Fisher's Tests", *J.A.S.A.* 1930. Therefore, one can conclude that all the four tests are inconsistent and of minor importance.

19. See: Frisch (1930).
20. Fisher (1923), P.271.
21. *Ibid.*, P.64.
22. Fisher (1923), P.72.
23. *Ibid.*, P.75.
24. Davis, H.T. (1947) Pointed out that "some economists, especially Haberler, feel that this test has no economic significance", Davis (1947), *The Theory of Econometrics*, P. 328. For more critics on Factor Reversal tests see: Winkler, W. (1954), "Older and Newer Ways of Solving the Index Numbers Problem", *Bulletin of the International Statistical Institute*, Vol. 34, 2nd part.
25. See Stone, R. (1956) *Quantity and Price Index in National Accounts*, O.E.E.C., Paris; PP. 38-39.
26. See: Geary, R.C. and Pratschke, J.L. (1968), *Some Aspects of Price Inflation in Ireland*, The Economic and Social Research Institute, paper no. 40, Dublin, P.37.
27. See: Meeker, R. (1929), "On the Best Form of Index Number", *J.A.S.A.*, Vol. 17, P. 915.
28. See Kana'an, T.H. (1965) *Input-Output and Social Accounts of Iraq 1960-1963*, Ministry of Planning, Baghdad, Sept. 1965; PP. V-13&14.