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Paraconsistency and its Possibilities: a personalised and partial perspective of the past

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ABSTRACT

I take paraconsistent logic to be one of the most important and significant developments in logic and metaphysics in the last 100 years, challenging, as it does, one of the deepest dogmas entrenched in Western philosophy: that consistency is a sine qua non of rational thought. This paper looks back at the modern development of the subject. It is partial, in that it concerns only what one might call the formative years of the subject. It is personal, in that I describe my own involvement in some areas. The first part of the paper concerns paraconsistency as such. It starts with the name itself, its origin and its meaning. After a brief look at the history before the 20th century, it notes the work of the early pioneers: Orlov, Jaśkowski, Halldén, Smiley, Val and Richard Routley, and da Costa. For each of these, it describes both what motivated them, and the techniques they used in their constructions. It then describes the early reception of paraconsistency, and the way that it became a global movement. Next, the paper turns to dialetheism, again starting with the name itself and its meaning. After a brief look at the history of the notion before the 20th century, it describes the origin of the modern subject in the work of Richard Routley/Sylvan and myself. Finally, it turns to one of the major applications of paraconsistency: inconsistent mathematics—and specifically set-theory, arithmetic, and geometry. It notes the work of da Costa, Routley/Sylvan, Meyer, Mortensen, and others, describing some of the results concerning the mathematical theories they investigated.

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Indeed, even at this stage, I predict a time when there will be mathe- matical investigations of calculi containing contradictions, and people will actually be proud of having emancipated themselves from con- sistency. (Wittgenstain, 1964, 332)

Introduction

Contemporary paraconsistency and its entourage of ideas is, in my view, one of the most important and significant developments in logic and metaphysics in the last 100 years, challenging, as it does, one of the deepest dogmas entrenched in Western philosophy: that consistency is a *sine qua non* of rational thought. What follows is a somewhat personal glance back at some of its history. It is not an attempt at a systematic history: that would require something inordinately longer. Neither is there anything essentially novel in its contents. However, I hope that having the observations together in one place performs some useful function.

The essay is broken up into two parts. The first concerns paraconsistency it- self, and especially what one might call its modern formative years. The second concerns two of the significant things it makes possible: dialetheism and incon- sistent mathematics.[1]

1. Paraconsistency

1.1. The Name

So let us start with paraconsistency and, first, the name itself. As is now well known, this was coined by the Peruvian philosopher Miró Quesada in a letter to Newton da Costa of 1975. In the 1960s and early 1970s, da Costa had been devel- oping paraconsistent logics, and was now seeking a name for this kind of logic. He wrote to his old friend Quesada, who suggested several possible names. His preferred one, which da Costa adopted, was ‘paraconsistent’. The name was used in public for the first time at the 3rd Latin American Symposium on Mathemat- ical Logic, held at the University of Campinas, July 1976.[2] After considering ‘ultraconsistent’ and ‘metaconsistent’, Quesada writes (Bartolo Alegre, 2020, 169):

There is, however, another possibility: use [the prefix] ‘para’, which in Greek means *next to*. ‘Paraconsistent logics’ sounds nice, a bit esoteric, it gives a more or less precise idea of what it is about (logics that are not like the classical ones, but are a little *next to* them because they can be applied to inconsistent systems) and it has the advantage that there is no deforming semantic load.

[1] This is a written-up version of a lecture given in February 2023 at SPLogiC, Sa˜o Paulo School of Advanced Science on Contemporary Logic, Rationality and Information, University of Camp- inas.

[2] For the letter itself and a brief discussion thereof, see Bartolo Alegre (2020).

In fact, the Greek prefix ‘para’ can be translated in a number of different ways. One of these is as *beyond*—as in *paradox*: beyond belief. Until I was informed otherwise by da Costa, I had always assumed that this was the sense that Quesada had in mind, since the logic indeed allows one to reason beyond the consistent. I confess that I still prefer this reading.

1.2. The Meaning

When Quesada suggested the name ‘paraconsistent’ it was simply a name for the sort of logic that da Costa had been developing. Nowadays it has acquired a more precise meaning, namely for a logic in which the principle of Explosion is not valid. As far as I know, that definition was first given explicitly in Priest and Routley (1983).

I had been working on what I learned to call paraconsistent logic for a year or two before I moved to Australia in 1976. I met Richard Routley (Sylvan, as he later became) at the first meeting of the Australasian Association for Logic I attended, in Canberra, August 1976. Richard had attended the 3rd Latin American Symposium on Mathematical Logic in July, (D’Otaviano & Gomes, 2020) and (I presume) learned the word there. I learned it from him.

