Adv. Math. Fin. App., 2026, **11**(1), P. 1-16

Advances in Mathematical Finance & Applications
www.amfa.iau-arak.ac.ir
Print ISSN: 2538-5569
Online ISSN: 2645-4610
Doi: https:10.71716/amfa.2026.61199776

Original Research

Estimating VaR and CoVaR by Using Neural Network Quantile Regression in Iranian Stock Indices

Ali Yehea Nemer[a], Marjan Damankeshideh[a,] *, Amirreza Keyghobadi[b], Shahriar Nessabian[a]

[a] *Department of Economics, Faculty of Economy and Accounting, Central Tehran Branch, Islamic Azad University, Tehran, Iran*

[b] *Department of Industrial Management, Faculty of Economy and Accounting, Central Tehran Branch, Islamic Azad University, Tehran, Iran*

ARTICLE INFO

Article history:
Received 2025-03-03
Accepted 2025-07-17

Keywords:
Value at Risk
Conditional Value at Risk
Quantile Regression
Stock Market Indices

ABSTRACT

The financial markets are encountering uncertain conditions that are heightening their tail risk. This study analyzed eight stock market indices employing a neural network quantile regression methodology from 24 July 2017 to 22 August 2023. The findings demonstrated that the proposed model effectively estimated the tail risk by VaR and CoVaR of the sample indices of the Iranian stock market while considering oil and gold price fluctuations as risk factors. The results showed that the global crisis of the COVID-19 pandemic, which began in China in 2020, had significant impacts on global indices. However, the shock was relatively worse in the Iranian stock market, particularly in some industries such as Metals, Metal ores, and Chemicals, and the Overall indices had greater vulnerability than the rest of the indices. During the global crisis in 2022, which was triggered by the war in Ukraine, the Iranian capital market experienced a significant shock.

1 Introduction

It is important to investigate the behavior of stock exchanges not only in the capital market but also in other markets such as currency, oil, and gold markets. These markets often display similar behaviors as the stock exchange index changes. Additionally, macroeconomic variables that influence a country's economy should be examined by the index. By analyzing the effectiveness of major stock exchange indices relative to one another, a larger and more comprehensive framework can be established to predict financial crises [1]. Iran's capital market is partly owned by companies dependent on global prices. Investment and holding companies play a significant role in this market. Some companies operating in the market are not overly dependent on global prices. Therefore, some argue that crises do not directly impact the Tehran Stock Exchange due to the lack of mutual interactions with other exchanges and the limited presence of foreign investors. It will not be considered an international

* Corresponding author. Tel.: +989121757388
E-mail address: *m.damankeshideh@iau.ac.ir*

exchange. Economic integration has increased significantly in recent decades due to greater dependence between stock markets and international markets. It is now widely accepted that financial variables spread over time between assets and markets [2]. However, the global financial crisis indirectly affects the stock prices and income of companies that produce products such as metals, petrochemicals, and minerals through the increase in the price of gold and he decline in crude oil prices [24]. On the other hand, according to some experts, Iran's capital market is protected from damage during global crises. They believe that the market has even turned threats into opportunities and grown significantly. Studying the factors that influence changes in stock indices at the Tehran Stock Exchange and how they interact with global markets can help identify the variables that affect fluctuations in the index. This can lead to better decision-making by investors and optimal allocation of resources [23].Volatility in financial markets reflects the stochastic variability of asset price movements induced by uncertainty. Its quantitative modeling is indispensable for investor risk management [16], as risk constitutes a primary decision variable alongside returns. Modern finance employs statistical measures to quantify this risk exposure formally [5]. However, there is limited empirical evidence on how catastrophic events influence the co-movements between global and Iranian markets [21]. This study aims to fill this gap by developing a novel model to quantify market risk in Iran’s stock market in the context of global commodity fluctuations. Specifically, we analyze the impact of global gold and oil price fluctuations on key indices within the Tehran Stock Exchange (TSE), incorporating the Value at Risk (VaR) and Conditional Value at Risk (CoVaR) frameworks. We adopt the neural network-based CoVaR model introduced by Keilbar and Wang (2022), which overcomes limitations of traditional linear risk modeling approaches. Neural networks are particularly effective in capturing nonlinear dependencies and tail risks, which are common in financial time series. By integrating neural networks with quantile regression, we aim to provide a more accurate estimation of systemic risk under extreme market conditions. VaR is defined as the maximum loss over a fixed time horizon at a certain level of confidence. VaR is interpreted as a certain quantile of the change in the values of the portfolio in the determined interval [14]. The Basel II Accord introduced VaR as the preferred measure for market risk. The limitation of VaR is that it measures the critical risk level of each market and does not systematically account for the dependency matrix [22]. CoVaR, proposed by Adrian and Brunnermeier (2016), extends the traditional Value at Risk (VaR) framework to assess systemic risk. While VaR measures a quantile of the return distribution, CoVaR specifically estimates this quantile conditional on a financial distress event. This distinction enables CoVaR to capture systemic linkages, making it a valuable tool for identifying systemically important global stock markets. The issue of nonlinearity is significant for accurately predicting risk measures in financial markets due to their complex dependency channels [5]. Neural networks have emerged as an effective tool for modeling complex nonlinear relationships, making them well-suited for accurately estimating Value at Risk (VaR). VaR quantifies the maximum potential loss over a specified time horizon at a given confidence level, representing the threshold that losses are unlikely to exceed with a certain probability. This study aims to comprehensively analyze the impact of global gold and oil price fluctuations on Iran’s capital market.

