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(Research Paper)

A Novel Simulation Optimization Model for the Project Portfolio Selection and Scheduling Problem Regarding the Risks of the Projects

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Abstract

Purpose: Timely project completion within the predicted budget is one of the primary goals of project-oriented organizations. The inherent risks associated with projects have made achieving these goals challenging for most organizations. In many cases, the interdependence of risks across different projects leads to undesirable consequences, further exacerbating this challenge. To address such challenges, this article introduces a novel formulation for the two-objective problem of project portfolio selection and scheduling. The first objective is to maximize the profit of the project portfolio by accounting for the aggregated cost impacts of risks on projects' activities. The second objective aims to minimize the implementation time of the project portfolio.

Design/methodology/approach: To evaluate the expected impacts of risks and their interactions, the objective functions are developed based on the Bayesian Networks approach. Current mathematical formulations for the integrated problem of project portfolio selection and scheduling exhibit significant limitations, including the absence of incorporating risk factors and their interdependencies within and across projects on the duration and cost of activities. As a solution approach, the simulation-based approximation approach is developed to establish linear modelling of the problem. Since this approach provides multi-objective mathematical modelling, augmentedϵ-constrained programming is employed to solve the presented model. For the validation of the formulations, the proposed model and solution approach are applied to a sample instance.

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Findings: The results of the model implementation, while confirming the validity of the proposed approach, demonstrated that selecting projects with fewer interference effects with other projects can yield more favourable outcomes for project-oriented organizations.

Research limitations/implications: Project portfolio selection/scheduling is a critical decision-making problem for every organization, particularly project-based organizations, as it significantly affects organizational performance. This paper investigated a novel approach to the joint problem of project selection and scheduling by assessing the aggregated individual and intensifying the risk impacts on the duration and cost of the projects' activities. Due to the uncertain environment of projects, the duration and cost estimation of the activities is not a straightforward task.

Originality/value: This study provided a new bi-objective mathematical model for the project selection and scheduling problem.

Keywords: Project portfolio selection; Project scheduling; Risk assessment; Bayesian Networks; Multi-objective mathematical model

1. Introduction

Choosing a project portfolio establishes the tactics of the upper management for the organization's future (Cooper et al., 2001; Roussel et al., 1991). The project selection approaches typically involve two steps: first, individual evaluation of all projects, and then selecting the eligible project portfolio utilizing a greedy algorithm. The projects are chosen and assessed based on a predefined set of indicators (Linton et al., 2002; Meade & Presley, 2002). In other ways, the combined problem of project portfolio selection and scheduling has turned into a very active research topic and has attracted many researchers (Carazo et al., 2010; Dixit & Tiwari, 2020; Ghorbani & Rabbani, 2009; Hesarsorkh et al., 2021; Liu & Wang, 2011; A Namazian et al., 2019; Namazian & Yakhchali, 2016; Ranjbar et al., 2022; Zolfaghari & Mousavi, 2021). In practical scenarios, projects may adversely impact each other due to existing interactions among their associated risks. In two tunnel-digging projects, wherein the first is located beneath the second, a common station is shared at which, passengers can switch lines. The possibility of a tunnel collapse is a significant risk for both projects. However, if these projects are executed simultaneously, the occurrence of this risk in the first project increases the likelihood and severity of the same risk in the second project. As a result, the duration and cost of activities in one project can be influenced by the risks associated with the other. To our knowledge, there is a lack of academic research exploring such practical cases in the context of project selection and scheduling, despite the importance of considering risk-related objectives in these problems. The more projects selected, the higher the overall risk level becomes. Also, interactions among risks can amplify their impact. Therefore, if the total risk level exceeds the organization's risk tolerance, some selected projects may need to be partially implemented. Thus, the duration and cost of an activity in one project are affected by the risk of another project. Diverse methodologies have been suggested for assessing project risks. In many of these approaches, risks are examined individually without considering their interactions. However, one method that considers the interactions across project risks is the Bayesian Networks approach, which allows for modelling the possibility of such interactions during project risk assessment.

In this article, a mathematical model is presented for the joint problem of project selection and scheduling with the objectives of maximizing the profit of the project portfolio and minimizing the period of portfolio implementation by evaluating the effects of projects' risks on the cost or duration of the activities. By applying Bayesian Network logic, this research proposes a novel approach for assessing project portfolio risk. This approach simultaneously considers the consequences, probabilities, and interactions among risks. Objective functions are developed using the Bayesian Networks method to evaluate the expected aggregated risk. To solve the problem, a simulation-based approximation approach is introduced. This study presents a mathematical model for the described problem and employs an optimization simulation method to determine the strategies for project selection and scheduling. Since these formulations provide multi-objective mathematical modelling, augmentedϵ-constraint programming is applied to solve the presented model. In summary, the innovations of the study include: 1) simultaneously addressing project selection and scheduling problems; 2) incorporating risk factors into the project portfolio selection and scheduling problem; 3) developing mathematical modelling and simulation approaches for the problem; 4) considering the interference effects of risks in modelling, and 5) applying Bayesian networks to construct and evaluate the project risk network.

The subsequent sections of the paper are organized as follows: Section 2 provides an extensive literature review, while Section 3 introduces the Bayesian Belief Network (BBN) approach. Section 4 demonstrates the mathematical formulation developed to address the combined problem of project portfolio selection and scheduling. Sections 5 and 6 explain the solution approach employed to handle and solve the formulated model. Section 7 serves as an illustrative application of the developed model and solution approach. Section 8, the last section, concludes the study by presenting the quintessential findings and mapping future research directions.

2. Literature review

This section provides a brief review of studies related to the problem of project portfolio selection and scheduling. Two main categories of project portfolio selection models are employed: The first one is Multi-Criteria Decision-Making (MCDM) approaches, which rank projects, and the other one is mathematical programming models. Researchers proposed utilizing a rating technique for project selection (Lootsma et al., 1990). The rating technique encompasses all the significant factors involved in the project selection process and offers a theoretical indicator to aid in choosing among various projects. In a study by De Reyck et al. (2005), an adjective three-step approach was introduced to tackle the Information Technology project portfolio selection problem. This approach recognizes the strong connection between improving portfolio processes and reducing project-related issues. According to Ghasemzadeh et al., (1999), a pre-qualification process is recommended, wherein each project is initially evaluated individually. If the project meets the basic criteria, it can move on to the next stage. A project selection approach was developed by Rathi et al., (2016) that combines fuzzy logic and Multiple Attribute Decision Making (MADM) techniques. This approach aims to identify suitable Six Sigma projects. The weights of the assessment indicators were determined by the Modified Digital Logic (MDL) method, and the ultimate ranking was computed based on the primacy index, which is derived from fuzzy-based VIKOR and TOPSIS methodologies.

In addition to MCDM approaches, several studies have employed mathematical programming to address the project portfolio selection problem. Schmidt (1993) suggested a nonlinear programming model to investigate the interdependencies among the candidate projects. In this model, three types of interactions are considered: outcome interactions, output interactions, and resource interactions. Additionally, a branch and bound algorithm has been proposed as a solution method for this model. Namazian & Yakhchali (2016) addressed the project portfolio selection problem with a focus on project schedules, aiming to achieve the minimum expected profit. Due to the inherent uncertainty in activity durations, these durations were modelled using semi-trapezoidal fuzzy numbers. Consequently, the problem was formulated as a fuzzy linear programming model. Schaeffer & Cruz-Reyes (2016) proposed a mathematical model framework for R&D project portfolio selection. Within this framework, each project proposal includes tasks characterized by distinct expenses. The MILP (Mixed Integer Linear Programming) model framework effectively handles task dependencies and incorporates the effects of these dependencies by including synergies. Nielsen et al., (2024) proposed a multi-objective optimization program designed to select portfolio projects through a transparent and selective process. This method prioritizes projects based on maximizing NPV and minimizing risk, all within specified constraints. The approach enables the case company to allocate its time and resources with the greatest impact on business and portfolio value.

Additionally, certain researchers have utilized a blend of the addressed approaches to highlight the challenge of project selection. In their study, put forward a hybrid method consisting of three stages to choose the optimal combination of projects (Tavana et al., 2015). The model they suggested had three stages with multiple steps and procedures. For the initial screening, they employed Data Envelopment Analysis (DEA), followed by the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) to rank the projects. Finally, linear Integer Programming (IP) was utilized to select the most appropriate project portfolio.