Soon after that, he and I decided to edit a collection of essays written by all the people we knew to be working in the area.[1] We wrote the essays in Priest and Routley (1983) as the introductory chapters of that volume. The definition of paraconsistency is given (p. 108), essentially as follows. A consequence relation,

$\vdash$, on a language $\mathcal{L}$ is paraconsistent iff:

D1 for some A and B of $\mathcal{L}$: $A, \neg A \nvdash B$

where $\neg$ is the negation of $\mathcal{L}$. There is another natural definition:

D2 for some Σ, A, B: $\Sigma \vdash A$, $\Sigma \vdash \neg A$ and $\Sigma \nvdash B$

D1 clearly entails D2 (assuming that if $A \in \Sigma$ then $\Sigma \vdash A$). Just let $\Sigma = \{A, \neg A\}$. The converse does not hold. There are logics which validate Explosion, but which still allow there to be sets of sentences whose logical consequences are inconsis- tent but non-trivial. The reason is that transitivity of deducibility, aka Cut, fails. (Cut is: if $\Sigma \vdash \mathrm{A}$ and $\Pi, A \vdash \mathrm{B}$ then $\Sigma \cup \Pi \vdash \mathrm{B}$.) As our essay goes on to note, if Cut holds, D2 entails D1.[2]

D2 is perhaps more in the “spirit” of paraconsistency—which is to allow the use inconsistent information sensibly; but when Richard and I wrote the essay, logics with Explosion but without Cut were not very significant; and D1 is sim- pler. Such logics have now come to assume greater significance. Indeed, they have been advocated as solutions to the paradoxes of semantic self-reference.[3] Notwithstanding, I am not disposed to change the

[1] This appeared as Priest, Routley, and Norman (1989).

[2] One can see this by proving the contrapositive. Suppose that D1 fails. Then for all A, B: $A, \neg A$ s B. We show that D2 fails. That is: for all Σ, A, B: if Σ s A and Σ s $\neg A$ then Σ s B. Now, suppose that Σ s A and Σ s $\neg A$. The result follows by two applications of Cut (and Contraction).

[3] For discussion and references, see Priest (2024).

definition. The reason is that I now think that Cut should be built into the definition of a logic intended as an account of good reasoning since, if Cut fails, the closure of a set under sentences under the consequence relation of the logic need not be a theory (in the logicians' sense) (Priest, 2025b, 2.2.).

1.3. History Before the 20th Century

One might think that paraconsistency is a relatively recent phenomenon. It is not. It goes back to the origins of logic in the West. Syllogistic is paraconsistent. Thus, the following syllogism is invalid, though its premises are contradictory:

- Some men are animals. No animals are men. So, all men are men. Indeed, Aristotle himself points out the paraconsistency of syllogistic:[1] In the first figure [of syllogisms] no deduction whether affirmative or negative can be made out of opposed propositions: no affirmative de- duction is possible because both propositions must be affirmative, but opposites are the one affirmative, the other negative. In the middle figure a deduction can be made both of opposites and of contraries. Let A stand for good, let B and C stand for science. If then one as- sumes that every science is good, and no science is good, A belongs to every B and to no C, so that B belongs to no C; no science, then is science. Similarly, if after assuming that every science is good one assumed that the science of medicine is not good; for A belongs to every B but to no C, so that a particular science will not be a science... Consequently, it is possible that opposites may lead to a conclusion, though not always or in every mood.

In fact, the first recorded appearance of Explosion in the history of logic is from Paris in the 12th century, in the Parvipontinian School of Adam of Balsham. It is thought that the principle is due to one of its members, William of Soissons (Martin ,1986).

After that time, the principle has a mixed fate in Medieval logic. Medieval ac- counts of logical consequence are numerous, though it is standard to distinguish between formal consequence and various kinds of material consequence.[2] Dif- ferent logicians give somewhat different accounts of formal conference and the various kinds of material consequence, however. Explosion figures in some of these accounts, but not others. In any case, virtually all knowledge of Medieval logic was lost in early Modern philosophy; and the standard account of logical consequence thereafter is again syllogistic.

Explosion becomes entrenched in standard logic after the development of so-called classical logic around the turn of the 20th Century. This account of logical consequence was produced

[1] *Prior Analytics* 63^b31–64^a16. The translation is from Barnes (1984). Note also that there is nothing suspicious about taking some of the terms of a syllogism to be the same. As this quote shows, Aristotle explicitly allows for this.

[2] On Medieval accounts of consequence, see Dutilh Novaes (2020).

initially, and most notably, by Frege and Russell, and subsequently polished by Hilbert, Tarski, Gentzen, and others.

The history of the term 'classical logic' is, incidentally, itself an interesting one. The theory was not called 'classical logic' by its founders: it was just logic. However, early in the new century, a new kind of mathematics, intuitionist math- ematics, was proposed by Brouwer. This rejected the validity of certain orthodox mathematics of the time. Brouwer needed a term to refer to the kind of mathemat- ics he rejected; and he used the term 'classical'. This was picked up by the ortho- dox. Quite some time later, the epithet was transferred from the mathematics to the logic that came to be held by the orthodox to underlie such mathematics—that of Frege and Russell. The term was certainly in use in this way by about 1930, but I do not know where it is first used in print with this meaning.

1.4. Some 20th Century History

1.4.1. Orlov and Relevant Logics

Modern non-classical logics (intuitionist, modal, many-valued) started to appear very early in the new century. Paraconsistent logics are certainly non-classical. These started to appear somewhat later. Paraconsistent logics are not a natural kind. Virtually any technique for constructing a logic can be used to construct a paraconsistent logic. In this section let us look at the earliest appearances of various techniques that were so used, together with their motivations.[1]

The earliest known modern paraconsistent logic appears in a paper by the Russian polymath Ivan Orlov in 1928, 'The Calculus of Compatibility of Propo- sitions'.[2] He gives an axiomatization of the relevant logic which we would now call the intensional fragment of R. That is, the primitive symbols are negation and fusion, ¬, ◦. The conditional, →, and fission, +. are defined. ◦ is understood as joint compatibility. Orlov is not clear about the distinction between the conditional and deducibility, and appears to identify the two. This being so, the logic is paraconsistent. Neither of the following is valid in the system:[3]

- $A \rightarrow (\neg A \rightarrow B)$
- $(A \circ \neg A) \rightarrow B$

Orlov's motivation is a curious one.[4] He wishes to add a modal provability opera- tor, □, to the machinery; and he holds that if one uses the material conditional and Excluded Middle, modal collapse occurs. (That is, one may prove that $\Box A \leftrightarrow A$.) This is a mistake, but it leads him to

[1] Neither is this a comprehensive account of such techniques and motivations. For example, there are the adaptive logics developed and explored by Diderik Batens and his colleagues. See, e.g., Batens (1986).