2 Literature

Academics have engaged in extensive discussions regarding the repercussions of the financial crisis on Iran's capital market. Although oil has historically played a central role in Iran’s economy, influencing both government revenues and investor sentiment, its impact on the domestic stock market is not always straightforward. Despite the widespread assumption that fluctuations in global oil prices directly affect stock market performance, empirical patterns show that this connection is neither stable nor consistent over time. Volatility in the oil market does not necessarily translate into parallel movements in Iran’s capital market, suggesting the presence of other structural or behavioral factors that moderate this relationship. This disconnect highlights the unique nature of Iran’s financial system and underscores the need for more nuanced models to assess systemic risk transmission [13]. The expansion of financial markets and the creation of new financial markets at various levels and strata of society have increased attention toward the stock exchange. This has led to higher public participation in the capital market, which is evident today. The periods of financial distress suggest that there is a high level of risk interconnectivity among financial markets [20]. Previous studies have mostly focused on examining tail risks in specific countries and industries and at the firm level [10,19,23]. However, these studies have failed to address the measurement of tail risks associated with indices of the stock market. These studies have shown that unexpected macroeconomic events, fluctuations in gold and oil prices, and sudden economic and financial crises can lead to tail risks. Investors can effectively manage these risks by implementing robust portfolio diversification strategies designed to minimize idiosyncratic risk exposure across specific markets or market segments. This approach necessitates a thorough understanding of cross-market linkages to properly evaluate tail risk dynamics and distinguish between markets demonstrating either heightened vulnerability during downturns or optimal positioning during market upswings [21]. Given the critical importance of this issue, as well as the interconnectedness of global markets and commodity prices with Iran's financial landscape, this study seeks to develop a more robust and context-sensitive model for calculating Value at Risk (VaR). It is a widely used quantitative risk measure that provides an estimate of the maximum potential loss an investment portfolio could incur over a specified time frame, given a certain confidence level. This tool became prominent when the Basel II Accord was introduced, as it was recognized as a preferred method for quantifying market risk across financial institutions. VaR helps banks and investors make informed decisions by illustrating the level of risk they are undertaking. The subprime mortgage crisis of 2008 exposed significant shortcomings in the use of VaR and highlighted the need for more robust risk management practices. In response to this global financial turmoil, the Basel Committee on Banking Supervision undertook a comprehensive review of its regulatory framework. This led to the development of Basel III, which aims to enhance the resilience of banks and the overall financial system. Basel III places a stronger emphasis on effective risk governance and management practices, requiring financial institutions to maintain higher capital reserves and implement stricter risk assessment procedures. Its primary goal is to mitigate systemic risk by ensuring that banks are better equipped to withstand financial shocks and protect the stability of the entire financial ecosystem [15]. However, while Basel III improved upon its predecessor, it still relies heavily on conventional risk measures that may fall short in capturing the dynamic and interconnected nature of global financial systems, especially in emerging markets like Iran. To address these limitations, this study proposes a novel model that integrates global oil and gold price dynamics into the VaR estimation process for the Tehran Stock Exchange (TSE). By employing neural network-based techniques, the model is better equipped to identify complex, nonlinear relationships that traditional models often overlook. This approach not only enhances the predictive accuracy of risk estimation under volatile conditions but also reflects a more realistic understanding of market behavior. As such, the research bridges the gap between regulatory requirements and empirical market complexities, offering both methodological advancement and practical implications for investors and policymakers navigating systemic uncertainty.

Value at Risk (VaR) represents the conventional nonlinear risk measure, whereas Conditional Value at Risk (CoVaR) captures nonlinear dependencies in financial data, reflecting cross-market risk spillover effects. However, both VaR and CoVaR exhibit inherent limitations when assessing tail risk. Recent methodological advances by Keilbar and Wang (2021) and Naeem et al (2023) address these shortcomings through neural network approaches, which provide enhanced tail risk analysis by employing sophisticated nonlinear modeling techniques for extreme risk forecasting. Quantile regression has emerged as a prominent statistical tool since its foundational development by Koenker and Bassett (1978). This methodology has gained widespread adoption across diverse academic fields and practical applications due to its nonparametric nature and precision in estimating conditional quantiles, even for complex, high-dimensional predictor spaces. The approach has been theoretically validated through proofs of algorithmic consistency. Empirical studies confirm the method's competitive predictive performance [18]. They have extended standard sample quantiles to regression contexts, offering more comprehensive information about the conditional distribution of response variables given predictors compared to classical mean regression. This development proves particularly valuable during extreme market conditions when financial variables like returns typically exhibit skewness, outliers, or asymmetries. Early methodologies assumed linear relationships between conditional quantiles and predictors - an assumption that simplified computation and theoretical analysis but imposed significant limitations. Blom et al (2023) proposed ensemble models that provide accurate estimates for all quantiles. A more recent strand of the literature relaxed the linearity assumption and considered non-parametric estimators for the conditional quantile, that is, based on different methods, see for example, Belloni et al (2019) and references therein. Quantile regression has become a widely adopted approach for Value-at-Risk estimation, with several methodological advancements demonstrating its effectiveness. Engle and Manganelli's (2004) CAViaR model established a framework for direct quantile estimation without requiring full distribution modeling. Empirical studies have consistently shown the superiority of quantile regression methods, with Chen and Chen (2002) demonstrating better performance for Nikkei 225 index VaR and CoVaR estimation compared to traditional variance-covariance approaches. Further extending these applications, Shim et al (2012) developed semiparametric support vector quantile regression (SSVQR) models to estimate risk measures across major indices, including the S&P 500, Nikkei 225, and KOSPI 200, showcasing the versatility of quantile regression techniques in financial risk assessment. The analysis revealed superior performance of the proposed models compared to both conventional variance-covariance methods and standard linear quantile regression approaches. While quantile regression techniques have been extensively employed in risk measurement applications, their utilization for volatility forecasting in equity markets remains relatively underexplored in the existing literature [4]. The findings of some researchers showed that there is a significant relation between the stock market uncertainty changes in an economic boom and the investment risk in general, which is not significant in terms of the economic turndown. The Investment risk during both economic boom and recession is decreased by the unexpected increase in profit of each share and propagation of positive news. Although the risk is increased by the spread of negative forecasts in relation to shares [25].