Beyond project selection, project scheduling has captured the attention of numerous researchers. Kellenbrink & Helber (2015) examined the problem of project scheduling with a dynamic framework where the activities were not fully predetermined. The researchers extended the traditional resource-constrained project scheduling problem by integrating a flexible project structure with endogenous decision-making. They introduced a genetic algorithm to tackle this scheduling problem and conducted a numerical analysis to assess its performance (Li & Womer, 2015) formulated dynamic programming algorithms incorporating the rollout policy to address project scheduling problems entailing resource constraints and activities with stochastic durations. To boost the efficiency of the foundational policy derived from a priority-rule heuristic, they employed constraint programming. Vahedi-Nouri et al., (2024) introduced an integrated problem concerning project scheduling and workforce planning within a Reconfigurable Manufacturing System (RMS), leveraging reconfigurable machines and human-robot collaboration. To address this, a new Mixed-Integer Linear Programming (MILP) model and an efficient Constraint Programming (CP) model were devised to formulate the problem and to minimize the makespan as the performance metric.

Furthermore, the challenge of selecting and scheduling projects has garnered significant attention from numerous researchers. Tofighian & Naderi (2015) devised an integer linear mathematical model for addressing the integrated problem of project portfolio selection and scheduling with multiple objective functions. The objectives were to simultaneously maximize the total expected profit and minimize the variation in resource usage. Also as a solution approach, they introduced a multi-objective optimization algorithm. Tavakolan et al., (2024) introduced a model aimed at simultaneous project portfolio selection and scheduling to maximize the project portfolio's net present value (PPNPV) while minimizing resource fluctuations (RF). The study accounted for resource constraints and integrated the selection and scheduling processes efficiently to construct the portfolio from the contractor's perspective in construction management.

On the contrary, numerous methods have been devised for evaluating project risk. For example Multiple Criteria Decision Making, (Aminbakhsh et al., 2013; Awodi et al., 2023; Dikmen et al., 2007; Kuo & Lu, 2013; Mousavi et al., 2023; Rodríguez et al., 2016; Yu et al., 2023; Zavadskas et al., 2010); Failure Mode and Effective Analysis, (Alvand et al., 2023; Cheng & Lu, 2015; Jamshidi et al., 2015; Liu et al., 2023; Zhu et al., 2024); Fault Tree Analysis, (Gierczak, 2014; Hyun et al., 2015; Zeng & Skibniewski, 2013); Monte Carlo Simulation, (Erfani & Tavakolan, 2023; Ali Namazian et al., 2019; Senova et al., 2023) and Bayesian Networks, (Hu et al., 2013; Khan & Faisal, 2023; Leu & Chang, 2013; Namazian & Haji Yakhchali, 2018; Osama et al., 2023; Sun et al., 2023); are widely used to assess project risks.

In previous studies, project selection, scheduling, and risk assessment have typically been examined separately. This approach has led to the selection and scheduling of projects without accounting for their interconnected risk networks, which include individual and interactive risk influences within and across projects. Given that these influences significantly affect key operational objectives, such as the time and cost of activities, they are crucial to addressing the project selection and scheduling problem effectively.

In other words, the results of the project selection and scheduling problem are influenced by existing risks, which can significantly limit the applicability of such models in real-world conditions and result in outcomes that lack sufficient validity. Therefore, integrating project selection and scheduling problems with risk assessment will bring the results closer to real-world conditions during project implementation. In this study, innovative risk-driven simulation-based formulations of the joint bi-objective problem of project selection and scheduling are suggested. To address the challenge, the Bayesian network method is employed to model this interconnected network. These formulations integrate the undesirable effects of project risks and their interactions by utilizing the Bayesian Networks approach to select and schedule projects. In addition to merging risk assessment with project selection and scheduling, this research also examines the interference effects of risks. These problems, which closely resemble real-world conditions, have not been considered in previous research, making this study the first in this context.

3. Bayesian Networks

In this paper, the objective functions have been developed based on the Bayesian Networks logit to evaluate the expected time and cost impacts of risks. This section describes this approach briefly. Bayesian Networks are visual depictions of information used in making decisions in uncertain situations. They serve as effective tools for decision analysis, encompassing tasks such as primary and secondary analyses, (Sousa & Einstein, 2012). The Bayesian Network provides a comprehensive modelling approach, providing a concise representation of the relationships within a stochastic system. It achieves this by visually illustrating the system variables and their interdependencies (Nordgård & Sand, 2010). The foundation of the Bayesian Network method lies in the Bayesian formula. It captures the connection between the prior and posterior probability and posterior probability, allowing for deriving specific accident probabilities using existing prior probabilities. This approach finds widespread application in uncertainty analysis (Tang et al., 2016). For the n independent and exclusive hypotheses$(j = 1,2, \dots . n)$, the relationship presented by Bayes' theorem is as follows:

$$P(H_j|E) = \frac{P(E|H_j) \times P(H_j)}{\sum_{i=1}^{n} P(E|H_i) \times P(H_i)}$$

Where$P(H_j|E)$ is the posterior or conditional probability for the hypothesis$H(j = 1,2, \dots ,n)$, regarding the obtained evidence(E); $P(H_j)$ denotes the prior probability; $P(E|H_j)$ represents the conditional probability, assuming t$H_j$rue, and the denominator represents the total probability which is a constant value, (Shabarchin & Tesfamariam, 2016). A Bayesian Network comprises two primary components: a qualitative part and a quantitative part. The qualitative part is presented by a Directed Acyclic Graph (DAG), where the nodes correspond to system variables, and the edges depict the conditional dependencies between variables. On the other hand, the quantitative part consists of a set of conditional probability functions that establish the relationships between the nodes in the graph, (Nordgård & Sand, 2010). The parent nodes denoted as the initial nodes without inward arrows, are referred to as the starting nodes. Also, the child nodes are the remaining inward arrows connected to them. To perform the calculations, it is essential to specify the states and probabilities associated with each node, (Khodakarami & Abdi, 2014). By accounting for the conditional dependencies among variables, Bayesian Network encapsulates the joint probability distribution $P(U)$ of variables$U = A_1,A_2, \dots ,A_n$, as:

$$P(U) = \prod_{i=1}^{n} P(A_i|Pa(A_i))$$

Where$Pa(A_i)$ is the parent set of variableA_is? Then, the probabilityA_i is calculated as:

$$P(A_i) = \sum_{U \neq A_i} P(U)$$

Where the sum is taken over all the variables exceptA_i.

4. Problem statement and formulation

This section presents a novel mathematical formulation of project selection and scheduling problems with the objectives of maximizing the profit of the project portfolio and minimizing the period of portfolio implementation by evaluating the effects of projects' risks on the cost and duration of the activities. Extant mathematical models of the integrated project selection and scheduling problem display significant limitations, including the absence of incorporating risk factors and their interdependencies within and across projects on the duration and cost of activities which often lead to delay in completion time of the activities and also increase in their completion cost. In the problem of project selection and scheduling, risks and their interactions have twofold negative effects on project portfolios, the costs and the duration of the activities. These negative effects reduce the profit of the project portfolio and also increase its completion time. In this research, a simulation-based optimization model is presented for the project selection and scheduling problem wherein the consequences and interactions of risks are simultaneously considered.

4.1 Problem statement and notation

Assume there are N projects, where each project j comprises n_j activities, nr_j risks and releases benefit of p_j after its completion time. For implementing the activity r of project l, the initial estimated duration is denoted as d^0_{rl}, where the impact of existing risks is not taken into account. Upon incorporating the respective risk effects, the actual duration becomes d_{rl}. Alternatively, to calculate the expected value of aggregated time and cost risks arising from the interactions of various risks, it is essential to assess their probabilities and impacts. The probability component of the expected value is determined by utilizing Bayesian Networks, wherein the collective risk probability is evaluated regarding the expressions of risks and their corresponding parent risks. Also, the respective time and cost impacts are determined considering both the initial and cumulative effects of risks on the duration and cost of activities. The cumulative impact of risks represents the total impact for each risk, encompassing both the initial impact and the amplified impact stemming from its parent risks. The required parameters are mentioned in Table (1) to formulate the problem.