[2] English translation, Orlov (2024). A discussion can be found in Dosˇen (1992)

[3] And when an extensional conjunction is added, neither is $(A \wedge \neg A) \rightarrow B$.

[4] On what follows, see Priest (2021)

replace the material conditional with the rel- evant →, where A → B is defined as ¬(A ∘ ¬B). That is, A is not compatible with ¬B.

Orlov's paper fell into oblivion; but drawing on the work of Church (1951) and Ackermann (1956), the inadequacy of the material conditional as an account of a legitimate conditional was taken up in the late 1950s by the US logicians Alan Anderson and Nuel Belnap, who founded the development of contemporary relevant logics (Anderson & Belnap, 1975).

Before we leave Russia, let us pause for a second on Nikolai Vasil'év. In a number of papers, most notably his 'Logic and Metalogic' of 1912/13, he cons- structed a logic he called Imaginary Logic.[1] This is an extension of syllogistic, augmenting its two forms of predication, 'is P' and 'is not P', with a third, 'is and is not P'. Vasil'év is sometimes cited as having produced an early paraconsistent logic. This, I think, is a confusion. Imaginary logic is not a modern logic: it is simply an extension of syllogistic which, as we have noted, is already paraconsis- tent. And as a reading of the paper cited above demonstrates, Vasil'év holds that things can satisfy the new predicate only in imaginary (i.e., non-actual) worlds. If anything, his work is therefore a harbinger of the theory of impossible worlds.[2]

1.4.2. Jas´kowski and Non-Adjunctive Logics

Moving a few miles to the West, the first well-known paper on contemporary paraconsistent logic is the Polish logician Stanisław Jas´kowski's 'Propositional Calculus for Contradictory Deductive Systems' of 1948.[3] Jas´kowski was moti- vated by the thought that our information might be provided by different sources, which therefore naturally break it into chunks. He formalised this by applying modal logic. He was writing before the world semantics for these logics, but what his approach comes to in terms of these is that each chunk can be thought to be what holds in a possible world of S 5 (and so is consistent). Given an S 5 inter- pretation $\mathfrak{M}$, A is true in $\mathfrak{M}$ iff A is true in some world in M. Validity is defined in terms of truth-preservation in all interpretations. Hence Explosion fails since A and ¬A can hold at different worlds of an interpretation, whilst B holds at none.

However, since the worlds are consistent, a conjoined contradiction, A ∧ ¬A, holds at none. So, the conjoined form of Explosion, A ∧¬A ⊢ B, is valid. It follows that the principle of Adjunction fails: A, B ⊬ A ∧ B. This is, in fact, little more than a consequence of the information being chunked. The idea of chunking, and so of non-adjunctive paraconsistency, was subsequently taken up in different ways by the US logicians Nicholas Rescher and Ruth Manor, and the Canadian logicians Peter Scotch and Ray Jennings.

[1] English translation, Vasil'e´v (1993).

[2] For further discussion of Vasil'e´v, see Priest (2000a).

[3] English translation, Jas´kowski (1969). For further details of this subsubsection, see Priest (2002), 4.2.

1.4.3. Halldén and Many-Valued Logics

Moving a few miles West again, the next paraconsistent logic was published by the Swedish logician Soren Halldén, in his essay of 1949, The Logic of Nonsense (Hallde´n, 1949). This was a 3-valued logic. Two values are the usual true, t, and false, f. Call the third value b. t and b are designated, and b is a fixed point for negation. Hence if the value of A is b, both it and its negation are designated. Hence, Explosion fails. What is distinctive about Halldén's logic is that b is infectious. That is, if the value of any subformula of A has the value b, so does b. Halldén justifies this by taking sentences with the value b to be some sort of nonsense. He gives as ex- amples of nonsensical sentences, paradoxical sentences such as the liar—though why the values of such sentences should be designated if they are nonsense is unclear.[1]

Subsequent writers, such as Florencio Asenjo, J. Michael Dunn, and myself, were to suggest other many-valued paraconsistent logics, including a 3- (LP) or 4- (FDE) valued logic where one of the values is designated, not infectious, and interpreted as both true and false (or at least, informed so). This at least makes it clear why it should be designated.

1.4.4. Smiley and Non-Transitive Logics

Moving East again, the next published paraconsistent logic was given by the British Logician Timothy Smiley in his essay 'Entailment and Deducibility', of 1959. (Smiley, 1959) This defines validity by starting with classical validities, and then filtering out the ones not wanted. Smiley was moved by the thought that validity should not hold vacuously. So, the inference from $A_1, ..., A_n$ to B is valid if B is a classical consequence of $A_1, ..., A_n$, $A_1 \wedge ... \wedge A_n$ is not a classical contradiction, and B is not a classical tautology. Or, to be precise, it is a substitution instance of such an inference, so that validity is closed under uniform substitution. Obviously, Explosion fails this test.