3 Methodology and Data

The methodology consists of two steps. First, we estimate VaR using linear quantile regression with risk factors as explanatory variables. Next, these results are used to estimate CoVaR for each index via a quantile regression neural network. This section addresses the study's primary research question:

how to provide a novel approach for estimating VaR and CoVaR using quantile regression techniques. This is particularly important for improving risk measure predictions given financial markets' complex dependency structures. The main question of this study:
The methodology employed in this study is structured into two distinct steps, each designed to enhance the accuracy and reliability of risk measurement in financial markets. In the first step, our analysis centers on the estimation of Value at Risk (VaR). This is accomplished through the application of linear quantile regression, which allows us to investigate how various risk factors such as market volatility, interest rates, and other economic indicators—serve as explanatory variables. By analyzing the relationship between these factors and the quantiles of asset returns, we can derive a more nuanced understanding of potential financial risks. Step two consists of building on the results obtained from the first phase. Here, through a sophisticated analytical approach, we estimate the Conditional Value-at-Risk (CoVaR) for each index: a quantile regression neural network. This advanced method enables us to capture the intricate and often nonlinear dependencies that arise in financial markets, thus providing a more robust measure of systemic risk.In this section, we address a pivotal question that arises from our research objectives: How can we generate a new perspective for estimating both VaR and CoVaR through the lens of quantile regression? This question is significant, as it seeks to improve the predictive performance of risk measures in light of the complex interrelations and dependency channels inherent to financial markets. Ultimately, our study aims to contribute valuable insights to the field of risk management by refining how these critical financial metrics are measured and understood.
1. Is it possible to construct a robust model for measuring value at risk based on various risk factors?
Addressing this central research question, our study makes multiple contributions to the literature. Following Keilbar and Wang's (2021) empirical methodology, we examine tail risk across various indices including the Overall, OTC Overall, Total Equal Weight, Chemical, Petroleum, Metals, Metal Ores, and Metal Products Indices using a methodology estimating VaR and CoVaR. considering daily logarithmic returns spanning the period from 24 July 2017 to 22 August 2023. We collected daily index data from fipiran and macroeconomic data (oil and gold prices) from Yahoo Finance. Before model training, we conducted data preprocessing to ensure data quality and consistency. Missing values were addressed using linear interpolation. In cases where missing observations exceeded a reasonable threshold, the corresponding entries were removed to avoid bias in model estimation.

Data archived from:

- Financial Information Processing of Iran

www.fipiran.ir

- Yahoo Finance

https://finance.yahoo.com

3.1 Neural Network Quantile Regression

There is a growing interest in the application of neural networks for predicting a wide range of outcomes, reflecting their increasing popularity in various fields, particularly in finance and risk management. Neural networks have been effectively utilized in a variety of studies focused on modeling value-at-risk (VaR), a critical measure for assessing the potential loss in value of an asset or portfolio at a given confidence level over a specified time period. In a notable study conducted by Petneházi (2021), convolutional neural networks (CNNs) were employed to forecast value-at-risk. By making specific modifications to the traditional algorithm, these convolutional networks were able to estimate various quantiles of the distribution rather than limiting themselves to predicting only the mean. This flexibility allows for a more comprehensive application of neural networks in the context of VaR forecasting, making it possible to evaluate potential extreme losses more effectively. In addition, a significant methodological contribution to this area of research is the neural network quantile regression developed by Keilbar and Wang (2021). This approach specifically modeled systemic risk spillover effects among banks, highlighting the interconnectedness of financial institutions and how risk can propagate through the system. The model utilizes marginal effects within the quantile regression framework to capture these relationships accurately. According to Keilbar and Wang (2021), the formulation of the linear quantile regression equation for a predetermined quantile level τ builds upon the foundational work of Koenker and Bassett (1978, 1982). This method provides a robust statistical framework for analyzing the effects of various predictors on different points of the outcome distribution, thereby offering deeper insights into the dynamics of systemic risk. The integration of these advanced techniques into quantile regression demonstrates the potential of neural networks to enhance risk forecasting and inform decision-making processes in finance and beyond.

$$Yt = Xt\beta + \varepsilon t\ ,\ t = 1, \ldots, n \tag{1}$$

The conditional quantile function satisfies $Q_t(\varepsilon_t|X_t) = 0$, where the dependent variable Y_t is expressed as a linear combination of predictors X_t. The linear quantile regression estimator is obtained by solving the following optimization problem:

$$\min_{\beta} \sum_{t=1}^{n} \rho_t \begin{pmatrix} Y_t \\ -X_t\,\beta \end{pmatrix} \tag{2}$$

where $\rho\tau\ (z) = |z| \cdot |\tau - \mathbf{I}(z < 0)|$ is the quantile loss function. This minimization problem can be formulated as a linear program and can be solved with a simplex or interior point algorithm.

3.2 Measurement of VaR

Value-at-Risk (VaR) serves as a fundamental metric for determining regulatory capital requirements in financial institutions. Formally, it represents the τ-th quantile of the projected loss distribution, expressed as:

$$P\left(X_{i.j} \le VaR_{i.t}^{\tau}\right) = \tau\ . \tag{3}$$

where $Xi,\ t$ is the return of a financial firm i at time t and $\tau \in (0,\ 1)$ is the quantile level. There exist numerous ways to estimate VaR. We refer to Kuester et al. (2006) for an extensive overview. There are several ways to estimate the latent volatility process, including using the GARCH model or directly by estimating the conditional quantiles. VaR is estimated by linear quantile regression.we consider the approach of Keilbar and Wang (2021), and Naeem et al (2023). The VaR of each firm I is estimated by linear quantile regression using a set of macro-state variables $Mt-1$.

$$X_{i.t} = \alpha_i + \gamma_i\, M_{i-1} \tag{4}$$

where the conditional quantile of the error term $Q\tau\ (\varepsilon i\ ,t\ |Mt-1) = 0$. The VaR estimate is the fitted value of the quantile regression.

$$VaR_{i.t}^{L\,Q\,R.\tau} = \hat{\alpha}_i + \hat{\gamma}_i\, M_{i-1} \cdot \tag{5}$$

3.3 Neural Network Quantile Regression for CoVaR Estimation

In the context of our distress scenario, we operate under the assumption that all other firms are positioned at their Value at Risk (VaR) levels. This approach draws upon the frameworks established by Hautsch et al. (2014) and Härdle et al. (2016), which emphasize the importance of VaR as a critical measure for assessing the potential risks faced by firms. By adopting this assumption, we aim to provide a clearer understanding of the market dynamics during periods of financial strain and how various firms might react under such circumstances. This analytical perspective allows us to appropriately model the interconnectedness of financial markets when they are each operating at their risk thresholds.

$$P\left(X_{j.t} \le CoVaR_{j.t}^{\tau}\,|X_{-j.t} = VaR_{-j.t}^{\tau}\right) = \tau. \tag{6}$$

