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Original Research

A New Method of Sensitivity Analysis of Returns to Scale in Two-Stage Network; A Case Study of the Insurance Industry in Iran

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ABSTRACT

One important issue in data envelopment analysis (DEA) which has been studied by many researchers is returns to scale (RTS). The authors developed DEA models to evaluate the efficiency of two-stage networks in returns to scale variable and introduced a new definition to determine return to scale classification in two-stage networks. The current article proposed an approach for determining the stability region of returns to scale classification in two-stage network DEA. The data were collected from insurance companies in Iran in 2019. We consider the insurance industry process as a two-stage network; the stage of marketing and that of investment. The effectiveness of insurance companies was evaluated, and, after determining the classification of returns to scale, we found a sustainability interval to classify returns to their scale.

1 Introduction

Data envelopment analysis (DEA) is one of the methods used to measure relative efficiency of peer decision-making units (DMUs) that have multiple homogeneous inputs and outputs introduced by Charnes et al. [3]. They discussed a non-parametric approach to identify the best performance in a set of DMUs and presented CCR model. BCC model offered by Banker et al [1]. This model with production frontiers measured the existing decision-making units by the convex hull. Later on the DEA was used as a measurement tool in different fields such as management, economics etc. During this period of model development, the economic concept of returns to scale (RTS) has also been widely studied within different frameworks provided by these methods. In the literature of classical economics, returns to scale describes the behavior of the rate of increase in single output relative to the associated increase in the inputs. If output increases by the same proportional change as all inputs change, then there are constant returns to scale (CRS). If output increases by less than the proportional change in inputs, there are decreasing returns to scale (DRS). If output increases by more than the proportional change in inputs,

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there are increasing returns to scale (IRS). Banker et al [2] described RTS concept for multiple-output cases using DEA. They focused on the sign of u_0 from BCC model for returns to scale classification. Färe and Grosskopf [6] provided a two-stage method to recognize returns to scale classification based on CCR and BCC models. Banker and Lovell [4] prepared a new method to determine returns to scale in DEA based on the envelopment form of the BCC model. Consequently, many scholars studied returns to scale in DEA. For example, Khodabakhshi et al [13] provided an Additive model method to estimate returns to scale in both stochastic and fuzzy data envelopment analyses. One of the interesting research topics is network DEA and many studies have been conducted in this regard. In traditional DEA models, DMUs were considered as a black box and the efficiency evaluation was limited by the final outputs and initial inputs. Färe and Grosskopf [5] presented a network model that measured the efficiency of the entire system due to its sub-units. Subsequently, the researchers recruited many networks in areas such as intermediate products, allocation of budgets, fixed factors, dynamic systems and etc. Kao and Hwang [8] examined the structure of the two-stage networks and provided a new model to evaluate the efficiencies of the whole process. This model described series of relationships between the whole process and the two sub-processes. Khaleghi et al. [12] studied the structure of the two-stage systems. The aim of their study was to determine the returns to scale (RTS) classification and scale elasticity (SE) in two-stage DEA. Sarparast et al. [28] presented diverse approaches to deal with two-stage networks which evaluated the efficiency of two-stage networks in variable returns to scale and introduced a new definition of the types of returns to scale in two-stage networks and also methods to determining the type of returns to scale of efficient units. Researchers provided DEA models for sensitivity analysis of returns to scale in the two-stage network DMUs. Peykani et al [25] introduced novel robust data envelopment analysis models capable of being investigated in the presence of discrete and continuous uncertainties. A year laer, in 2019, they [27] presented a new approach, FDEA, for scale efficiency and stock ranking. Put differently, the very model was offered to measure the efficiency of stocks when negative data and uncertainties within input/output parameters exist (Peykani et al. [23, 24, 26]. Khodakaram et al. [14], in their article “concurrent estimation of efficiency, effectiveness, and returns to scale” studied the efficiency, effectiveness and return to scale of DMUs simultaneously. Neralic and Wendell [22], also, provided an algorithm approach to sensitivity in DEA for the CCR and additive models that provide sufficient conditions that preserve the efficiency of the input and/or outputs of DMUs. Nastion and et al [21] prepared an article entitled “sensitivity analysis in data envelopment analysis for interval data remains insure and improve the efficiency of DEA modeling and presented a model to calculate the lower and upper limits for each DMU. Kang et al [7] presented an article in this study proposes a hybrid two-stage network model and a mixed network DEA with the shared-inputs model to jointly measure the efficiency and effectiveness of a metro transport system. Performance is determined by the hybrid two-stage network DEA with the shared-inputs model to account for the non-storable service feature. To solve the problems of non-linearity, obtaining a total return greater than one, the need to assign variable weights to combine the divisive returns, adopt a fixed weight to combine the divisive returns, and Inability to find efficient two-stage DMUs in the network contribute network DEA, Khoveynia et al [15] have proposed an input-output-oriented linear model to measure the overall efficiency of two-stage DMUs with shared resources.

Tavassoli et al [29] formulated a Fuzzy Network DEA (FNDEA) model to assess the efficiency of Iran's EDNs components with sustainability, considerations and uncertain data. In order to utilize all input and output criteria, this study also proposes a fuzzy linear programming model to determine the optimal lower bound to all input and output weights. Furthermore, some appropriate policies are suggested based on the

strengths and weaknesses of each EDN to improve its efficiency. Michalia et al. [19] examined the applicability of the subsampling bootstrap procedure in the approximation of the asymptotic distribution of the DEA estimator when the production process has a network structure, and in the presence of undesirable factors. Evidence on the performance of subsampling bootstrap is obtained through Monte Carlo experiments for the case of two-stage series structures, where overall and stage efficiency scores are calculated using the additive decomposition approach. Results indicate great sensitivity both to the sample and subsample size, as well as to the data generating process. Lianga et al [16] provided, for the first time, the production process of manufacturing industry is modeled as a network system integrated by AI technology development stage, AI application stage and AI upgrade stage because by optimizing production and industrial structure, artificial intelligence (AI) is considered to play a key role in low carbon manufacturing. Understanding the performance of AI driven low carbon manufacturing is of great significance to achieve carbon emission reduction targets and sustainable development of resources. Then, an interactive three stage network DEA model with ratio data is developed to evaluate the manufacturing industry in China from 2016 to 2019. Show that many regions perform well in the AI application stage while most of them have low AI technology development and AI upgrade performance.
In this study we propose a new method for sensitivity analysis of returns to scale and present a new model based on Kao and Hwang's model [8] and concepts presented by Sarparast et al. [28]. The rest of the paper is organized as follows: Section 2 presents the basic DEA model and the generic two-stage process and a review of studies conducted by Kao and Hwang [8] and Sarparast et al. [28], then we proceed to introduce a new method for returns to scale classification and provide a new model for sensitivity analysis of returns to scale classification in two-stage network systems in Section 3. In section 4, we use data from Sarparast et al.'s [28] article and compare two methods. After that, two methods are used to analyze the data of insurance companies in Iran in 2019, and the results of the two methods are compared together.

2 Background
2.1. The Basic Concepts of DEA

Suppose that there is a set of DMUs consisting of DMU_1, DMU_2, …, DUU_n, where each DMU_j produces s outputs $\mathbf{y_{rj}}$ $(\mathbf{r = 1 \dots s})$ using m inputs $\mathbf{x_{ij}}$ $(\mathbf{i = 1 \dots m})$. The CCR input-oriented radial efficiency of DMU_o (x_o, y_o) is obtained by solving the following model:

$$\begin{aligned} &\mathbf{Min} \quad \theta \\ &\mathbf{s.t}: \\ &\sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io} && i = 1, \dots, m. \\ &\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} && r = 1, \dots, s. \\ &\lambda_j \ge 0 && j = 1, \dots, n. \end{aligned} \tag{1}$$

The dual form of the model (1) is obtained from the same data which then is used in the model (2). Where u and v are non-negative variables corresponding to constraints. Model (1) and (2) are respectively called envelopment form and multiplier form of CCR model (Charnes et al. [3]). DMU_o is CCR-efficient if and only if the optimal solutions obtained from model (1) and (2) are equal to 1. Then, on the basis of all optimal lambda solutions to (1), the CCR RTS method can be expressed as (Banker and Thrall. [2]):

$$
\begin{aligned}
&\mathbf{Max} \quad \sum\nolimits_{r=1}^{s} u_r y_{rp} \Big/ \sum\nolimits_{i=1}^{m} v_i x_{ip} \\
&\mathbf{s.t:} \\
&\sum\nolimits_{r=1}^{s} u_r y_{rj} \Big/ \sum\nolimits_{i=1}^{m} v_i x_{ij} \le 1 \qquad j = 1, \dots, n. \\
&u_r \ge 0 \qquad r = 1, \dots, s. \\
&v_i \ge 0 \qquad i = 1, \dots, m.
\end{aligned}
\tag{2}
$$

Where u and v are non-negative variables corresponding to constraints. Model (1) and (2) are respectively called envelopment form and multiplier form of CCR model (Charnes et al. [3]). DMU_o is CCR-efficient if and only if the optimal solutions obtained from model (1) and (2) are equal to 1. Then, on the basis of all optimal lambda solutions to (1), the CCR RTS method can be expressed as (Banker and Thrall. [2]):

If $\sum_{j=1}^{n} \lambda_j^* = 1$ in any alternate optimum, then DMU_o has constant returns to scale (CRS).

If $\sum_{j=1}^{n} \lambda_j^* > 1$ for all alternate optimum, then DMU_o has decreasing returns to scale (DRS).

If $\sum_{j=1}^{n} \lambda_j^* > 1$ for all alternate optimum, then DMU_o has increasing returns to scale (IRS).

