A DYNAMIC INTER-INDUSTRY MODEL OF PRICE DETERMINATION A TEST OF THE NORMAL PRICE HYPOTHESIS*

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In their recent paper Nordhaus and Godley [4] have carried out a test of the Normal Price Hypothesis (NPH) for U.K. manufacturing. The present study is a development out of their work and relies heavily upon it.

Their testing procedure differs from that adopted by other investigators in this field in two main respects.[1] Firstly, they employ a more comprehensive and satisfactory method of computing 'normal' costs, and secondly, they follow the very unorthodox procedure of estimating the distributed lag between costs and price changes from the individual industry data for costs, sales and stock/output ratios. The use of disaggregated data to arrive at this aggregate lag structure inevitably involves an underlying multi - sectoral model. One main object of this paper is to propose and discuss such a dynamic multi - sectoral model of price determination and show how it can be employed to test the NPH at an industry level.

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1. In particular see Neild[2].

The first section of the paper will be devoted entirely to the development of the model and the discussion of suitable procedures for its estimation and solution. We show that an aggregate lag structure between manufacturing prices and costs as a whole exists only if the unit normal costs of each industry can be assumed to be fixed fractions of the corresponding unit normal costs of total manufacturing.

In the second section, in order to obtain estimtes of unit normal costs, we shall discuss the results of econometric investigations which we have carried out for the explanation of hours, earnings and employment in manufacturing industries. Although these relations have already been estimated by Nordhaus and Godley, it was thought necessary to have a fresh look at the earnings and employment relations since they were not satisfactorily estimated, both because of a high degree of auto-correlation in the residuals of the estimated relations, and also because, in the case of the employment relations, the coefficients of the response of employment to changes in lagged values of output and hours were imposed.

In the Nordhaus-Godley study the value assumed for the proportion of materials which enters the production process at the beginning is arbitrarily chosen. Given that the conditions for the existence of an aggregate lag structure between prices and exogenous costs are approximately satisfied, we compute manufacturing prices in section 3, assuming alternative values for this proportion within the range zero and unity, and investigate the significance of the appropriate choice of this parameter for the prediction of prices .

In the fourth section we shall derive a suitable relation for testing the NPH within the Nordhaus-Godley framework of computing prices. This relation clearly shows that the Nordhaus-Godley procedure for testing the NPH by regression of the actual price on the computed price and the demand variable is not valid. We shall also report on further tests of NPH investigating in particular the effects of lagged demand variables on the mark ups.

In order to compare the Nordhaus-Godley procedure of estimating lag structures with the more orthodox'regression-method', we shall estimate a linear relation between prices and costs, assuming separate lag structures on unit normal and non-material costs. Finally, by comparing the predictive performance of our 'regression method' with the Nordhaus-Godley procedure we highlight the importance of the averaging nature of the 'regression method' for the explanation and prediction of manufacturing price movements.

1. The model

The dynamic price formation model which we shall outline here emerges as a generalization of Leontief's work in the sense that prices will now be assumed to be determined by unit 'normal historical' costs rather than unit actual current costs. The two fundamental concepts of 'normal' and 'historical' are fully defined and discussed in the Nordhaus-Godley paper: "The normal value of a variable is defined as the value that variable would take, other things equal, if output were on its trend path". The historical value of a variable, on the other hand, is simply its distributed lag value.

Consider now the problem of price determination in the manufacturing sector which is composed of n separate industries. Under the NPH each price will equal the unit normal historical cost of labour and materials of the relevant industry multiplied by a fixed mark up factor.[1] Some of the materials purchased by each industry originate outside manufacturing, and prices of these commodities will be assumed to be given independently of the manufacturing prices. We shall refer to this category of costs as exogenous material costs. The remaining materials are purchased from other manufacturing industries and form the category of endogenous material costs. Furthermore we shall distinguish between those inputs which enter the productive process at the beginning of a production period (i.e. the initial entry inputs) and the remainder, such as fuel, labour, and some of the ma-

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1_ The mark up includes overhead costs as well as profits.

terials which will be added progressively throughout the period (i.e. the progressively added inputs). Such a division of inputs palys an important role in the Nordhaus -Godley method of estimating the individual industry lag profiles.

In the light of what has been said above, the following pricing equations emerge.

$$p_j(t) = (1+\pi_j)\ (w_j^*\ (t)+ m_j^*\ (t) + z_j^*\ (t)) \qquad (1.1)$$

$$j=1,2,\ldots,n$$

where

$p_j\ (t)$ = the j-th industry output price.

π_j = the j-th industry mark up factor.

$w_j^*\ (t)$ = the j-th industry unit normal historical costs of the exogenous progressively added inputs.

$m_j^*\ (t)$ = the j-th industry unit normal historical costs of the exogenous initial entry material inputs.

$z_j^*\ (t)$ = the j-th industry unit normal historical costs of the endogenous material inputs.

But according to the concept of historical value we can write

$$w_j^*\ (t) = \sum_{s=1}^{q_j} g_{js}\ w_j\ (t-s+1) \qquad (1.2)$$

where $w_j(t)$ is the j-th industry unit normal costs of exogenous progressively added inputs. Now employing the lag operator D defined $D^s w_j(t) = w_j(t-s)$, relation (1.2) can also be written as

$$w_j^*(t) = G_j(D)w_j(t)\ ;\ G_j(D) = \sum_{s=1} g_{js}\ D^{s-1} \qquad (1.3)$$

where $G_j(D)$ is the j-th industry lag operator function with respect to the exogenous progressively added inputs.

Similarly we have

$$m_j^*(t) = H_j(D)m_j(t) \quad ; \quad H_j(D) = \sum_{s=1} h_{js} D^{s-1} \tag{1.4}$$

and, assuming that of all the material inputs used in the production process of the j-th industry the proportion u_j enters at the beginning and the rest enters progressively, we also have

$$z_j^*(t) = K_j(D)\ z_j(t) \ ; \ K_j(D) = u_j H_j(D) + (1-\mu_j) G_j(D) \tag{1.5}$$

Furthermore $z_j(t)$, which is the unit normal endogenous material cost, is itself composed of the sales of all other manufacturing industries to the j-th industry and, assuming fixed production coefficients, can be written as

$$z_j(t) = \sum_{r=1}^{n} a_{rj}\ p_r(t) \tag{1.6}$$

where a_{rj} denotes the inputs coefficient and is the amount of the r-th industry output needed as an input to produce one unit of the j-th industry output.

With the help of relations (1.3) - (1.6) the individual industry pricing equations can now be written as

$$p_j(t) = (1+\pi_j)\ [G_j(D)\ w_j(t) + H_j\ (D)m_j(t) + K_j\ (D) \sum_{r=1}^{n} a_{rj}\ p_r(t)] \tag{1.7}$$

$$j = 1,2,\ldots,n$$

As we have already pointed out, these pricing equations are obtained under NPH and to test for the effect of variations in demand (or any other variable) upon prices we only need to add appropriate demand variables to each of the above pricing relations.

The problem of estimating the industry lag (i.e. G_j (D), H_j(D)) will be dealt with in section 3. Here we are mainly concerned with two possible method of solving system (1.7) for prices in terms of exogenous normal costs, mark - ups , input-output coefficients and industry lag profiles.