Let $X_{j,t}$ denote the vector of returns for all financial institutions excluding firm j at time period t, and let $VaR^{t}_{-j,t}$ represent the corresponding vector of Value-at-Risk estimates. The Conditional Value-at-Risk (CoVaR) is then derived as the fitted conditional quantile function, utilizing the VaR estimates obtained in the initial estimation phase. The τ-quantile of bank j's return distribution is modeled as a function of peer bank returns using the neural network framework previously defined in Section 3.3:

$$\begin{gathered} X_{j.t} = h_\theta\left(X_{-j.t}\right) + \varepsilon_{j.t}\,. \\ X_{j.t} = \sum_{m=1}^{M_n} \omega_m^o\, \psi\left(\sum_{k\neq 1}^{K} \omega_{k.m}^h\, X_{k.t} + b_m^h\right) + b^o + \varepsilon_{j.t}\,. \end{gathered} \tag{7}$$

with the conditional quantile of error term $Q\tau\ (\varepsilon\, j\ ,t\ |X-j\ ,t\ ) = 0$. To calculate the CoVaR of firm j , the fitted neural network has to be evaluated at the distress scenario: CoVaRτ

$$CoVaR_{j.t}^{\tau} = \hat{h}_\theta\left(VaR_{-j.t}^{\tau}\right). \tag{8}$$

The parameter θ represents the trained neural network's estimated weights. Nonlinear transformations are achieved through activation functions in the network architecture. The CoVaR measure corresponds to the τ-quantile of the potential loss distribution under a specified financial distress condition. In our case, this distress scenario is all other firms being at their VaR.

3.4 Model selection and out-of-sample performance

For estimating CoVaR based on Keilbar and Wang (2021), we utilize a neural quantile regression network by Python. To employ this network, it is necessary to specify the activation function, the number of neurons in the network, and its structure beforehand. We reset these hyperparameters at the beginning of each new year. For this purpose, we use the moving-window method. In this approach, we have subsets of data arranged in sequence as training (τ_1), evaluation (τ_2), and test (τ_3) sets. Each business year typically comprises an average of 250 days. It approximates a standard fiscal year (excluding weekends/holidays), ensuring consistency with annual risk assessment frameworks (e.g., Basel III requirements for market risk measurement). We consider the first 200 days of each year as training data, the next 50 days as evaluation data, and the subsequent 250 days of the next year as test data. The weight and bias parameters of the neural network for each selected model are obtained by training the neural network on the training data. The performance of the trained model on evaluation data is evaluated, and hyperparameters are adjusted to minimize the network's loss function during training. Additionally, the evaluation set helps prevent overfitting of the neural network. Overfitting occurs when the network learns the training data well but performs poorly on new (out-of-sample) data, leading to a significant decrease in accuracy. Therefore, the evaluation data are used to tune the hyperparameters. In summary, at first, We divide the data into three subsets: training (τ_1), validation (τ_2), and test (τ_3) for each window. Then, for each index stock and each window, we estimate the conditional quantile of X_j based on X_{-j} using the training set (τ_1). Next, we select the model specification that minimizes the average quantile loss using the validation set (τ_2). Finally, we calculate the average quantile loss based on the tuned neural network using the test set (τ_3). This process allows us to iteratively refine our model and evaluate its performance on unseen data, ensuring robustness and accuracy in our predictions. Finally, the test set is used to assess the performance of the unbiased network. The predictive accuracy of the model is obtained by calculating the average quantile loss function (AQL^{OOS}) on the test data (out-of-sample) according to the formula provided.

$$A\,Q\,L^{oos} = \frac{1}{|\mathcal{T}_3|} \sum_{t\,\in\,\mathcal{T}_3} \rho_\tau \{X_{j.t} - \hat{Q}^\tau\left(X_{j.t}\,|X_{-j.t}\right)\} \tag{9}$$

Given that our data span from January 4, 2012, to May 5, 2022, and we evaluate the model's performance on the data of the next year as the test set, we have a total of 10 to 5 windows. For each stock index, we need to adjust the neural network hyperparameters at the beginning of each year, including the number of neural network nodes, the number of epochs, batch size. L_2 penalty term on weight parameters, with a dropout rate of p. Finally, we evaluate the forecasting accuracy of our neural network quantile regression approach against a conventional linear quantile regression benchmark. The architecture of the neural network consists of three hidden layers with 64, 32, and 16 neurons, respectively. We use the ReLU activation function for all hidden layers and a linear activation function for the output layer to appropriately model the quantile outputs.

$$X_{j.t} = \beta_0 + \sum_{i\neq j}^{K} X_{i.t}\,\beta_i + \varepsilon_{j.t} \tag{10}$$

The fundamental model is calibrated using both training (τ_1) and validation (τ_2) data subsets, tuning parameters are used in this estimation, allowing for the use of the entire dataset. The out-of-sample prediction performance is then tested using the unseen test dataset (τ_3), with the condition Q^{τ} (ϵ_t|X-j, t) = 0.

4 Findings

Table 1 provides a comprehensive overview of summary statistics for the stock market indices in Iran, highlighting important measures such as average returns, volatility, skewness, and kurtosis. Among the various indices analyzed, the Total Equal Weight Index stands out as having the highest average return, recorded at 0.0023. This suggests that, on average, investments in this index yield more favorable returns compared to the other indices. In contrast, the Metal Products Index reflects the lowest average return at 0.0011, indicating a less profitable investment opportunity in this sector. When examining volatility, which is represented by the standard deviation of returns, the Petroleum Index exhibits the highest level of volatility with a standard deviation of 0.021. This high volatility may imply that returns in the petroleum sector are subject to significant fluctuations, possibly due to external factors such as global oil prices or geopolitical events. Conversely, the Total Equal Weight Index demonstrates the lowest standard deviation at 0.012, suggesting more stable and consistent returns. The Overall Index and OTC Overall Index also show similar standard deviation values, indicating a comparable level of volatility in these markets.