Banker et al [1] introduced the BCC model which separates technical efficiency and scale efficiency. Unlike CCR model that assumes constant returns to scale, BCC model assumes returns to scale as variable. Envelopment form (3) of BCC model is model (3).

$$
\begin{aligned}
&\mathrm{Min} \quad \theta \\
&\mathbf{s.t:} \\
&\sum\nolimits_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io} \qquad i = 1, \dots, m. \\
&\sum\nolimits_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} \qquad r = 1, \dots, s. \\
&\sum\nolimits_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \ge 0 \qquad j = 1, \dots, n.
\end{aligned}
\tag{3}
$$

The dual (multiplier) form of the BCC model was represented in (3):

DMU_o is BCC-efficient if and only if the optimal solutions obtained from model (3) and (4) are equal to 1. Banker and Thrall [2] offered a theorem based on the optimal solution obtained from multiple forms of BCC model to identify RTS with the sign of u_0. IIf $u_0^* = 0$ in any alternate optimum, then DMU_o has constant returns to scale (CRS). If $u_0^* < 0$ for all alternate optimum, then DMU_o has decreasing returns to scale (DRS). If $u_0^* > 0$ for all alternate optimum, then DMU_o has increasing returns to scale (IRS).

$$
\begin{aligned}
&\mathbf{Max} \sum_{r=1}^{s} u_r y_{rp} + u_0 \\
&\quad \mathbf{s.t}: \\
&\quad \sum_{i=1}^{m} v_i x_{ip} = 1 \\
&\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \le 0 \qquad j = 1, \dots, n. \\
&\quad u_r \ge 0 \qquad r = 1, \dots, s. \\
&\quad v_i \ge 0 \qquad i = 1, \dots, m.
\end{aligned} \tag{4}
$$

2.2. Review of the Literature

Figure 1 shows a two-stage network process. The overall system produces the final output (y) with the consumption of primary input (x). The production process is composed of two sub-processes with D intermediate produces z so that z is the output of stage 1 as well as the input of stage 2. Let Z_{dp} (d=1,

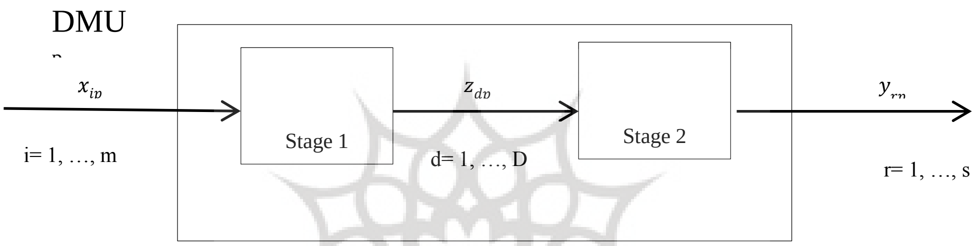

Fig. 1: two-stage network system.

…, D) is d th output of the first stage, that is produced with inputs x_{ip} (i= 1, …, m), and i th input is the second stage which is consumed for the output production y_{rp} (r = 1, …, s). Kao and Hwang [8] presented the following model to calculate the overall efficiency taking into account the series relationship of the two sub-processes.

$$
\begin{aligned}
&\theta_o^{overal} = \mathbf{Max} \quad \sum_{r=1}^{s} u_r y_{ro} \Big/ \sum_{i=1}^{m} v_i x_{io} \\
&\quad \mathbf{s.t}: \\
&\quad \sum_{d=1}^{D} w_r z_{dj} \Big/ \sum_{i=1}^{m} v_i x_{ij} \le 1 \qquad j = 1, \dots, n. \\
&\quad \sum_{r=1}^{s} u_r y_{rj} \Big/ \sum_{d=1}^{D} w_r z_{dj} \le 1 \qquad j = 1, \dots, n \\
&\quad \sum_{r=1}^{s} u_r y_{rj} \Big/ \sum_{i=1}^{m} v_i x_{ij} \le 1 \qquad j = 1, \dots, n. \\
&\quad v_i \ge 0 \qquad i = 1, \dots, m. \\
&\quad w_d \ge 0 \qquad d = 1, \dots, D. \\
&\quad u_r \ge 0 \qquad i = 1, \dots, s.
\end{aligned} \tag{5}
$$

Note that they used the multipliers associated with Z_{dj} to be the same no matter whether it plays the role of output or input. The dual version of (5) is

$$\text{Max} \quad \theta_p = \left(\sum_{r=1}^{s} u_r y_{rp} + u_0\right) \Big/ \left(\sum_{i=1}^{m} v_i x_{ip} - v_0\right)$$

$$\begin{aligned}
&\text{s.t:} \\
&\left(\sum_{r=1}^{s} u_r y_{rj} + u_0\right) \Big/ \sum_{d=1}^{D} w_d z_{dj} \le 1 && j = 1, \dots, n. \\
&\sum_{d=1}^{D} w_d z_{dj} \Big/ \left(\sum_{i=1}^{m} v_i x_{ij} - v_0\right) \le 1 && j = 1, \dots, n. \\
&\left(\sum_{r=1}^{s} u_r y_{rj} + u_0\right) \Big/ \left(\sum_{i=1}^{m} v_i x_{ij} - v_0\right) \le 1 && j = 1, \dots, n. \\
&v_i \ge \epsilon && i = 1, \dots, m. \\
&w_d \ge \epsilon && d = 1, \dots, D. \\
&u_r \ge \epsilon && r = 1, \dots, s.
\end{aligned} \tag{6}$$

$$\begin{aligned}
&\theta_o^{*} = \text{Min} \quad \theta \\
&\text{s.t:} \quad \sum_{j=1}^{n} (\lambda_j + \delta_j) x_{ij} + si_i = \theta x_{io} && i = 1, \dots, m. \\
&\sum_{j=1}^{n} \delta_j z_{dj} - sw_d = 0 && d = 1. \dots . D. \\
&\sum_{j=1}^{n} \lambda_j y_{rj} - so_r = y_{ro} && r = 1. \dots . s. \\
&\lambda_j \ge 0 && j = 1. \dots . n
\end{aligned} \tag{7}$$

DMUo is efficient if and only if the optimal solutions obtained from model (5) and (6) are equal to 1. This means that DMUo is efficient if and only if both stages are efficient in model (3). In this article, we used model (6) to identify returns to scale in the two-stage networks and sensitivity analysis of returns to scale. Sarparast et al. [28] provided model (7) to deal with two-stage networks which evaluated the efficiency of two-stage networks in variable returns to scale with regard to relationship of the two sub-processes and on the basis of presentation of a new method to identify returns to scale in a two-stage network. They used Production Possibility Set (PPS) for a network system defined by Fare and Grosskopf [5] and gave a new definition to the returns to scale in a two-stage network as follows:

$$PPS_{n\text{-}v} = \left\{(x, z, y) \mid x \le \sum_{j=1}^{n} \lambda_j x_j \;\&\; z \ge \sum_{j=1}^{n} \lambda_j z_j \;\&\; z \le \sum_{j=1}^{n} \mu_j z_j \;\&\; y \ge \sum_{j=1}^{n} \mu y_j \;, \lambda_j \& \mu_j \ge 0; j = 1, \dots, n.\right\}$$

Definition 1: Network P is on the efficiency frontier $T_{n\text{-}v}$, so:

1. P th network has increasing returns to scale if and if $\boldsymbol{\delta^{*} > 0}$ exits, so

$$\forall\, \delta;\, 0 < \delta < \delta^{*} \rightarrow \left((1+\delta)x_p, (1+\delta)z_p, (1+\delta)y_p\right) \in \text{int}\, T_{n-v}$$

2. P th network has decreasing returns to scale if and if $\boldsymbol{\delta^{*} > 0}$ exits, so

$$\forall\, \delta;\, 0 < \delta < \delta^{*} \rightarrow \left((1-\delta)x_p, (1-\delta)z_p, (1-\delta)y_p\right) \in \text{int}\, T_{n-v}$$

3. P th network has constant returns to scale if and if $\boldsymbol{\delta}^* > \mathbf{0}$ exits, so

a) $\forall\, \boldsymbol{\delta}; \mathbf{0} < \boldsymbol{\delta} < \boldsymbol{\delta}^* \to \left((\mathbf{1}+\boldsymbol{\delta})\mathbf{x_p}, (\mathbf{1}+\boldsymbol{\delta})\mathbf{z_p}, (\mathbf{1}+\boldsymbol{\delta})\mathbf{y_p}\right) \in \boldsymbol{\partial}\, \mathbf{T_{n-v}} \,\&\, \left((\mathbf{1}-\boldsymbol{\delta})\mathbf{x_p}, (\mathbf{1}-\boldsymbol{\delta})\mathbf{z_p}, (\mathbf{1}-\boldsymbol{\delta})\mathbf{y_p}\right) \in \boldsymbol{\partial}\, \mathbf{T_{n-v}}$

or

b) $\forall\, \boldsymbol{\delta}; \mathbf{0} < \boldsymbol{\delta} < \boldsymbol{\delta}^* \to \left((\mathbf{1}+\boldsymbol{\delta})\mathbf{x_p}, (\mathbf{1}+\boldsymbol{\delta})\mathbf{z_p}, (\mathbf{1}+\boldsymbol{\delta})\mathbf{y_p}\right) \notin \mathbf{T_{n-v}} \,\&\, \left((\mathbf{1}-\boldsymbol{\delta})\mathbf{x_p}, (\mathbf{1}-\boldsymbol{\delta})\mathbf{z_p}, (\mathbf{1}-\boldsymbol{\delta})\mathbf{y_p}\right) \notin \mathbf{T_{n-v}}$

or

c) $\forall\, \boldsymbol{\delta}; \mathbf{0} < \boldsymbol{\delta} < \boldsymbol{\delta}^* \to \left((\mathbf{1}+\boldsymbol{\delta})\mathbf{x_p}, (\mathbf{1}+\boldsymbol{\delta})\mathbf{z_p}, (\mathbf{1}+\boldsymbol{\delta})\mathbf{y_p}\right) \in \boldsymbol{\partial}\, \mathbf{T_{n-v}} \,\&\, \left((\mathbf{1}-\boldsymbol{\delta})\mathbf{x_p}, (\mathbf{1}+\boldsymbol{\delta})\mathbf{z_p}, (\mathbf{1}-\boldsymbol{\delta})\mathbf{y_p}\right) \notin \mathbf{T_{n-v}}$

or

d) $\forall\, \boldsymbol{\delta}; \mathbf{0} < \boldsymbol{\delta} < \boldsymbol{\delta}^* \to \left((\mathbf{1}+\boldsymbol{\delta})\mathbf{x_p}, (\mathbf{1}+\boldsymbol{\delta})\mathbf{z_p}, (\mathbf{1}+\boldsymbol{\delta})\mathbf{y_p}\right) \notin \mathbf{T_{n-v}} \,\&\, \left((\mathbf{1}-\boldsymbol{\delta})\mathbf{x_p}. (\mathbf{1}-\boldsymbol{\delta})\mathbf{z_p}, (\mathbf{1}-\boldsymbol{\delta})\mathbf{y_p}\right) \in \boldsymbol{\partial}\mathbf{T_{n-v}}$””

They provided a theorem for returns to scale which identifies RTS with the sign of $\mathbf{u_0^*} + \mathbf{v_0^*}$ in model (7) as follows:

Theorem 1: Suppose that $\mathrm{DMU_o}$ is called efficient under model (7), and $(\mathbf{v^*}.\mathbf{w^*}.\mathbf{u^*}.\mathbf{v_0^*}.\mathbf{u_0^*})$ is an obtained optimal solution from model (7); then:

i) If in any optimal solution; $\mathbf{u_0^*} + \mathbf{v_0^*} > \mathbf{0}$ then $\mathrm{DMU_P}$ has IRS.

ii) If in any optimal solution; $\mathbf{u_0^*} + \mathbf{v_0^*} < \mathbf{0}$ then $\mathrm{DMU_P}$ has DRS.

iii) If in some optimal solution; $\mathbf{u_0^*} + \mathbf{v_0^*} = \mathbf{0}$ then $\mathrm{DMU_P}$ has CRS.