One obvious method is to solve system (1.7) for prices in a recursive manner. Given the values of costs and prices up to and including the period '0' we can compute the prices for period '1' by solving the linear system in period '1' prices. These computed prices can then be used to arrive at period '2' prices which will lead us to period '3' prices and so on. Although this method provides us with values of predicted prices, it does not yield any information with respect to the implied lag structures between costs and prices. Furthermore, such a direct procedure has the disadvantage of requiring at least some knowledge of actual prices to start the computations.[1]

On the other hand, by evaluating certain lag operator functions within a given range, it is possible to compute both the 'final form' lag profiles and prices. In order to show how this can be done we first write system (1.7) in matrix form.

$$\underset{\sim}{p}(t)=G(D)\underset{\sim}{w}(t) + H(D)\,\underset{\sim}{m}(t) + K(D)A'\underset{\sim}{p}(t) \qquad (1.8)$$

where $\underset{\sim}{p}'(t)=(p_1(t),p_2(t),\ldots, p_n(t))$;

$\underset{\sim}{w}'(t)=(w_1(t),w_2(t),\ldots, w_n(t))$

$\underset{\sim}{m}'(t)=(m_1(t),m_2(t),\ldots, m_n(t))$;

$A = (a_{ij})$ is the matrix of input-output coefficients and $G(D)$, $H(D)$ and $K(D)$ are diagonal matrices with their j - th diagonal elements $(1+\pi_j)G_j(D)$, $(1+\pi_j)H_j(D)$ and $(1+\pi_j)\,K_j\,(D)$ respectively.

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1. But note that since the dynamic process which we are dealing with is not explosive, one could start the computations with any set of initial conditions provided that one started the computations at some time long before period'1'.

Now assuming that the above dynamic system is stable and has started a long time ago, the solution of (1.8)can be written as[1]

$$\underset{\sim}{p}(t) = [I-K(D)A']^{-1} [G(D)\underset{\sim}{w}(t) + H(D)\underset{\sim}{m}(t)]$$

which implies the following <u>final form</u> solution for the j-th industry price in terms of the exogenous costs.

$$p_j(t) = \sum_{s=1}^{n} \lambda_{js}(D)G_s(D)(1+\pi_s)\,w_s(t) + \sum_{s=1}^{n} \lambda_{js}(D)H_s(D)(1+\pi_s)m_s(t) \qquad (1.9)$$

$$j = 1,2,\ldots.,n$$

where $\lambda_{ij}(D)$ is the i-j element of the inverse of $I - K(D)A'$ which can suitably be called the Leontief lag operator matrix of dynamic inter-industry analysis.

Since the final form lag operator functions such as $\lambda_{js}(D)G_s(D)$ and $\lambda_{js}(D)H_s(D)$ are very complicated functions of D, any analytic solution of lag coefficients is out of the question and some numerical method should be employed. One such procedure is to approximate the lag operator functions by a high degree polynomial in the neighbourhood of $D = 0$. But because of the high degree of multicollinearity present in the estimation of fifth or higher order polynomial regressions, it was found necessary to fit the values of the lag operator functions first in terms of Chebyshev's polynomials and then to transform the estimated values back to the parameters of an ordinary polynomial which are in fact the lag coefficients. Such a transformation of data enables much more accurate estimates of the lag coefficients.

It is clear from the explicit solution given by (1.9) that $p_j(t)$ depends not only on its own historical unit normal

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1. The system of difference equations (1.8) is stable if and only if all the roots of the determinantal polynomial $\phi(x) = |I-K(x)A'| = 0$, lie outside the unit circle. See, for example, Dhrymes [1, pp.507-525].

costs, but also on the historical unit normal costs of all other industries seen from the viewpoint of its own productive process. Moreover, if we insist on the existence of an aggregate lag structure between manufacturing prices and costs as a whole, we need to make the following further restrictive assumptions:

$$w_j(t) = \alpha_j w(t) \tag{1.10}$$
$$m_j(t) = \beta_j m(t)$$

where $w(t)$ and $m(t)$ denote the total manufacturing industries' unit normal costs of 'progressively added' and 'initial entry' exogenous inputs, respectively.

Now given that these conditions for the existence of the aggregate lag structure are satisfied, the pricing relations given by (1.9) become

$$p_j(t) = \left[\sum_{s=1}^{n} \lambda_{js}\,(D)G_s(D)(1+\pi_s)\,\alpha_s\right] w(t) \; +$$
$$\left[\sum_{s=1}^{n} \lambda_{js}\,(D)H_s(D)(1+\pi_s)\quad {}_s\right] m(t) \tag{1.11}$$
$$j = 1,2,\ldots,n$$

which implies the following aggregate relation for the explanation of manufacturing prices.

$$p(t) = \left[\sum_{j=1}^{n} \sum_{s=1}^{n} \lambda_{js}(D)G_s(D)(1+\pi_s)\alpha_s\, f_j\right] w(t) \; +$$
$$\left[\sum_{j=1}^{n} \sum_{s=1}^{n} \lambda_{js}\,(D)H_s(D)(1+\pi_s)\beta_s f_j\right] m(t) \tag{1.12}$$

where $p(t) = \sum_{j=1}^{n} f_j p_j(t)$ and f_j is the weight of the j-th industry in total manufacturing.

Note that the lag operator functions of $w(t)$ and $m(t)$

in the above relation are unscaled in the sense that the sum of the lag coefficients is not unity. Denoting the scaled lag operator functions of $w(t)$ and $m(t)$ by $B_1(D)$ and $B_2(D)$ respectively, relation (1.12) can also be written as

$$p(t) = \gamma_1 B_1(D) w(t) + \gamma_2 B_2(D) m(t) \tag{1.13}$$

where[1]

$$B_1(D) = \left[\sum_{j=1}^{n} \sum_{s=1}^{n} (1+\pi_s)\alpha_s f_j \lambda_{js}(D) G_s(D) \right] \Big/ \gamma_1$$

$$B_2(D) = \left[\sum_{j=1}^{n} \sum_{s=1}^{n} (1+\pi_s)\beta_s f_j \lambda_{js}(D) H_s(D) \right] \Big/ \gamma_2$$

with

$$\gamma_1 = \sum_{j=1}^{n} \sum_{s=1}^{n} \lambda_{js}(1)(1+\pi_s)\alpha_s f_j \quad \text{and}$$

$$\gamma_2 = \sum_{j=1}^{n} \sum_{s=1}^{n} \lambda_{js}(1)(1+\pi_s)\beta_s f_j$$

Hence the aggregate pricing relation for manufacturing will be[2]

$$p(t) = \gamma_1 w^*(t) + \gamma_2 m^*(t)$$

The above result also indicates that, contrary to the Nordhaus-Godley procedure of predicting prices, there is no need to impose the weights by which the two kinds of historical unit costs are added, as they are fully determined by the underlying pricing model.

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1. Note that $G_s(1) = H_s(1) = 1$ for all $s=1,2,\ldots,n$

2. As before, the historical value of a variable is denoted by an asterisk(*).

2. The computation of unit normal costs-A different econometric approach

In order to obtain estimates of normal costs we require a reasonable econometric explanation of hours, earnings and employment in manufacturing, so that costs can be corrected for cyclical changes in output. Such estimates have already been made by Nordhaus and Godley. On several counts these are not completely satisfactory. Firstly, in estimating their hours and earnings equations the authors have utilized data which refer to total manufacturing and do not exclude food, drink and tobacco. Secondly, they have used current weighted indices of hours and earnings, while the of base weighted indices is more appropriate.1 Thirdly, without any strong justification, they have imposed the lag structures of output and hours in their employment relations. Fourthly, the disturbances of their earnings and employment relations were strongly autocorrelated, and the method of first-differencing employed to deal with this problem cannot be regarded as very satisfactory. In fact, in the case of their employment relations, in spite of the first-differencing, the values of the Durbin-Watson statistics were still too low.

In this section we shall present alternative estimates of hours, earnings and employment relations which do not suffer from the above-mentioned shortcomings. In re-estimating earnings and hours relations we have also abandoned the disaggregation by sex and estimated these relations for male and female employees as a whole. Furthermore, we have corrected the basic hourly rated for the important engineering settlements over the period 1967-69. These corrections have been made on the basis of figures kindly supplied by the Department of Employment and productivity (D.E.P.). The use of corrected basic hourly rates in the earnings relation did significantly better than when the uncorrected basic hourly rates were used.