Table 1: Descriptive Statistics of Iran Stock Market Indices

	Overall Index	**OTC Overall Index**	**Total Equal Weight**	**Chemical Index**	**Petroleum Index**	**Metals Index**	**Metal Ores Index**	**Metal Products Index**
Mean	0.0019	0.0020	0.0023	0.0021	0.0022	0.0022	0.0021	0.0011
Max	0.0472	0.0516	0.0427	0.0572	0.0774	0.0627	0.0872	0.0985
Min	-0.0543	-0.0466	-0.0460	-0.0601	-0.0893	-0.0599	-0.0581	-0.1133
JB statistic	110.74	138.65	68.24	104.41	8.67	51.57	134.94	117.75
Std.Dev	0.0139	0.0123	0.0122	0.0163	0.0219	0.0185	0.0187	0.0199
Skewness	-0.0118	-0.0073	-0.0299	0.0413	-0.0773	0.1736	0.2774	-0.1809
Kurtosis	1.3212	1.4781	1.0364	1.2805	0.3383	0.8329	1.3478	1.3132
ARCH Statistic	465.14	364.08	362.55	639.34	1141.73	817.81	839.27	948.64
Q (20) statistic	450.41	413.99	791.68	310.86	277.51	268.43	263.85	300.88

Additionally, it is noteworthy that the Metals Index and Metal Ores Index have virtually identical standard deviations, approximately 0.018, indicating similar levels of volatility. The Chemical Index, with a standard deviation of 0.016, falls in between these ranges. These figures reveal varying degrees of risk associated with different sectors within the Iranian stock market. The analysis also delves into the distribution characteristics of these indices. The Overall Index, OTC Overall Index, Total Equal Weight Index, and Petroleum Index demonstrate negative skewness, signifying that their return distributions are left-skewed. This left skewness implies a tendency for these indices to have a higher probability of extreme negative returns, which can be a concern for investors. In contrast, the Metals Index, Metal Ores Index, and Chemical Index present positive skewness, suggesting that these indices may offer a greater likelihood of experiencing extremely positive returns. Furthermore, all indices exhibit negative kurtosis, which indicates that their return distributions are flatter and have lighter tails compared to a normal distribution. The OTC Overall Index shows the highest kurtosis while maintaining the lowest skewness among the indices, which may reflect its unique distribution characteristics and investor sentiments. Skewness and kurtosis provide essential information about the risk characteristics of stock returns. Negative skewness indicates a higher likelihood of extreme negative returns, which increases downside risk for investors. Positive skewness suggests a greater chance of unusually high positive returns but also creates asymmetry in return expectations. Negative kurtosis implies fewer extreme outliers, yet volatility clustering means periods of calm can be interrupted by sudden spikes in risk. Understanding these distributional features is crucial for accurate risk assessment and developing effective risk management strategies, especially in markets subject to structural and external shocks. The slight skewness values observed in these markets highlight their susceptibility to unexpected external economic shocks or irregular market conditions. The Jarque-Bera test for normality reveals that all markets exhibit significantly higher values than what would be expected for a normal distribution, indicating that the return distributions deviate from normality. Lastly, the presence of the ARCH (Autoregressive Conditional Heteroskedasticity) effect within the sampled returns suggests that the markets are subject to volatility clustering, where periods of high volatility tend to be followed by higher volatility. Additionally, the Ljung-Box Q test has confirmed the presence of autocorrelation in the data, indicating that past return values may have a predictive relationship with future values, which is a critical consideration for investors and analysts working within these markets.

4.1 The Correlation Matrix

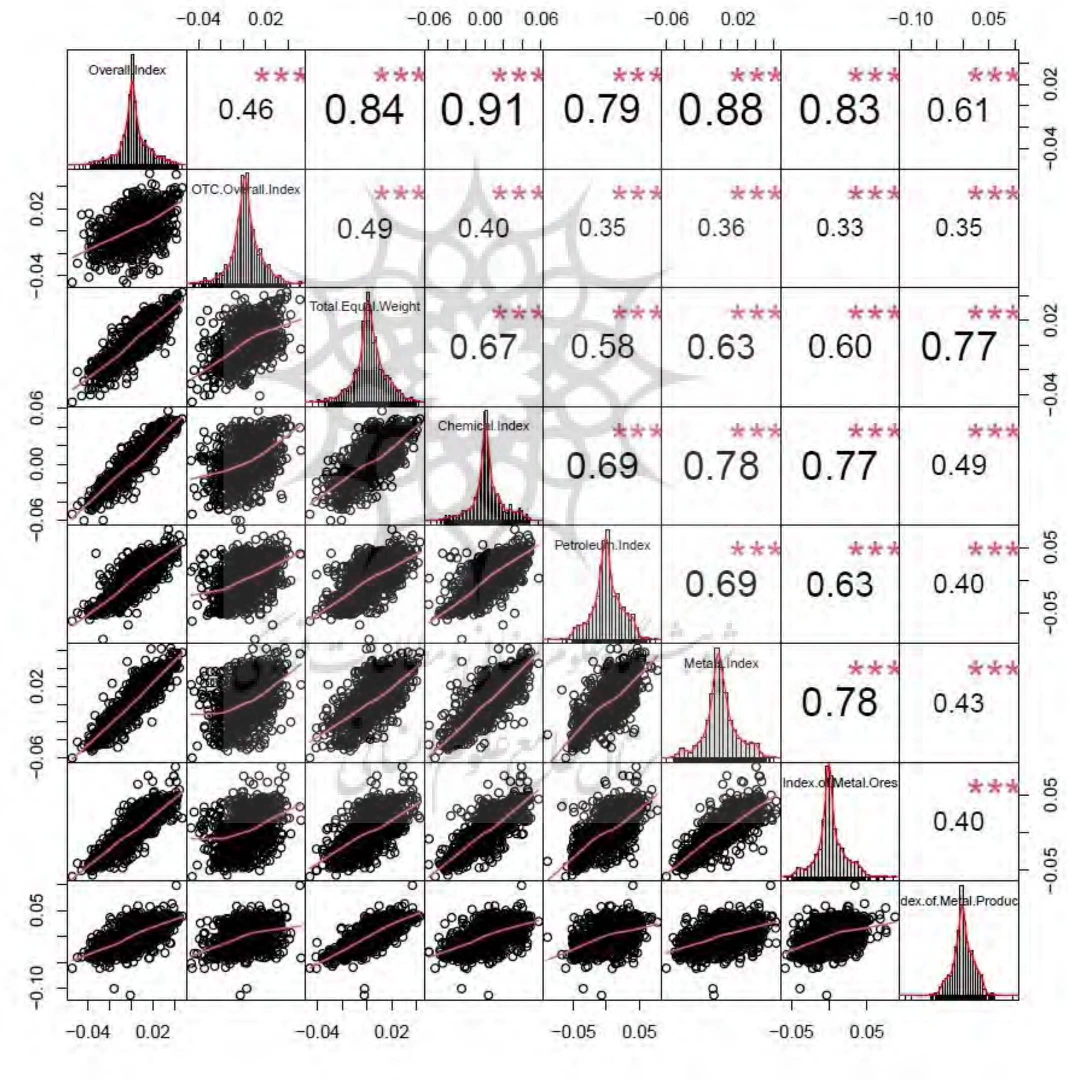

Fig. 1: Correlation Plot Among Iran Stock Market Indices

The correlation matrix illustrated in Figure 1 provides a comprehensive analysis of the relationships between various stock market indices. It reveals that the Chemical, Metal, and Total Equal

