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OPEC Crude Oil Price Prediction Based on Chaos Theory and GMDH-GA

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Highlights

- Crude oil price prediction using chaos theory;
- Proving that the crude oil price time series is chaotic;
- Converting crude oil time series to wavelets;

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Abstract

The price of crude oil is exposed to various factors that cause random, sudden, and chaotic price fluctuations. Accurate forecasting of oil price has a central impact on the macro economy. The aim of this study is to predict the fluctuations of Organization of Petroleum Exporting Countries (OPEC) crude oil in the long-term using the chaos theory and the GMDH-GA algorithm. First, the daily oil price time series is decomposed by wavelet transformation. Then, chaos is tested using the embedding dimension, the Lyapunov power, and GA tests. Finally, time series noises are reduced by reconstructing the wavelet phase space. Three nonlinear models, namely the GMDH-GA model, the GMDH-GA wavelet model, and the GMDH-GA extended model, were used to forecast time series. Although the results showed that all three models were favorable in terms of the root mean square error (RMSE) and the correlation coefficient, the developed GMDH-GA neural network model with a low RMSE and high correlation coefficient was most effective in predicting the daily price of OPEC crude oil.

Keywords: Chaos theory, Correlation dimension, GMDH-GA, OPEC crude oil price forecast, Renyi dimension.

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1. Introduction

Due to the rapid changes happening in today's world, there is much emphasis on economic forecasts. Energy, as the driver of many economic efforts, has an important place in the economic progress of countries (Yuan, Zhao, and Umair, 2023). Currently, oil and gas account for approximately 60%, coal for 15%, and nuclear along with other sources for the remaining 25% of energy supply (Xiuzhen, Zheng, and Umair, 2022). Crude oil plays an important role in people's daily life and industrial economic development and is considered an important strategic resource worldwide (Yuan, Zhao, and Umair, 2023). The oil market is volatile due to its high volatility, and the crude oil market is different from other energy markets (Xiuzhen, Zheng, and Umair, 2022). Accurate forecasting of oil price has a central impact on the macro economy. The research of the World Bank shows that a 30% drop in oil price

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causes a 0.5% increase in GDP in importing countries. Since crude oil constitutes a significant percentage of the exports of some countries, a rapid change in the price of crude oil can have extensive financial consequences. The fall in the price of crude oil can slow down the economic activity of exporting countries (Ullah, Chishti, and Majeed, 2020). In recent years, the complexity and diversity of crude oil price have had an increasing impact on the economic development of countries. Therefore, accurate forecasting of crude oil price is useful for maintaining economic stability and avoiding risks (Zhao et al., 2015). Governments can prepare for unexpected jumps in oil price with the help of accurate forecasts. While supply and demand are the primary drivers of crude oil price, other variables such as stock market, economic activity, political situation, and other external factors also play a role (Wu et al., 2024). The accurate prediction of oil price is necessary for economic planning of exporting and importing countries. The correct estimation of the future price of crude oil is one of the most important research questions in the field of forecasting (Pan et al., 2023). Due to the nonlinear nature of the oil market, accurately predicting price in the oil market is not an easy task. Crude oil price forecast errors are mainly caused by the complexity of the supply and demand structure and the existence of many unexpected factors that disturb the market balance. Both exogenous factors, such as the state of the world economy, and endogenous components of the oil market, such as oil consumption, inventory, and supply, have an effective role in crude oil price (Liu and Chen, 2022).

Due to the erratic nature of oil price and the impact of various environmental factors on the crude oil market, finding a suitable forecasting tool has always been a challenge. Forecasting crude oil future price is very challenging due to the three characteristics of crude oil price, namely lag, nonlinearity, and their interrelationship among different oil markets, most traditional crude oil price forecasting models cannot handle simultaneously (Cheng et al., 2019). Researchers have conducted extensive studies in this field and have proposed various models. However, most contemporary forecasting models consider oil price fluctuations stochastic processes and rely on statistical methodology (Wu et al., 2024). Most of the existing literature has relied on the autoregressive integrated moving average (ARIMA) and artificial neural network (ANN) for forecasting (Iqbal et al., 2020). Prediction based on past prices is difficult due to nonlinearity, uncertainty, and dynamics in these prices (Iram, Jabbar, and Bhatti, 2022). Crude oil price is often nonstationary time series, making it challenging to achieve acceptable forecasting accuracy using time series-based models. In addition, these methods depend on the assumptions of linear and normal distribution, which does not adequately reflect the specific characteristics of crude oil price series (Dong et al., 2024).

In general, energy prices, like crude oil price, are affected by certain events such as seasonal changes, as well as uncertain events such as geopolitical events. Uncertain events cause prices to change randomly and make price prediction a difficult task. These random changes act as noise, which affects the definite changes in prices (Behradmehr and Ahrari, 2014). In recent years, the exploration of nonlinear time series has gained significant traction, especially in the use of the chaos theory to understand the properties of fuzzy systems. The chaos theory provides an alternative framework to explain seemingly random behavior in deterministic systems. Several nonlinear measures such as fractal dimension, the Lyapunov power, Poincaré sections, and entropies have been devised for time series analysis. Currently, the time series analysis of chaotic nonlinear systems using correlation dimensions and the largest Lyapunov exponent is increasingly used for system classification and modeling. This approach holds promise for examining crude oil price time series and facilitates the identification of underlying behavioral patterns.

In this study, the behavior of Organization of Petroleum Exporting Countries (OPEC) oil price time series is analyzed using the chaos theory. The characteristics of this time series are extracted using wavelet transform, and the developed GMDH-GA model is used to predict the time series. The procedure is as follows. First, the general time series is selected. The chaos test is performed on the wavelets, and the time series noises are reduced by determining the phase parameters. OPEC crude oil prices are nonlinear due to chaotic behavior in the time series. Therefore, the GMDH-GA algorithm is used to predict it. TISEAN software, which is suitable for analyzing nonlinear time series (Pan et al., 2023), and MATLAB software are utilized to numerically solve the models.

Our study is organized as follows. The second section describes a comprehensive literature review on oil prices and forecasting methods, and the third part explains the proposed research model. Following this, in section four, turbulence tests are conducted on daily OPEC crude oil price series to confirm the chaotic properties of the time series. Finally, section five includes a summary, conclusions, and suggestions for future research efforts.

2. Literature review

Research in the field of crude oil price forecasting can be divided into three main areas: 1) econometric methods, 2) machine-learning methods, and 3) integrated segmentation methods (J. L. Zhang, Zhang, and Zhang, 2015). Kaboudan (2001) used two genetic meta-heuristic algorithms, genetic algorithm (GA) and single-layer artificial neural network for short-term crude oil price forecasting. The predictions produced by these two calculation methods were compared and evaluated with a random prediction method. The results showed that GA had an advantage over random predictions, but ANN prediction was inferior (Kaboudan, 2001). Oil price changes are very complex and therefore unpredictable. One of the main challenges of econometric models is to predict such seemingly unpredictable economic series. Traditional linear mathematical models used for forecasting are not efficient, especially for complex series such as crude oil price. Moshiri and Foroutan (2004) modeled and forecasted daily oil price futures using the autoregressive integrated moving average and GARCH models. Then, they tested the chaos using embedding dimension tests, BDS, the Lyapunov power, and neural networks. Also, they set up a nonlinear and flexible ANN model for series forecasting. Chaos tests showed that the price of oil in futures follows a chaotic process, and the ANN model makes better predictions (Moshiri and Foroutan, 2004). Xie et al. (2006) introduced a support vector machine (SVM) method for crude oil price forecasting, which was better than ARIMA and back-propagation neural network (BPNN) models (Xie et al., 2006).