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1. The use of base weighted indices has been suggested by Mr. Godley himself who has also supplied the appropriate data.

Our preferred estimate of the hours relation is[1]

$$H_t = \underset{(21.30)}{16.54} + \underset{(38.09)}{0.69790}\, HS_t + \underset{(11.34)}{12.33}\, CU_t + \hat{u}_t \qquad (2.1)$$

$$\bar{R}^2 = 0.9789 \; ; \; DW = 1.65 \; ; \; \hat{\sigma}_u = 0.1683$$

wher

H ≡ Actual base weighted index of hours worked per week in manufacturing excluding food, drink and tobacco .

HS ≡ Standard or normal hours per week.

CU ≡ The degree of capacity utilization measured as the deviation of actual from trend output.

The above relation is estimated by the Ordinary Least Squares method and the estimated parameters are very similar to those estimated by Nordhaus and Godley. We also tried lagged values of CU in (2.1), but did not find any significant improvement in the fit of the relation.

The functional form adopted by Nordhaus and Godley for the explanation of earnings can be written as

$$Log_e\,(AWE) = b_o + b_1T + b_2\, Log_e\,(BHR) + b_3 Log_e(\bar{\bar{H}}) + u$$

where

$\bar{\bar{H}} = HS + \lambda(H-HS)$; λ denotes the overtime premium coefficient

AWE ≡ Average Weekly Earnings

BHR ≡ Basic Hourly Wage Rates

1. The bracketed figures refer to the ratio of parameters to their asymptotic standard errors. $\bar{R}^2$ is the multiple correlation coefficient adjusted for loss of degrees of freedom . DW refers to Durbin-Watson statistics. σ stands for the estimated standard error of u. The estimate of a variable will be denoted by a cap (^). These conventions will be observed throughout.

and T ≡ Time

The experiments which we carried out for the estimation of the above relation have already been fully discussed elsewhere[7]. The results of the computations for the present data are summarized in Table 1 where estimated log likelihood and chi-square values are given for values of the overtime premium coefficient in the range $1.0 \leq \lambda \leq 2.5$ under the following four specifications of the error process.[1]

Ordinary Least Squares (OLS)	$u_t = v_t$
First order Moving Average (MA) error specification	$u_t = v_t + \gamma v_{t-1}$
First order Autorogressive (AR1) error specification with stochastic initial value	$u_t = \rho u_{t-1} + v_t$
Second Order Autoregressive (AR2) error specification with stochastic initial values	$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + v_t$

where u_t is the disturbance of the relation and v_t is assumed to be normally distributed with zero mean and variance σ^2.

It is clear from the chi-square values given in this Table that the MA, AR1 and AR2 error specifications are all significently more appropriate than the OLS specification and that, if there is a choice to be made, the AR1 error specification seems to be the most appropriate. This is in fact true for all the tabulated values of the overtime premium. Furthermore it is easily seen from Table 1 that the maximum likelihood estimate of the overtime premium in the case of the AR1 error specification is 2.1, which is not significantly different from the normally assumed value of 1.5 (The chi-square value for the test of the null hypothesis that the

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1. The derivation of the likelihood functions of these error specifications and the methods of computation can be found in Chapters 3 and 4 of the author's Ph. D. Thesis[5].

overtime premium is equal to 1.5 is 2.64, which is not significant even at 10% level). Consequently we have selected the following estimate of the earnings equation to correct the earnings figures for cyclical changes in hours.

$$\underset{}{\mathrm{Log}_e}\,(AWE) = \underset{(-4.36)}{-2.62465} + \underset{(6.11)}{0.02203T} + \underset{(9.88)}{0.77037}\ \mathrm{Log}_e\,(\,BHR\,)\ +$$

$$\underset{(8.40)}{0.94531}\ \mathrm{Log}_e\ (\bar{H}) + \hat{u}_t \qquad (2.2)$$

where $\bar{H} = HS + 1.5(H-HS)$; $\hat{u}_t = \underset{(3.47)}{0.5437}\ \hat{u}_{t-1} + \hat{v}_t$;

$\hat{\sigma} = 0.62\%.$; $\bar{R}^2 = 0.9995$

Table 1* The estimated log likelihood and chi-square values of the earnings relation for the overtime premium coefficient (λ) within the range $1.0 \leq \lambda \leq 2.5$ in the case of all for error specifications.

The overtime premium λ	$\hat{L}_{OLS}$	$\hat{L}_{MA}$	$\hat{L}_{AR1}$	$\hat{L}_{AR2}$	$\chi^2_{MA,OLS}$	$\chi^2_{AR1,OLS}$	$\chi^2_{AR2,AR1}$
1.0	158.63	165.50	167.66	169.08	13.74	18.06	2.84
1.1	160.56	166.93	168.99	170.17	12.74	16.86	2.30
1.2	162.34	168.20	170.13	171.03	11.72	15.58	1.80
1.3	163.93	169.28	171.06	171.76	10.70	14.26	1.40
1.4	165.28	170.18	171.82	172.36	9.80	13.08	1.08
1.5	166.39	170.91	172.42	172.83	9.04	12.06	0.82
1.6	167.27	171.47	172.88	173.19	8.40	11.22	0.62
1.7	167.94	171.89	173.23	173.46	7.90	10.58	0.46
1.8	168.43	172.19	173.46	173.64	7.52	10.06	0.36
1.9	168.76	172.37	173.62	173.75	7.22	9.72	0.26
2.0	168.99	172.48	173.71	173.80	6.98	9.44	0.18
2.1	169.06	172.51†	173.74†	173.81†	6.90	9.37	0.13
2.2	169.08†	172.50	173.73	173.78	6.84	9.31	0.09
2.3	169.03	172.44	173.70	173.73	6.82	9.34	0.06
2.4	168.95	172.35	173.64	173.66	6.80	9.38	0.04
2.5	168.82	172.25	173.56	173.57	6.86	9.48	0.02

* $\hat{L}_{OLS}$ = The estimated log likelihood value for the independently distributed disturbances.

$\hat{L}_{MA}$ = The estimated log likelihood value for the MA error specification.

$\hat{L}_{AR1}$ = The estimated log likelihood value for the AR1 error specification.

$\hat{L}_{AR2}$ = The estimated log likelihood value for the AR2 error specification.

$\chi^2_{MA,OLS} = 2(\hat{L}_{MA} - \hat{L}_{OLS})$; $\chi^2_{AR1,OLS} = 2(\hat{L}_{AR1} - \hat{L}_{OLS})$;

$\chi^2_{AR2,AR1} = 2(\hat{L}_{AR2} - \hat{L}_{AR1})$

† indicates the overall maximum of the log likelihood values

Note that our estimates of the proportional effects of the basic hourly rates and the standard hours equivalent upon earnings are much larger than those obtained by Nordhaus and Godley. This in turn indicates that the D.E.P. published earnings corrected for overtime are not as unreliable as was suggested by Nordhaus and Godley. (1) This is not, however, to say that one should abondon relation (2.2) in favour of the D.E.P. published figures.