Table 1. Definitions of the parameters

Parameter	Definition
T_{Por}	Project portfolio completion time
R_{kj}	Risk k of the project j
$P\left(R^C_{kj}\right)$	The Bayesian Networks-based probability function for risk k in project j
P^C_{Por}	The Bayesian Networks probability function of the risks in the project portfolio
T^C_{Por}	Cost impact of the risks on the project portfolio
E^C_{Por}	Expected cost impact of the risks on the project portfolio
$Pa^I(R_{kj})$	The set of internal parent risks associated with risk k in project j
$Pa^{E_l}(R_{k_j})$	The set of external parent risks associated with risk k in project j from project l
$Pa^{I_m}(R_{kj})$	The m^{th} internal parent risk of the risk k in project j
$Pa^{E_{ln}}(R_{kj})$	The n^{th} external parent risk of the risk k in project j from project l
$\alpha^C_{Pa^{I_m}(R_{kj})}$	Percentage of increase in cost impact of m^{th} internal parent risk of the risk k of project j
$\alpha^C_{Pa^{E_{ln}}(R_{kj})}$	Percentage of increase in cost impact of n^{th} external parent risk of the risk k of project j

Parameter	Definition
	from project l
$\alpha^T_{Pa^{I_m}(R_{kj})}$	Percentage of increase in time impact of m^{th} internal parent risk of the risk k of project j
$\alpha^T_{Pa^{E_{ln}}(R_{kj})}$	Percentage of increase in time impact of n^{th} external parent risk of the risk k of project j from project l
$R^C_{r'lkj}$	The initial cost impact of risk k in project j on activity r in project l
$R^T_{r'lkj}$	Initial time impact of risk k in project j on activity r in project l
R^C_{rlkj}	Cumulative cost impact of risk k in project j on activity r in project l
R^T_{rlkj}	Cumulative time impact of risk k in project j on activity r in project l
R^C_{kj}	Cumulative cost impact of risk k in project j
W^C_{rl}	Estimated cost for performing activity r of project l
R^T_{rl}	Percentage of increase in duration of activity r of project j
$Pr(r,r,'l)$	Precedence relationship between activities r and $r$the project l
Br_j	The set of risks of Project j
A_{kj}	The set of the projects and their risks affect the risk k of project j
NR_{kj}	The number of the projects and their risks affect the risk k of project j
G_{rl}	The set of risks that impact activity r in project l
M	Positive large number

Finally, the decision variables of the problem consist of y_j and x_{rl} for which y_j, binary decision variable of the problem, takes value one if project j is selected to place in project portfolio and otherwise is equal to zero. Also x_{rl} represents the start time of activity r of project l.

4.2 Objective functions

Two objective functions are considered for the problem. The first objective, as shown in equation (1), is to maximize the profit of the project portfolio by taking into account the negative consequences of contingent risks on activities' costs. These consequences are intensified due to interactive relationships among risks in different projects.

$$Max\ Z_1 = \sum_{j=1}^{n} p_j y_j - \frac{E^C_{Por}}{2^{(\sum_{l=1}^{n} nr_l(1-y_l))}} \quad (1)$$

$$= \sum_{j=1}^{n} p_j y_j - \frac{\sum_{R_{11}=0}^{1} \sum_{R_{21}=0}^{1} \cdots \sum_{R_{kj}=0}^{1} \cdots \sum_{R_{nr_jN}=0}^{1} (P^C_{Por})(T^C_{Por})}{2^{(\sum_{l=1}^{n} nr_l(1-y_l))}}$$

The fraction denominator stated in this objective function is considered for calculating the number of effective states of risks. For example, if we have three projects, each consists five risks, and these projects are selected to place in the project portfolio, all states (2^{15} states) would be effective regarding the binary states for each risk including the occurrence or nonoccurrence of that risk. However, in cases where merely two projects are selected as project portfolios,2^5 states would not be effective anymore. As a results the expected cost impact of the risks , E^C_{Por}, should be divided by 2^5 to measure the actual impacts of effective risks.

The second objective function, as shown in equation (2), is to minimize the required time to perform projects by considering the negative consequences of risks on the activities' duration.

$$Min\ Z_2 = T_{Por} \tag{2}$$

4.3 Constraints

Regarding the Bayesian Networks approach, the joint probability function of the nodes is obtained by multiplying the marginal probability function given their parent nodes. Thus the probability function of the risks of the project portfolio, P_{Por}^{C}, as for the probability function of the individual risks, $P(R_{kj}^{C})$, is calculated by equation (3).

$$P_{Por}^{C} = \prod_{k,j} P\left(R_{kj}^{C}\right) = \prod_{k,j} \left(P({}'R_{kj}) y_j + (1 - y_j)\right) \tag{3}$$

In which, $P({}'R_{kj})$ is calculated by equation (4):

$$\begin{aligned} P({}'R_{kj}) &= P(R_{kj}|Pa^{I}(R_{kj})) \prod\nolimits_{h \in A_{kj}} (1 - y_h) \\ &+ \sum_{l_1 \in A_{kj}} P(R_{kj}|Pa^{I}(R_{kj}), Pa^{E_{l_1}}(R_{kj})) y_{l_1} \\ &+ \left(\sum_{l_1, l_2 \in A_{kj}, l_1 \neq l_2} \left(P\left(R_{kj}|Pa^{I}(R_{kj}), Pa^{E_{l_1}}(R_{kj}), Pa^{E_{l_2}}(R_{kj})\right) - \sum_{i=1}^{2} P\left(R_{kj}|Pa^{I}(R_{kj}), Pa^{E_{l_i}}(R_{kj})\right) \right) \right) y_{l_1} y_{l_2} \prod_{h \in A_{kj}, \neq l_1 . l_2} (1 - y_h) \\ &+ \ldots + \left(P(R_{kj}|Pa^{I}(R_{kj}), Pa^{E_{l_1}}(R_{kj}), Pa^{E_{l_2}}(R_{kj}), \ldots, Pa^{[E_{l_{NR_{kj}}}]}(R_{kj}))\right) \prod_{h=1}^{NR_{kj}} (1 - y_h) \end{aligned} \tag{4}$$

Equations (3) and (4) guarantee that if a project is selected in the project portfolio, the effects of its risks on that project and also on the risk of the other selected projects are applied to evaluate the profit of the portfolio.

Two methods can be employed to evaluate the influence of risk on the duration and cost of an activity: the risk-driven approach and the activity-based approach. According to research conducted by (Creemers et al., 2014), the effectiveness of analyzing risks is higher with the risk-driven approach than with the activity-based approach. Thus, assessing the impacts of risks individually is a distinct approach compared to estimating the distribution of activities' duration and cost depending on the impacts of uncertain factors. Equation (5) represents the aggregated effect of risks on the project portfolio, stated by the effect of risks on the activities' cost (equations (6) and (7)). As can be seen, parent risks may intensify the impacts on their offspring owing to the existing interactions among them.

$$T^C_{Por} = \sum_{k,j} R^C_{kj} \tag{5}$$

$$R^C_{kj} = \sum_{r,l} R^C_{rlkj} W^C_{rl} \tag{6}$$

$$R^C_{rlkj} = y_j R^C{}_{rlkj} \left(1 + \sum_m \alpha^C_{Pa^{Im}(R_{kj})} + \sum_{l,n} \alpha^C_{Pa^{Eln}(R_{kj})} y_l \right) \tag{7}$$

The actual duration of each activity is evaluated considering the initial estimated duration and its increased percentage due to the time impacts of risks as equation (8).

$$d_{rl} = d^0_{rl}(1 + \frac{\sum^1_{R_{11}=0} \sum^1_{R_{21}=0} \cdots \sum^1_{R_{kj}=0} \cdots \sum^1_{R_{nr_jN}=0} (P^C_{Por})(\sum_{R_{kj} \in G_{rl}} R^T_{rlkj})}{2^{(\sum^n_{l=1} nr_l(1-y_l))}}) \tag{8}$$

Where the accumulated time effect of risk k in project j on activity r in project l, R^T_{rlkj}, is calculated based on equation (9). Risk cumulative time impact includes the initial impact and the intensifying impact caused by its parent risks.

$$R^T_{rlkj} = y_j R^T{}_{rlkj} \left(1 + \sum_m \alpha^T_{Pa^{Im}(R_{kj})} +) \sum_{l,n} \alpha^T_{Pa^{Eln}(R_{kj})} y_l \right) \tag{9}$$

On the other hand, the precedence relationships between activities determine the order in which activities should be performed (Equation 10).

$$x_{rl} \geq x_{rl} + d_{rl} - M(1 - y_l) \quad \forall r,r \in {}'Pr(r,r,'l) \tag{10}$$

$$x_{rl} \leq M y_l \qquad \forall r,l \tag{11}$$

Equation (11) states that if a project has not been selected, its activities will not have a starting time. Finally, the required time for performing projects should be greater than the last completion time of all activities. This constraint is shown in equation (12).

$$T_{Por} \geq x_{n_j+1,j} \quad \forall j \in \{1,2, \ldots .n\} \tag{12}$$

5. Problem-solving approach

A simulation-based approximation approach has been developed to linearize the structure of the formulation. Having formulated as a multi-objective linear mathematical model, augmented *epsilon*-constraint programming is applied to achieve the Pareto frontier of the mentioned problem.