Weight indices have the strongest correlations with the Overall Index. This indicates that movements in these indices are closely aligned with the overall performance of the market. Conversely, the OTC Overall Index and the Metal Products Index exhibit a notably lower correlation with the Overall Index when compared to their counterparts. This suggests that these indices may operate somewhat independently of the overall market trends. Furthermore, the OTC Overall and Metal Products indices have the lowest correlation across the board with all other indices, which points to their unique behavior within the broader market context. In examining the other indices, the Chemical, Petroleum, and Metal Ores indices consistently demonstrate relatively high correlations with one another and with most other indices. However, this trend does not extend to the OTC Overall and Metal Products indices, which remain outliers in this regard. Of particular interest is the strong correlation observed between the Metal Products Index and the Total Equal Weight Index, indicating that fluctuations in Metal Products are likely to influence or reflect changes in the Total Equal Weight Index. However, the Total Equal Weight Index itself is characterized by the lowest correlation with all indices, which may suggest that it represents a more diversified or distinct segment of the market. Overall, the analysis indicates a significant degree of correlation among the various indices of the stock market. This suggests potential interlinkages and relationships among the Overall Index, OTC Overall, Total Equal Weight, and the Chemical, Petroleum, Metals, Metal Ores, and Metal Products indices, highlighting the interconnected nature of stock market dynamics. From a risk management and portfolio construction perspective, the correlation matrix offers critical insights. High positive correlations, such as those observed between the Metals and Metal Ores indices, imply that these sectors tend to move together, limiting the benefits of diversification if included in the same portfolio. In contrast, the relatively low correlation of the OTC Overall and Metal Products indices with the rest of the market indicates their potential as diversification instruments. Including these indices in a portfolio could help reduce systematic risk, particularly in periods of market turbulence. Moreover, understanding these correlation structures can inform strategies for hedging and sector rotation, especially in emerging markets like Iran, where market inefficiencies and external shocks play a significant role.

4.2 Estimating VaR and CoVaR

According to the method of calculation that was explained in formulas 1,2, and 3, the linear quantile regression method was used to calculate VaR for each index To address potential non-stationarity issues in our dataset, we implemented a 250-day rolling window estimation approach. As illustrated in Figure 2, the graphical representation includes: actual returns (depicted as black data points), Value-at-Risk estimates (shown in blue), and Conditional Value-at-Risk measures (displayed in red), all derived through quantile regression methodology. The returns of the Petroleum and Metal products indices exhibit significantly greater fluctuations compared to other stock market indices, highlighting their inherent volatility. A thorough analysis of time-varying trends related to returns, Risk assessment metrics such as VaR, and CoVaR reveals which indicates that these indices experience heightened volatility during crisis periods. For example, the financial crisis that unfolded in 2020 due to the outbreak of COVID-19 was closely followed by the geopolitical tensions arising from the Ukraine crisis in 2022. During the COVID-19 pandemic, specific indices, particularly those related to Metals, Metal ores, and Chemicals, displayed considerable vulnerability, indicating that they were more adversely affected than other indices in the market.