Nguyen and Nabney (2008) presented a forecasting technique for energy prices on a daily basis. Their technique is a combination of wavelet transform and prediction models such as multilayer perceptron, linear regression, or GARCH. Their results showed that the prediction accuracy improved significantly when the wavelet transform was used (Nguyen and Nabney, 2008). Yu et al. (2008) proposed a neural network ensemble learning model based on empirical mode decomposition (EMD) for crude oil price forecasting. They used a three-layer feedforward neural network (FNN) model in modeling to increase the prediction accuracy. They combined the forecast results with an adaptive linear neural network (ALNN) to formulate an output set for the original crude oil price series. The found that the experimental results showed the attractiveness of the EMD-based neural network ensemble learning model (Yu, Wang, and Lai, 2008).

Kang et al. (2011) investigated the effect of structural changes in volatility on information transmission in two crude oil prices. They used the iterated cumulative sums of squares (ICSS) algorithm to evaluate the effect of these structural changes. Incorporating these changes into GARCH reduced the degree of volatility in the model, indicating that ignoring structural changes might distort the direction of the information flow and transmission of fluctuations between crude oil markets (Kang, Cheong, and Yoon, 2011). Azadeh et al. (2012) presented a flexible algorithm based on artificial neural network and fuzzy regression (FR) to deal with optimal long-term oil price forecasting in uncertain and complex environments. They showed that selected ANN models significantly outperformed FR models in terms of mean absolute percentage error (MAPE) (Azadeh et al., 2012). Guo, Li, and Zhang (2012) developed an improved oil price forecasting model that used SVM. The new model, called the GA-SVM prediction model, was based on GA optimization parameters. GA was used to optimize SVM parameter selection methods according to training data and improve the prediction accuracy of SVM. Their results showed that the prediction efficiency of GA-SVM was better than the traditional SVM (Guo, Li, and Zhang, 2012). Behradmehr and Ahrari (2014) used wavelet transform as a tool to smooth and minimize the noise introduced in crude oil prices. They investigated the effect of wavelet smoothing on oil price forecasting while the GMDH neural network was used as a forecasting model. In addition, the generalized autoregressive conditional heteroskedasticity model was used to capture the time-varying variance of crude oil prices. Their results showed that the prediction performance was enhanced by more than 40% when the effect of noise was minimized, and the variance was regressed by the conditional heteroscedasticity model (Behradmehr and Ahrari, 2014).

Zhang et al. (2015) proposed a new hybrid method for crude oil price forecasting. They used the ensemble empirical mode decomposition (EEMD) method to analyze the international price of crude oil. Further, they developed least square support vector machine with particle swarm optimization (LSSVM–PSO) and the GARCH model to forecast the nonlinear and time-varying components of crude oil price. Their results showed that the proposed new hybrid method had a strong predictive capability for crude oil prices due to its excellent performance in adapting to random sample selection, data abundance, and structural failures in the samples. In addition, the comparison results showed that the new method was superior in prediction accuracy compared to the known methods for crude oil price forecasting (J. L. Zhang, Zhang, and Zhang, 2015). Cheng et al. (2015) proposed a new hybrid model of vector error correction and nonlinear autoregressive neural network (VEC–NAR) model. The results showed that the VEC–NAR model had superior forecasting accuracy compared to traditional models such as GARCH, VAR, VEC, and NAR in multi-stage short-term forecasting (Cheng et al., 2019). Ghazi Salah et al. (2019) investigated the robustness, efficiency, and accuracy of multiscale forecasting in crude oil markets. They explicitly defined an automatic hybrid ARMA wavelet model to detect the inherent nonlinear dynamics of crude oil returns with a hierarchical structure. Entropic estimation was used to calculate the optimal level of decomposition. The wavelet-based forecasting method takes into account the turbulent behavior of the oil series, while capturing drifts, spikes, and other nonstationary effects that common frequency-domain methods completely miss. These results opened a new horizon on the predictability of crude oil markets in unstable conditions (Uddin et al., 2019).

Ghoddusi et al. (2019) reviewed the growing literature of energy or financial economics applications by machine learning (ML). Their review included applications in energy price forecasting such as crude oil, demand forecasting, risk management, business strategies, data processing, and macro energy trend analysis. Their analysis showed that SVM, ANN, and GA were among the most used (Ghoddusi, Creamer, and Rafizadeh, 2019).

Bekiros et al. (2020) investigated the potential of multi-scale forecasting in the crude oil market using multi-scale wavelet analysis on returns and volatility of crude oil indices. The analysis was based on an invariant discrete wavelet transform augmented by an entropy-based method to determine the optimal time scale decomposition under different market regimes. The experimental results showed that the five-stage wavelet forecast based on volatility was better than the random walk forecast compared to the wavelet forecast based on returns. Their results might have important implications for market efficiency and price predictability in crude oil markets (Bekiros et al., 2020). Yusheng Huang and Yong Deng (2021) introduced variable mode decomposition (VMD) to forecast crude oil prices. They used a rule based on improved signal energy (ISE) to select the VMD parameter. Finally, a prediction model

(VMD–LSTM–MW model) was built by combining VMD, long-term short-term memory (LSTM) network, and moving window strategy. The superiority of the VMD–LSTM–MW model was demonstrated by conducting monthly and daily crude oil price forecasting experiments (Huang and Deng, 2021).

Jiang et al. (2021) combined a group-decomposition approach, optimized by the seagull algorithm, with sentiment analysis to address this problem. An ensemble empirical mode decomposition (EEMD) method was employed to decompose crude oil futures data and reduce the effect of noise. A seagull optimization algorithm (SOA) was introduced to tune the meta-parameters of gated regression units (GRUs). Multiple linear regression (MLR) integrated the prediction results of each component. This approach performed significantly better than some other comparison models in predicting fluctuations in oil prices. Zhang et al. (2023) presented a new hybrid forecasting model for oil price forecasting based on recurrent neural networks using VMD, sample entropy (SE), and gated regression units. The proposed VMD–SE–GRU model obtained a root mean square error value of 0.6735, a mean absolute error of 0.4585, a mean absolute percent error of 0.8059, and a value of 0.9272. Therefore, the proposed hybrid VMD–SE–GRU framework had several advantages over previous models and produced highly accurate predictions with shorter execution time (Zhang et al., 2023). Dong et al. (2024), based on the chaotic nature of crude oil price series, proposed a model for crude oil price forecasting that included the VMD algorithm, the phase space reconstruction (PSR) technique, the convolutional neural network (CNN), and the bidirectional long- and short-term memory network (BiLSTM). Specifically, noises in the original data were removed using the VMD algorithm. In the next step, crude oil prices were reconstructed using the PSR technique. The reconstructed and removed phase space was then fed into the CNN–BiLSTM model for multi-stage predictions. The experimental results showed that the proposed model obtained the lowest MAPE and MSE (Dong et al., 2024).