5.1 Mathematical model extension

In the present section, a nonlinear mathematical approach is proposed to solve the model. As mentioned before, the joint probability function of risks is obtained by multiplication of the marginal probability function given their parent risks. The marginal probability function of each risk is a function of its project decision variable and the decision variables of those projects affecting that risk. Thus the joint probability function of risks in the project portfolio is a multiplication of the problem decision variables which would be a nonlinear function. In a nonlinear format, there is no assurance of finding the optimal solution to the problem, which necessitates its linearization. The marginal probability function of each risk not affected by other projects' risks is mentioned in equation (13).

$$P(R_{kj}^{C}) = \prod_{k \in Br_j} \left[P(R_{kj}|Pa^{I}(R_{kj}))y_j + 1 - y_j\right] \quad (13)$$

$$= \prod_{k \in Br_j} ([P(R_{kj}|Pa^{I}(R_{kj})) - 1]y_j + 1) = \left(\sum_{k \in Br_j} [P(R_{kj}|Pa^{I}(R_{kj})) - 1]\right) y_j$$

$$+ \left(\sum_{k_1,k_2 \in Br_j} [P(R_{k_1 j}|Pa^{I}(R_{k_1 j})) - 1][P(R_{k_2 j}|Pa^{I}(R_{k_2 j})) - 1]\right) y_j^2$$

$$+ \ldots + \left(\prod_{k \in Br_j} [P(R_{kj}|Pa^{I}(R_{kj})) - 1]\right) y_j^{nr_j} + 1$$

Given the binary nature of the variable y_j and the possibility of replacing its powers;

$$P(R_{kj}^{C}) = (\sum_{k \in Br_j} [P(R_{kj}|Pa^{I}(R_{kj})) - 1] \quad (14)$$

$$+ \sum_{k_1,k_2 \in Br_j} [P(R_{k_1 j}|Pa^{I}(R_{k_1 j})) - 1][P(R_{k_2 j}|Pa^{I}(R_{k_2 j})) - 1]$$

$$+ \ldots + \prod_{k \in Br_j} [P(R_{kj}|Pa^{I}(R_{kj})) - 1])y_j + 1$$

In the case that risk $R_{k_i j}$ is affected by the risk(s) of the project l_t, equation (14) is modified as equation (15).

$$(15)$$

$$P(R_{kj}^{C}) = \left(\prod_{k \in Br_j, \neq k_i} \left[P(R_{kj}|Pa^{I}(R_{kj}))y_j + 1 - y_j\right]\right) (P(R_{k_i j}|Pa^{I}(R_{k_i j}), Pa^{E_{l_t}}(R_{k_i j}))y_j y_{l_t}$$

$$+ P(R_{k_i j}|Pa^{I}(R_{k_i j}))y_j(1 - y_{l_t}) + 1 - y_j)$$

$$= \prod_{k\in Br_j, \neq k_i} ([P(R_{kj}|Pa^I(R_{kj})) - 1]y_j + 1)(P(R_{k_ij}|Pa^I(R_{k_ij}) - 1)y_j + 1$$
$$+[P(R_{k_ij}|Pa^I(R_{k_ij}), Pa^{E_{l_t}}(R_{k_ij})) - P(R_{k_ij}|Pa^I(R_{k_ij}))]y_j y_{l_t})$$

This equation can be simplified as equation (16).

$$P(R_{kj}^C) = (\sum_{k\in Br_j} [P(R_{kj}|Pa^I(R_{kj})) - 1] + \sum_{k_1,k_2\in Br_j} [P(R_{k_1j}|Pa^I(R_{k_1j})) - 1][P(R_{k_2j}|Pa^I(R_{k_2j})) - 1] + \ldots + \prod_{k\in Br_j} [P(R_{kj}|Pa^I(R_{kj})) - 1])y_j + 1 + [P(R_{k_ij}|Pa^I(R_{k_ij}), Pa^{E_{l_t}}(R_{k_ij})) - P(R_{k_ij}|Pa^I(R_{k_ij}))](B_{k_ij} + 1)y_j y_{l_t} \quad (16)$$

Where B_{k_ij} is gained by equation (17).

$$B_{k_ij} = \sum_{k\in Br_j, \neq k_i} [P(R_{kj}|Pa^I(R_{kj})) - 1] + \sum_{k_1,k_2\in Br_j, \neq k_i} [P(R_{k_1j}|Pa^I(R_{k_1j})) - 1][P(R_{k_2j}|Pa^I(R_{k_2j})) - 1] + \ldots + \prod_{k\in Br_j, \neq k_i} [P(R_{kj}|Pa^I(R_{kj})) - 1] \quad (17)$$

Generally, considering the above equations, the joint probability function of risks associated with the project portfolio would be as equation (18).

$$P(R_{Por}^C) = \sum_{i=1}^{n} \alpha_i y_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \alpha_{ij} y_i y_j + \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} \alpha_{ijk} y_i y_j y_k + \ldots + \alpha_{ijk\ldots n} y_i y_j y_k \ldots y_n + 1 \quad (18)$$

On the other hand, to calculate the cost effect of risks in the project portfolio, the effect of each risk including the initial effect and the intensifying effect caused by its parent risks should be assessed. The impact of each risk not affected by risk(s) of other projects, is calculated by equation (19).

$$R_{kj}^C = \sum_{r,l} R_{rlkj}^C W_{rl}^C = y_j \sum_{r,l} \left[{R^C}_{rlkj} \left(1 + \sum_{m} v\alpha^C_{Pa^{I_m}(R_{kj})} \right) \right] W_{rl}^C = \beta_j y_j \quad (19)$$

Where β_j will be as equation (20):

$$\beta_j = \sum_{r,l} [R^C{}_{rlkj}(1 + \sum_m \alpha^C_{Pa^{I_m}(R_{kj})})]W^C_{rl} \tag{20}$$

When risk is affected by risk(s) of external projects, this impact is assessed by equation (21) whichβ_j is calculated as before and β_{jl} is calculated by equation (22).

$$R^C_{kj} = \sum_{r,l} R^C_{rlkj} W^C_{rl} \tag{22}$$

$$= y_j \sum_{r,l} \left[R^C{}_{rlkj} \left(1 + \sum_m \alpha^C_{Pa^{I_m}(R_{kj})} + \sum_{l,n} \alpha^C_{Pa^{E_{ln}}(R_{kj})} y_l \right) \right] W^C_{rl}$$

$$= \beta_j y_j + \beta_{jl} y_j y_l \quad (21)$$

$$\beta_{jl} = \sum_{r,l} [R^C{}_{rlkj}(\sum_{l,n} \alpha^C_{Pa^{E_{ln}}(R_{kj})})]W^C_{rl}$$

Consequently, the cost effect of risks in the project portfolio will be as equation (23).

$$T^C_{Por} = \sum_{k,j} R^C_{kj} = \sum_{i=1}^{n} \beta_i y_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \beta_{ij} y_i y_j \tag{23}$$

Therefore, the expected cost impact of risks on the project portfolio, i.e., the probability functions, and the cost impacts of risks are mentioned in equation (24).

$$E^C_{Por} = \sum_{R_{11}=0}^{1} \sum_{R_{21}=0}^{1} \cdots \sum_{R_{kj}=0}^{1} \cdots \sum_{R_{nr_jN}=0}^{1} (P^C_{Por})(T^C_{Por}) \tag{24}$$

$$= \sum_{i=1}^{n} \gamma_i y_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \gamma_{ij} y_i y_j$$

$$+ \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} \gamma_{ijk} y_i y_j y_k + \ldots + \gamma_{ijk\ldots n} y_i y_j y_k \ldots y_n$$