Proof: Refers to [28].

In the theorem (1), efficient DMUs were divided into three categories.

$\mathbf{E} = \langle\ \mathbf{j}\ \mid\ \mathbf{DMU_j} \textbf{ is efficient under model } (\mathbf{7})\ \rangle$

$\mathbf{E^1} = \langle\ \mathbf{j}\ \mid \mathbf{j} \in \mathbf{E}\ \&\ \mathbf{u_{0+}v_0} > \mathbf{0}\ \rangle$

$\mathbf{E^2} = \langle\ \mathbf{j}\ \mid\ \mathbf{j} \in \mathbf{E}\ \&\ \mathbf{u_{0+}v_0} < \mathbf{0}\ \rangle$

$\mathbf{E^3} = \langle\ \mathbf{j}\ \mid\ \mathbf{j} \in \mathbf{E}\ \&\ \mathbf{u_{0+}v_0} = \mathbf{0}\ \rangle$

If p belongs to the $\mathrm{E^1}$ set, then $\mathrm{DMU_p}$ has IRS; therefore, increasing outputs can change its returns to scale. By taking this concept into account, they offered the following non-linear programming**.** Let us suppose that $\mathrm{DMU_p}$ (x_p, z_p, y_p) has increasing returns to scale, and z_p is the first-stage output produced by using x_p as input. In the second stage, it is used as an input to produce y_p. They are interested in the sensitivity analysis of the classification of returns to scale in $\mathrm{DMU_p}$. For $\mathrm{DMU_p}$, therefore, we have the following perturbed data:

$x_p = x_p + \alpha; 0 \le \alpha \le \min_j \left\{\min_i \{|x_{ij} - x_{ip}|\}\right\}$

$y_p = y_p + \beta; 0 \leq \beta \leq \min_j \left\{\min_r \{|y_{rj} - y_{rp}|\}\right\}$

If DMU_p has increasing returns to scale, according to the concept above, a model for sensitivity analysis is presented as follows:

$$
\begin{aligned}
&\mathbf{Max} \quad \alpha + \beta \\
&\mathbf{s.t:} \\
&\sum_{i=1}^{m} v_i + \sum_{r=1}^{s} u_r = 1. \\
&\sum_{r=1}^{s} u_r(y_{rp} + \beta) - \sum_{i=1}^{m} v_i(x_{ip} + \alpha) + u_0 + v_0 = \theta_p^* - 1 \\
&\sum_{r=1}^{s} u_r(y_{rp} + \beta) - \sum_{d=1}^{D} w_d z_{dp} + u_0 \leq 0 \\
&\sum_{d=1}^{D} w_d z_{dp} - \sum_{i=1}^{m} v_i(x_{ip} + \alpha) + v_0 \leq 0 \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} + u_0 \leq 0 \qquad j = 1, \dots, n. \,\&\, j \neq p \\
&\sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} + v_0 \leq 0 \qquad j = 1, \dots, n. \,\&\, j \neq p \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 + v_0 \leq 0 \qquad j = 1, \dots, n. \,\&\, j \neq p \\
&u_0 + v_0 \geq \epsilon. \\
&0 \leq \alpha \leq \min_j \left\{\max_i \{|x_{ij} - x_{ip}|\}\right\}, \\
&0 \leq \beta \leq \min_j \left\{\min_r \{|y_{rj} - y_{rp}|\}\right\} \\
&v_i \geq \epsilon \qquad i = 1, \dots, m. \\
&w_d \geq \epsilon \qquad d = 1, \dots, D. \\
&u_r \geq \epsilon \qquad r = 1, \dots, s.
\end{aligned}
\tag{8}
$$

They have used the restriction $\sum_{i=1}^{m} v_i + \sum_{r=1}^{s} u_r = 1$, and assumptions; $k'_p = \min_j \left\{\max_i \{|x_{ij} - x_{ip}|\}\right\}$, and $k_p = \min_j \left\{\max_r \{|y_{rj} - y_{rp}|\}\right\}$, so $0 \leq v_i\beta \leq v_i k_p$ (for all i), hence $0 \leq \sum_{i=1}^{m} v_i\beta \leq \sum_{i=1}^{m} v_i k_p$, and also $0 \leq \sum_{r=1}^{s} u_r\alpha \leq \sum_{r=1}^{s} u_r k'_p$; to convert a nonlinear model (8) to a linear model (9).

The following assumptions were made: $k'_p = \min_j \left\{\max_i \{|x_{ij} - x_{ip}|\}\right\}$ and $k_p = \min_j \left\{\max_r \{|y_{rj} - y_{rp}|\}\right\}$. Accordinally, if (α^*, β^*) is an optimal solution of model (9), increasing inputs per α; $0 \leq \alpha \leq \alpha^*$ and increasing outputs per β; $0 \leq \beta \leq \beta^*$ cannot change the type of constant returns to scale in the evaluated unit.

$$
\begin{aligned}
&\mathbf{Max} \quad \alpha+\beta \\
&\mathbf{s.t:} \\
&\sum_{r=1}^{s} u_r y_{rp} - \sum_{i=1}^{m} v_i x_{ip} + \beta + \sum_{r=1}^{s} u_r k'_p - \alpha + u_0 + v_0 \le \theta_p^* - 1 \\
&-\sum_{r=1}^{s} u_r y_{rp} + \sum_{i=1}^{m} v_i x_{ip} - \beta + \alpha + \sum_{i=1}^{m} v_i k_p - u_0 - v_0 \le 1 - \theta_p^* \\
&\sum_{r=1}^{s} u_r y_{rp} - \sum_{d=1}^{D} w_r z_{dp} + \beta + u_0 \le 0 \\
&\sum_{d=1}^{D} w_r z_{dp} - \sum_{i=1}^{m} v_i x_{ip} - \alpha + \sum_{r=1}^{s} u_r k'_p + v_0 \le 0 \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_r z_{dj} + u_0 \le 0 \qquad j=1,\dots,n.\ \&\ j \ne o \\
&\sum_{d=1}^{D} w_r z_{dj} - \sum_{i=1}^{m} v_i x_{ij} + v_0 \le 0 \qquad j=1,\dots,n.\ \&\ j \ne o \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 + v_0 \le 0 \qquad j=1,\dots,n.\ \&\ j \ne o \\
&u_0 + v_0 \ge \epsilon. \\
&0 \le \alpha \le k'_p \\
&0 \le \beta \le k_p \\
&v_i \ge \epsilon \qquad i=1,\dots,m. \\
&w_d \ge \epsilon \qquad d=1,\dots,D. \\
&u_r \ge \epsilon \qquad r=1,\dots,s.
\end{aligned} \tag{9}
$$

Also, they provided another linear programming problem (10) when p belongs to the E^2 set. Note that if DMU_p has DRS, then increasing outputs cannot change its returns to scale.

$$
\begin{aligned}
&\mathbf{Max} \quad \alpha+\beta \\
&\mathbf{s.t:} \\
&\sum_{r=1}^{s} u_r y_{rp} - \sum_{i=1}^{m} v_i x_{ip} - \beta + \sum_{i=1}^{m} v_i k_p + \alpha + u_0 + v_0 \le \theta_p^* - 1 \\
&-\sum_{r=1}^{s} u_r y_{rp} + \sum_{i=1}^{m} v_i x_{ip} + \beta - \alpha + \sum_{r=1}^{s} u_r k'_p - u_0 - v_0 \le 1 - \theta_p^* \\
&\sum_{r=1}^{s} u_r y_{rp} - \sum_{d=1}^{D} w_r z_{dp} - \beta + \sum_{i=1}^{m} v_i k_p + u_0 \le 0 \\
&\sum_{d=1}^{D} w_r z_{dp} - \sum_{i=1}^{m} v_i x_{ip} + \alpha + v_0 \le 0 \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_r z_{dj} + u_0 \le 0 \qquad j=1,\dots,n.\ \&\ j \ne p \\
&\sum_{d=1}^{D} w_r z_{dj} - \sum_{i=1}^{m} v_i x_{ij} + v_0 \le 0 \qquad j=1,\dots,n.\ \&\ j \ne p
\end{aligned} \tag{10}
$$

$$\mathbf{u_0 + v_0 \le \epsilon}.$$

$$\mathbf{0 \le \alpha \le k'_p}$$

$$\mathbf{0 \le \beta \le k_p}$$

$$\mathbf{v_i \ge \epsilon} \qquad \mathbf{i = 1, \dots, m}.$$

$$\mathbf{w_d \ge \epsilon} \qquad \mathbf{d = 1, \dots, D}.$$

$$\mathbf{u_r \ge \epsilon} \qquad \mathbf{r = 1, \dots, s}.$$

In this case, they regarded $k'_p = \min_j \left\{ \max_i \left\{ \left| x_{ij} - x_{ip} \right| \right\} \right\}$ and $k_p = \left\{ \min_r \left\{ \left| y_{rp} \right| \right\} \right\}$. And, finally, they catered model (11) and (12) when DMU_p has CRS because increasing or decreasing outputs can change its returns to scale.