A general feature of filter logics[2] is that transitivity fails. Thus, on Smiley's filter, $A, \neg A \vdash A \wedge (\neg A \vee B)$ (since it is a substitution instance of $p, q \vdash p \wedge (q \vee r)$) and $A \wedge (\neg A \vee B) \vdash B$, but it is not the case that $A, \neg A \vdash B$.[3] Subsequently, non- transitive substructural logics (paraconsistent according to Definition D2) were advocated by Neil Tennant, Paul É gré, Ellie (David) Ripley, and others—perhaps most notably as providing solutions to various paradoxes including the liar and the sorites.

1.4.5. Routley and Routley, and Relevant Logics

The relevant logics of Orlov, and Anderson and Belnap, were specified purely axiomatically. A suitable semantics was found only later. A key step in this con- cerned the treatment of negation discovered by the Australasian logicians Richard and Val Routley in their 'Semantics

[1] Further on Hallde´n, see Priest (2025a), 4.3.
[2] Though not an invariable one. See Dunn (1980).
[3] Further on filter logics, see Priest (2002), 4.1.

of First-Degree Entailment', of 1972. (Routley & Routley, 1972) First Degree Entailment (FDE) is the core of all standard relevant logics, but itself has no conditional connective.

The semantics is a world semantics. Each world, w, comes with a mate, w*, such that w** = w. And:

- ¬A is true at w if w is not true at w*
- —not w, as would be the case in standard modal logic. This allows for worlds where both A and ¬A are true, or neither A not ¬A is true. In particular, if p is true at w but not w*, then p and ¬p are both true at w. Hence there can be worlds where p and ¬ p is both true, but q is not, giving a counter-example to Explosion. The semantics was extended later with an appropriate semantics for the conditional by Richard and Bob Meyer.

That modal operators require a world-shift for their evaluation is, of course, standard. That negation might also require one was novel. The idea was later extended to more complex semantics for negation by Dunn, Greg Restall, and others (Horn and Wansing, 2020, 2.2.).

1.4.6. Da Costa and Non-Truth-Functional Logics

Finally, let us stay in the southern hemisphere, but go to the other side of the globe. The Brazilian logician Newton da Costa started to work on paraconsistent logics in his doctoral dissertation of 1963, Systemas Formais Inconsistentes. His aim appears to have been, loosely, to dualise the intuitionistic notion of negation, so allowing for statements to be inconsistent, rather then incomplete.[1] A summary of his logical systems appeared in English in his 1974 essay 'On the Theory of Inconsistent Formal Systems'.[2] The treatment was purely axiomatic, but when a semantics was found a few years later by himself and Elias Alves, this had a non-deterministic account of negation.[3] If A is false in an interpretation, ¬A is true. But if A is true, ¬A can be true or false. Hence it is possible to have both A and ¬A true, and so a counterexample to Explosion. Another significant feature of the logics was the ability to express the consistency of a sentence in the language itself. There is a monadic operator, $^\circ$, such that A° expresses the consistency of A. Da Costa's motivation for his logics was clearly to make it possible to develop and explore interesting inconsistent theories. The 1974 essay cites both set theory and a Meinongian theory of objects in this regard. More of this in due course.

The idea of a non-deterministic semantics for a paraconsistent logic was later taken up and developed by others, such as Diderik Batens and Arnon Avron. And the use of a consistency (or, more generally, "classicality") operator was taken up and developed into a theory of LFIs (Logics of Formal Inconsistency) by other Brazilian logicians, such as Walter Carnielli, Marcelo Coniglio, and Joaõ Marcos.

[1] There are other ways of dualising intuitionistic logic. See, e.g., Priest (2009). In fact, the first person who realised that this was possible was Popper in 1948. See Binder and Piecha (2017).

[2] da Costa (1974).

[3] da Costa and Alves (1977).

1.4.7. And...

Let me end this review with a comment on two papers from the foundational pe- riod of the subject which I have not so far mentioned—those by two US logicians, David Nelson and William Cooper.

Nelson (1949) provides a critique of the intuitionistic account of negation. Thus, just as one should not be able to claim to have proved A v B unless one has a proof of A or a proof of B, one should not be able to claim to have proved that ¬ (A ∧ B) unless one has a proof of ¬A or ¬B (which is not the case in intuitionist logic. Hence, one should not expect ¬ (A ∧¬A) to be a logical truth. On the basis of this idea, Nelson proposed a logic with "strong negation". This is a paraconsistent logic, now usually called N_4, which has models in which both A and ¬A may hold, for some A. One such model he constructed was an inconsistent arithmetic (the first known).

Cooper (2019) was concerned with infelicities of the classical material condi- tional. On the basis of this, he proposed a 3-valued logic, OL, in which ¬ (A → B) is equivalent to A → ¬B. Two of the three values are classical. The non-classical value is a fixed point for negation, interpreted as some sort of defective state- ment—that of conditionals with a false antecedent. This is designated. (As for Halldén, no conceptual ground for this is provided.) Hence the logic is paracon- sistent.