In which the coefficients for individual variables, the multiplication of two, three and generally for n variables are obtained by equations (25),(26),(27) and (28) respectively.

$$\gamma_j = \alpha_j \beta_j \quad \forall j \in \{1,2, \ldots . n\} \tag{25}$$

$$\gamma_{ij} = \alpha_i(\beta_j + \beta_{ij}) + \alpha_j(\beta_i + \beta_{ij}) + \alpha_{ij}(\beta_i + \beta_j) + \alpha_{ij}\beta_{ij} \quad \forall i,j \in \{1,2,\ldots n\}, i \neq j \tag{26}$$

$$\gamma_{ijk} = \alpha_i \beta_{jk} + \alpha_j \beta_{ik} + \alpha_k \beta_{ij} + \alpha_{ij}(\beta_k + \beta_{ik} + \beta_{jk}) \tag{27}$$

$$+\alpha_{ik}(\beta_j + \beta_{ij} + \beta_{jk}) + \alpha_{jk}(\beta_i + \beta_{ij} + \beta_{ik})$$

$$+\alpha_{ijk}(\beta_i + \beta_j + \beta_k + \beta_{ij} + \beta_{ik} + \beta_{jk}) \quad \forall i,j,k \in \{1,2, \ldots . n\}, i \neq j \neq k$$

$$\gamma_{1,2,\ldots,n} = \sum_{\substack{\{i\} \subseteq \{1,2,\ldots,n\}, \{k\} \subseteq \{i\}, \\ \{j\} = \{\{1,2,\ldots,n\} - \{i\}\} \cup \{k\}}} \alpha_{\{i\}} \beta_{\{j\}} \tag{28}$$

It is crucial to conduct a thorough assessment of risk impacts to determine the time impacts of risks on activity durations. This includes evaluating both the initial impacts and the intensifying effects caused by their parent risks. The increased percentage in duration of activity r of project j due to the impacts of risks, which are not affected by risk(s) of other projects, is calculated by equation (29).

$$R_{rl}^{T} = \sum_{R_{kj}\in G_{rl}} R_{rlkj}^{T} = y_j \sum_{R_{kj}\in G_{rl}} [R^{T}{}_{rlkj}(1 + \sum_{m} \alpha_{Pa^{Im}(R_{kj})}^{T})] \tag{29}$$

$$= \eta_j y_j$$

Where η_j will be as equation (30);

$$\eta_j = \sum_{R_{kj}\in G_{rl}} [R^{T}{}_{rlkj}(1 + \sum_{m} \alpha_{Pa^{Im}(R_{kj})}^{T})] \tag{30}$$

When risks are affected by risk(s) of external projects, these impacts are assessed by equation (31) which η_j is calculated as before and η_{jl} is calculated by equation (32).

$$R_{rl}^{T} = \sum_{R_{kj}\in G_{rl}} R_{rlkj}^{T} \tag{32}$$

$$= y_j \sum_{R_{kj}\in G_{rl}} \left[R^{T}{}_{rlkj}\left(1 + \sum_{m} \alpha_{Pa^{Im}(R_{kj})}^{T} + \sum_{l,n} \alpha_{Pa^{Eln}(R_{kj})}^{T} y_l\right)\right]$$

$$= \eta_j y_j + \eta_{jl} y_j y_l \quad (31)$$

$$\eta_{jl} = \sum_{R_{kj}\in G_{rl}} [R^{T}{}_{rlkj}(\sum_{l,n} \alpha_{Pa^{Eln}(R_{kj})}^{T})]$$

Consequently, the time impacts of effective risks on the duration of activity r of project l will be as equation (33).

$$R_{rl}^{T} = \sum_{R_{kj}\in G_{rl}} R_{rlkj}^{T} = \sum_{i=1}^{n} \eta_i y_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \eta_{ij} y_i y_j \tag{33}$$

Thus, the expected time impact of risks on the duration of activity r of project l which is stated by the probability functions and the time impacts of effective risks are mentioned in equation (34).

$$E_{rl}^{T} = \sum_{R_{11}=0}^{1} \sum_{R_{21}=0}^{1} \dots \sum_{R_{kj}=0}^{1} \dots \sum_{R_{nr_jN}=0}^{1} (P_{Por}^{C}) \left(\sum_{R_{kj}\in G_{rl}} R_{rlkj}^{T} \right) \tag{34}$$

$$= \sum_{i=1}^{n} \zeta_i y_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \zeta_{ij} y_i y_j + \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} \zeta_{ijk} y_i y_j y_k + \cdots$$

$$+ \zeta_{ijk\dots n}\, y_i y_j y_k, \dots . y_n$$

In which the coefficients for individual variables, the multiplication of two, three and generally for n variables are obtained by equations (35),(36),(37) and (38) respectively.

$$\zeta_j = \alpha_j \eta_j \quad \forall j \in \{1,2, \ldots .n\} \tag{35}$$

$$\zeta_{ij} = \alpha_i(\eta_j + \eta_{ij}) + \alpha_j(\eta_i + \eta_{ij}) + \alpha_{ij}(\eta_i + \eta_j) + \alpha_{ij}\eta_{ij} \quad \forall i,j \in \{1,2, \ldots .n\}, i \neq j \tag{36}$$

$$\zeta_{ijk} = \alpha_i \eta_{jk} + \alpha_j \eta_{ik} + \alpha_k \eta_{ij} + \alpha_{ij}(\eta_k + \eta_{ik} + \eta_{jk})$$
$$+\alpha_{ik}(\eta_j + \eta_{ij} + \eta_{jk}) + \alpha_{jk}(\eta_i + \eta_{ij} + \eta_{ik})$$
$$+\alpha_{ijk}(\eta_i + \eta_j + \eta_k + \eta_{ij} + \eta_{ik} + \eta_{jk}) \quad \forall i,j,k \in \{1,2, \ldots .n\}, i \neq j \neq k \tag{37}$$

$$\zeta_{1,2,\ldots,n} = \sum_{\substack{\{i\} \subseteq \{1,2,\ldots,n\}, \{k\} \subseteq \{i\}, \\ \{j\} = \{\{1,2,\ldots,n\} - \{i\}\} \cup \{k\}}} \alpha_{\{i\}} \eta_{\{j\}} \tag{38}$$

As a result, the actual duration of each activity will be as equation(39).

$$d_{rl} = d_{rl}^0 \left(1 + \frac{\sum_{i=1}^{n} \zeta_i y_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \zeta_{ij} y_i y_j + \ldots + \zeta_{ijk\ldots n} \quad y_i y_j y_k \cdots y_n}{2^{(\sum_{l=1}^{n} nr_l (1-y_l))}}\right) \tag{39}$$

The denominator of the fraction stated in the first objective function of the problem and also in the equation related to the actual duration of the activities, equations (1) and (39), is used to calculate the number of effective states of risks. This value is a nonlinear function of problem decision variables for which due to the binary nature of decision variables, it can be modified as equation (40).

$$2^{(\sum_{l=1}^{n} nr_l (1-y_l))} = 2^{(\sum_{l=1}^{n} nr_l)} + \sum_{i=1}^{n} \vartheta_i y_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \vartheta_{ij} y_i y_j$$
$$+ \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} \vartheta_{ijk} y_i y_j y_k + \ldots + \vartheta_{ijk\ldots n} \quad y_i y_j y_k \cdots y_n \tag{40}$$

The first objective function of the problem, equation (1), will be obtained by replacing the nominator and denominator of its fraction with the equations (24) and (40) respectively, as shown in equation(41);

$$\begin{aligned} &Max\ Z_1 \\ &= \sum_{j=1}^{n} p_j y_j \\ &- \frac{\sum_{i=1}^{n} \gamma_i y_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \gamma_{ij} y_i y_j + \ldots + \gamma_{ijk\ldots n} \quad y_i y_j y_k \cdots y_n}{2^{(\sum_{l=1}^{n} nr_l)} + \sum_{i=1}^{n} \vartheta_i y_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \vartheta_{ij} y_i y_j + \ldots + \vartheta_{ijk\ldots n} \quad y_i y_j y_k \cdots y_n} \\ &= \frac{\sum_{i=1}^{n} d_i y_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} d_{ij} y_i y_j + \ldots + d_{ijk\ldots n} \quad y_i y_j y_k \cdots y_n}{2^{(\sum_{l=1}^{n} nr_l)} + \sum_{i=1}^{n} \vartheta_i y_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \vartheta_{ij} y_i y_j + \ldots + \vartheta_{ijk\ldots n} \quad y_i y_j y_k \cdots y_n} \end{aligned} \tag{41}$$