Table 1 summarizes a number of past studies on the subject of crude oil forecasting according to the concepts and tools they used.

Table 1

Summary of past research in the field of crude oil price forecasting

Year	**Author(s)**	**Meta Heuristic**	**Time series**	**Statistics**	**Chaos**	**Reference**
2001	Kaboudan	ANN and GA				(Kaboudan, 2001)
2004	Moshiri and Froutan	ANN	ARIM and GARCH		Embedding dimension, BDS, and the Lyapunov exponent	(Moshiri and Foroutan, 2004)
2006	Xie et al.	SVM and BPNN	ARIM and GARCH			(Xie et al., 2006)
2008	Nguyen and Nabney		GARCH		Wavelet transform	(Nguyen and Nabney, 2008)

Year	Author(s)	Meta Heuristic	Time series	Statistics	Chaos	Reference
2008	Yu et al.	EMD, ANN, FNN, and ALNN				(Yu, Wang, and Lai, 2008)
2011	Kang et al.		GARCH			(Kang, Cheong, and Yoon, 2011)
2012	Azadeh et al.	ANN		FR and ANOVA		(Azadeh et al., 2012)
2012	Guo and Zhang	SVM and GA				(Guo, Li, and Zhang, 2012)
2014	Behradmehr and Ahrari	GMDH		Auto-regressive		(Behradmehr and Ahrari, 2014)
2015	Zhao et al.	ANN, GA, and SVM	CGARCH			(Zhao et al., 2015)
2015	Zhang et al.	PSO		LSSVM	EEMD	(J. L. Zhang, Zhang, and Zhang, 2015)
2019	Cheng et al.	NAR	GARCH	VEC and VAR		(Cheng et al., 2019)
2019	Ghazi Salah et al.		ARMA		Hybrid wavelet	(Uddin et al., 2019)
2020	Bekiros et al.				Wavelet transform	(Bekiros et al., 2020)
2021	Yusheng Huang				VMD and ISE	(Huang and Deng, 2021)
2022	Jiang et al.	SOA		MLR	EEMD	(Jiang et al., 2022)
2023	Zhang et al.			MAPE, MAE, and RMSE		(S. Zhang et al., 2023)
2024	Dong et al.	PSR, CNN, and BILSTM		MAPE and MSE	VMD	(Dong et al., 2024)

Year	Author(s)	Meta Heuristic	Time series	Statistics	Chaos	Reference
2024	This research	GMDH–GA			Wavelet transform, embedding dimension, BDS, and the Lyapunov exponent	-

The study of the published articles on crude oil price forecasting shows that the price of crude oil is nonlinear due to the chaotic behavior in the time series. Therefore, most studies have used metaheuristic algorithms to predict it. Some studies also paid attention to the chaotic nature of crude oil prices and used the tools of this branch of physics to eliminate noise. In the present study, the behavior of OPEC oil price time series is analyzed using the chaos theory, and the features of this time series are extracted using wavelet transformation. The developed GMDH–GA model is employed to predict time series.

3. Methodology

3.1. Proposed model

The conceptual framework of the present study is depicted in Figure 1.

3.2. Time series selection

Given the paramount significance of time series data in ensuring the precision of current study's outcomes, we utilized the maximum available dataset spanning a period of 15 years. Employing Rosenstein and Collins algorithms, we calculated the Lyapunov exponent to discern potential chaotic behaviors (Rosenstein, Collins, and De Luca, 1993).

3.3. Wavelet transforms and noise reduction

In the oil market, there are noisy behaviors that disrupt the study of the system. The noise of time series must be reduced to identify the correct behavior of the time series and the precise results of the chaotic tests. Noisy behavior is detected by a discrete wavelet transform and is removed from the time series.

3.3.1. Wavelet transforms

Some characteristics and features of time series cannot be observed on a time basis, so by transferring time series to another basis, these properties are observed and examined. There are several ways to achieve this goal. One of these methods is wavelet transformation (Misiti et al., 1996; Percival and Walden 2000). The continuous wavelet transform (CWT) of a function is defined by the sum of the multiplication of function in the scaled wavelet function and shifted in the whole time period. Therefore, Equation (1) can be written as follows:

$$C(scale, position) = \int_{-\infty}^{+\infty} f(t)\, \psi(scale, position)\, dt$$
$$C(a, b) = \int_{-\infty}^{+\infty} f(t) \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) dt \qquad (1)$$