Both of these papers fell largely into oblivion, and so had no immediate impact on the development of the subject. However, in the last ten years there have been significant investigations of both ideas. Indeed, they have come together in the now blossoming study of connexive logics (Wansing, 2023).

1.4.8. Some Final Remarks

The development of formal paraconsistent logics has continued apace since the 1970s, of course. We have seen many novel developments in semantics, proof the- ory, algebraicization, applications, and other things. But let us finish our history here. We have covered what one might call the "first generation" of paraconsistent logicians. As is clear, they were spread widely across the globe; and each devel- oped their ideas largely in ignorance of the work of the others. Each had quite different motivations and deployed different techniques. However, the more or less simultaneous and independent discoveries of paraconsistent logic show that it was a subject whose time had come.

In the late 1970s Richard and I decided to make the paraconsistency movement "self-conscious". To this end we set out to edit the first collection of essays with contributors from all over the world. The result was Paraconsistent Logic: Essays on the Inconsistent. (Priest, Routley & Norman ,1989) The ms was sent off to the press about 1983; but due to a financial miscalculation by the press, the book appeared (as a very expensive volume!) only in 1989. After this, Richard and I decided that it was time to have an international conference on the subject. Australia was not a good place for this because of its relative geographical isolation. So, we approached Diderik Batens, who kindly agreed to host it in Gent. Thus, the First World Congress on Paraconsistency took place there in 1997. Sadly, Richard died the year before the conference was held.

Subsequent World Congresses have since appeared all over the world—as be- fits the international nature of the subject: in Brazil, France, Australia, India, Poland, and Mexico. There is no body that organises the conferences. In good anarchist fashion, it is left to whoever wants to pick up the baton to organise the next one.

The early years of modern paraconsistent logics were not easy ones. The log- ics were generally not well received, since they made it clear, with all the weight of modern logic, that contradictions could be tolerated, in some sense. Hence, they challenged the deeply entrenched Aristotelian horror contradictionis in Western philosophy. (More of this in a moment.) However, the merits of the formal work could not be gainsaid, and the official acceptance of paraconsistent logic can be dated notionally to 1991, when Mathematical Reviews and the Zentralblatt für Mathematik added it to their official taxonomy of categories (the Mathematics Subject Classification, 03B53)—though some older papers do appear with that classification. One of the editors of Math Reviews, Andrés Caicedo, explains as follows:[1]

The class 03B53 was added in 1991 (by both Math Reviews and zb- MATH), with the description changing slightly ... in 2000. (We revise and update the MSC every ten years.) The reason for the class ap- pearing in old papers in zbMATH is that they continuously change the classification of old papers to reflect the current version of the MSC. At Math Reviews, on the other hand, we try not to add "new" classes to old papers already in the database, or to change the classes used originally, to try and preserve the historical record of how the paper was originally classified by our staff. There are some exceptions to this policy: mainly, when we add to the database a paper published years ago but that for some reason we did not originally cover. In that case, we use the current classification system rather than the one in place at the time of publication.

Hence, by 1991, mathematicians had no problems with paraconsistent logic—even if many philosophers still did.

2. Dialetheism

2.1. The Name

Let us now turn to the subject of dialetheism—essentially, the view that some contradictions are true. And the first thing to note about this is following. If one is a dialetheist, Explosion is valid, and valid inference preserves truth, then everything is true. Da Costa called a theory which entails everything a trivial theory. Hence the view that everything is true has come to be

[1] Email communication.

called trivialism. This is a view, one more outrageous than which, cannot be thought. Hence if one is a dialetheist and wishes not to be a trivialist, one must endorse the correctness of a paraconsistent logic. As is clear, however, the mere endorsement or use of a paraconsistent logic does not commit one to dialetheism. Indeed, none of the originators of paraconsistency we met in 2.4 endorsed dialetheism (at least at the times in question: Richard did come to endorse it a little later). As we saw, they were motivated by quite different considerations.

The name 'dialetheism' was coined by Richard and myself when we were writing the introductory chapters of Paraconsistent Logic. At the time, the word 'paraconsistent' was in use for both what is now called paraconsistency and for di- aletheism. (Sometimes the latter was called strong paraconsistency.) We wanted to make it clear that these were quite different beasts. We searched for an appro- priate word, mining the dictionaries of all the languages we could find. Failing to find one, we decided to coin a neologism: di/aletheism—from the Ancient Greek:

> Two-way/truth; something that faces both truth and falsity. We were motivated by a comment in Wittgenstein's Remarks on the Foundations of Mathematics con- cerning Russell's paradox: (Wittgenstein, 1956, III 59, 131e)

Why should Russell's contradiction not be conceived as something supra-propositional, something that towers above the propositions and looks in both directions like a Janus head?... The proposition that con- tradicts itself would stand like a monument (with a Janus head) over the propositions of logic.

We forgot to agree how to spell the word, however. I wrote it with an 'e'; Richard without. The 'e' has now become standard.[1]

2.2. The Meaning

The first time a clean definition of the meaning of the word was given was in In Contradiction.[2] Here it is (p. 4):

The notion of true contradiction is at the heart of this book. Awkward as neologisms are, it will therefore be convenient to have a word for it. I will use 'dialetheia'. So, to avoid any confusion, let me say, right at the start, that a dialetheia is any true statement of the form: α and it is not the case that α.