Where the coefficients for individual variables, the multiplication of two, three and generally for n variables are obtained by equations (42),(43),(44) and (45) respectively.

$$d_j = p_j(\vartheta_j + 2^{(\sum_{l=1}^{n} nr_l)}) - \gamma_j \quad \forall j \in \{1,2, \dots .n\} \tag{42}$$

$$d_{ij} = p_i(\vartheta_i + \vartheta_{ij}) + p_j(\vartheta_i + \vartheta_{ij}) - \gamma_{ij} \quad \forall i,j \in \{1,2, \dots .n\}, i \neq j \tag{43}$$

$$d_{ijk} = p_i(\vartheta_{jk} + \vartheta_{ijk}) + p_j(\vartheta_{ik} + \vartheta_{ijk}) + p_k(\vartheta_{ij} + \vartheta_{ijk}) - \gamma_{ijk} \quad \forall i,j,k \in \{1,2, \dots .n\}, i \neq j \neq k \tag{44}$$

$$d_{1,2,\dots,n} = p_1(\vartheta_{2,3,\dots,n} + \vartheta_{1,2,\dots,n}) + \sum_{t=2}^{n} (\vartheta_{1,2,\dots,t-1,t+1,\dots,n} + \vartheta_{1,2,\dots,n}) - \gamma_{1,2,\dots,n} \tag{45}$$

5.2 Approximation linearization

In the previous section, an approach was presented to model the problem exactly. As can be seen, this approach needs many calculations to assess the coefficients of the problem and linearize it. This section proposes a two-step approach to solve the problem with fewer calculations. In the first step, based on the main modelling of the first objective function and the actual duration of an activity, a revised formulation is presented to calculate the profit of the project portfolio and the required duration for performing each activity. Then, by applying linearization of the multiplication of variables, linear mathematical modelling is obtained. In the second step, a simulation structure is proposed to calculate the coefficient of the modelling developed in the previous step. After these two steps, the problem becomes a linear programming model that can be easily solved.

Step 1: Revised problem formulation

1-1: Problem modelling

Regarding equations (24) and (34) the expected time and cost impacts of risks on the duration of activities and project portfolio can be evaluated for which all risk scenarios are considered and their probabilities are computed based on the Bayesian Networks approach. For large-size problems, it will be time-consuming to consider all scenarios and calculate respective probabilities. As a result, scenario generating based on the prior and conditional probabilities can be used to reduce calculations and estimate the impacts of risks. Thus, by substituting the right part of the first objective function in equation (1) with equation (23) and the right part of equation (8) with equation (33), the profit of the project portfolio and the actual duration of each activity is respectively evaluated for different scenarios, as shown in equations (46) and (47).

$$Max\ Z_1 = \sum_{j=1}^{n} p_j y_j - \sum_{j=1}^{n} \beta_j y_j - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \beta_{ij} y_i y_j \quad (46)$$

$$= \sum_{j=1}^{n} (p_j - \beta_j) y_j - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \beta_{ij} y_i y_j$$

$$d_{rl} = d_{rl}^0 (1 + \sum_{j=1}^{n} \eta_j y_j + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \eta_{ij} y_i y_j + \ldots + \eta_{ijk\ldots n} y_i y_j y_k, \ldots . y_n) \quad (47)$$

1-2: Linearization

In this part, a linear model is obtained to solve the problem by applying the linearization of the multiplication of variables method. Equations (46) and (47) can be linearized as equations (48) and (49) by taking into account the constraints which are given later.

$$Max\ Z_1 = \sum_{j=1}^{n} (p_j - \beta_j) y_j - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \beta_{ij} y_{ij} \quad (48)$$

$$d_{rl} = d_{rl}^0 (1 + \sum_{i=1}^{n} \eta_j y_j + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \eta_{ij} y_{ij} + \cdots + \eta_{ijk\ldots n} y_{ijk,\ldots,n}) \quad (49)$$

$$y_{ij} \geq 1 + M(y_i + y_j - 2) \quad \forall i,j \in \{1,2, \ldots ., n\}, i \prec j \quad (50)$$

$$y_{ij} \leq \frac{1}{2}(y_i + y_j) \quad \forall i,j \in \{1,2, \ldots, n\}, i \prec j$$

$$y_i, y_{ij} \in \{0,1\}$$

Step 2: Simulation (Scenario generation)

In this section, the proposed approach for generating a scenario is presented to calculate the coefficients of the formulations in the previous step i.e. β_j, β_{ij}, η_j and η_{ij}. This approach consists of the following steps.

2-1: Risk clustering: Risk clustering pertains to grouping risks at various levels based on their interactive relationships. These levels are as follows:

- First level: Risks without any parent risks.
- k^{th} level: Risks whose parent risks fall between the first level and the $(k-1)^{th}$level (from the second level to the last level).

2-2: Random number generation: First-level risks are unaffected by other risks and act as independent factors influencing other risks. Secondary risks are influenced by other risks (such as child risks) and can, in turn, influence additional risks (such as parent risks). At this step, random numbers with uniform distribution (between 0 and 1) are produced for all these risks.

2-3: Risk state determination: At this step, the state of each risk, i.e., the risk occurrence/nonoccurrence is determined based on the generated numbers for risks and their respective prior and conditional probabilities. If the generated random number is less than the specified probability of the associated risk, the risk is simulated; otherwise, risk nonoccurrence is considered. This process is performed sequentially from the first level to the last one.

2-4: Coefficient calculation: At this step, according to equations (22),(24),(32) and (34), the aggregated effect of each risk on the corresponding activity is computed. This effect signifies the impacts of each risk, considering both its primary effect and the intensifying effects of its parent risks. By identifying the aggregated effect, the calculation of the problem coefficients would be possible.

2-5: Average coefficient calculation: Repeat steps 2 and 4 according to the number of predetermined iterations. To ensure a significant degree of dependability in the problem outcome, it is crucial to repeat the addressed steps extensively, as this guarantees the dependability of the acquired outcomes. Thus, the addressed processes are carried out from the generation of random numbers to the calculation of problem coefficients with the acceptable predetermined number of iterations. The flowchart of the proposed algorithm can be depicted in Figure (1).