In order to arrive at a satisfactory employment relation for the operative employees we estimated a log-linear relation between employment and lagged values of output without imposing any of the coefficients. As expected, because of the extreme multicollinearity among the successive lagged values of output, the OLS estimates were very poorly determined. But when more appropriate methods such as MA, ARI or AR2 were used, the results were found to be very satisfactory. The main reason for this is that by making appropriate allowance for the autocorrelation of the disturbance the multicollinearity between lagged values of output is weakened, and in this way one kills two birds with one stone. In fact we found that the AR2 error specification performed significantly better than any of the other error processes and hence our preferred relation:

$$\begin{aligned}
\mathrm{Log}_e\,(E_{op}) = {} & \underset{(8.49)}{7.30} - \underset{(-2.16)}{0.00147T} - \underset{(-6.99)}{0.0004T^2} - \underset{(-8.24)}{0.00502S_1} - \\
& \underset{(-9.78)}{0.0073S_2} - \underset{(-8.40)}{0.0050S_3} + \underset{(2.72)}{0.07672}\,\mathrm{Log}_e\,(X_t) + \\
& \underset{(4.44)}{0.12328}\,\mathrm{Log}_e\,(X_{t-1}) + \underset{(5.17)}{0.14299}\,\mathrm{Log}_e\,(X_{t-2}) + \\
& \underset{(2.83)}{0.07778}\,\mathrm{Log}_e\,(X_{t-3}) + \underset{(1.60)}{0.04054}\,\mathrm{Log}_e\,(X_{t-4}) - \\
& 0.26685\,\mathrm{Log}_e\,(HC_t) + \hat{u}_t \qquad (2.3)
\end{aligned}$$

(1) See the footnote on p. 858 of [4]

with $\hat{u}_t = \underset{(9.50)}{1.251}\hat{u}_{t-1} - \underset{(-3.13)}{0.3907}\hat{u}_{t-2} + \hat{v}_t$; $\hat{\sigma} = 0.29\%$; $\bar{R}^2$ =

0.9835

where

E_{op} ≡ Operatives Employment

X ≡ Output

Hc ≡ Customary Hours ; HC = 16.54 + 0.6979 HS

and S_1, S_2 and S_3 stand for seasonal dummies.

The lag operator function of output in the above relation is

$$0.16631 + 0.26724D + 0.30997D^2 + 0.16861D^3 + 0.08788D^4$$

which has a reasonable shape and implies an average lag of 1.75 quarters, with a peak occurring after two quarters. The long-run elasticity of employment with respect to output is 0.46131, which is slightly less than that obtained by Nordhaus and Godley. We also found up significant effect of lagged Customary Hours (HC) on operatives' employment.[1]

Finally, to get a satisfactory relation for Adminstrative, Technical and Clerical Workers' (ATC) employment, we estimated a relation assuming a geometrically declining lag structure for output. We also assumed the disturbances of this relation to follow a first-order autoregressive scheme. The following result was obtained.[2]

$$\text{Log}_e(E_{ATC}) = \underset{(5.06)}{4.92} + \underset{(1.77)}{0.00597}T - \underset{(-2.53)}{0.00004}T^2 - \underset{(-1.08)}{0.00129}S_1 - \underset{(-1.89)}{0.00257}S_2 - \underset{(-1.80)}{0.00211}S_3 + \underset{(2.31)}{0.04253} \sum_{i=0}^{\infty} \underset{(12.76)}{0.9}^i$$

1) We also estimated relation (2.3) assuming a declining geometric lag structure for the hours, but found it to be insignificant. In another experiment we imposed the output lag coefficients on hours, and this too proved to be insignificant.

2) The method of estimation and other related matters are fully discussed elsewhere [6].

$$\text{Log}_e\ (X_{t-i}) + \hat{\hat{u}}_t \tag{2.4}$$

$$\text{with } \hat{u}_t = \underset{(7.76)}{0.7993}\hat{u}_{t-1}\ \hat{v}_t\ ;\ \hat{\sigma} = 0.50\%;\quad \bar{R}^2 = 0.9987$$

As expected, the lag structure of output in the above relation has rather a long tail. The fitted equation implies a mean lag of nine quarters. The overall effect of output upon ATC employment is estimated at 0.4253 which is much higher than that estimated by Nordhaus and Godley.

Now using the estimated relations (2.1) - (2.4) we can obtain an estimate of the unit normal labour costs [1] except for the employers' national insurance contribution which should be added as a separate item. Non-labour costs were taken to be the same as those employed by Nordhaus and Godley.

3. The estimation of industry lag profiles and the computation of prices

In order to estimate the individual industry lag profiles we can employ either of the two following procedures. Firstly, we can adopt the orthodox method which requires the time series data on prices as well as normal costs for each individual industry and estimate the unknown parameters of the pricing model developed in the first section by some 'suitable' simultaneous estimation technique. Alternatively, we can follow the Nordhaus-Godley procedure which gives estimates of individual industry lag profiles for the two types of costs using only a census year data on costs and stock/output ratios.

Due to the limited scope of the present study we shall restrict ourselves to the latter procedure and estimate the individual industry lag profiles assuming the proportion μ of all materials enters the productive process at the beginning (i.e. $\mu_{\underline{j}} = \mu$). [2] In their study Nordhaus and Godley

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1) For further details see [4 , p.861].

2) The simultaneous estimation of the unknown parameters of the pricing model given by (1.7) is in progress as part of a more comprehensive study of pricing in manfucturing being carried out under the general direction of Messrs. Nordhaus and Godley.

that $\mu = 2/3$ which, as they themselves agree, is arbitrary. Here, both for the purpose of comparison and demonstration, we shall first give the results of our computations for $\mu = 2/3$. Later we shall investigate the sensitivity of the computed prices to different values of μ within the range $0 \leq \mu \leq 1$.

Table 2 gives the estimates of g_{ij} and h_{ij}, the lag coefficients for progressively added and initial entry inputs respectively, assuming that $\mu = 2/3$. [1]

Furthermore, given that the conditions for the existence of an aggregate lag structure between costs and prices can be assumed to hold through time, [2] we have used the 1963 Input-Output Table of the U.K. economy published by the Central Statistical Office to obtain estimates of a_{ij}, α_i, β_i and π_i for all $i,j = 1, ..., n$. These estimates, together with estimates of g_{ij}, h_{ij}, enable us to compute the 'final form' of the industry and manufacturing lag profiles as defined in relations (1.11) and (1.12) respectively. The results of these computations are given in Table 3.

(1) The data used for the computation reported here have been prepared by Mr. K. Courts who has also been responsible for the calculation of individual industry lag profiles.

(2) These conditions are given by (1.10).

Table 2* Individual industry lag profiles for progressively added and initial entry inputs to nine manufacturing industries, excluding food, drink and tobacco.

(a) Progressively added inputs

	Quarters			
	0	1	2	3
1. Chemicals and Allied Trades	0.3559	0.5858	0.0583	0.0
2. Steel	0.3011	0.5675	0.1313	0.0
3. Mechanical Eng.	0.1725	0.3451	0.3433	0.1391
4. Electrical Eng.	0.1673	0.3345	0.3345	0.1637
5. Textiles	0.2135	0.4269	0.3346	0.0250
6. Clothing and footwear	0.4404	0.5516	0.0081	0.0
7. Timber	0.4217	0.5638	0.0145	0.0
8. Paper and printing	0.2940	0.5616	0.1444	0.0
9. Others	0.1917	0.3834	0.3540	0.0709

* It is assumed that 2/3 of materials enter the productive process at the beginning.

(b) initial entry inputs

	Quarters			
	0	1	2	3
1. Chemicals and Allied Trades	0.0	0.5952	0.4048	0.0
2. Steel	0.0	0.3396	0.6604	0.0
3. Mechanical Eng.	0.0	0.0	0.1022	0.8978
4. Electrical Eng.	0.0	0.0	0.0108	0.9892
5. Textiles	0.0	0.0	0.6578	0.3422
6. Clothing and footwear	0.0	0.8646	0.1354	0.0
7. Timber	0.0	0.8143	0.1857	0.0
8. Paper and printing	0.0	0.2992	0.7008	0.0
9. Others	0.0	0.0	0.3918	0.6082

Table 3 Final lag profiles for nine manufacturing industries and total manufacturing, excluding food, drink and tobacco. It is assumed that 2/3 of materials enter the productive process at the beginning.