$$
\begin{aligned}
&\mathbf{Max} \quad \boldsymbol{\alpha^1 + \beta^1} \\
&\text{s.t:} \\
&\sum_{r=1}^{s} \mathbf{u_r y_{rp}} - \sum_{i=1}^{m} \mathbf{v_i x_{ip}} + \boldsymbol{\beta^1} + \sum_{r=1}^{s} \mathbf{u_r k'_p} - \boldsymbol{\alpha^1} + \mathbf{u_0 + v_0} \le \boldsymbol{\theta_p^* - 1} \\
&- \sum_{r=1}^{s} \mathbf{u_r y_{rp}} + \sum_{i=1}^{m} \mathbf{v_i x_{ip}} - \boldsymbol{\beta^1} + \boldsymbol{\alpha^1} + \sum_{i=1}^{m} \mathbf{v_i k_p} - \mathbf{u_0 - v_0} \le \boldsymbol{1 - \theta_p^*} \\
&\sum_{r=1}^{s} \mathbf{u_r y_{rp}} - \sum_{d=1}^{D} \mathbf{w_r z_{dp}} + \boldsymbol{\beta^1} + \mathbf{u_0} \le \mathbf{0} \\
&\sum_{d=1}^{D} \mathbf{w_r z_{dp}} - \sum_{i=1}^{m} \mathbf{v_i x_{ip}} - \boldsymbol{\alpha^1} + \sum_{r=1}^{s} \mathbf{u_r k'_p} + \mathbf{v_0} \le \mathbf{0} \\
&\sum_{r=1}^{s} \mathbf{u_r y_{rj}} - \sum_{d=1}^{D} \mathbf{w_r z_{dj}} + \mathbf{u_0} \le \mathbf{0} \qquad \mathbf{j = 1, \dots, n. \;\&\; j \ne p} \\
&\sum_{d=1}^{D} \mathbf{w_r z_{dj}} - \sum_{i=1}^{m} \mathbf{v_i x_{ij}} + \mathbf{v_0} \le \mathbf{0} \qquad \mathbf{j = 1, \dots, n. \;\&\; j \ne p} \\
&\sum_{r=1}^{s} \mathbf{u_r y_{rj}} - \sum_{i=1}^{m} \mathbf{v_i x_{ij}} + \mathbf{u_0 + v_0} \le \mathbf{0} \qquad \mathbf{j = 1, \dots, n. \;\&\; j \ne p} \\
&\mathbf{u_0 + v_0 = 0}. \\
&\mathbf{0 \le \boldsymbol{\alpha^1} \le k'_p} \\
&\mathbf{0 \le \boldsymbol{\beta^2} \le k_p} \\
&\mathbf{v_i \ge \epsilon} \qquad \mathbf{i = 1, \dots, m}. \\
&\mathbf{w_d \ge \epsilon} \qquad \mathbf{d = 1, \dots, D}. \\
&\mathbf{u_r \ge \epsilon} \qquad \mathbf{r = 1, \dots, s}.
\end{aligned}
\tag{11}
$$

And

$$
\begin{aligned}
&\mathbf{Max} \quad \alpha^2 + \beta^2 \\
&\quad \mathbf{s.t} \\
&\sum_{r=1}^{s} u_r y_{rp} - \sum_{i=1}^{m} v_i x_{ip} - \beta^2 + \sum_{i=1}^{m} v_i k_p + \alpha^2 + u_0 + v_0 \le \theta_p^* - 1 \\
&-\sum_{r=1}^{s} u_r y_{rp} + \sum_{i=1}^{m} v_i x_{ip} + \beta^2 - \alpha^2 + \sum_{r=1}^{s} u_r k'_p - u_0 - v_0 \le 1 - \theta_p^* \\
&\sum_{r=1}^{s} u_r y_{rp} - \sum_{d=1}^{D} w_r z_{dp} - \beta^2 + \sum_{i=1}^{m} v_i k_p + u_0 \le 0 \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_r z_{dj} + u_0 \le 0 \qquad j = 1, \dots, n. \ \& \ j \ne p \\
&\sum_{d=1}^{D} w_r z_{dj} - \sum_{i=1}^{m} v_i x_{ij} + v_0 \le 0 \qquad j = 1, \dots, n. \ \& \ j \ne p \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 + v_0 \le 0 \qquad j = 1, \dots, n. \ \& \ j \ne p \\
&u_0 + v_0 = 0. \\
&0 \le \alpha \le f'_p, \\
&0 \le \beta \le f_p \\
&v_i \ge \epsilon \qquad i = 1, \dots, m. \\
&w_d \ge \epsilon \qquad d = 1, \dots, D. \\
&u_r \ge \epsilon \qquad r = 1, \dots, s.
\end{aligned} \tag{12}
$$

Where $\mathbf{k_p} = \min_j \left\{ \min_r \{ |y_{rj} - y_r| \} \right\}$, $\mathbf{k'_p} = \min_j \left\{ \max_i \{ |x_{ij} - x_{ip}| \} \right\}$, $\mathbf{f_p} = \left\{ \min_r \{ |y_{rp}| \} \right\}$ and $\mathbf{f'_p} = \min_j \left\{ \min_i \{ |x_{ij} - x_{ip}| \} \right\}$ By choosing $\alpha^* = \min\{\alpha^1 . \alpha^2\}$ and $\beta^* = \min\{\beta^1 . \beta^2\}$, per α; $0 \le \alpha \le \alpha^*$ increasing and decreasing in inputs and per β; $0 \le \beta \le \beta^*$increasing and decreasing in outputs cannot change type of constant returns to scale evaluation unit. In all above situations, [0, β^*] is best interval to increase or decrease outputs for DMU_O so that its returns to scale does not change. Actually [0, β^*] is the stability region of the RTS classifications.

3 Methodology

3.1 Classification of Returns to Scale

In reality, most productions are manufactured based on multi-stage nature. Identification of such technologies by the concept like network returns to scale is of utmost importance to firm managers for the stage-specific analysis of their business decisions concerning development or relationships so as to improve their firm's overall behavior; therefore, determination of stability region of returns to scale classification is very important. For this reason, we proposed a new method to detect returns to scale in two-stage networks. We employed model (6) and (7), then we used the obtained optimal solutions from model (6) to offer a new method for identification of returns to scale.

It is supposed that DMU_p is efficient under model (7) and $(\boldsymbol{\theta}^*, \boldsymbol{\lambda}^*, \boldsymbol{\delta}^*)$ is an optimum solution from model (6) to assess the DMU_p. We claim that the following theorem is always true. **Theorem2:** Supposing that DMU_o is called efficient under model (7) and $(\boldsymbol{\theta}^*, \boldsymbol{\lambda}^*, \boldsymbol{\delta}^*)$ is an obtained optimal solution from model (6), then

i) If in any optimal solution; $\mathbf{1}\boldsymbol{\lambda}^* + \mathbf{1}\boldsymbol{\delta}^* < \mathbf{1}$ then DMU_P has IRS.

ii) If in any optimal solution; $\mathbf{1}\boldsymbol{\lambda}^* + \mathbf{1}\boldsymbol{\delta}^* > \mathbf{1}$ then DMU_P has DRS.

iii) If in some optimal solutions; $\mathbf{1}\boldsymbol{\lambda}^* + \mathbf{1}\boldsymbol{\delta}^* = \mathbf{1}$ then DMU_P has CRS.

Proof: If we suppose that $(\mathbf{v}^*, \mathbf{w}^*, \mathbf{u}^*)$ is the optimal value of (5) and $(\boldsymbol{\theta}^*, \boldsymbol{\lambda}^*, \boldsymbol{\delta}^*)$ is optimal value for (6) and $(\hat{\mathbf{v}}, \hat{\mathbf{w}}, \hat{\mathbf{u}}, \mathbf{u_0^*}, \mathbf{v_0^*})$ is the optimal value of (7), So:

$$\hat{\mathbf{u}}\mathbf{y_p} + \mathbf{u_0^*} = \mathbf{1} \ , \hat{\mathbf{v}}\mathbf{x_p} - \mathbf{v_0^*} = \mathbf{1} \tag{13}$$

$$\hat{\mathbf{u}}\mathbf{y_j} - \hat{\mathbf{v}}\mathbf{x_j} + \mathbf{u_0^*} + \mathbf{v_0^*} = -\mathbf{G_j^\circ} \qquad \forall \ \mathbf{j} \tag{14}$$

$$\hat{\mathbf{w}}\mathbf{z_j} - \hat{\mathbf{v}}\mathbf{x_j} + \mathbf{v_0^*} = -\mathbf{G_{1j}^\circ} \qquad \forall \ \mathbf{j} \tag{15}$$

$$\hat{\mathbf{u}}\mathbf{y_j} - \hat{\mathbf{w}}\mathbf{z_j} + \mathbf{u_0^*} = -\mathbf{G_{2j}^\circ} \qquad \forall \ \mathbf{j} \tag{16}$$

$$\boldsymbol{\theta}^*\mathbf{x_p} - \mathbf{s_v^*} = \textstyle\sum_j (\boldsymbol{\lambda}_j^* + \boldsymbol{\delta}_j^*)\mathbf{x_j} \tag{17}$$

$$\mathbf{y_p} + \mathbf{s_u^*} = \textstyle\sum_j \boldsymbol{\lambda}_j^* \mathbf{y_j} \tag{18}$$

$$\mathbf{s_w^*} = \textstyle\sum_j \boldsymbol{\delta}_j^* \mathbf{z_j} \tag{19}$$

It is supposed that $\sum_{j=1}^{n} \boldsymbol{\lambda}_j + \sum_{j=1}^{n} \boldsymbol{\mu}_j = \mathbf{1} + \boldsymbol{\varepsilon}$. Multiplying the sides by constraints: (16), (17) and (18) in $\boldsymbol{\phi}_j^*$, $\boldsymbol{\alpha}_j^*$ and $\boldsymbol{\beta}_j^*$, respectively that $\boldsymbol{\phi}_j^*$, $\boldsymbol{\alpha}_j^*$ and $\boldsymbol{\beta}_j^* \geq \mathbf{0}$:

$$\hat{\mathbf{u}}\left(\textstyle\sum_j \boldsymbol{\phi}_j^* \mathbf{y_j}\right) - \hat{\mathbf{v}}\left(\textstyle\sum_j \boldsymbol{\phi}_j^* \mathbf{x_j}\right) + \textstyle\sum_j \boldsymbol{\phi}_j^* \mathbf{u_0^*} + \textstyle\sum_j \boldsymbol{\phi}_j^* \mathbf{v_0^*} = -\textstyle\sum_j \boldsymbol{\phi}_j^* \mathbf{G_j^\circ} \tag{20}$$

$$\hat{\mathbf{w}}\left(\textstyle\sum_j \boldsymbol{\alpha}_j^* \mathbf{z_j}\right) - \hat{\mathbf{v}}\left(\textstyle\sum_j \boldsymbol{\alpha}_j^* \mathbf{x_j}\right) + \textstyle\sum_j \boldsymbol{\alpha}_j^* \mathbf{v_0^*} = -\textstyle\sum_j \boldsymbol{\alpha}_j^* \mathbf{G_{1j}^\circ} \tag{21}$$

$$\hat{\mathbf{u}}\left(\textstyle\sum_j \boldsymbol{\beta}_j^* \mathbf{y_j}\right) - \hat{\mathbf{w}}\left(\textstyle\sum_j \boldsymbol{\beta}_j^* \mathbf{x_j}\right) + \textstyle\sum_j \boldsymbol{\beta}_j^* \mathbf{u_0^*} = -\textstyle\sum_j \boldsymbol{\beta}_j^* \mathbf{G_{2j}^\circ} \tag{22}$$