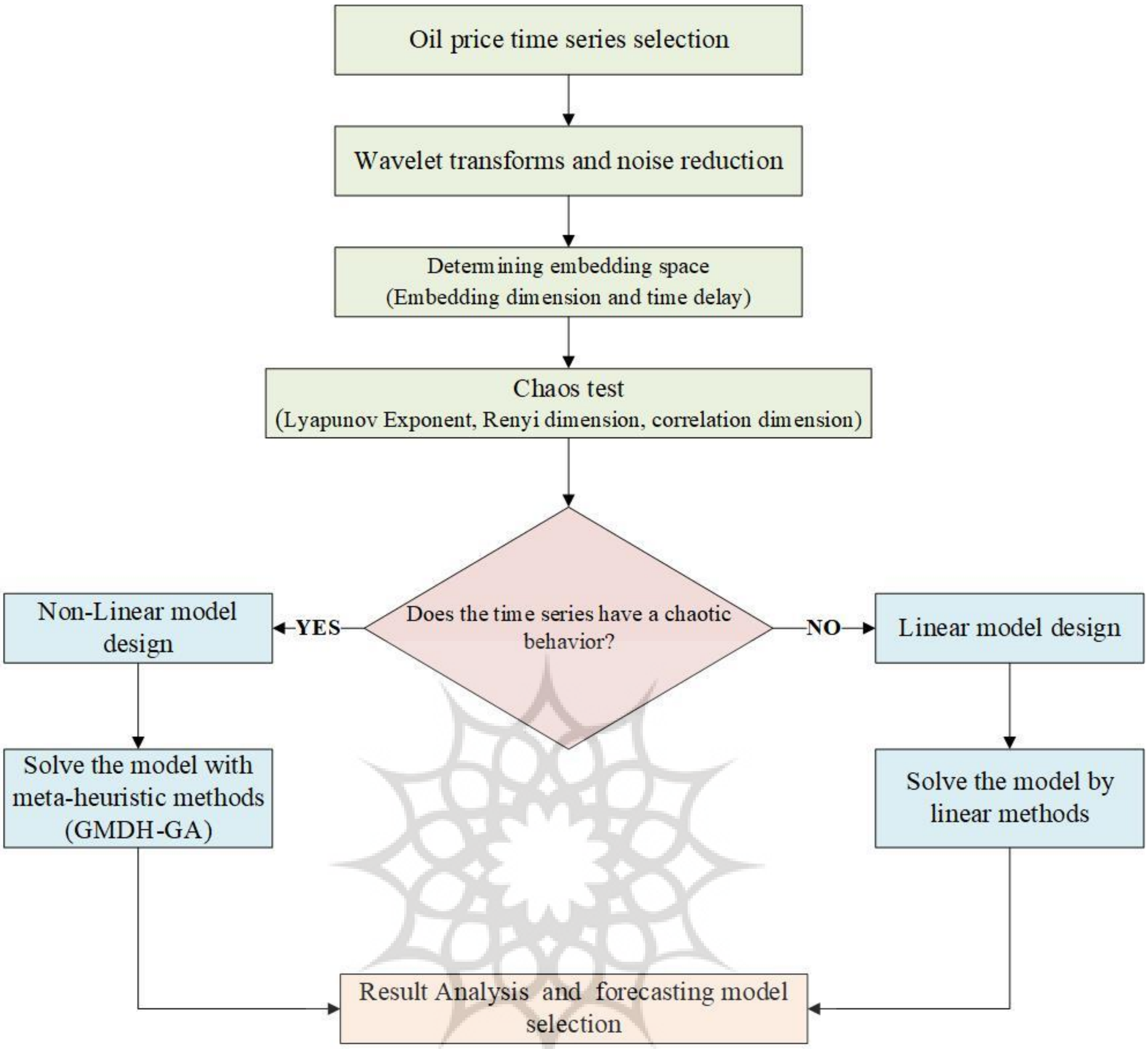

Figure 1

The conceptual framework of research

The result is the wavelet coefficients (C), which is a function of scale and position. The multiplication of each of these coefficients in the corresponding scaled and shifted wavelet determines its share in the main signal. The shift means the movement of the wavelet along the time axis, and scale means the amount of wavelet extension along the time axis. The large scale of the wavelet is equal to the low frequencies, and the small scale equals the high frequencies. Each function used as a wavelet must have an average of zero, have a non-zero norm, and be limited in time properly. These characteristics are presented in Equations (2)–(4), respectively.

$$\int_{-\infty}^{+\infty} \Psi(t)\, dt = 0 \quad (2)$$

$$0 < \int_{-\infty}^{+\infty} \Psi^2(t)\, dt < \infty \quad (3)$$

There are the largest value of LB and the smallest value of UB when the following is met:

$$\forall t \leq LB \qquad \Psi(t) = 0 \quad (4)$$

$\forall t \geq UB \qquad \Psi(t) = 0$

The wavelet-based transmission signal can be returned to the time-based signal by Equation (5).

$$f(t) = \frac{1}{K_\Psi} \iint C(a,b) \frac{1}{\sqrt{a}} \Psi\left(\frac{t-b}{a}\right) \frac{da\, db}{a^2} \tag{5}$$

Signals are usually discrete, so the discretization of the wavelet transform is inevitable.

3.3.2. Phase space reconstruction

In a nonlinear system, a single-variable time series can bring information about the whole multivariable system (Sivakumar, 2002). Therefore, in systems where all dynamic variables are not available, the dynamics of the system can be achieved by reconstructing the phase space of the system using the single-variable time series. The method used for this purpose is the Takens method (Takens, 2006). Takens' theory states that if a time series is obtained from a given dynamic system, it is possible to reconstruct the phase space of this system with the help of the same time series by creating time delays as large as τ in m dimensions. In this definition, τ and m are called time delay and the embedding dimension, respectively. Selecting the amount of τ affects the structure of the attractor. If a very small τ is selected, the delay vectors will be highly interdependent so that all the points around the diameter axis will be in the phase space and will cause the loss of the characteristics of the attractor structure. Further, if the size τ is chosen too large, the time delay vectors become dynamically independent. In this case, the reconstructed phase space will be very complex, even if the actual attractor of the system is simple (Casdagli et al., 1991). In general, two methods of autocorrelation function (ACF) and average mutual information (AMI) are employed to estimate time delay (Fraser and Swinney, 1986). In strange attractors, if they are surrounded by the right dimension, the paths of the state do not intersect. The most common method of determining the optimal embedding dimension for chaotic time series is the nearest-neighbor counting method.

3.4. Chaos test

3.4.1. The Lyapunov exponent test

The Lyapunov exponent provides a criterion for the level to which two points converge in the phase space and presents information about the system's dependence on initial conditions over time (Ott, 2002). In a state of a stable fixed point, all Lyapunov's powers are negative. If the system is sensitive to the initial conditions, it will have at least one positive the Lyapunov exponent. One of the important features of chaotic systems is that the system is sensitive to changes in initial conditions. A Lyapunov exponent is a tool for studying this feature of dynamic systems. When a small change occurs in the initial state of the chaotic system, the effect of this change will become clearer over time. It creates a time path that is completely different from the previous time path. The Lyapunov exponents are the average convergence or divergence rate of adjacent paths in the phase space. Numerous algorithms have been proposed to calculate the Lyapunov exponent. In this study, Rosenstein and Collins algorithm was employed to calculate the Lyapunov exponent (Gencay and Dechert, 1992). In this algorithm, first, a Δn point of the time series in the phase space and all its neighbors with a distance less than ε is selected. Then, the average distances of all the larger neighbors, greater than the movement path of the reference point, are calculated as a function of the relative time according to Equation (6).

$$\lambda = Lim_{N\to\infty} \sum_{n_0=1}^{N} ln\left(\frac{1}{|v(s_{n_0})|} \sum_{s_n \in v(s_{n_0})} \left|s_{n_0+\Delta n} - s_{n+\Delta n}\right|\right) \tag{6}$$

where S_{n0} expresses reference points, $V(S_{n0})$ is the neighborhood of S_{n0} with a diameter of ε, and $S_{n0+\Delta n}$ denotes time covered by the delay vector S_{n0}. $S(\Delta n)$ is calculated for both values of minimum embedding dimension (M) and optimal distance (ε). If function $S(\Delta n)$ shows a strong linear increase for Δn intervals, its slope at each stage estimates the largest Lyapunov exponent (λ).

3.4.2. Fractal dimension

Chaotic dynamic systems show their dynamism in dependence on initial conditions. Paths are spaced apart exponentially over time. This behavior of the chaotic dynamic systems is consistent with the geometry of the system attract. Considering a small part of the attractor the initial condition, we know that after a while the images will fill the whole attractor. In fact, despite the distance between the paths and the instability of the system in a small part of the attractor, eventually, the paths are absorbed by the attractor, and they are spaced apart in the range of attraction. Fractals do not have the correct dimension like other Euclidean geometric shapes, and the concept of fractional dimension is used to describe them. Therefore, the concept of dimensions can be expanded, and the distribution of the set of points in the phase space can be examined. Box counting dimension, information dimension, correlation dimension, and other methods have been defined for this purpose (Panigrahy et al., 2019). Finally, the chaotic time series is studied by a hybrid neural network with a genetic algorithm.

3.5. Definition parameters, indices, and variables of model

Based on the chaos theory and related tests, the variables and parameters used in the current research are described in Table 2.

Table 2

The variables and parameters of the model

Variable	Description	Range
τ	Time delay	By ACF or AMI
FNN	False nearest neighbors	
M	Embedding dimension	Number of false neighbors equal to zero
ACF	Autocorrelation function	Zero or a threshold value of $1/e$
AMI	Average mutual information	
λ_I	Lyapunov exponent	i=1, m

3.6. GMDH–GA neural network

In this framework, an evolutionary design approach is adopted to optimize the structure of the neural network. Specifically, the genetic algorithm is employed for this purpose (Ünal and Başçiftçi, 2022). In the genetic algorithm, solutions to the problem are represented by a list of parameters referred to as chromosomes or genomes. These chromosomes typically consist of a simple set of data [36]. In the GMDH–GA neural network, each chromosome signifies the structure of a GMDH neural network and comprises a symbolic string of letters or numbers, with each symbol representing an input variable of the network. The length of each chromosome is $2K + 1$, where K denotes the number of layers in the neural network.