(a) Progressively added inputs

	Quarters									
	0	1	2	3	4	5	6	7	8	9
1. Chemicals and Allied Trades	0.2928	0.5016	0.0994	0.0507	0.0284	0.0132	0.0067	0.0034	0.0020	0.0019
2. Steel	0.2210	0.4350	0.1594	0.0777	0.0538	0.0254	0.0133	0.0068	0.0040	0.0036
3. Mechanical Eng.	0.1212	0.2492	0.2627	0.1645	0.0931	0.0543	0.0254	0.0144	0.0100	0.0051
4. Electrical Eng.	0 1143	0.2350	0.2465	0.1768	0 0985	0.0656	0.0299	0.0162	0.0120	0.0051
5. Textiles	0.1867	0.3774	0.3158	0.0639	0.0287	0.0133	0.0068	0.0036	0.0020	0.0018
6. Clothing and footwear	0.2600	0.3936	0.1503	0.1195	0.0416	0.0175	0.0084	0.0043	0.0031	0.0017
7. Timber	0.3405	0.4868	0.0788	0.0503	0.0221	0.0105	0.0052	0.0027	0.0018	0.0012
8. Paper and printing	0.2502	0.4909	0.1664	0.0466	0.0235	0.0107	0.0057	0.0029	0.0018	0.0013
9. Others	0.1487	0.3035	0 3031	0.1196	0.0619	0.0292	0.0156	0.0088	0 0063	0.0032
Total Manufacturing	0.1808	0.3396	0.2387	0.1152	0.0600	0.0320	0.0158	0.0087	0.0061	0.0032

(b) Initial entry inputs

	Quarters									
	0	1	2	3	4	5	6	7	8	9
1. Chemicals and Allied Trades	0.0	0.5021	0.3555	0.0463	0.0535	0.0240	0.0083	0.0049	0.0033	0.0020
2. Steel	0 0	0.2384	0.4769	0.0594	0.1001	0.0755	0.0201	0.0144	0.0103	0.0048
3. Mechanical Eng.	0.0	0.0048	0.0645	0.4212	0.1157	0.1796	0.1199	0.0398	0.0345	0.0200
4. Electrical Eng.	0.0	0.0043	0.0236	0.5131	0.0974	0.1512	0.1791	0.0320	0.0250	0.0242
5. Textiles	0 0	0.0023	0.5993	0.3323	0.0303	0.0190	0.0081	0.0041	0.0028	0.0018
6. Clothing and footwear	0.0	0.3194	0.1100	0.2892	0.2042	0.0426	0.0173	0.0081	0.0062	0.0029
7. Timber	0.0	0.6929	0.1805	0.0569	0.0442	0.0131	0.0061	0.0030	0.0020	0.0013
8. Paper and printing	0.0	0.2677	0.6359	0.0341	0.0325	0.0172	0.0056	0.0035	0.0020	0.0015
9. Others	0.0	0.0041	0 3132	0.5025	0 0698	0.0573	0.0221	0.0135	0.095	0.0080
Total Manufacturing	0.0	0.1571	0.3203	0.3249	0.0758	0.0593	0.0323	0.0130	0 0090	0.0082

In order to compute prices, we also need to know the weights by which to add the two types of input costs. These weights, which are implied by the pricing model itself, are computed and given in Table 4 below.

Table 4 Implied weights for adding the two types of input costs of manufacturing industries, excluding food, drink and tobacco.

	Progressively added inputs(γ_1)	Initial entry inputs(γ_2)
1. Chemical and allied trade	1.4951	1.9227
2. Steel	1.5805	1.4109
3. Mechanical Eng.	1.6691	0.8800
4. Electrical Eng.	1.6376	1.0689
5. Textiles	1.4444	2.2268
6. Clothing and footwear	1.5684	1.4833
7. Timber	1.4331	2.2941
8. Paper and printing	1.4336	2.2910
9. Others	1.5468	1.6129
Total manufacturing	1.5525	1.5785

These estimates of lag structures and implicit weights completely summarize the solution of the dynamic pricing model in terms of unit exogenous normal costs. Together with the unit normal costs derived in the previous section, they provide us with estimates of the total manufacturing price over time. Other estimates of the manufacturing price can also be computed by the same procedure for any value of μ in the range $0 \leq \mu \leq 1$.

Denoting all these price estimates by $\hat{p}(t;\mu)$, we have under the NPH(1)

$$\Delta \text{ Log } P(t) = \Delta \text{ Log } \hat{P}(t;\mu) + \varepsilon_t \quad t=1,\ldots,T \qquad (3.1)$$

where $P(t)$ is the observed price at time t and ε_t is the disturbance of the relation and is assumed to be distributed as $N(0,\sigma^2_\varepsilon)$.

At this stage it is important to note that the value of μ has two seperate and opposing influences upon predicted prices. On the one hand, the effect of increasing μ is to reduce the estimates of the production periods and hence of the mean lags (2); on the other hand, increasing μ implies imposing the 'initial entry' lag profile (which has a longer mean lag as compared with the 'progressively added' type) on a larger proportion of the material costs. Consequently, it is not possible to tell, on a priori grounds, what effect changing μ would have on the predicted prices.

In order to determine the significence of μ for price predictions, we have computed estimates of σ_ε and the log likelihood values of relation (3.1) for μ 0.0, 0.1,..., 1.0. The results of these computations are summarized in Table 5 below.

(1) $\Delta(\equiv 1-D)$ denotes the first difference operator.

(2) Note that the production period is given by $2S/(X + \mu M)$ where $S \equiv$ Stocks; $X \equiv$ Output and $M \equiv$ Total material costs [4, pp.862-65].

Table 5 Estimates of σ_ε and the log likelihood values of relation(3.1) for the proportion of materials entering at the beginning of the productive process (i.e. μ) within the range $0 \leq \mu \leq 1$.

μ	$\hat{\sigma}_\varepsilon$	$\hat{L} = -T \operatorname{Log}(\hat{\sigma}_\varepsilon)$
0.0	0.39%	326.60
0.1	0.40%	326.14
0.2	0.40%	325.56
0.3	0.41%	324.88
0.4	0.41%	324.24
0.5	0.42%	323.53
0.6	0.42%	322.79
0.66666	0.42%	322.30
0.7	0.43%	322.01
0.8	0.43%	321.21
0.9	0.44%	320.50
1.0	0.44%	319.61

These estimates indicate that as we increase μ from 0.0 to 1.0 the log likelihood values dicline uniformly, and that the Maximum Likelihood (ML) estimate of μ is $\hat{\mu} = 0$ which differs significantly from the value of 2/3 assumed by Nordhaus and Godley. This implies that as far as the prediction of prices is concerned the separation of the inputs into the 'initial entry' and 'progressively added' types is of no significance.

In Table 6 we have computed the aggregate lag structure. of material and non-material costs for values of μ= 0.0, which is the ML estimate of μ 1.

Table 6 Aggregate lag structures for material and non-material costs

	The proportion of materials entering at the beginning, μ = 0.0		The proportion of materials entering at the beginning, μ = 2/3	
quarters	Materials costs	Non-material costs	Material costs	Non-material costs
0.	0.1507	0.1507	0.0603	0.1808
1.	0.3098	0.3098	0.2179	0.3396
2.	0.2502	0.2502	0.2931	0.2387
3.	0.1746	0.1746	0.2550	0.1152
4.	0.0614	0.0614	0.0705	0.0600
5.	0.0279	0.0279	0.0502	0.0320
6.	0.0138	0.0138	0.0268	0.0158
7.	0.0064	0.0064	0.0116	0.0087
8.	0.0053	0.0053	0.0145	0.0093
mean lags	1.89	1.89	2.46	1.79

Note that by increasing μ from 0.0 to 2/3 we have increased the mean lag of material costs and at the same time decreased the mean lag of non-material costs. The overall result clearly depends upon the proportion of material costs (excluding fuel) in the total costs. Using the 1963 estimate of this proportion (i.e. 0.21) it is easily seen that for μ = 2/3 the mean lag of all costs is approximately 1.93 quarters which is slightly larger than the comparable estimate for μ =0.0 which is 1.89 quarters.