The sum of constraints (20) and (21) and (22) is equal

$$\begin{gathered}\hat{\mathbf{u}}\left(\textstyle\sum_j (\boldsymbol{\phi}_j^* + \boldsymbol{\beta}_j^*)\mathbf{y_j}\right) - \hat{\mathbf{v}}\left(\textstyle\sum_j (\boldsymbol{\phi}_j^* + \boldsymbol{\alpha}_j^*)\mathbf{x_j}\right) + \hat{\mathbf{w}}\left(\textstyle\sum_j (\boldsymbol{\alpha}_j^* - \boldsymbol{\beta}_j^*)\mathbf{z_j}\right) + \textstyle\sum_j (\boldsymbol{\phi}_j^* + \boldsymbol{\beta}_j^*)\,\mathbf{u_0^*} + \textstyle\sum_j (\boldsymbol{\phi}_j^* + \boldsymbol{\alpha}_j^*)\,\mathbf{v_0^*} + \\ \left(\textstyle\sum_j \boldsymbol{\phi}_j^* \mathbf{G_j^\circ} + \textstyle\sum_j \boldsymbol{\alpha}_j^* \mathbf{G_{1j}^\circ} + \textstyle\sum_j \boldsymbol{\beta}_j^* \mathbf{G_{2j}^\circ}\right) = 0\end{gathered} \tag{23}$$

Let $\boldsymbol{\phi}_j^* + \boldsymbol{\beta}_j^* = \boldsymbol{\lambda}_j^*$ and $\boldsymbol{\alpha}_j^* - \boldsymbol{\beta}_j^* = \boldsymbol{\delta}_j^*$. Hence (24) can be expressed like this:

$$\hat{u}\left(\sum_j \lambda_j^* y_j\right) - \hat{v}\left(\sum_j (\lambda_j^* + \delta_j^*) x_j\right) + \hat{w}\left(\sum_j \delta_j^* z_j\right) + \sum_j \lambda_j^* u_0^* + \sum_j (\lambda_j^* + \delta_j^*) v_0^* + \left(\sum_j \phi_j^* G_j^{\circ} + \sum_j \alpha_j^* G_{1j}^{\circ} + \sum_j \beta_j^* G_{2j}^{\circ}\right) = 0 \tag{24}$$

It is clear that $\sum_j \boldsymbol{\phi}_j^* \mathbf{G}_j^{\circ} + \sum_j \boldsymbol{\alpha}_j^* \mathbf{G}_{1j}^{\circ} + \sum_j \boldsymbol{\beta}_j^* \mathbf{G}_{2j}^{\circ} > \mathbf{0}$, so:

$$\hat{u}\left(\sum_j \lambda_j^* y_j\right) - \hat{v}\left(\sum_j (\lambda_j^* + \delta_j^*) x_j\right) + \hat{w}\left(\sum_j \delta_j^* z_j\right) + \sum_j \lambda_j^* u_0^* + \sum_j (\lambda_j^* + \delta_j^*) v_0^* < 0 \tag{25}$$

Due to the (17), (18), (19) and (25), then we have $\hat{\mathbf{u}}(\mathbf{y_p} + \mathbf{s}_u^*) - \hat{\mathbf{v}}(\boldsymbol{\theta}^* \mathbf{x_p} - \mathbf{s}_v^*) + \hat{\mathbf{w}}(\mathbf{s}_w^*) + (\mathbf{1} + \boldsymbol{\varepsilon})\mathbf{u}_0^* + (\boldsymbol{\theta}^* + \boldsymbol{\varepsilon})\mathbf{v}_0^* < \mathbf{0}$, So $\hat{\mathbf{u}}\mathbf{y_p} - \hat{\mathbf{v}}\boldsymbol{\theta}^*\mathbf{x_p} + \mathbf{u}_0^* + \mathbf{v}_0^* + \hat{\mathbf{u}}\mathbf{s}_u^* + \hat{\mathbf{v}}\mathbf{s}_v^* + \hat{\mathbf{w}}\mathbf{s}_w^* + \boldsymbol{\varepsilon}\mathbf{u}_0^* + \boldsymbol{\varepsilon}\mathbf{v}_0^* < \mathbf{0}$, Hence, $\boldsymbol{\varepsilon}(\mathbf{u}_0^* + \mathbf{v}_0^*) < \mathbf{0}$ if $\boldsymbol{\varepsilon} < \mathbf{0}$, then $\mathbf{u}_0^* + \mathbf{v}_0^* > \mathbf{0}$. As a result, if DMU$_p$ was efficient, then if $\mathbf{1}\boldsymbol{\lambda}^* + \mathbf{1}\boldsymbol{\delta}^* < \mathbf{1}$, and then DMU$_P$ has IRS.Other cases can be proved, similarly. □ Assume that $\mathbf{Q} = \langle\, \mathbf{j} \mid \mathbf{DMU_j} \textbf{ is efficient under model } (\mathbf{7}) \rangle$, Then the RTS classification for DMU$_o$ is identified as IRS if and only if $\mathbf{o} \epsilon \mathbf{Q}^1 = \{\mathbf{j} \in \mathbf{Q}\ \&\ \mathbf{1}\boldsymbol{\lambda}^* + \mathbf{1}\boldsymbol{\delta}^* < \mathbf{1}\}$ in all optimal solutions, DRS if and only if $\mathbf{o} \epsilon \mathbf{Q}^2 = \{\mathbf{j} \in \mathbf{E}\ \& \mathbf{1}\boldsymbol{\lambda}^* + \mathbf{1}\boldsymbol{\delta}^* > \mathbf{1}\}$ in all optimal solutions, and CRS if and only if $\mathbf{o} \epsilon \mathbf{Q}^3 = \{\mathbf{j} \in \mathbf{E}\ \& \mathbf{1}\boldsymbol{\lambda}^* + \mathbf{1}\boldsymbol{\delta}^* = \mathbf{1}\}$ in some optimal solutions. The classification is used for sensitivity analysis of returns to scale.

3.2. Stability Regions for Maintaining Returns to Scale in Two Stage Networks

Returns to scale is an economic and important concept in DEA which explains the behavior of the increase rate in output relative to the associated increase in the inputs. In this section, we are looking for degree of changes in the outputs and inputs of efficient DMU$_o$, on the condition that its returns to scale classification remains constant. If $\mathbf{o} \epsilon \mathbf{Q}^1$, then DMU$_o$ has IRS; hence, decreasing outputs cannot change its returns to scale. Consider the following disturbed at the inputs and outputs of the DMU$_o$:

$$\mathbf{x_o} = \mathbf{x_o} + \boldsymbol{\alpha}$$
$$\mathbf{y_o} = \mathbf{y_o} + \boldsymbol{\beta}$$

The following model obtains the best interval between increasing outputs and inputs that DMU$_o$ is still IRS.

$$\begin{aligned}
&\mathbf{Max}\ \ \boldsymbol{\alpha} + \boldsymbol{\beta} \\
&\mathbf{s.t}: \\
&\qquad \sum_{\substack{j=1\\ j\neq o}}^{n} \left(\lambda_j + (\delta_j' - \delta_j'')\right) x_{ij} + (\lambda_o + (\delta_o' - \delta_o''))(x_{io} + \alpha) \le \theta^*(x_{io} + \alpha) \quad i = 1, \dots, m. \\
&\qquad \sum_{j=1}^{n} (\delta_j' - \delta_j'') z_{dj} \ge 0 \qquad\qquad d = 1, \dots, D. \\
&\qquad \sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj} + \lambda_o(y_{ro} + \beta) \ge (y_{ro} + \beta) \qquad\qquad r = 1, \dots, s. \\
&\sum_{j=1}^{n} \lambda_j + \sum_{j=1}^{n} (\delta_j' - \delta_j'') \le 1 + \epsilon \\
&\qquad 0 \le \alpha \le k_o'
\end{aligned} \tag{26}$$

$$0 \le \beta \le k_o$$
$$\lambda_j, \delta_j{}', \delta_j{}'' \ge 0 \qquad\qquad j = 1, \dots, n.$$

The θ^* is efficiency score of the DMU$_o$ under Model (6). $\epsilon > 0$ is a non-Archimedean constant? Model (26) is a non-linear programming problem. Let $0 \le \alpha \le k'_o$ and $0 \le \beta \le k_o$ that $k'_o = \min_j \left\{\max_i\{|x_{ij} - x_{ip}|\}\right\}$ and $k_o = \min_j \left\{\max_r\{|y_{rj} - y_{rp}|\}\right\}$ (Sarparast et al.[28]). Thus,

$$\begin{cases} 0 \le \lambda_o\alpha \le k'_o\,\lambda_o \to \alpha\lambda_o = k'_o\,\lambda_o - S^1 & ;k'_o\,\lambda_o - S^1 \ge 0\,, S^1 \ge 0 \\ 0 \le \delta_j{}'\alpha \le k'_o\delta_j{}' \to \alpha\delta_j{}' = k'_o\delta_j{}' - S^2 & ;k'_o\delta_j{}' - S^2 \ge 0, S^2 \ge 0 \\ 0 \le \alpha\delta_j{}'' \le k'_o\delta_j{}'' \to \alpha\delta_j{}'' = k'_o\delta_j{}'' - S^3 & ;k'_o\delta_j{}'' - S^3 \ge 0, S^3 \ge 0 \\ 0 \le \beta\,\lambda_o \le k_o\,\lambda_o \to \beta\,\lambda_o = k_o\,\lambda_o - S^4 & ;k_o\,\lambda_o - S^4 \ge 0\,, S^4 \ge 0 \end{cases} \tag{27}$$

Based on the above factors (27), model (26) can be changed as follows:

$$\begin{aligned} &\text{Max } \alpha + \beta \\ &\text{s.t:} \\ &\sum_{j=1}^{n} \left(\lambda_j + (\delta_j{}' - \delta_j{}'')\right) x_{ij} + (k'_o\,\lambda_o - S^1) + (k'_o\delta_j{}' - S^2) - (k'_o\delta_j{}'' - S^3) \le \theta^*(x_{io} + \alpha), \quad i = 1, \dots, m \\ &\sum_{j=1}^{n} (\delta_j{}' - \delta_j{}'') z_{dj} \ge 0 \qquad d = 1, \dots, D. \\ &\sum_{j=1}^{n} \lambda_j y_{rj} + k_o\,\lambda_o - S^4 \ge (y_{ro} + \beta) \qquad r = 1, \dots, s. \\ &\sum_{j=1}^{n} \lambda_j + \sum_{j=1}^{n} (\delta_j{}' - \delta_j{}'') \le 1 + \epsilon \\ &k'_o\,\lambda_o - S^1 \ge 0 \\ &k'_o\delta_j{}' - S^2 \ge 0 \\ &k'_o\delta_j{}'' - S^3 \ge 0 \\ &k_o\,\lambda_o - S^4 \ge 0 \\ &0 \le \alpha \le k'_o \\ &0 \le \beta \le k_o \\ &S^1, S^2, S^3, S^4 \ge 0 \\ &\lambda_j, \delta_j{}', \delta_j{}'' \ge 0 \qquad j = 1, \dots, n. \end{aligned} \tag{28}$$

Model (28) is a linear programming problem. Theorem 3 shows that its optimal solution is equal to the optimal solution obtained from the model (26).