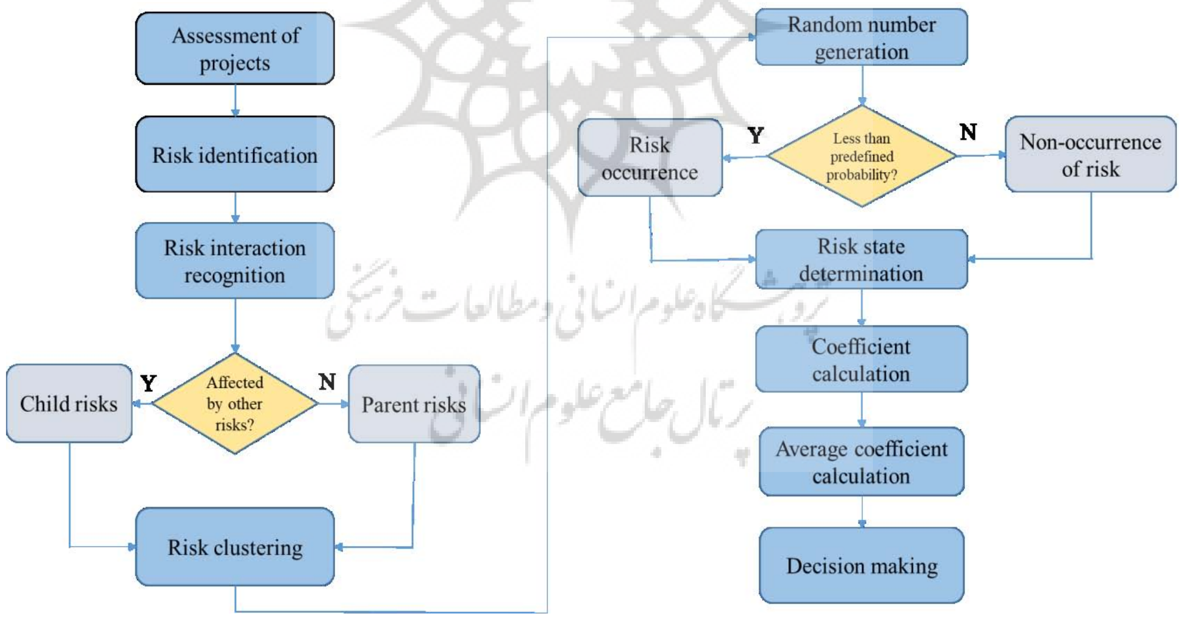

Fig 1. Flowchart of the algorithm

6. Multi-objective solution method

In multi-objective mathematical programming (MMP), there are multiple objectives, and no single optimal solution can optimize all objective functions simultaneously. Consequently, decision-makers seek the "most preferred" solution rather than an optimal one, and the notion of optimality is replaced with efficiency or Pareto optimality. An efficient solution cannot be improved in one objective without deteriorating its performance in at least one of the other objectives. Several techniques have been proposed to deal with MMP problems. The ϵ-constraint method is a recently introduced approach where one of the objective functions is selected as the primary objective, and the remaining objective functions are included as constraints within the feasible solution space of the problem. This method enables the optimization of the main objective function while considering others as constraints. This technique offers several advantages over the weighting method, which combines objective functions into a single weighted sum objective function. These advantages are discussed in (Mavrotas, 2009).

6.1 Conventional ϵ-constraint technique

Consider an MMP problem with p competing objective functions denoted as $f_i(x)$,$(i = 1,2,\dots.p)$, in which S is the feasible solution space defined by the constraints of the MMP problem and $x \in S$ is the vector of decision variables. Without sacrificing generality, we assume that all objective functions are designed to maximize their respective criteria. Typically, the ϵ-constraint technique seeks to maximize or minimize the main objective function f_1 while treating the remaining objective functions as constraints (Chankong & Haimes, 2008; Cohon, 2004):

$$\begin{aligned} &Maximize\ f_1(x) \\ &\quad Subject\quad to \\ &\quad\quad f_2(x) \geq e_2, \\ &\quad\quad f_3(x) \geq e_3, \\ &\quad\quad f_p(x) \geq e_p, \\ &\quad\quad x \in S. \end{aligned} \tag{51}$$

To apply the ϵ-constraint method, it is necessary to have the ranges of at least $p-1$ objective functions, which are employed as additional objective function constraints. These ranges are essential for determining grid points corresponding to e_2,e_3, … ,e_p values. The prevalent method involves deriving these ranges from the payoff table, which is commonly employed. To calculate the payoff table for a multi-objective optimization problem with p competing objective functions, the process involves initially determining the individual optima for each objective functionf_i. Subsequently, for each objective function f_i, the values of the remaining objective functions f_1,f_2, … ,f_{i-1},f_{i+1}, … ,f_p are computed using the solution that optimizes f_i. The payoff table has $p$rows and columns. Each j column represents the values obtained for the objective function f_j. The minimum and maximum values in each column represent the objective function $f_j$range, considering a constraint.

The ϵ-constraint method does not optimize the confine of objective functions upon the entire Pareto set. A lexicographic optimization technique can be used to solve this problem. The lexicographic optimization of a sequence of objective functions involves optimizing the first objective function initially, and subsequently optimizing the second objective function among the potential alternative optimal solutions, and so forth. Practically, The ϵ-constraint method proceeds by optimizing each objective function, one at a time, while fixing the values of the previously optimized objective functions. The Pareto optimal solutions generated by The ϵ-constraint method may contain dominated or inefficient solutions. The augmented ϵ-constraint technique has been proposed to address this issue.

6.2 Augmented ϵ-constraint technique

In the conventional ϵ-constraint method, there is no assurance that the solutions obtained will be efficient, and it may produce inefficient solutions that can be dominated by other feasible solutions. To address this limitation, (Mavrotas, 2009) proposes the augmented ϵ-constraint method. In this approach, the inequality constraints of the objective functions in equation (66) are initially converted into equality constraints by introducing positive surplus variables, also known as slack variables. Subsequently, the primary objective function is modified by adding the sum of slack values. Thus, the augmented ϵ-constraint method can be expressed as follows:

$$Maximize\ (f_1(x) + eps \times (s_2/r_2 + s_3/r_3 + \ldots + s_p/r_p)) \quad (52)$$

$$Subject\ to$$

$$f_2(x) - s_2 = e_2,$$

$$f_3(x) - s_3 = e_3,$$

$$f_p(x) - s_p = e_p,$$

$$x \in S, s_i \in R^+.$$

Where eps is an adequately small number usually between $10^{-3} 10^{-6}$ and $s_2, s_3, \ldots, s_p$ are the introduced slack variables for the constraints (51) of the MMP problem. In the conventional ϵ-constraint approach, the slack variables s_i are typically set to zero by generating solutions as $f_i = e_i$. In contrast, within the augmented ϵ-constraint method, the primary objective function is expanded to incorporate the total slack variables. This approach effectively mitigates the generation of inefficient solutions. It can be demonstrated that the augmented ϵ-constraint method exclusively produces efficient solutions. The proof for this claim can be found in the work of Mavrotas (2009).

7. Numerical example

In the sample instance, as shown in Figure (2), three candidate projects, P1, P2 and P3, containing 6, 7 and 5 activities are considered.

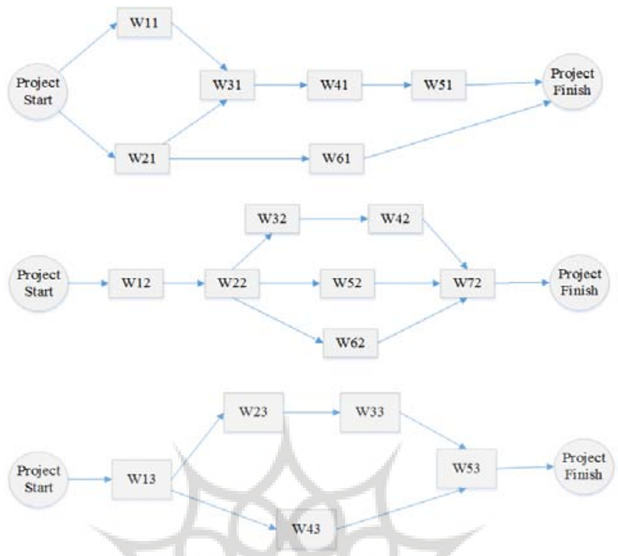

Fig 2. Activity networks of the projects

The presumed risk network of the projects, encompassing the risks and their interactions, is shown in Figure (3):

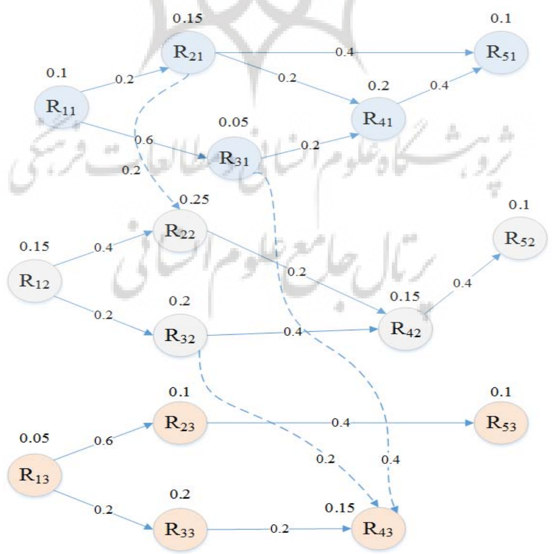

Fig 3. Risk network of the projects

Figure (3) illustrates that, beyond interactions among risks within each project, certain risks from one project can influence the risks in other projects. For instance, risk R_{21} from project 1 has an impact on the risk R_{22} of project 2. This implies that if both Projects 1 and 2 are chosen for inclusion in the project portfolio, the occurrence of the parent risk, R_{21} in

Project 1 can elevate the probability or consequence of the child risk, R_{22} in Project 2. Values shown in this figure, present the primary and intensifying effects of risks which will be described in the next section. To estimate the coefficient of the model, risks should be clustered. Table (2) shows the risks' clustering concerning their interactions.