4. Results

This study first performed chaos tests on the OPEC crude oil daily price series and presented the nonlinear model of the neural network by confirming the chaotic behavior of the time series to predict this time series.

4.1. Data set

4.1.1. Time series selection

This work uses weekly data on OPEC oil price. The data belong to a period of time from the beginning of January 2003 to the end of August 2017 and are available on www.opec.org, www.eia.gov, and www.quandl.com websites (Jabalameli, Ghorbani, and Ahmadian, 2020; Bekheet, 2020).

4.1.2. Wavelet transformation and noise reduction

The time series was decomposed by the Daubechies mother wavelet to five levels, and the results are shown in Figure 2.

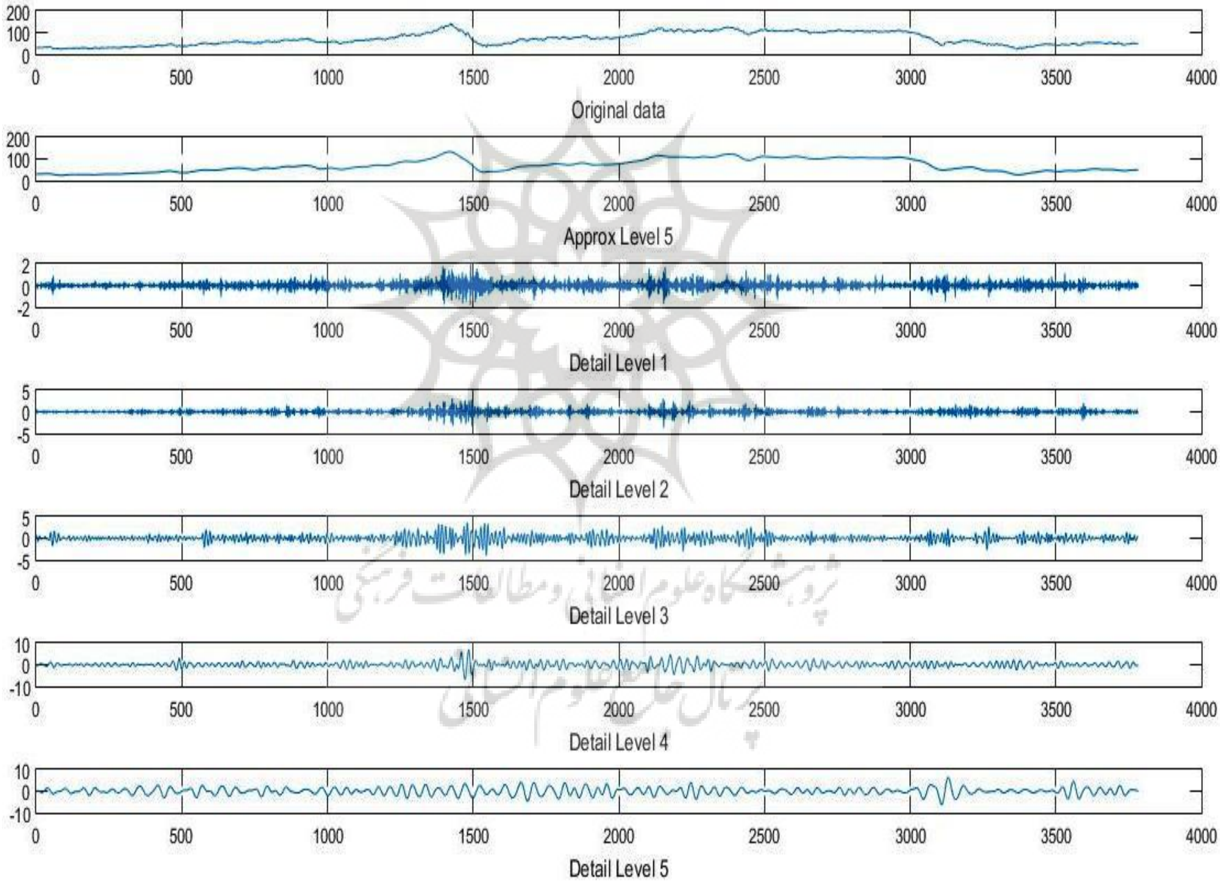

Figure 2

The transformation of the time series to discrete wavelet at five levels

According to Figure 2, the main time series is decomposed into six time series due to wavelet transformation: five high-frequency time series and one low-frequency time series. The high-frequency time series have drastic and rapid changes and are the main ones called time-series details. The low-frequency time series indicates slow and soft changes and is approximate to the original time series.

The histogram diagrams of the time series data are plotted and compared in Figure 3 to further identify and investigate their distribution.

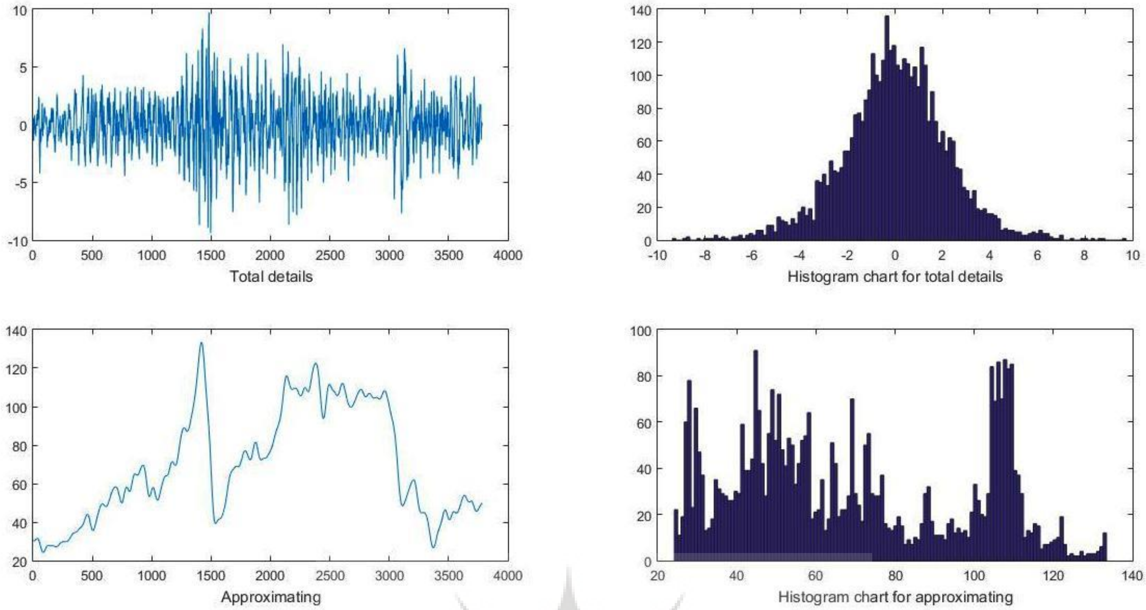

Figure 3

The histogram of the detail and approximate time series

Figure 3 demonstrates that the rapid changes in the main time series have a normal distribution and can be studied by statistical methods. However, the slow time series, which is an approximation of the original series, does not have a specific distribution, and its behavior is examined by chaotic theory tools.