Theorem 3. The optimal solution obtained from the model (28) is equal to the optimal solution obtained from the model (26).

Proof: It is assumed that $\left(\alpha^*, \beta^*, \lambda^*, \delta'^*, \delta''^*\right)$ is an optimal solution obtained from non-linear programming problem (26) and $\left(\hat\alpha, \hat\beta, \hat\lambda, \delta', \delta'', \hat S^1, \hat S^2, \hat S^3, \hat S^4\right)$ is an optimal solution obtained from model (28). $\left(\hat\alpha, \hat\beta, \hat\lambda, \hat\delta', \hat\delta'', k'_o\,\hat\lambda_o - \hat\alpha\hat\lambda_o, k'_o\hat\delta_o{}' - \hat\alpha\hat\delta_o{}', k'_o\hat\delta_o{}'' - \hat\alpha\hat\delta_o{}'', k_o\,\hat\lambda_o - \hat\beta\,\hat\lambda_o\right)$is a feasible solution for model (28), so it is true in the constraint of the model (28) as follows:

$$
\begin{aligned}
&\sum_{j=1}^{n}\left(\hat\lambda_j + \left(\hat\delta_j{}' - \hat\delta_j{}''\right)\right)x_{ij} + (k'_o\,\hat\lambda_o - (k'_o\,\hat\lambda_o - \hat\alpha\hat\lambda_o)) + (k'_o\hat\delta_j{}' - (k'_o\hat\delta_o{}' - \hat\alpha\hat\delta_o{}')) - (k'_o\delta_j{}'' - (k'_o\hat\delta_o{}'' - \hat\alpha\hat\delta_o{}''))\\
&\qquad\qquad \le \theta^*(x_{io} + \alpha) \qquad\qquad i = 1, \dots, m.\\
&\sum_{j=1}^{n}\left(\hat\delta_j{}' - \hat\delta_j{}''\right)z_{dj} \ge 0 \qquad\qquad d = 1, \dots, D.\\
&\sum_{j=1}^{n}\hat\lambda_j y_{rj} + k_o\,\hat\lambda_o - (k_o\,\hat\lambda_o - \hat\beta\,\hat\lambda_o) \ \ge (y_{ro} + \beta) \qquad\qquad r = 1, \dots, s.\\
&\sum_{j=1}^{n}\hat\lambda_j + \sum_{j=1}^{n}\left(\hat\delta_j{}' - \hat\delta_j{}''\right) \le 1 + \epsilon\\
&0 \le \hat\alpha \le k'_o\\
&0 \le \hat\beta \le k_o\\
&k'_o\,\hat\lambda_o - \left(k'_o\,\hat\lambda_o - \hat\alpha\hat\lambda_o\right) \ge 0\\
&k'_o\hat\delta_j{}' - \left(k'_o\hat\delta_o{}' - \hat\alpha\hat\delta_o{}'\right) \ge 0\\
&k'_o\hat\delta_j{}'' - \left(k'_o\hat\delta_o{}'' - \hat\alpha\hat\delta_o{}''\right) \ge 0\\
&k_o\,\hat\lambda_o - \left(k_o\,\hat\lambda_o - \hat\beta\,\hat\lambda_o\right) \ge 0\\
&k'_o\,\hat\lambda_o - \hat\alpha\hat\lambda_o, k'_o\hat\delta_o{}' - \hat\alpha\hat\delta_o{}', k'_o\hat\delta_o{}'' - \hat\alpha\hat\delta_o{}'', k_o\,\hat\lambda_o - \hat\beta\,\hat\lambda_o \ge 0\\
&\hat\lambda_j, \hat\delta_j{}', \hat\delta_j{}'' \ge 0
\end{aligned} \tag{29}
$$

We rewrite (29) as follows:

$$
\begin{aligned}
&\sum_{j=1}^{n}\left(\hat\lambda_j + \left(\hat\delta_j{}' - \hat\delta_j{}''\right)\right)x_{ij} + \hat\alpha\hat\lambda_o + \hat\alpha\hat\delta_o{}' - \hat\alpha\hat\delta_o{}'' \le \theta^*(x_{io} + \alpha) \qquad i = 1, \dots, m.\\
&\sum_{j=1}^{n}\left(\hat\delta_j{}' - \hat\delta_j{}''\right)z_{dj} \ge 0 \qquad\qquad d = 1, \dots, D.\\
&\sum_{j=1}^{n}\hat\lambda_j y_{rj} + \hat\beta\,\hat\lambda_o \ \ge (y_{ro} + \beta) \qquad\qquad r = 1, \dots, s.\\
&\sum_{j=1}^{n}\hat\lambda_j + \sum_{j=1}^{n}\left(\hat\delta_j{}' - \hat\delta_j{}''\right) \le 1 + \epsilon\\
&0 \le \hat\alpha \le k'_p\\
&0 \le \hat\beta \le k_p\\
&\boldsymbol{\hat\lambda_j, \hat\delta_j{}', \hat\delta_j{}'' \ge 0}
\end{aligned} \tag{30}
$$

Model (30) Shows that $(\hat\alpha, \hat\beta, \hat\lambda, \delta', \delta'')$ is a feasible solution for model (26), so $\alpha^* + \beta^* \ge \hat\alpha + \hat\beta$; (i). If $\left(\alpha^*, \beta^*, \lambda_j{}^*, \delta_j{}'^*, \delta_j{}''^*\right)$ is an optimal solution for model, (26) then according to (27), $\left(\alpha^*, \beta^*, \lambda_j{}^*, \delta_j{}'^*, \delta_j{}''^*, S^{1^*} = k'_o\,\lambda_o{}^* - \alpha^*\lambda_o{}^*, S^{2^*} = k'_o\delta_j{}'^* - \alpha^*\delta_j{}'^*, S^{3^*} = k'_o\delta_j{}''^* - \alpha^*\delta_j{}''^*, S^{4^*} = k_o\,\lambda_o{}^* - \beta^*\,\lambda_o{}^*\right)$ which is a feasible solution for model (26), so $\alpha^* + \beta^* \le \hat\alpha + \hat\beta$;(ii).

According to (i) and (ii), we prove $\alpha^* + \beta^* = \hat\alpha + \hat\beta$□ □

To maintain the efficiency of the unit under evaluation, at least one of the constraints

$$\sum_{j=1}^{n}\left(\lambda_j + \left(\delta_j{}' - \delta_j{}''\right)\right)x_{ij} + (k'_o\,\lambda_o - S^1) + (k'_o\delta_j{}' - S^2) - (k'_o\delta_j{}'' - S^3) \le \theta^*(x_{io} + \alpha), i = 1, \dots, m.$$

is satisfied**.** That is why we propose the following model.

$$
\begin{aligned}
&\text{Max } \alpha+\beta\\
&\text{s.t:}\\
&\sum_{j=1}^{n}\left(\lambda_j+\left(\delta_j{}'-\delta_j{}''\right)\right)x_{ij}+(k'_o\,\lambda_o-S^1)+(k'_o\delta_j{}'-S^2)-(k'_o\delta_j{}''-S^3)+so_i\\
&\qquad\qquad =\theta^*(x_{io}+\alpha) \qquad\qquad i=1,\dots,m.\\
&\sum_{j=1}^{n}(\delta_j{}'-\delta_j{}'')z_{dj}\geq 0 \qquad\qquad d=1,\dots,D.\\
&\sum_{j=1}^{n}\lambda_j y_{rj}+k_o\,\lambda_o-S^4\ \geq (y_{ro}+\beta) \qquad\qquad r=1,\dots,s.\\
&\sum_{j=1}^{n}\lambda_j+\sum_{j=1}^{n}(\delta_j{}'-\delta_j{}'')\geq 1+\epsilon\\
&\quad k'_o\,\lambda_o-S^1\geq 0\\
&\quad k'_o\delta_j{}'-S^2\geq 0\\
&\quad k'_o\delta_j{}''-S^3\geq 0\\
&\quad k_o\,\lambda_o-S^4\geq 0\\
&\quad 0\leq\alpha\leq k'_o\\
&\quad 0\leq\beta\leq k_o\\
&\quad so_i\leq t_i M\\
&\quad \sum_{i=1}^{m}t_i\leq m-1;\\
&\quad t_i\epsilon\{0.1\}\\
&\quad so,S^1,S^2,S^3,S^4\geq 0\\
&\quad \lambda_j,\delta_j{}',\delta_j{}''\geq 0 \qquad\qquad j=1,\dots,n.
\end{aligned}
\tag{31}
$$

Now, imagine DMU_o has DRS and $o\epsilon Q^2$. In this case, decreasing outputs may change returns to scale classification of DMU_o. We consider chaos in amount of inputs and outputs as follows:

$$x_o = x_o-\alpha$$
$$y_o = y_o-\beta$$

Applying this model yields the most appropriate amount of reduction in inputs and outputs where DMU_o is still DRS.

$$
\begin{aligned}
&\text{Max } \alpha+\beta\\
&\text{s.t:}\\
&\sum_{j=1}^{n}(\delta_j{}'-\delta_j{}'')z_{dj}\geq 0 \qquad\qquad d=1,\dots,D.\\
&\sum_{\substack{j=1\\ j\neq o}}^{n}\lambda_j y_{rj}+\lambda_o(y_{ro}-\beta)\geq (y_{ro}-\beta) \qquad\qquad r=1,\dots,s.\\
&\sum_{j=1}^{n}\lambda_j+\sum_{j=1}^{n}(\delta_j{}'-\delta_j{}'')\geq 1+\epsilon\\
&0\leq\alpha\leq k'_o, \qquad\qquad 0\leq\beta\leq k_o\\
&\lambda_j,\delta_j{}',\delta_j{}''\geq 0 \qquad\qquad j=1,\dots,n.
\end{aligned}
\tag{32}
$$