1. Fuel costs are included in non-material costs.

4. Tests of the normal price hypothesis[1]

Under the weakest version of the NPH it is specified that the mark up of price over unit 'normal historical' costs is insensitive to cyclical changes in demand. One simple functional relation for the alternative hypothesis that the mark up does depend on demand is

$$P(t)/\hat{P}(t;\mu) = \gamma_0 (X/XN)_t^{\gamma_1} (X/XN)_{t-1}^{\gamma_2} \ldots (X/XN)_{t-s}^{\gamma_s} \qquad (4.1)$$

where X/XN denotes the deviation of output from its exponential trend and is used as a proxy for the demand variable.

$P(t) \equiv$ Actual price

$\hat{P}(t;\mu) \equiv$ Computed price based on Nordhaus-Godely metnod of estimating individual industry lag profiles.

A suitable relation for estimation (4.1) can now be written as

$$\Delta \mathrm{Log}\ P(t) - \Delta \mathrm{Log}\ \hat{P}(t;\mu) = \sum_{i=1}^{s} \gamma_i \ \Delta \mathrm{Log}\ (X/XN)_{t-i} + \mu_t \qquad (4.2)$$

where the disturbance μ_t is assumed to have the AR1 error specification discussed in section 2.

Note that the relation employed by Nordhaus and Godley to test the NPH is

$$\Delta \mathrm{Log}\ P(t) = \sigma_o + \alpha_1 \ \Delta \mathrm{Log}\ \hat{P}(t;\mu) + \alpha_2 \ \Delta \mathrm{Log}(X/XN)_t \qquad (4.3)$$

which makes no allowance for the possible influence of lagged variables upon the mark up, and yet assume that the mark up

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(1) I should like to thank Prodessor Mirrlees for his helpful comments with respect to this section.

depends upon the variations in normal costs.(2)

Table 7* Regression estimates for tests of Normal Price Hypothesis within the Nordhaus-Godley frame work of estimation of individual industry lag profiles.

Part A The proportion of materials entering at the beginning: $\mu = 2/3$.

Constant	$\Delta Log(X/XN)_t$	$\Delta Log(X/XN)_{t-1}$	$\Delta Log(X/XN)_{t-2}$	$\hat{\rho}$	$\hat{\sigma}$
	-0.01032 (-0.30)			0.4216 (3.30)	0.39%
	0.00305 (0.09)	0.06602 (1.96)		0.4350 (3.51)	0.37%
		0.06326 (1.86)	-0.00932 (-0.27)	0.4326 (3.41)	0.38%
-0.00143 (-1.95)	-0.00633 (-0.19)			0.3387 (2.58)	0.37%
-0.00141 (-1.93)	0.00159 (0.05)	0.06375 (1.94)		0.3553 (2.79)	0.36%
-0.00143 (-1.94)		0.06146 (1.86)	-0.01211 (-0.37)	0.3525 (2.71)	0.37%

Part B The proportion of materials entering at the beginning: $\mu = 0.0$.

Constant	$\Delta Log(X/XN)_t$	$\Delta Log(X/XN)_{t-1}$	$\Delta Log(X/XN)_{t-2}$	$\hat{\rho}$	$\hat{\sigma}$
	-0.02118 (-0.68)			0.4453 (3.55)	0.35%
	-0.00766 (-0.25)	0.06002 (1.94)		0.4573 (3.71)	0.34%
		0.06167 (1.97)	-0.00131 (-0.04)	0.4548 (3.60)	0.35%
-0.00146 (-2.15)	-0.01812 (-0.59)			0.3490 (2.69)	0.34%
-0.00144 (-2.14)	-0.01017 (-0.34)	0.05785 (1.93)		0.3621 (2.84)	0.33%
-0.00143 (-2.09)		0.05880 (1.94)	-0.00398 (-0.13)	0.3622 (2.79)	0.34%

(*) The dependent variable of the above regressions is $\Delta Log\ P(t) - \Delta Log\ \hat{P}(t;\mu)$. The regressions are fitted to the period 1955 (I) - 1969 (IV). $\hat{\rho}$ denotes the Maximum Likelihood estimator of the parameter of the first order autoregressive specification assumed for the disturbances. Other symbols are defined in the text.

depends upon the variations in normal costs.[1]

The estimation of (4.2) over the period 1955(1) - 1969(4) for $\mu = 2/3$ and $\mu = 0.0$ gives the results summarized in parts A and B respectively of Table 7. It is clear that the current level of demand does not have any significant effect upon the mark up. But this is not so when we consider lagged values of the demand variable. Of several alternative lag structures which we assume for the demand variable, a simple one quarter lagged demand variable proves to be the most appropriate and gives a <u>marginally significant</u> coefficient. The quantitative effect of this lagged demand upon the mark up is, however, rather small and implies only a 0.06 percent increase in the mark up for every one percent increase in the demand variable.

A more restrictive version of the NPH is that the mark up is not only insensitive to demand changes but is also fixed over time. To test this version of the NPH we have re-estimated relation (4.2) with an intercept. The results are again summarized in Table 7. The fact that the intercept is negative and significant indicates that this latter version

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(1) Using relation (4.3) we get

$$\text{The mark up} = A\, e^{\alpha_o t} \,[\, \hat{P}(t;\mu)\,]^{1-\alpha_1}\, (X/XN)_t^{\alpha_2}$$

It is clear that unless $\alpha_1 = 1$, the mark up will be affected by $P(t;\mu)$ and hence the normal costs.

of the NPH cannot be accepted and that there has been a secular decline of the mark up of price over costs. This fact, which has also been pointed out by Nordhaus and Godley, requires further investigation.

5. The regression method

The tests of the NPH that we have so far carried out are all based on the Nordhaus-Godley procedure of estimating individual industry lag profiles. This procedure assumes that the only source of lag between prices and costs is the widespread practice of historical cost pricing among manufacturing firms and rules out other important sources of lags such as 'behavioural' and 'adustment' lags.(1)

As has already been mentioned, an alternative approach for testing the NPH which does not have the above-mentioned shortcomings is to estimate a distributed lag relation between unit normal costs and prices together with an index of demand. This is the method adopted by Neild [2] and it allows an 'average' estimate of 'institutional','behavioural' and 'adjustment' lags. However, Neild's estimating relation and his estimation procedure are rather restrictive and have attracted a number of criticisms especially from Rushdy and Lund [8] (2). In order to avoid the problems connected with Neild's approach here we estimate a linear relation between prices and costs, imposing two separate geometrically declining lag structures on material and non-material costs with differing lag coefficients. Furthermore, we assume the disturbances of this relation follow a first order autoregressive process. The estimation method employed is similar to that explained in [6] except that instead of one we now have <u>two</u> variables which are subject to geometric lag structures. In order to test for the effect of demand on prices we have also included current and lagged values of the demand variable in this aggregate pricing relation.

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(1) This also means that the estimates of lag profiles are assumed to be independent of whether the NPH is rejected or not.