[1] Actually, we nearly shot ourselves in the foot when looking for an appropriate word. We were in the reference section of the library at the ANU, consulting dictionaries, and we found a word in Ancient Greek that we liked. I now forget what it was: it was translated as something like *inconsistent*, *contradictory*, or some cognate of these. When we were walking out of the library, it occurred to us that we ought to go back and check it in the Greek to English direction. It said something like: 'contradictory, absurd, stupid'. Needless to say, we dropped the idea.

[2] Priest (1987). This was written when I was on leave in Pittsburgh in 1983. It did not appear till 1987, however, due to problems finding a publisher who was prepared to accept it. (See the Preface to the 2nd edn.)

If I were to write this again, I would just say that it is a pair of statements, α and ¬α, which are both true, since conjunction is not of the essence of the notion. But, of course, given a standard account of conjunction (which In Contradiction does), these are equivalent formulations.[1] Notice, however, that there is nothing about contradictions being in the world—unless the world is the Wittgensteinian one of everything that is the case.

Let me take this opportunity to comment on a misunderstanding of dialetheism that I frequently hear. Here, for example, is Carnielli and Rodrigues: (Carnielli and Rodrigues, 2019, 3790)

The dialetheist claims that some contradictions are ontological in the sense that they are due to some 'inner contradictory essence of real- ity'—or in other words, that reality, in order to be correctly described, demands pairs of contradictory propositions.

Now, if proposition is just a generic name for a truth bearer, what comes after the dash is fine. But the rest is just plain false; and why the phrase is in quotation marks I have no idea. It is not something I have ever said or would say.

As In Contradiction points out,[2] it is not even clear that the claim that there are contradictions in reality makes sense—except in the entirely banal one that for a sentence to be true generally requires the cooperation both of words and of the world. To endorse it in any more robust sense, one has to endorse some kind of correspondence theory of truth. I have never endorsed such a view. Indeed, the only theory of truth I have ever advocated (In Contradiction, 4.5—the "teleo-logical theory of truth"), is anything but such a realist theory. Dialetheism is not committed to any particular view of truth. That this is so is spelled out at length in Doubt Truth to be a Liar, ch. 2. (Priest, 2006) There, I run through all the standard views of truth and point out that all of them are compatible with dialetheism.

So let me say it one more time: dialetheism is the view that some statements of the form A and ¬A are both true. As to what truth means, you can choose your own favorite view.

2.3. History Before the 20th Century

Let us now turn to the history of the view. As we noted, paraconsistency has been the orthodox view in Western philosophy for most of its history. This is not the case with dialetheism—quite the opposite (Priest, 2018 & Deguchi et al, 2021).

There were dialetheists before Aristotle—at least according to Aristotle. In Metaphysics Γ, he cites Heraclitus and Protagoras as examples. However, in that text he enunciates and defends the Principle of Non-Contradiction (PNC). He states this as follows (5^b18–5^b22) (Kirwan, 1993):

[1] And given the assumption that to say that α is false is just to say that ¬α is true (which *In Contradiction* also makes), this is also to say that some α is both true and false.

> For the same thing to hold good and not hold good simultaneously of the same thing and in the same respect is impossible (given any further specifications which might be added against dialectical diffi- culties).

The qualification is there to rule out spurious counterexamples, such as ‘Hypatia is a man’ (that is, a human), and ‘Hypatia is not a man’ (that is, a woman).

Now, there have been a few dialetheists between Aristotle and the 20th cen- tury. The most obvious example is Hegel. Thus, alluding to Zeno, he says in the Logic (Miller, 1959, 440):

> Common experience ... says that there is a host of contradictory things, contradictory arrangements, whose contradiction exists not merely in external reflection, but in themselves... External sensuous motion is contradiction’s immediate existence. Something moves, not because at one moment it is here and at another there, but because at one and the same moment it is here and not here, because in this “here”, it at once is and is not.

However, such examples are striking because of their rarity. Not to put too fine a point on the matter, the PNC has been high orthodoxy in Western philosophy since Aristotle. Nearly every philosopher has simply taken it for granted—indeed, so much so that there is, as far as I am aware, no sustained defence of it in the history of Western philosophy since Aristotle.

Aristotle’s arguments are hopeless, however, as scholars since Łukasiewicz have noted (Priest 2006, ch. 1.). Aristotle has one main argument which is long, tangled, and ob- scure. Commentators cannot even agree about how it is meant to work; let along that it works. There are then about half a dozen brief throw-away arguments. And the most obvious thing about these is that they are beside the point, since their conclusion is to the effect that it is not the case that all contradictions are true—or even that no one can believe that all contradictions are true. The illicit slide be- tween some and all is patent. The orthodoxy of the PNC has, then, simply been dogma.

Let me now briefly return to the fraught reception of contemporary paracon- sistent logic in its early years. This is due, I think, entirely to the orthodoxy of the PNC. Logicians failed to distinguish between paraconsistency and dialethe- ism. And dialetheism does run hard against orthodoxy. Consistency was taken to be a sine qua non of core philosophical notions, such as truth, rationality—and sometimes validity. So much was this so, that anything that endangered this bit of dogma was rejected out of hand.[1] I leave the reader to reflect on what this says about the rationality of philosophers.

[1] In fact, perhaps surprisingly, when one factors consistency out of these notions, what remains is not much difference from what was there beforehand. See Priest (2006), 12.14.