$\epsilon > 0$ is a non-Archimedean constant and $k'_p=\min_j\left\{\max_i\{|x_{ij}-x_{io}|\}\right\}$ and $k_p=\min_r\{|y_{ro}|\}$ (Sarparast et al. [28]. Using the technique (27), model (32) can be transformed into the following form:

$$
\begin{aligned}
&\text{Max } \alpha+\beta\\
&\text{s.t:}\\
&\sum_{j=1}^{n}\left(\lambda_j+\left(\delta_j{}'-\delta_j{}''\right)\right)x_{ij}-(k'_o\,\lambda_o-S^1)-\left(k'_o\delta_j{}'-S^2\right)+(k'_o\delta_j{}''-S^3)\leq\theta^*(x_{io}-\alpha)\\
&\qquad i=1,\dots,m.\\
&\sum_{j=1}^{n}(\delta_j{}'-\delta_j{}'')z_{dj}\geq 0 \qquad\qquad d=1,\dots,D.\\
&\sum_{j=1}^{n}\lambda_j y_{rj}-k_o\,\lambda_o+S^4\ \geq (y_{ro}+\beta) \qquad\qquad r=1,\dots,s.
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
&\sum_{j=1}^{n}\lambda_j+\sum_{j=1}^{n}\left(\delta_j{}'-\delta_j{}''\right)\geq 1+\epsilon\\
&k_o'\,\lambda_o-S^1\geq 0\\
&k_o'\delta_j{}'-S^2\geq 0\\
&k_o'\delta_j{}''-S^3\geq 0\\
&k_o\,\lambda_o-S^4\geq 0\\
&0\leq\alpha\leq k_o'\\
&0\leq\beta\leq k_o\\
&S^1,S^2,S^3,S^4\geq 0\\
&\lambda_j,\delta_j{}',\delta_j{}''\geq 0 \qquad\qquad j=1,\dots,n.
\end{aligned}
$$

Theorem 4. The optimal solution obtained from the model (33) is equal to the optimal solution obtained from the model (32).

Proof. The proof is analogous to theorem 3.

Because ($o\epsilon Q^1$), we propose the following model to maintain the efficiency of the unit under assessment.

$$
\begin{aligned}
&\text{Max}\quad \alpha+\beta\\
&\text{s.t:}\\
&\sum_{j=1}^{n}\left(\lambda_j+\left(\delta_j{}'-\delta_j{}''\right)\right)x_{ij}-(k_o'\,\lambda_o-S^1)-\left(k_o'\delta_j{}'-S^2\right)+(k_o'\delta_j{}''-S^3)+so_i=\theta^*(x_{io}-\alpha)\\
&i=1,\dots,m.\\
&\sum_{j=1}^{n}(\delta_j{}'-\delta_j{}'')z_{dj}\geq 0 \qquad\qquad d=1,\dots,D.\\
&\sum_{j=1}^{n}\lambda_j y_{rj}-k_o\,\lambda_o+S^4\quad\geq(y_{ro}+\beta) \qquad\qquad r=1,\dots,s.\\
&\sum_{j=1}^{n}\lambda_j+\sum_{j=1}^{n}\left(\delta_j{}'-\delta_j{}''\right)\geq 1+\varepsilon\\
&k_o'\,\lambda_o-S^1\geq 0\\
&k_o'\delta_j{}'-S^2\geq 0\\
&k_o'\delta_j{}''-S^3\geq 0\\
&k_o\,\lambda_o-S^4\geq 0\\
&0\leq\alpha\leq k_o'\\
&0\leq\beta\leq k_o\\
&so_i\leq t_i M\qquad i=1,\dots,m.\\
&\sum_{i=1}^{m}t_i\leq m-1;\\
&t_i\epsilon\{0.1\}\qquad i=1,\dots,m.\\
&S^1,S^2,S^3,S^4\geq 0\\
&\lambda_j,\delta_j{}',\delta_j{}''\geq 0 \qquad\qquad j=1,\dots,n.
\end{aligned}
\tag{34}
$$

Model (34) is a linear programming problem and has a stability region that maintains returns to scale classification of DMU_o. In the last case, if DMU_o has CRS, then both increasing and decreasing outputs can change its returns to scale classification. We propose model (35) and model (36) for sensitivity analysis of its returns to scale classification.

$$
\begin{aligned}
&\text{Max} \quad \alpha^1 + \beta^1 \\
&\text{s.t:} \\
&\sum_{j=1}^{n} \left(\lambda_j + (\delta_j{}' - \delta_j{}'')\right) x_{ij} + (k'_o\,\lambda_o - S^1) + (k'_o\delta_j{}' - S^2) - (k'_o\delta_j{}'' - S^3) + so_i \\
&\qquad\qquad = \theta^*(x_{io} + \alpha^1) \qquad\qquad i = 1, \dots, m. \\
&\sum_{j=1}^{n} (\delta_j{}' - \delta_j{}'') z_{dj} \geq 0 \qquad\qquad d = 1, \dots, D. \\
&\sum_{j=1}^{n} \lambda_j y_{rj} + k_o\,\lambda_o - S^4 \quad \geq (y_{ro} + \beta^1) \qquad\qquad r = 1, \dots, s. \\
&\sum_{j=1}^{n} \lambda_j + \sum_{j=1}^{n} (\delta_j{}' - \delta_j{}'') = 1 \\
&\quad k'_o\,\lambda_o - S^1 \geq 0 \\
&\quad k'_o\delta_j{}' - S^2 \geq 0 \\
&\quad k'_o\delta_j{}'' - S^3 \geq 0 \\
&\quad k_o\,\lambda_o - S^4 \geq 0 \\
&0 \leq \alpha^1 \leq k'_p = \max_j \left\{ \max_i \{ |x_{ij} - x_{io}| \} \right\} \\
&0 \leq \beta^1 \leq k_o = \max_j \left\{ \max_r \{ |y_{rj} - y_{rp}| \} \right\} \\
&so_i \leq t_i M \qquad i = 1, \dots, m. \\
&\sum_{i=1}^{m} t_i \leq m - 1; \\
&T_i \epsilon \{0.1\} \qquad i = 1, \dots, m. \\
&S^1, S^2, S^3, S^4 \geq 0 \\
&\lambda_j, \delta_j{}', \delta_j{}'' \geq 0 \qquad\qquad j = 1, \dots, n.
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
&\text{Max} \quad \alpha^2 + \beta^2 \\
&\text{s.t:} \\
&\sum_{j=1}^{n} \left(\lambda_j + (\delta_j{}' - \delta_j{}'')\right) x_{ij} - (k'_o\,\lambda_o - S^1) - \left(k'_o\delta_j{}' - S^2\right) + (k'_o\delta_j{}'' - S^3) + so_i \\
&\qquad\qquad = \theta^*(x_{io} - \alpha^2) \qquad\qquad i = 1, \dots, m. \\
&\sum_{j=1}^{n} (\delta_j{}' - \delta_j{}'') z_{dj} \geq 0 \qquad\qquad d = 1, \dots, D. \\
&\sum_{j=1}^{n} \lambda_j y_{rj} - k_o\,\lambda_o + S^4 \quad \geq (y_{ro} + \beta^2) \qquad\qquad r = 1, \dots, s. \\
&\sum_{j=1}^{n} \lambda_j + \sum_{j=1}^{n} (\delta_j{}' - \delta_j{}'') = 1 \\
&\quad k'_o\,\lambda_o - S^1 \geq 0 \\
&\quad k'_o\delta_j{}' - S^2 \geq 0 \\
&\quad k'_o\delta_j{}'' - S^3 \geq 0 \\
&\quad k_o\,\lambda_o - S^4 \geq 0 \\
&0 \leq \alpha^2 \leq \; k'_p = \min_j \left\{ \max_i \{ |x_{ij} - x_{io}| \} \right\} \\
&0 \leq \beta^2 \leq \; k_p = \left\{ \min_r \{ |y_{ro}| \} \right\} \\
&so_i \leq t_i M \qquad\qquad i = 1, \dots, m. \\
&\textstyle\sum_{i=1}^{m} t_i \leq m - 1; \\
&t_i \epsilon \{0.1\} \qquad\qquad i = 1, \dots, m. \\
&S^1, S^2, S^3, S^4 \geq 0 \\
&\lambda_j, \delta_j{}', \delta_j{}'' \geq 0 \qquad\qquad j = 1, \dots, n.
\end{aligned}
\tag{36}
$$

The decrease or increase of the input of α; $0 \leq \alpha \leq \alpha^* = \min\{\alpha^1 . \alpha^2\}$; and the decrease or increase of the output of β; $0 \leq \beta \leq \;\; \beta^* = \min\{\beta^1 . \beta^2\}$; keeps the classification of DMU_o.

4 Illustrative Example

4.1 Case of Virtual Data

In this section, we will apply a set of synthetic data to demonstrate the proposed method in section 3 which was proposed by Sarparast et al. [28] for the first time. Table 1 shows the data of 10 two-stage DMUs with single input, single intermediate value and single final output.

Table 1: A Set of Synthetic Data for Ten Two-Stage DMUs

DMU	1	2	3	4	5	6	7	8	9	10
Input for stage 1 (x)	2.5	4	5.5	8	11	6.5	9	4	5	2
Intermediate measure(z)	3.5	5.5	6.5	8	6	4.5	8	5.5	4	2
Final output(y)	7	10	5.5	13	9	7	8	9	6	1

Sarparast et al. [28] appraised 10 DMUs by model (7); the efficiency scores of DMUs under model (7) can be found in the second column of the Table 2. $\mathrm{E} = \{1,2,4,10\}$ is a set of index efficient DMUs. The third and fourth columns show min and max $u_0^* + v_0^*$ that (u_0^*, v_0^*) is an optimal solution obtained from model (7) to evaluate the score efficiency of DMUs. According to theorem 1 and the third and fourth columns, it can be said that DMU_1 has CRS. DMU_4 has DRS, DMU_{10} has IRS and DMU_2. The fifth column shows returns to scale classification of the efficient DMUs based on Sarparast et al. [28].
In the evaluation of DMUs in model (6), DMU_1 is efficient, and its efficiency score is equal to 1. Model (6) is one of the constant returns to scale models that assesses two-stage networks. The sixth column of the Table 2 shows scores of efficiency of two-stage network system under model (6).