(2) But see Neild's reply [3]

Table 8* Distributed lag relations between manufacturing price and costs over the period 1953(1) - 69(4). The regression approach

F1:
$$P(t) = \underset{(7.24)}{26.95} + \underset{(4.66)}{7.09} \sum_{i=0}^{\infty} \underset{(4.01)}{0.5^i}\, mc(t-i) + \underset{(4.05)}{36.47} \sum_{i=0}^{\infty} \underset{(2.70)}{0.4^i}\, nmc(t-i) \underset{(-0.95)}{-2.53} (X/XN)_t + \hat{u}_t$$

$$\hat{u}_t = \underset{(8.72)}{0.92}\, \hat{u}_{t-1} + \hat{v}_t \; ; \; \hat{\sigma} = 0.2933 \; ; \; \bar{R}^2 = 0.9991$$

F2
$$P(t) = \underset{(6.26)}{25.49} + \underset{(4.12)}{6.57} \sum_{i=0}^{\infty} \underset{(3.52)}{0.5^i}\, mc(t-i) + \underset{(3.77)}{36.62} \sum_{i=0}^{\infty} \underset{(2.51)}{0.4^i}\, nmc(t-i) \underset{(-0.97)}{-2.80} (X/XN)_t \underset{(0.89)}{+2.63} (X/XN)_{t-1} + \hat{u}_t$$

$$\hat{u}_t = \underset{(8.08)}{0.92}\, \hat{u}_{t-1} + \hat{v}_t \; ; \; \hat{\sigma} = 0.2983 \; ; \; \bar{R}^2 = 0.9990$$

F3
$$P(t) = \underset{(6.18)}{23.63} + \underset{(3.66)}{5.51} \sum_{i=0}^{\infty} \underset{(2.91)}{0.5^i}\, mc(t-i) + \underset{(3.91)}{37.37} \sum_{i=0}^{\infty} \underset{(2.59)}{0.4^i}\, nmc(t-i) + \underset{(1.13)}{3.30} (X/XN)_{t-1} \underset{(-0.20)}{-0.56} (X/XN)_{t-2} + \hat{u}_t$$

$$\hat{u}_t = \underset{(7.71)}{0.899}\, \hat{u}_{t-1} + \hat{v}_t \; ; \; \hat{\sigma} = 0.2935 \; ; \; \bar{R}^2 = 0.9991$$

F4
$$P(t) = \underset{(1.92)}{23.34} \underset{(-0.02)}{- 0.00344}T + \underset{(3.61)}{5.49} \sum_{i=0}^{\infty} \underset{(2.09)}{0.5^i}\, mc(t-i) + \underset{(3.83)}{37.63} \sum_{i=0}^{\infty} \underset{(1.89)}{0.4^i}\, nmc(t-i) + \underset{(0.97)}{3.3197} (X/XN)_{t-1}$$
$$- 0.18\, (X/XN)_{t-2} + \hat{u}_t$$

$$\hat{u}_t = \underset{(7.66)}{0.90}\, \hat{u}_{t-1} + \hat{v}_t \; ; \; \hat{\sigma} = 0.2959 \; ; \; \bar{R}^2 = 0.990$$

(*)

P(t)	≡	Actual price (1963=100)
mc(t)	≡	Index of unit normal material costs (1963=1.0)
nmc(t)	≡	Index of unit normal non-material costs (1963=1.0)
T	≡	Time trend

For the other notations see the footnote to page 9.

The results of estimating such a distributed lag structure between price and costs are summarized in Table 8 and clearly substantiate the finding that the current level of demand does not have any significant effect upon the mark up in manufacturing industries. The results in this Table also show that the mark up is insensitive to changes in the demand variable. This is in direct contrast to our previous conclusion. One reason for this is that the estimates of lag profiles given in Table 8 are markedly different from those which we obtained employing the Nordhaus-Godley procedure. (see Table 6). The mean lags of material and non-material costs are now estimated to be one quarter and 2/3 repectively which are much smaller than those obtained previously. Furthermore, the inclusion of a time trend in the regression of price on the normal costs proved insignificant. This together with the fact that the constant is significant indicates that although a proportional mark up of price over costs has been declining over the period 1953-69,an additive mark up has remained more or less unchanged.

consequently, our 'preferred' relation for the explanation of manufacturing prices in terms of costs will be

$$p(t) = \underset{(8.53)}{24.99} + \underset{(4.63)}{6.72} \sum_{i=o}^{\infty} \underset{(3.83)}{0.5^{i}}\, mc(t-i) + \underset{(4.08)}{36.62} \sum_{i=o}^{\infty} \underset{(2.72)}{0.4^{i}}\, nmc(t-i) + \hat{u}_t \qquad (5.1)$$

$$\hat{u}_t = \underset{(8.67)}{0.92}\, \hat{u}_{t-1} + \hat{v}_t \ ; \ \hat{\sigma} = 0.2932 \ ; \ \bar{R}^2 = 0.9991$$

where $mc(t) \equiv$ Index of unit normal material costs (1963=1.0)
$nmc(t) \equiv$ Index of unit normal non-material costs (1963=1.0)

This relation can also be written as

$$p(t) = 24.99 + 13.44\ mc^*(t) + 61.03\ nmc^*(t)$$

where 13.44 and 61.03 are the estimates of the long-term response of prices to a change in material and non-material costs respectively. It is worth nothing that the sum of these long-term effects and the weight attached to the other costs (i.e. 24.99) add, as they should (see, [2, p.19]), to approximately 100 which is the price index in 1963. Furthermore, the proportion of material costs in the total material and non-material costs is equal to 18% which is very close to the estimate of this proportion (i.e. 21%) obtained from the 1963 Input-Output Tables.

more, the proportion of material costs in the total and non-material costs is equal to 18% which is very close to the estimate of this proportion (i.e. 21%) obtained from the 1963 Input-Output Tables.

In order to check that the estimates given in Table 8 do in fact refer to the global maximum of the likelihood function, we have drawn the contour of the concentrated log likelihood surface in terms of the geometric lag coefficients of the two types of costs. In every case the log likelihood surface proved to be unimodal. Such a contour for relation (5.1) is given in Figure 1, which demonstrates the unimodal nature of the likelihood function.(1)

Table 9* Actual and predicted prices for manufacturing, excluding food, drink and tobacco over the period 1967(I) - 1969(IV).

	Actual Price	The Regression Method	The Nordhaus - Godley Procedure	
	$P(t)$	$\hat{P}(t)$	$P(t;2/3)$	$\hat{P}(t;0.0)$
1967(1)	107.60	107.77	111.84	111.66
(2)	107.57	107.94	112.07	111.86
(3)	108.40	108.74	112.40	112.21
(4)	109.17	109.84	113.11	113.14
1968(1)	111.40	111.66	114.52	114.87
(2)	112.53	112.86	116.56	116.96
(3)	112.73	113.66	118.44	118.57
(4)	113.13	114.70	119.82	119.88
1969(1)	114.07	115.83	121.00	121.12
(2)	115.10	116.73	122.31	122.41
(3)	116.47	117.61	123.56	123.69
(4)	118.00	118.40	124.73	124.87

(*) $\hat{P}(t)$ denotes price predictions based on regression estimates.
$\hat{P}(t;\mu)$ denotes price predictions employing the Nordhaus-Godley procedure of estimating individual industry lag profiles: $\mu = 2/3$ and $\mu = 0.0$.

(1) I should like to thank B. Shearey and G. Rendle for providing me with a suitable subroutine in order to draw this contour.

One method of evaluating the two procedures for estimating lag structures discussed in this paper is to compare their predictive performance. For this purpose we have re-estimated relation (5.1) over the period 1953(1)- 1966(IV) Employing this pricing relation we have used only data on unit normal costs to predict prices for the period 1967(1) - 1969(IV). These price predictions are directly comparable with the predicted prices obtained by using the Nordhaus-Godley estimates of individual industry lag profiles. The results of these predictions together with actual prices are given in Table 9 .

Clearly, as far as the prediction of the level of prices is concerned, the 'regression method' has proved to be far superior to the Nordhaus-Godlev procedure. This is not, however, very important if we are primarily interested in the prediction of the percentage change in prices which does not depend upon an accurate estimate of the mark up. We have plotted the percentage change in actual and predicted prices over the period 1967(2) - 1969(4) in Figure 2. Again a casual look at this Figure indicates that the 'regression method' has also been much better in predicting percentage changes[1]

These results again emphasize the averaging nature of the 'regression method' which is of great importance for explanig and predicting price movements especially when properly specified dynamic models are simply not available.