2.4. Some 20th Century History

So let us now turn to the 20th Century. There were harbingers of dialetheism in the first half of the century, such as Łukasiewicz and Wittgenstein; but I think it fair to say that contemporary dialetheism comes out of Australia in the 1970s, with the work of Richard and myself. I had written a draft of 'Logic of Paradox', (Priest, 1979) which endorses dialetheism, before I moved to Australia in 1976. When I then met him, Richard had been working on relevant logic, and was already playing with the idea of dialetheic solutions to the semantic and set theoretic paradoxes (Routley, 1977). We quickly joined forces to advocate the view. Its first major outing was in my *In Contradiction*. I think it fair to say that the view would have got nowhere had it not been able to draw on substantial and incontestable work in paraconsistent logic, showing that the idea was not simply moonshine.

Dialetheism is often taken simply to be a view about the paradoxes of self- reference. And indeed, that was its initial major driving-force. But dialetheism is not intrinsically connected with any one application. It is simply a view about some contradictions being true, not about which ones. Paradoxical assertions of self-reference are prime candidates for such things; but dialetheism has many possible applications, and does not depend on any one of them. Thus, *In Contra- diction* discusses applications of dialetheism to the paradoxes of self-reference, change and motion, and the law.

Indeed, for me, the most transparent examples of dialetheias come from law, where it is clear how they are the results of our actions. The law cannot make many things true (e.g., that pigs can fly); but it can make certain things true by an act of social fiat. For example, a duly constituted legislature can make it the case that certain groups of people do or do not have certain legal rights. Thus, there may be a law which says that:

- All people in group *X* have the right to do *Z*
- No person in group *Y* has the right to do *Z*.

All is consistent unless and until someone in both groups turns up. Call them *a*. Then, when subject to the relevant jurisdiction, *a* both does and does not have the right to do *Z* (Priest, 1987, ch. 13.).

Actually, once one sees this, the possibility of other cases of dialetheism also becomes more transparent. Truth is a function of two things: the meanings of our words and the reality which they describe. In general, these have to cooper- ate to deliver a true sentence (though perhaps sometimes language on its own is enough). 'Grass is green' is true because of the meaning of 'grass' and and the colour of grass. Now, in an obvious sense, language is a human construction, and even if reality is not, it may well behave in such a way as to render true contra- dictory statements in the language we have constructed—as the legal case shows. There was no divine guardian ensuring that the results of our linguistic produc- tions would deliver only consistency.

For example, we may well create boundaries with our linguistic partitions. But it is the continuity and symmetry of the reality partitioned that may cause a dialetheia at one (Weber, 2021).

Again, contemporary developments in dialetheism have continued apace since its early years. New applications have been espoused, concerning sorites para- doxes, aesthetics, the philosophy of mind, the philosophy of language, and the history of philosophy, East and West. But let us leave this bit of the story here, and move on to our third topic: one of the major applications of paraconsis- tency—inconsistent mathematics.

3. Inconsistent Mathematics

3.1. Set Theory

This takes us back to Newton da Costa. Da Costa was unique amongst the founders of paraconsistency whom we looked at in 2.4, in that his prime concern was the construction of inconsistent but non-trivial theories. Many such theories have since been constructed, concerning, *inter alia*, truth, motion, non-existent objects, boundaries. But being a mathematician, Da Costa was primarily concerned with mathematical theories. So let us stick with these here. First, though, note that da Costa was not interested in such theories because he thought they might be true. Da Costa was never a dialetheist (as far as I am aware). He was interested in the theories because they can have an interesting mathematical structure in their own right. One does not have to think that a mathematical theory is true to see that it is mathematically engaging. Thus, for example, the intuitionistic theory of smooth infinitesimals has a mathematical structure of clear interest, even if one is not an intuitionist.

Da Costa and his co-workers were the first logicians to investigate (intention- ally!) inconsistent mathematical theories. From the early 1960s, da Costa pub- lished notes on inconsistent set theories with his student and later colleague Ayda Arruda. And in his 1974 'On the Theory of Inconsistent Formal Systems', there is a long discussion of an inconsistent version of Quine's *NF*. Nor was paracon- sistency used merely to quarantine the set theoretic paradoxes. The properties of inconsistent sets were actively investigated. (Arruda and Batens, 1982)

The problem with the set theories developed was, however, this. The set theo- retic paradoxes are driven by the naive comprehension scheme (NCS):

- $\exists z \forall x (x \in z \leftrightarrow A(x))$ for every condition $A(x)$ (in which z is not free). And given a logic which contains both *modus ponens* and the *contraction principle* ($A \to (A \to B) \vdash A \to B$), the NCS leads to triviality, independently of the behaviour of negation, in the shape of Curry Paradoxes. Since da Costa's logics all validated the contraction principle, they could not contain NCS.

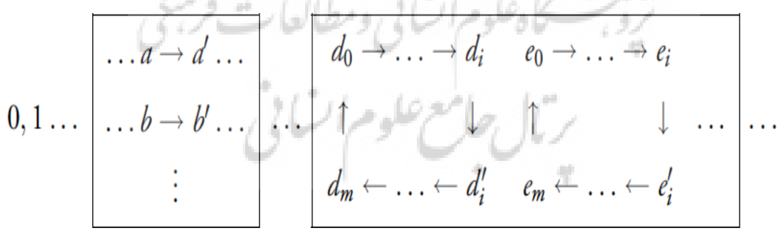
Moreover, the NCS appears to be true. That, after all, is what drives the set- theoretic paradoxes. Hence the holy grail of paraconsistent set theory was always a theory which could accommodate the NCS. True, da Costa was not a dialetheist.