4.2. Chaos tests

4.2.1. Determining the embedding phase space

Exploring the chaotic dynamics of the time series necessitates defining the embedding phase space, which involves selecting a time delay and an embedding dimension. This choice is crucial for accurately capturing the underlying dynamics of the time series.

Step 1) Determining the time delay of the time series: The determination of the embedding parameters, including the time delay and embedding dimension, can be achieved through methods such as the auto correlation function (ACF) or average mutual information (AMI). In the autocorrelation function method, the appropriate delay time is identified as the point where the function reaches zero or a threshold value of $1/e$. Figure 4a illustrates that the time series exhibits strong autocorrelation and fails to reach zero or the threshold value within 150 time delays. Alternatively, the average mutual information method can be employed. Figure 4b presents the AMI values plotted for various delays. Point 34, identified as the first local minimum, is selected as the optimal time delay value for the time series.

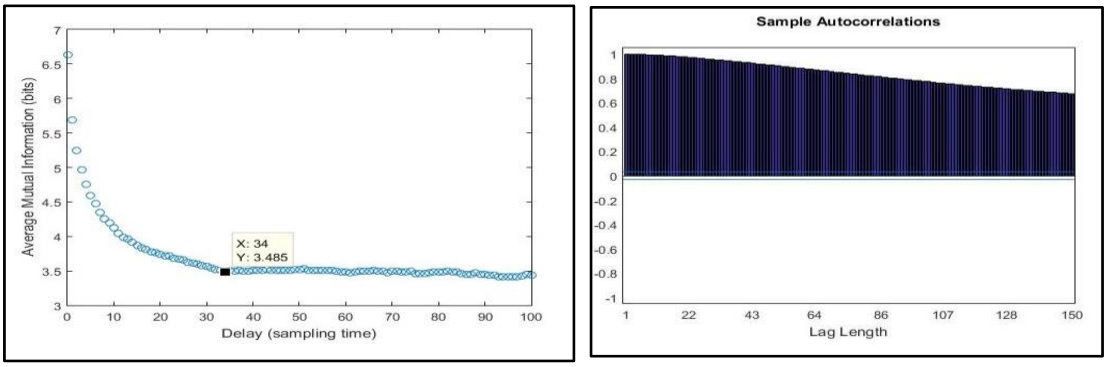

Figure 4

Autocorrelation function diagram and average mutual information

Step 2) Determining the embedding dimension: The false nearest neighbor algorithm is also used to obtain the embedding dimension, and Figure 5 shows the results of this method. The first point where the graph reaches zero is three, and this point is the size of the embedding dimension.

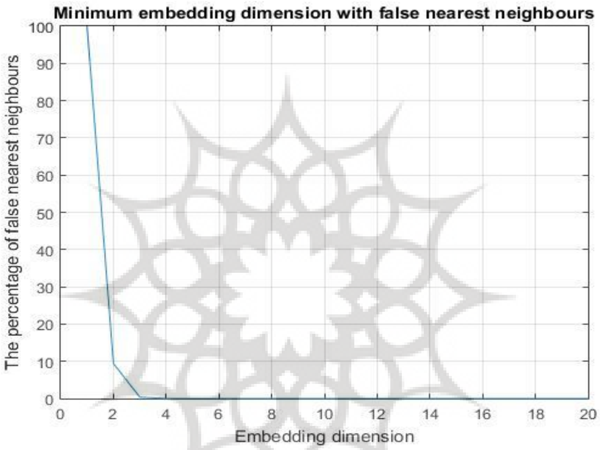

Figure 5

The diagram of the false nearest neighbor algorithm

The time series is displayed in Figure 6 to understand the degree of dependence on the initial conditions. The time series is drawn, taking into account a time delay (τ) of 34 and an embedding dimension (m) of 3. It is obvious that points in the form of lines, called paths, never intersect, and in parts of space, they are far apart and sometimes close together. This behavior indicates that the system is sensitive to the initial conditions.

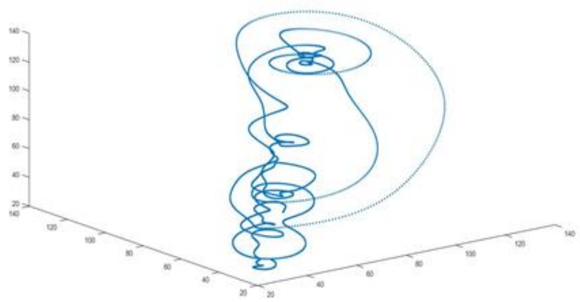

Figure 6

Time series display in the state space

The Lyapunov exponent

The Lyapunov exponent, one of the tests of investigating time series chaos, was used to examine the system more closely and demonstrate sensitivity to the initial conditions of the system. Figure 7 delineates the results of this test, and its slope is equal to the Lyapunov exponent.

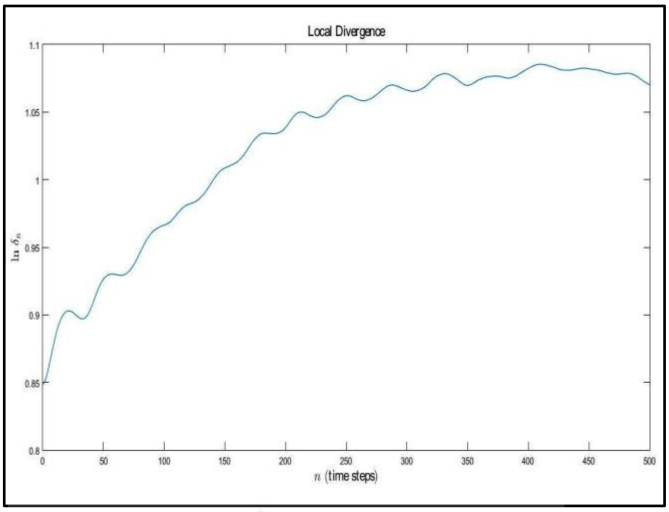

Figure 7

The Lyapunov exponent estimation on time series in the embedding space dimensions

Due to the positive slope, this system is sensitive to the initial conditions, one of the important features of chaotic systems. Given that the embedding phase space in this study equals three, we will have a Lyapunov exponent in each dimension. The Lyapunov exponent was calculated utilizing TISEAN software version 38. For a more accurate estimate, the value of the Lyapunov exponent can be obtained for several different initial points (*K*), as listed in Table 3.

Table 3

The Lyapunov exponents in line with the embedding space dimensions

After wavelet	λ_1	λ_2	λ_3
K = 40	0.394	0.170	0.010
K = 50	0.352	0.163	0.029
K = 100	0.327	0.175	–0.013
K = 160	0.320	0.156	–0.006

According to Table 3, the Lyapunov exponent is positive in two dimensions. In one dimension, the Lyapunov exponent is sometimes positive and sometimes negative by changing the value of *K*. However, in the two other dimensions, this value is always positive for different values of *K*: $\lambda_1 = 0.35 \pm 0.03$ and $\lambda_2 = 0.160 \pm 0.01$. The Lyapunov exponent concept on the time series diagram in the embedding space dimensions is also shown in Figure 8.