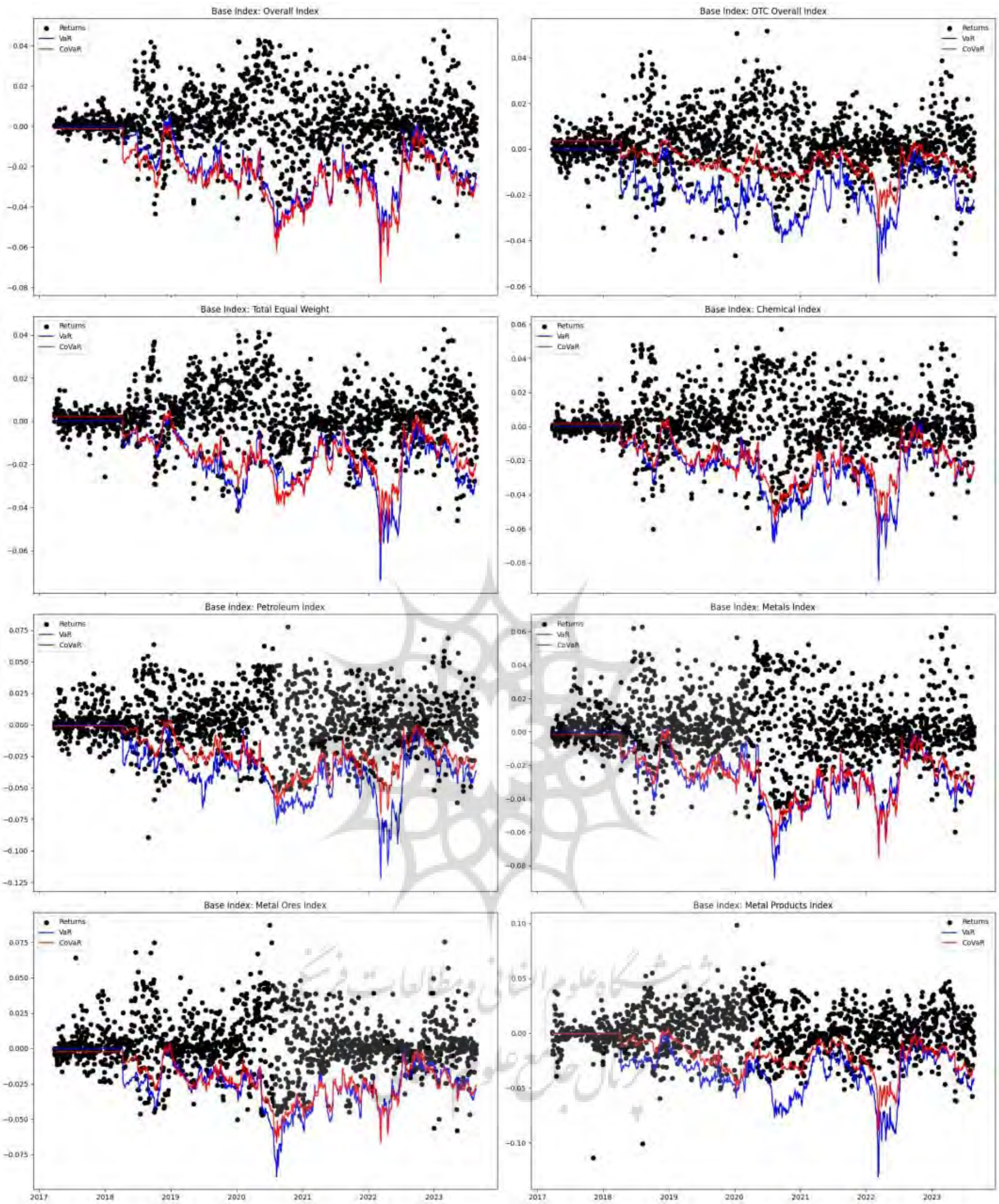

Fig. 2: displays the return series (in black dots), alongside the Value at Risk (blue curve) and Conditional Value at Risk (red curve), derived using quantile regression. The analysis covers indices such as Overall, OTC Overall, Total Equal Weight, Chemical, Petroleum, Metals, Metal Ores, and Metal Products, with Tau set at 5%.

This heightened sensitivity suggests that these indices are particularly responsive to fluctuations in the oil and gold markets, which serve as critical economic indicators. Moreover, the ongoing war in Ukraine has had a more pronounced impact on the stock market in Iran than the initial crisis posed by the COVID-19 pandemic. This finding underscores the complex interplay between geopolitical events and market performance. After enduring a prolonged period of market uncertainty, it is observed that the market generally tends to experience a phase of relatively rapid growth as conditions stabilize. A comprehensive examination of VaR and CoVaR valuations during these critical periods reveals that all indices, with the notable exceptions of Metal ores and Metal indices, faced more substantial declines as a direct consequence of the ongoing war in 2022. In essence, the overall landscape of stock market indices showcased notable spikes in activity and significant fluctuations during the year 2020. These pronounced spikes during periods of crisis indicate an increase in systemic risk. In conclusion, the war in Ukraine has demonstrated a greater influence on market dynamics than the disruptions caused by the COVID-19 pandemic, fundamentally altering the risk landscape for investors.

5 Discussion and Conclusions

The growing interconnectedness of global financial markets has become a critical focus for portfolio optimization strategies. Understanding the nature and intensity of linkages between leading commodity markets and other financial instruments is fundamental for achieving either enhanced portfolio returns or effective risk mitigation. This research introduces an innovative neural network-based approach for Conditional Value-at-Risk (CoVaR) estimation. The proposed methodology specifically addresses nonlinear dependencies in financial markets, which are crucial for accurate risk measurement given the complex interdependence patterns characterizing modern market dynamics. The study used neural network quantile regression to estimate the tail risk of Overall, OTC Overall, Total Equal Weight, Chemical, Petroleum, Metals, Metal Ores, and Metal Products Indices from 24 July 2017 to 22 August 2023. By estimating the VaR and CoVaR of the stock market indices of Iran, the study identified significant distressing events such as the global financial crisis during the period of this research, which were the COVID-19 pandemic in 2020 and the Ukraine crisis in 2022. By estimating the VaR and CoVaR of the selected indices, the study reported scattered plots for all stock market indices in the 8 indices. The findings demonstrated that the proposed model effectively estimated the value at risk of the sample indices of the Iranian stock market while considering oil and gold price fluctuations as risk factors. The results showed that the global crisis of the COVID-19 pandemic, which began in China in 2020, had significant impacts on global indices. However, the shock was relatively worse in the Iranian stock market, particularly in some industries such as Metals, Metal ores, and Chemicals, and the Overall indices had greater vulnerability than the rest of the indices. During the global crisis in 2022, which was triggered by the war in Ukraine, the Iranian capital market experienced a significant shock. Unlike in the COVID-19 pandemic crisis, where the price of oil decreased and the price of gold increased, in this crisis, the global prices of oil and gold both increased simultaneously. Based on these observations, we can conclude that Iran's stock market is relatively less impacted by the drop in world oil prices, and only certain industries are more sensitive to such fluctuations.It is imperative to note that the price fluctuations of gold have a direct and significant impact on Iran's stock market indices. This is in line with [13]. Given that the gold market is a parallel market with the stock market for investment, it is highly recommended that investors in Iran consider investing in the gold market as a defensive shield against inflation. Therefore, to effectively manage the risk of one's portfolio, it is crucial that investors carefully analyze the fluctuations of the global gold market and make accurate predictions of its value at risk. Conversely, tracking oil price fluctuations considering that it does not seem to have a direct effect on Iran's stock market, and the results of this study are in line with the results of [11]. These findings have important implications for policymakers, regulatory bodies, investors, portfolio managers, and other financial market participants. This research can be extended to develop structural financial and economic models that can help explain the factors behind the synchronization of returns phenomenon. Considering the varying impact of global crises on different sectors of Iran's stock market, this study offers valuable insights for portfolio optimization. Investors are encouraged to diversify their portfolios by including assets such as gold, which have proven to be effective hedges during market downturns. Notably, gold exhibited a stabilizing effect during both the COVID-19 pandemic and the Ukraine war, highlighting its importance as a defensive asset. Moreover, due to Iran's lower sensitivity of its overall stock market to oil price shocks, investors may prioritize monitoring gold market dynamics over oil. Policymakers and portfolio managers should consider these insights when designing investment strategies or regulatory frameworks, especially in volatile macroeconomic conditions. Future investment approaches would benefit from incorporating dynamic risk assessment models, such as the proposed neural network quantile regression, to enhance real-time risk forecasting and strategic allocation under uncertainty. It is recommended that future studies extend the proposed model to other emerging markets to enable cross-market comparisons and to analyze differences or similarities in how markets respond to global shocks, particularly concerning investors' behavioral variables.