But none the less, because of the apparently *a priori* truth of the NCS, such a set theory, if one can be found, has to be particularly interesting.

Richard, however, did take the NCS to be true. He had become a dialetheist by the late 1970s, and came to advocate a paraconsistent set theory containing the NCS as providing the correct solution to the paradoxes. He constructed a set theory with NCS based on an an underlying relevant logic which did not contain the contraction principle. The theory was shown to be non-trivial by Brady.[1] That showed that the theory did not do too much; but there was a real question about whether it did enough. Richard had shown that it could deliver elementary parts of set theory; (Routley, 1977) but it was unclear whether it could deliver, e.g., the standard results of transfinite arithmetic. That it could do so was shown only about 30 years later by Zach Weber. One notable thing about his proofs was that they did not simply reconstruct standard proofs; they made essential and novel use of the properties of inconsistent sets themselves. (Weber, 2021)

3.2. Arithmetic

A second kind of mathematical structure that has been intensively studied is arith- metic—particularly natural number arithmetic. Spinning off work by Bob Meyer on arithmetic based on relevant logics, it was discovered that there are inconsistent models of arithmetic with important properties. For example, they model every- thing provable in Peano Arithmetic, even though they can be finite. One might think of these as non-standard models of a certain kind. And just as the class of classical non-standard models has an intricate and interesting general structure, so do these. Generally speaking, they start with a sequence of consistent numbers, and then exhibit a cyclical structure of inconsistent numbers. (Priest, 2000b) One may depict the structure thus:



The models also have important connections with Gödel's theorems. Gödel's theorems show that any axiomatic theory of arithmetic (of appropriate strength) is either inconsistent or incomplete. Normally, the inconsistency option is ignored, since in classical logic inconsistency implies triviality. But the paraconsistent models can but used to show that there are axiomatic theories of arithmetic that are complete (that is, for every *A*, either *A* or ¬*A* is provable), though

[1] Brady (1989). Note, however, that the paper was written about 10 years before this.

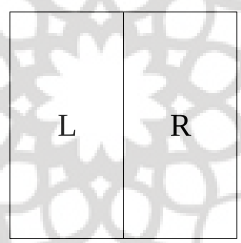
non-trivially inconsistent—the inconsistencies, indeed, being tightly circumscribed. In particular, one can construct, as usual, a sentence, G, that "says of itself that it is not provable", $\neg\exists x Prov\ (x, \mathbf{n})$, where $\mathbf{n}$ is the numeral of the gödel code of G. In con- sistent incomplete axiomatic arithmetics, neither G nor $\neg G$ is provable. In the inconsistent complete axiomatic arithmetics, both G and $\neg G$ are provable. (Priest 2019)

None of this shows that arithmetic might actually be dialetheic, of course—and that it is so certainly appears less plausible than that set theory is so. However, in 'Logic of Paradox' (Priest, 1979) I argued, explicitly on the basis of Gödel's theorem, that it is; and these mathematical developments certainly lend weight to the argument there.

3.3. Geometry

Many other kinds of inconsistent mathematical theories and structures have been studied, including topologies, categories, and linear algebras. (Mortensen, 1995) Let me conclude by considering one which itself has a very interesting application: geometry.

Start with a simple example. Consider (a bounded part of) the real plane. Divide it into two disjoint parts, left, L, and right, R. So, $L \cap R = \emptyset$. Let the line of division be, say, the line $x = 0$. Is that part of L or R? We may depict the situation thus:



$x = 0$

Of course, the description under-determines an answer to the question. But when the example is suitably fleshed out, considerations of symmetry might suggest that it is both. We would then have the following characterisation:

- if $x < 0$, $(x, y) \in L$ (consistently)
- if $x > 0$, $(x, y) \in R$ (consistently)
- if $x = 0$, $(x, y) \in L \cap R$.

Given an appropriate paraconsistent logic, this is entirely coherent. This simple idea can be employed in the mathematics of "impossible pictures". Thus, consider the following picture:

The three-dimensional content of the picture is impossible. How should one de- scribe that content mathematically? Any mathematical characterisation will spec- ify, amongst other things, the orientations of the various faces. Now, consider the left-hand face, and in particular its lighter shaded part. This is vertical. Next, consider the top of the lower step on the right-hand side of the picture. This is horizontal. Finally, consider the boundary between them (a vertical line on the illustration). That is a boundary that is on both planes. So, it is vertical and hor- izontal. That's a contradiction, since it cannot be both; but that's exactly what makes the content of the picture impossible. Note that the characterisation of the content must deploy a paraconsistent logic, since it should not imply, e.g., that the top of the higher step is vertical.[1]

Conclusion

The previous sections tell just a little of the history of paraconsistency and its possibilities to date. Future historians will have a lot more to say. As to the future of paraconsistency itself: I think we have only just started to explore the full richness of the idea; and what will emerge in years to come, only time will tell.

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[1] For further discussions of inconsistent geometries and impossible pictures, see Mortensen (2010).

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