4.2.2 Correlation dimension test

The correlation dimension is used to examine the dimensions of the time series in the phase space. The total correlation diagram for different dimensions is obtained using the TISEAN software.

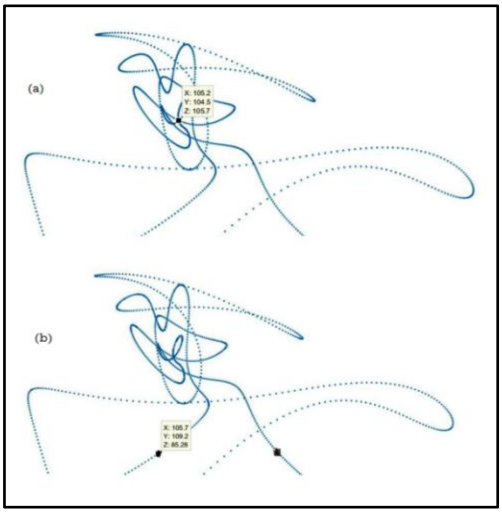

Figure 8

The Lyapunov exponent concept on the time series diagram in the embedding space dimensions

4.3. Neural network models

Three different models were used in the current study to predict the price of OPEC crude oil. According to the results, the efficiency of the models was examined, and the best model was finally selected.

4.3.1. GMDH–GA neural network model

The input to the neural network was the time series of OPEC oil price with a time delay of 34 and a dimension of 3. In this model, 70% of the data were considered for learning and 30% for network testing. In the internal structure of the network, the data selected for learning were divided into two categories: The first category was used to obtain the model coefficients, and the second category of data was employed to fit the model with the least square error (LSE) criterion. This approach strengthens the model's coefficients. The model was repeated based on different divisions to optimally divide the learning data, and the optimal division of the learning data was finally reported to be 50%.

There are three selection methods in the structure of the genetic algorithm for parent selection: the roulette wheel, random, and competitive. This model randomly selected one of these three methods in the neural network structure. The root mean square error (RMSE) of the first model with three layers was equal to 3.8333, and its correlation coefficient equaled 0.99552.

4.3.2. GMDH–GA wavelet neural network model

In this model, the inputs to the neural network were decomposed into five levels with wavelet transformations before entering the network. Then, the six obtained time series, each with a time delay of 34 and a dimension of 3, entered the neural network. Thus, there were 18 inputs to the neural network. The general structure of the GMDH–GA wavelet neural network model is shown in Figure 9.

The RMSE of the second model with two layers equaled 3.2864, and its correlation coefficient was equal to 0.99564.

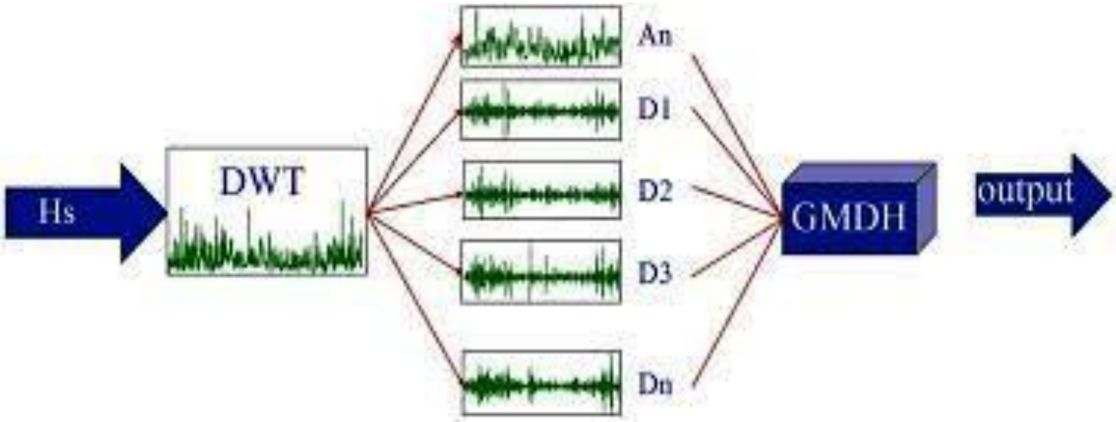

Figure 9

The structure of the GMDH–GA wavelet neural network

4.3.3. Developed GMDH wavelet neural network model

In this model, the time series was decomposed into five levels before entering the neural network. The optimal time delay for each of the time series levels was obtained by means of average mutual information and optimal dimension through the algorithm of the false nearest neighbor. The time series phase was specified by the time delay and dimension values. As an example, the graph of the first time series state with $\tau = 2$ and $m = 3$ is plotted in Figure 10a, and the second time series state with $\tau = 1$ and $m = 4$ is drawn in Figure 10b.

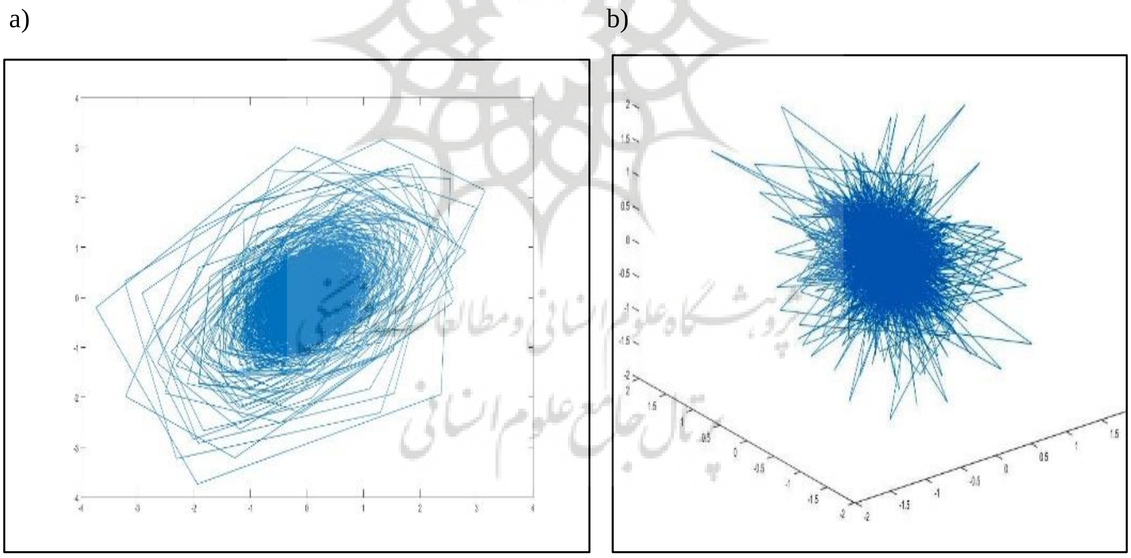

Figure 10

Time series phase diagram in the first and second states

Figures show that the state-space diagram of the first time series has noise, but that of the second time series has no noise. Data from time series with noise are removed from the calculations, and time series without noise enter the neural network with their time delay and dimension. The results of the time series are present in Table 4.

The RMSE of the third model with 3 layers is equal to 2.67333, and the correlation coefficient of the model equals 0.99733.