Table 2. Risk clustering

Level 1	*Level 2*	*Level 3*	*Level 4*	*Level 5*
R_{11}	R_{21}	R_{41}	R_{51}	R_{52}
R_{12}	R_{31}	R_{22}	R_{42}	
R_{13}	R_{32}	R_{43}		
	R_{23}	R_{53}		
	R_{33}			

Table 3 indicates the estimated time and cost for performing each activity and its effective risk(s).

Table 3. Estimated time and cost and affective risk(s) of each activity

Project	Activity	Estimated time	Estimated cost	Affective risk(s)
P1	W11	10	100	R_{11}
	W21	12	120	R_{11} R_{21}
	W31	8	80	R_{31}
	W41	5	50	R_{41}
	W51	5	50	R_{41}
	W61	10	100	R_{51}
P2	W12	5	50	R_{12}
	W22	5	50	R_{22}
	W32	10	100	R_{22} R_{32}
	W42	10	100	R_{32}
	W52	10	100	R_{42}
	W62	5	50	R_{52}
	W72	5	50	R_{52}
P3	W13	8	80	R_{13}
	W23	5	50	R_{23}
	W33	12	120	R_{33}
	W43	10	100	R_{43}
	W53	15	150	R_{53}

7.1 Probability and impact of risk assessment

The probability of each risk including prior and conditional probability and the impact of risk including primary and intensifying impact are described.

7.2 Risk prior and conditional probability assessment

In the context of Bayesian Networks, there are two categories of probabilities: prior probability and conditional probability. The term "prior probability" describes a probability $P(R_{11} = 1)$ since in a given model, it represents the probability of an event prior to any updates made to its probability using new information within that model's framework. On the other hand, a probability like $P(R_{12} = 1|R_{11} = 1)$ is referred to as a conditional or posterior probability (Neapolitan, 2004).

The primary probability can be interpreted as the assessed probability of a risk, unaffected by other activated risks. Conditional probability can be understood as the assessed probability of a risk, influenced by another risk within the network. Qualitative scales often consist of five to ten levels. They are commonly used to express such probabilities. To convey the prior and conditional probabilities of risks, linguistic variables can be utilized with corresponding numerical values, as detailed in Table (4).

Table 4. Scales of prior and conditional probabilities

Annual Frequency		Probability	
Descriptor	Definition	Descriptor	Value
Frequent	Up to once a month or more	Very high	0.9
Likely	Once a month up to once every 6 months	High	0.7
Possible	Once in 6 months up to once in 12 months	Medium	0.5
Unlikely	Once a year up to once every two years	Low	0.3
Rare	Once every two years or less	Very low	0.1

Prior and conditional probabilities of the risks in the presented instance are shown in Figure (4).

	0	1
R11	0.9	0.1

	0	1
R12	0.7	0.3

	0	1
R13	0.7	0.3

	R11	0	1
R21	0	0.9	0.7
	1	0.1	0.3

	R11	0	1
R31	0	0.7	0.5
	1	0.3	0.5

	R12	0	1
R32	0	0.5	0.7
	1	0.5	0.3

	R42	0	1
R52	0	0.7	0.5
	1	0.3	0.5

	R13	0	1
R23	0	0.9	0.7
	1	0.1	0.3

	R13	0	1
R33	0	0.3	0.2
	1	0.7	0.8

	R23	0	1
R53	0	0.3	0.2
	1	0.7	0.8

	R21	0		1	
	R31	0	1	0	1
R41	0	0.9	0.7	0.8	0.6
	1	0.1	0.3	0.2	0.4

	R21	0		1	
	R41	0	1	0	1
R51	0	0.7	0.4	0.6	0.3
	1	0.3	0.6	0.4	0.7

	R21	0		1	
	R12	0	1	0	1
R22	0	0.7	0.6	0.4	0.3
	1	0.3	0.4	0.6	0.7

	R22	0		1	
	R32	0	1	0	1
R42	0	0.7	0.6	0.4	0.3
	1	0.3	0.4	0.6	0.7

	R31	0				1			
	R32	0		1		0		1	
	R33	0	1	0	1	0	1	0	1
R43	0	0.8	0.7	0.6	0.5	0.6	0.5	0.4	0.3
	1	0.2	0.3	0.4	0.5	0.4	0.5	0.6	0.7

Fig 4. Prior and conditional probabilities

7.3 Risk primary and intensifying impact assessment

Impact or consequence pertains to the degree to which a risk event can influence the project. The primary effect or main impact of each risk is assessed using a value ranging from 0 to 1 (or 0 to 100 per cent) to indicate the level of effectiveness of that risk on the corresponding activity. Alongside the individual effects of each risk on the project's risk assessment criteria, the interactions among risks amplify these effects. Interactive effects denote the escalation of the impact of a child risk when its parent risk occurs. Table (5) is used to assess the interactive effects between risks.

Table 5. Interactive effect assessment of the parent risk on the child risk.

$\alpha^{C}_{Pa(R_{kj})}$	0	0.2	0.4	0.6	0.8	1
Descriptor	Insignificant cost increase	Very low (20 percent cost increase)	Low (40 percent cost increase)	Moderate (60 percent cost increase)	High (80 percent cost increase)	Very high (100 percent cost increase)

In Table (5), the value zero is associated with conditions when the risk occurrence affects the probability of another risk but has no impact on the effects of child risks.

7.4 Numerical example implementation results

To verify the developed solution approach, Table (6) is used to address both exact and approximate values of the problem objective functions for all possible states of the project portfolio. The number of iterations in the approximation or scenario generation approach is 1000.

Table 6. Results for each decision of the instance

Variables			MaxZ_1		MinZ_2	
y_1	y_2	y_3	Approximation	Exact	Approximation	Exact
1	0	0	36.19	35.76	31.08	30.89
0	1	0	0.84	1.89	39.05	38.71
0	0	1	11.19	12.798	43.68	43.25
1	1	0	36.48	35.27	39.05	38.71
1	0	1	46.36	46.55	43.68	43.25
0	1	1	11.36	12.55	43.68	43.25
1	1	1	44.98	43.67	43.68	43.25

As can be seen, the results confirm the efficiency and validity of the proposed simulation-based optimization algorithm compared to the exact solution. According to this table, the selection of the first project, $(y_1 = 1, y_2 = 0 y_3 = 0)$ and also the selection of the first and third projects, $(y_1 = 1,\ y_2 = 0,\ y_3 = 1)$ are the Pareto solutions to the problem. For the second case, activity scheduling of the selected projects for the proposed approach has been shown in Table (7).

Table 7. Activity scheduling of the solution

Project	Activity	Duration	Start time
P1	W11	10.1	0
	W21	12.42	0
	W31	8.21	12.42
	W41	5.26	20.63
	W51	5.23	25.89
	W61	10.52	12.42
P3	W13	8.20	0
	W23	5.21	8.20

W33	14.02	13.41
W43	11.25	8.20
W53	16.25	27.43

8. Discussion and conclusions

The problem of project portfolio selection and scheduling is a critical decision for every organization, particularly in project-based organizations, as it has a significant impact on organizational performance. This paper investigated a novel approach to the joint problem of project selection and scheduling by assessing the aggregated individual and intensifying risks' impacts on the duration and cost of the projects' activities. Due to the uncertain environment of projects, estimating activity duration/cost is not a straightforward task. Risks serve as the primary sources of indeterminacy in projects, influencing both the duration and cost of activities. This study provided a new bi-objective mathematical model for the project selection and scheduling problem. These objective functions are to maximize the profit of the project portfolio and to minimize the project portfolio implementation time by considering the aggregated cost and time impacts of risks on projects' activities. Taking into account the risk network of the projects, which outlines potential risks and their interactions, objective functions based on Bayesian Networks are developed to evaluate the expected impacts of these risks. For the nonlinear mathematical model, an approach was proposed to select and schedule projects in an approximation linear manner. Since this formulation provides a multi-objective mathematical model, augmented ϵ-constraint programming, one of the efficient multi-objective programming methods, is used to solve the presented model. The presented model was verified through a sample instance and the results confirm the efficiency and validity of the proposed simulation-based optimization algorithm compared to the exact solution. To enhance the practical applicability of the structure proposed in this paper, it is recommended to extend this problem either as a project selection and scheduling problem with considerations for technical constraints such as resources or budget constraints or as a problem involving the selection of risk response strategies to mitigate the impacts of risks, for further studies.

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