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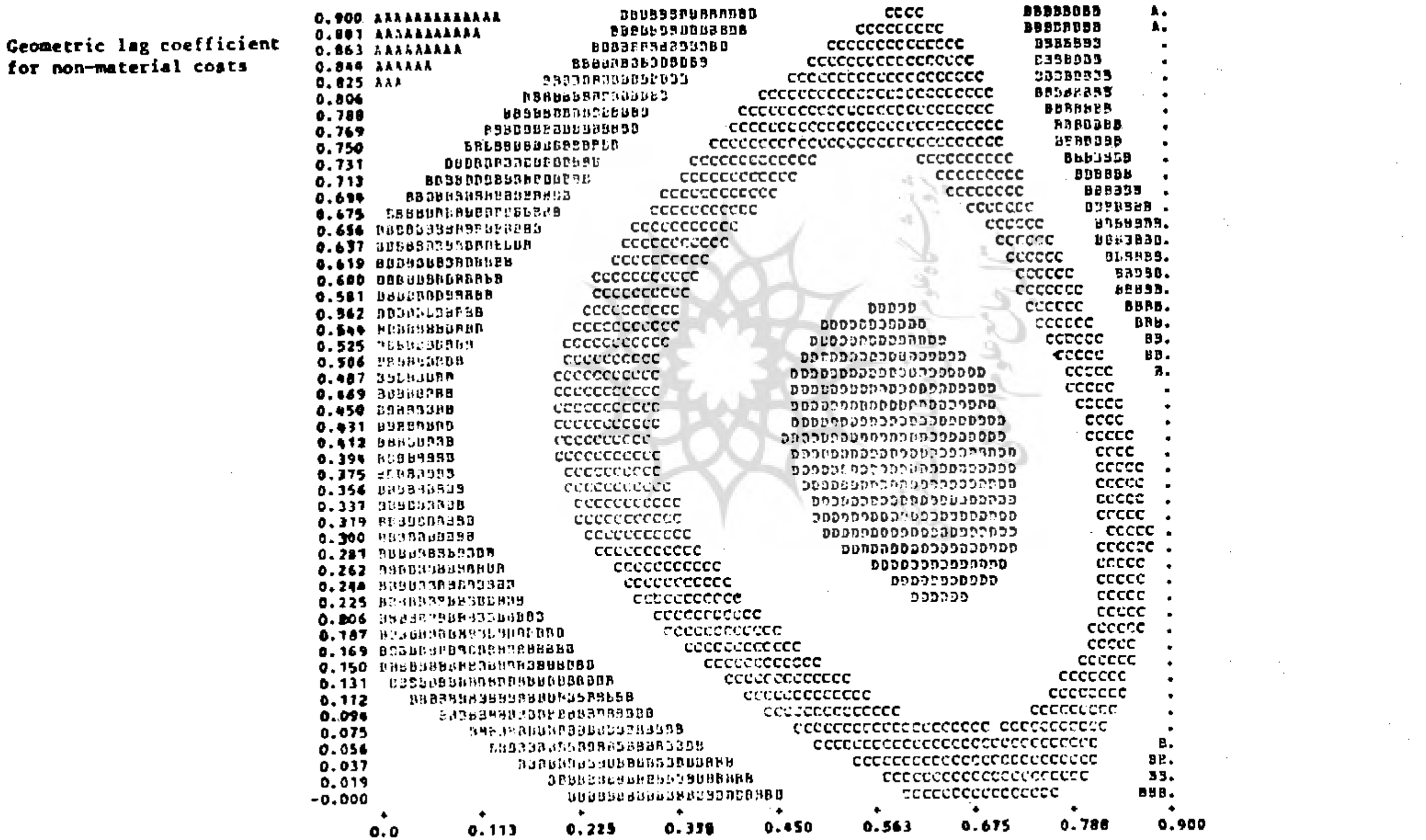
(1) The Root Mean Square Prediction Errors (RMSPE) of the alternative prediction of the percentage change in prices were

	The Regression Method	The Nordhaus-Godley Procedure	
		$\mu = 2/3$	$\mu = 0.0.$
RMSPE ($\Delta P/P$)	0.372%	0.635%	0.557%

Fig 1

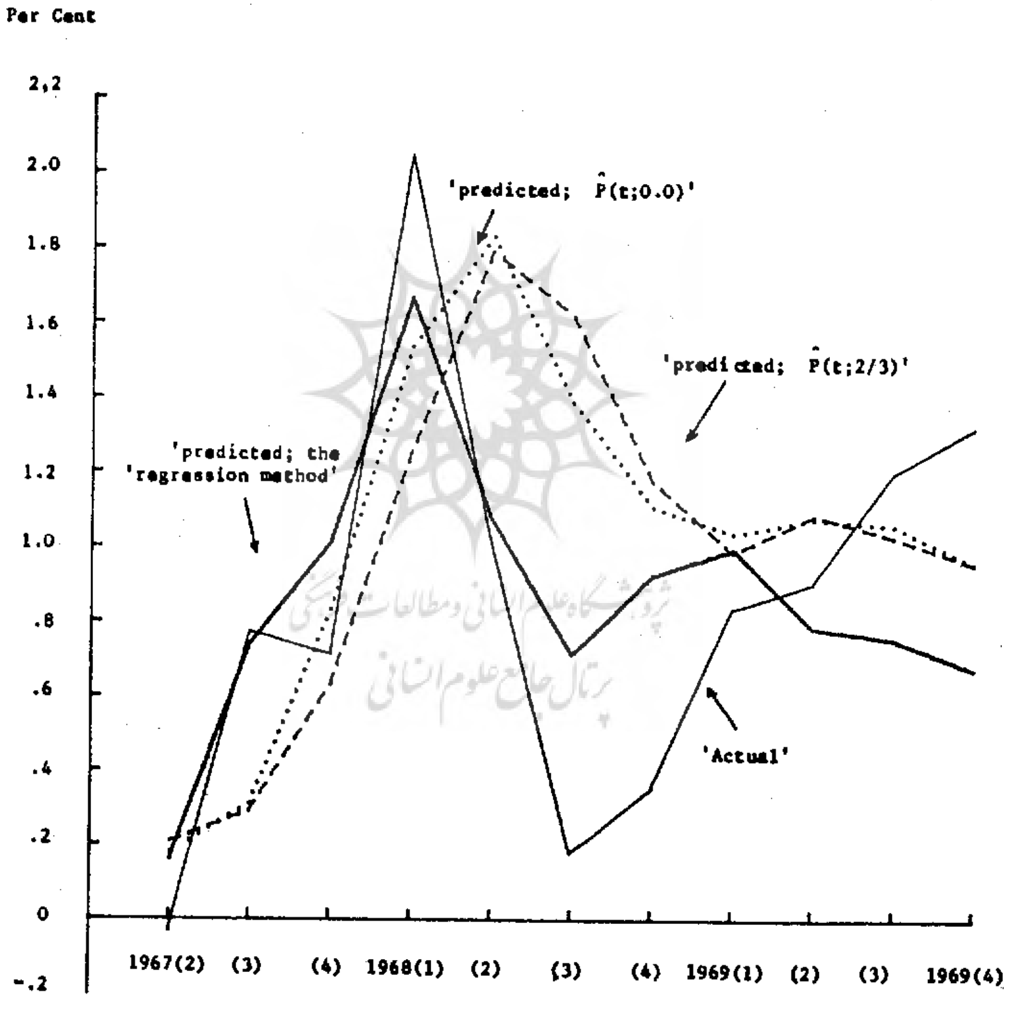
The contour of the concentrated log likelihood surface for the lag coefficients of material and non-material costs.



CHARACTERS	A		B		C		D
LEVELS		73.5	76.6	79.7	81.2	82.8	84.3

Fig. 2(*)

Actual and Predicted Percentage Change in Prices over the period 1967(2) - 1969(4)



(*) $\hat{P}(t;\mu)$ denotes price predictions employing the Nordhaus-Godley procedure of estimating individual industry lag profiles; $\mu = 2/3$ and $\mu = 0.0$.

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