Table 4

The chaos characteristics of time series

Time series	Time delay	Optimal dimension	Behavior in phase space	Frequency type	Result
1	2	3	Noisy	Slow and soft	Series removed
2	1	4	Noiseless	Fast	Series entered
3	3	4	Noiseless	Fast	Series entered
4	6	4	Noiseless	Fast	Series entered
5	11	4	Noiseless	Fast	Series entered

Finally, the overall results of the three models are presented in Table 5. The result is an optimal value of 30 repetitions for each model per specific layer number.

Table 5

The overall results of the three crude oil price forecast models by number of layers

	Number of layers	RMSE	Correlation coefficient
First model	1	3.9943	0.99609
	2	4.1488	0.9946
	3	3.8333	0.99552
	4	4.2599	0.99566
Second model	1	3.3565	0.99598
	2	3.2864	0.99564
	3	3.6725	0.99678
	4	3.4225	0.99718
Third model	1	3.1886	0.9957
	2	3.9838	0.99534
	3	2.6733	0.99733
	4	2.928	0.99726

Comparing the results of the models by the number of layers listed in Table 5 shows that all three models are suitable for predicting the price of OPEC crude oil due to the high correlation coefficient and low forecast error. However, the third model with a number of layers equal to three is more appropriate than the two other models based on the smaller root mean square error.

5. Discussion and policy implications

In today's world, there is much emphasis on economic forecasts. Crude oil plays an important role in the industrial economic development of countries and is considered an important strategic resource around the world. The accurate forecasting of crude oil price has a central impact on the macroeconomics and is necessary for the economic planning of exporting and importing countries. Moreover, prediction based on past prices is difficult due to nonlinearity, uncertainty, and dynamics in these prices. Crude oil prices are often nonstationary time series, making it challenging to achieve acceptable forecasting accuracy using time series-based models. Nondeterministic events cause prices to change randomly and affect deterministic price changes like noise. As a result, crude oil price has nonlinear and chaotic time series. Several nonlinear measures such as the fractal dimension, the Lyapunov power, Poincaré sections, and entropies have been devised for analyzing time series.

This study analyzed the behavior of OPEC oil price time series from 2003 to 2017 using the chaos theory. The features of this time series were extracted using the wavelet transform, and five wavelets plus one main wavelet were studied. The graphs demonstrated that the rapid changes in the main time series had a normal distribution and could be investigated using statistical methods. Nevertheless, the slow time series, an approximation of the main series, did not have a special distribution, and its behavior should be investigated with the tools of the chaos theory. TISEAN software, which is suitable for analyzing nonlinear time series, and MATLAB software were utilized to numerically solve the models.

Exploring the chaotic dynamics of the time series required the definition of the embedded phase space, including the choice of the time delay and the embedding dimension. The primary tool for determining the time delay parameter is the autocorrelation function method. The time delay in this method is the point where the function reaches zero or the threshold value of $1/e$. Nonetheless, the results showed that the time series had a strong autocorrelation and did not reach zero or the threshold value even in 150 time lags. As a result, it was not possible to determine the time delay parameter through the method. Consequently, the second method, average mutual information, should be used. The AMI values plotted for different delays showed that the time delay was equal to 34: the first local minimum detected) (m = 3). The embedding dimension was also calculated at three through the wrong nearest neighbor algorithm (τ = 34)).

The graphs were drawn a with time delay parameter of 34 and an embedding dimension of 3 to understand the dependence of the time series on the initial conditions. The points in the form of lines, called paths, never intersected and were far from each other and sometimes close in some parts of the space. This behavior showed that the system was sensitive to the initial conditions. To examine the time series more precisely and to show its sensitivity to the initial conditions, we employed the Lyapunov curve, which is one of the tests for investigating the chaos of time series. Because of the positive slope, the time series was sensitive to the initial conditions, which is one of the important characteristics of chaotic systems. Considering that the embedded phase space in this research was equal to three, we would actually have a Lyapunov exponent in each dimension. The Lyapunov exponent was positive in two dimensions: $\lambda_1 = 0.35 \pm 0.03$ and $\lambda_2 = 0.16 \pm 0.01$. However, it was sometimes positive and sometimes negative in the other dimension. This was due to the presence of noise behavior in the original time series, disturbing the identification of the system behavior.

The time series must be de-noised to correctly identify the behavior of the time series and the accurate results of the chaos tests. There are also noisy behaviors in the global oil market, which disrupts the study of systems. In this research, the noise behavior was detected by the wavelet transform and was removed from the time series. In the following, the correlation dimension was used to examine the dimensions of the time series in the phase space. The slope of the graph was 0.88 and equaled the correlation dimension. The tests performed to identify the behavior of the OPEC crude oil price time series showed that it had a chaotic behavior. Since chaos occurs in nonlinear systems, OPEC crude oil price behaved nonlinearly.

This study employed three neural network methods to predict the time series of OPEC crude oil price. The first model was GMDH–GA neural network. The inputs to this network were the time series resulting from the wavelet transform with one level, reduced by the wavelet transform. This time series was entered into the neural network with a time delay of 34, and the optimal structure of the neural network with 3 layers was obtained. The root mean square error prediction of this model was 3.8333, its correlation coefficient equaled 0.99552. The second model was GMDH–GA. In this model, the neural network inputs were the time series resulting from wavelet decomposition with five levels. The details related to level one as noise were removed from the input data. The resulting five time series, which included different frequencies, were entered into the neural network with a time delay of 34. The RMSE of this model was 3.2864, and its correlation coefficient equaled 0.99564. The third model was a GMDH–GA neural network. In this model, the inputs of the neural network were time series resulting from wavelet decomposition with five levels. The details related to level one as noise were removed from the input data, and three time series were entered into the neural network, each with a different time delay. The optimal model was obtained with three layers. The root mean square error of this model was 2.6733, and its correlation coefficient equaled 0.99733.

6. Conclusions

This study developed a hybrid model of chaotic concept and GMDH–GA neural network for more accurate prediction of crude oil price. The time series were de-noised using wavelet transform. The chaotic parameters of the time series were identified, and various tests were performed on the system. The tests performed to identify the behavior of the OPEC crude oil price time series showed that they had a chaotic behavior. Since chaos occurs in nonlinear systems, OPEC crude oil price behaves nonlinearly. Therefore, nonlinear prediction methods should be used to predict it. In this research, three neural network methods were utilized to predict the time series of OPEC crude oil price. All the three models were favorable in terms of the correlation coefficient, but the root mean square error of the third model was lower; the correlation coefficient of this model was also higher than that of the two other models. Considering the chaos of the OPEC crude oil price time series, we suggested removing the noise by the wavelet transformation. Chaotic time series indicate nonlinearity, so nonlinear models, especially other neural networks, should be employed to predict OPEC crude oil price.

Nomenclature

ANN	Artificial neural network
BPNN	Back-propagation neural network
EEMD	Ensemble empirical mode decomposition
GMDH–GA	Group method of data handling-genetic algorithm
MAPE	Mean absolute percentage error
OPEC	Organization of Petroleum Exporting Countries
RMSE	Root mean square error
SVM	Support vector machine
VEC-NAR	Vector error correction and nonlinear auto regressive
VMD	Variable mode decomposition

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