References

[1] Asayesh, K., Fallahshams, M., Jahangiryan, H., Gholami Jamkarani, R., Explaining the Systemic Risk Model Using the Marginal Expected Shortfall Approach (MES) for the Banks Listed on the Tehran Stock Exchange, *J Plan Budg*. 2020; 25(2):115-34. In Persian

[2] Behnam, S., Tehrani, R., Tabrizian, B., Investigating the Effects of Time Variables of Gold, Crude Oil and Foreign Exchange Markets on Herding Behavior in Tehran Stock Foreign Exchange, *Adv Math Financ Appl,* 2023; 8(4): 389-407. Doi:10.22034/amfa.2022.1964376.1778

[3] Belloni, A., Chernozhukov, V., Fernández-Val, I., Wei, Y., Conditional quantile processes based on series or many regressors, *Journal of Econometrics*, 2019; 213(1): 4–29.

[4] Blom, HM., de Lange, PE., Risstad, M., Estimating Value-at-Risk in the EURUSD Currency Cross from Implied Volatilities Using Machine Learning Methods and Quantile Regression, *J Risk Financ Manag*, 2023; 16(7):312. Doi:10.3390/jrfm16070312

[5] Chen, M.Y., Chen, J.E., Application of quantile regression to estimation of value at risk, *Review of Financial Risk Management*, 2002;1-16.

[6] De Bandt, O., Hartmann, P., What Is Systemic Risk Today? In: Risk Measurement and Systemic Risk, Proceedings of the Second Joint Central Bank Research Conference, *Tokyo: Bank of Japan*, 1998; 37-84. Doi:10.5089/9781589064409.071

[7] Engle, R.F., Manganelli, S., CAViaR: Conditional autoregressive value at risk by regression quantiles, *Journal of Business & Economic Statistics,* 2004; 22(4): 367-381. Doi:10.1198/073500104000000370

[8] Härdle, W.K., Wang, W., Yu, L., TENET: Tail-Event driven network risk, *Journal of Econometrics,* 2016;192(2):499–513. Doi: 10.1016/j.jeconom.2015.06.008

[9] Hautsch, N., Schaumburg, J., Schienle, M., Financial network systemic risk contributions, *Review of Finance.* 2015; 19(2): 685–738. Doi:10.1093/rof/rfu010

[10] Sharma, GD., Tiwari, AK., Talan, G., Jain, M., Revisiting the sustainable versus conventional investment dilemma in COVID-19 times, *Energy Policy*, 2021; 156: 112467. Doi: 10.1016/j.enpol.2021.112467

[11] Shahrestani, P., Rafei, M., The impact of oil price shocks on Tehran Stock Exchange returns: Application of the Markov switching vector autoregressive models, *Resources Policy*, 2020; 65:101579. Doi: 10.1016/j.resourpol.2020.101579

[12] Shim, J., Kim, Y., Lee, J., Hwang, C., Estimating value at risk with semiparametric support vector quantile regression*, Computational Statistics*. 2012; 27(4): 685–700. Doi:10.1007/s00180-011-0283-z

[13] Tehrani, M., Boghosian, A., Mirlohi, SM., Spillover between Tehran Stock Exchange and the International Oil Market, *Financial Research Journal*, 2021; 23(3): 466–81. Doi: 10.22059/frj.2021.312616.1007087

[14] Tahmasebi, M., Yari, GH., Risk Measurement and Implied Volatility Under Minimal Entropy Martingale Measure for Levy Process, *Advances in mathematical finance & applications*,2020; 5(4): 449-67. Doi:10.22034/amfa.2020.1880442.1310.

[15] Keilbar, G., Wang, W., Modelling systemic risk using neural network quantile regression, *Empirical Economics* ,2022; 62: 93–118. Doi.10.1007/s00181-021-02035-1

[16] Khodayari, M.A., Yaghobnezhad, A., Khalili Eraghi, M., A Neural-Network Approach to the Modeling of the Impact of Market Volatility on Investment, *Advances in mathematical finance & applications*, 2020; 5(4): 569-81. Doi: 10.22034/amfa.2020.1879320.1298.

[17] Koenker, R., Bassett, G. Jr., Robust tests for heteroscedasticity based on regression quantiles, *Econometrica,* 1982; 50: 43–61. Doi.10.2307/1912528

[18] Kuester, K., Mittnik, S., Paolella, MS., Value-at-risk prediction: a comparison of alternative strategies, *J Financ Econ*, 2006; 4(1): 53–89. Doi.10.1093/jjfinec/nbj002

[19] Liu, T., Hamori, S., Spillovers to renewable energy stocks in the US and Europe: Are they different? *Energies*, 2020; 13(12): 3162. Doi.10.3390/en13123162

[20] Mensi, W., Naeem, MA., Vo, XV., Kang, SH., Dynamic and frequency spillovers between green bonds, oil, and G7 stock markets: Implications for risk management, *Econ Anal Policy*, 2021; 73:331–44. Doi: 10.1016/j.eap.2021.11.015

[21] Naeem, MA., Karim, S., Yarovaya, L., Lucey, BM., Systemic risk contagion of green and Islamic markets with conventional markets, *Annals of Operations Research*, 2023; 347(1): 265-287. Doi.10.1007/s10479-023-05330-5

[22] Wang, J., Wang, S., Lv, M. et al. Forecasting VaR and ES by using deep quantile regression, GANs-based scenario generation, and heterogeneous market hypothesis, *Financial Innovation*, 2024;10(36):1-35. Doi.10.1186/s40854-023-00564-5

[23] Xuefeng, Z., Razzaq, A., Gokmenoglu, KK., Rehman, FU., Time-varying interdependency between COVID-19, tourism market, oil prices, and sustainable climate in the United States: Evidence from advance wavelet coherence approach, *Economic Research-Ekonomska Istraživanja,* 2022; 35(1): 3337–59. Doi:10.1080/1331677X.2021.1992642

[24] Zeinedini, S., Karimi, MS., Khanzadi, A., Impact of global oil and gold prices on the Iran stock market returns during the Covid-19 pandemic using the quantile regression approach, *Resources Policy,* 2022; 76(5):102602. Doi: 10.1016/j.resourpol.2022.102602

[25] Zanjirdar, M., Moslehi Araghi, M., The Impact of Changes in Uncertainty, Unexpected Earning of Each Share and Positive or Negative Forecast of Profit Per Share in Different Economic Condition, Quarterly Journal of Fiscal and Economic Policies,2016;4(13): 55-76.