Table 2: Results Table Corresponding To Artificial Data

	θ_p^* (7)	Max$u_0^*+v_0^*$ (7)	Min $u_0^*+v_0^*$ (7)	RTS	θ_p^* (6)	Max $1\lambda^* + 1\delta^*$ (6)	Min $1\lambda^* + 1\delta^*$ (6)	RTS
DMU_1	1.0000	1.6429	-0.3810	CRS	1.0000	1.0000	1.0000	CRS
DMU_2	1.0000	-0.2420	-0.0001	DRS	0.8929	1.4286	1.4286	DRS
DMU_3	0.4808	-0.1670	-0.2230	-	0.3571	0.7857	0.7857	IRS
DMU_4	1.0000	-0.7540	-2.0000	DRS	0.5804	1.8571	1.8571	DRS
DMU_5	0.6042	-0.0001	-0.0001	-	0.2922	1.2857	1.2857	DRS
DMU_6	0.4930	-0.2040	-0.6150	-	0.3846	0.9999	0.9999	IRS
DMU_7	0.5208	-0.0001	-0.0001	-	0.1190	1.1429	1.1429	DRS
DMU_8	0.8788	-0.2420	-0.7270	-	0.0893	1.2857	1.2857	DRS
DMU_9	0.5270	0.1760	0.1760	-	0.0893	0.8571	0.8571	IRS
DMU_{10}	1.0000	2.8750	2.8750	IRS	0.1786	0.1429	0.1429	IRS

The seventh and eighth columns of Table 2 show min and max $1\lambda^* + 1\delta^*$ where $(\theta^*, \lambda^*, \gamma^*)$ is an optimal solution obtained from model (6). The returns to scale of the classification of the DMUs according to theorem 2 can be found in the following Table. The results in the fifth and ninth columns indicate that both methods produce the same results. To investigate the sensitivity analysis classification RTS of DMUs, we apply models (28), (33), (35) and (36). DMU_2, DMU_4, DMU_5, DMU_7 and DMU_8 have DRS; therefore, we use model (34) to determine appropriate reduction in the inputs and outputs so that these DMUs are still DRS. DMU_3, DMU_6, DMU_9, DMU_{10} and DMU_8 have DRS, so we use model (31) to recognize perfect enhancement of the inputs and outputs so that these DMUs are still IRS. Table 3 shows values of α^* and β^* obtained from models (28), (33), (35) and (36).

Table 3: Table of Classification of Returns to Scale in Artificial Intelligence in The New Method

DMU	1	2	3	4	5	6	7	8	9	10
	CRS	DRS	IRS	DRS	DRS	IRS	DRS	DRS	IRS	IRS
α^*	3.214									
β^*	7.000									
α^*			5.500			1.500			6.000	7.000
β^*			5.499			1.615			3.601	7.037
α^*		2.000		7.000	7.000		7.000	7.000		
β^*		10.000		13.000	1.000		4.292	4.032		

For example, DMU_1 has CRS. For increasment or decreasment in the inputs and outputs as well as α and β; $0 \leq \alpha \leq \alpha^* = 3.214$and $0 \leq \beta \leq \beta^* = 7.000$; DMU_1 still has CRS. Or DMU_{10} has IRS, thus increase in the inputs and outputs as well as α and β; $0 \leq \alpha \leq \alpha^* = 7.000$ and $0 \leq \beta \leq \beta^* = 7.037$; DMU10 is also IRS. A similar interpretation can be found for DMU_2.

4.2 Case of the Iranian Insurance Industry

If organizations aim to determine their current situation and survive and progress in today's competitive world, they should constantly evaluate their performance against that of other companies, identify their strengths and weaknesses, and improve their operationsI In today's insurance industry, competition has become more stringent given the entry of numerous private sector companies. The longevity of enterprises will depend on high performance and competitiveness.

Insurance companies play a role in a society's economy, advancing the growth and development of an entire economic complex in a country by maintaining national wealth, compensating for economic losses, and guaranteeing large investments and their development. Development has been one of the main visions of the Islamic Republic of Iran, where the insurance industry enjoys favorable support expected to significantly contribute to goal achievement in the country. The primary mission of Iran's insurance industry is to eliminate uncertainty—a function that is particularly apparent in any social group and even the national economy in the region. The elimination of uncertainty can positively influence job security and the consistency of earnings, enhance quality of life, preserve national wealth, and ensure investment development. Such industry should, therefore, invest in a premium environment where insurance can be effectively received—an environment that can promote and increase capital growth, and, ultimately, national production. These issues have always revolved around the level of an insurance company's performance, and whether insurance companies are at a stage wherein they generate returns of scale from production. In other words, there is a need to determine at what stage insurance companies are in the medium-term expense landscape, and whether they can increase the value by maintaining existing facilities. Scientific methods of minimizing the errors caused using mental judgment can facilitate the identification of companies' positions with respect to one another and, consequently, give rise to appropriate solutions that strengthen the position of each institution. In this regard, the measurement of efficiency has been an important management issue. The economic unit strategies that strengthen companies identify advantages and shortcomings, eliminate defects, and enhance the strengths of an entire unit. The process that underlies insurance generally involves two stages. The first is marketing activity, which entails guaranteeing marketability and the capacity to attract premium-earning portfolios and the use of intermediary systems, agencies, and legal advisers. The second stage is concerned with profitability, which stems from surplus assets and liabilities. In this stage, the residual resources of an institution are invested in financial markets for the purpose of earning appropriate returns. In the first stage, insurance companies incur two types of expenses. The first set of expenses encompasses employee's salaries and different types of expenses from daily operations. Such expenditures are called operating expenses (x1). The second set of expenses comprises those paid to agencies, brokers, and lawyers and expenses associated with the recovery of insurance services, such as damage-related expenses (funding for the provision of physical damages, expenses incurred by law enforcement, and health ministery expenses), and expenses from net losses arising from other insurance expenses. These expenditures are referred to as insurance expenses (x2). Both expense types are considered inputs of the first stage of the insurance process. In this stage, an insurance company receives premiums directly from an insurer's customers and revenues from reinsurance, including remuneration of reliance benefits and damage remuneration received from reliance insurers. As previously stated, the second stage centers on profitability, originating from premium insurance, and insurance payments are reduced in situations of loss and damage.

The residual resources of an institute are also invested in investment markets. Correspondingly, the inputs of this stage are direct written insurance (z1) and reinsurance premiums (z2) from reliance insurers, which are the outputs of the first stage. The outputs of the second stage of the insurance process are underwriting profits (y1), profits from investment in secure exchange companies, and profits from non-stock exchange companies (investment profit (y2)). Figure 2 illustrates the two-step production process of insurance companies in Iran. On the basis of evaluated research documentation, we selected six of the indicators of 88 Iranian private insurance companies and used the insurance industry's salary information obtained from the codal.ir website. The salary data extracted were those issued from March 21, 2018 to March 20, 2019. Note that each of the 18 insurance companies have been fully involved in this fiscal year, and each of them was considered a DMU. Data on these units are presented in the Table 4.

The following Table shows the descriptive characteristics of the indices that served as descriptive parameters for each index. The value of the descriptive parameters is separate for each indicator, and includes information on mean, minimum, maximum, and median values. The second category of information includes distribution metrics, such as standard deviation, coefficient of variation, skidding, and stretching, which indicate the distribution of data around the middle axis. In the study of a statistical community, the value of a representative member of that community is called the central value; the amount of each factor is distributed around it, and each numeric criterion is called a center-oriented criterion. In other words, it is the criterion that represents the center of a dataset. The mean and median are the most common criteria for determining centrality. The mean is the most important central value as it is an indicator of the equilibrium point and the center-of-gravity distribution.

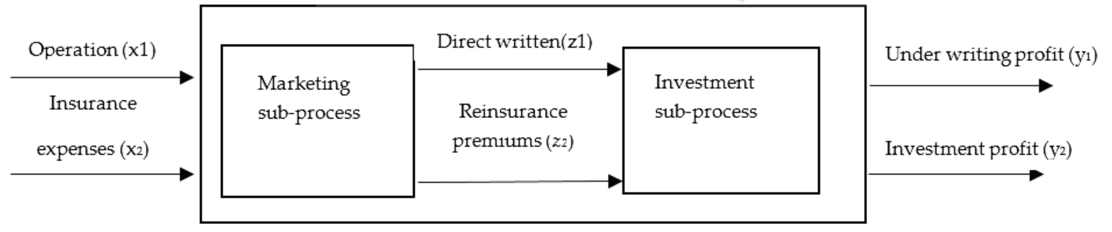

Fig. 2: Production System of Private Insurance Companies in Iran

Table 4 : Initial Data Related to Insurance Companies (Million USD)

Insurance Companies	Operation expenses (x_1)	Insurance expenses (x2)	Direct written premiums (z1)	reinsurance premiums(z2)	Underwriting profit (Y1)	Investment profit (Y2)
Novin	3652253	3181379	3652253	544178	517615	518163
Mihan	1526227	2829462	1526227	656401	49919	168853
Alborz	1287314	19564048	18878990	2881842	1718410	796590
Asia	2237186	28339909	27654591	4208193	2323425	1641675
Dey	372099	5322443	6057551	164535	641361	314728
Karafarin	586118	1761364	8260670	1147305	422171	1263308
Taavon	182788	1318596	1575029	163513	130016	88433
Dana	2092653	19483046	21922773	3078337	2342862	1240129
Kosar	452251	10702943	10163083	211880	388424	610140
Ma	477743	2894189	3663625	835892	525344	960720
Mellat	397058	4199581	3645680	703319	142510	1174096
Tejarateno	98878	292713	434200	21217	23989	542876
Khavarmianeh	13.406	10	95	7	29	26354
Parsian	1091593	15795614	17535105	269287	1330169	577331
Saman	289446	3288248	5098701	530469	677213	554935
Sarmad	289491	1721734	1936888	345447	354632	252932
Razi	643706	7067361	7971127	1024836	602678	510929
Pasargad	857454	8423501	11388798	1316439	935380	1633872

It also serves as a good index of data centrality. The main central index is the mean value of operating expenses in insurance companies; it is equal to 757,766, indicating that most data are centered around the point of mean value. Dispersion parameters serve as criteria for determining the degree of dispersion of each parameter or the level of dispersion toward the mean of the most highly dispersed parameters. These parameters are the coefficient of variation (dispersion coefficient) obtained from dividing the standard deviation by the mean of data; this coefficient is used as one of the dispersion modalities for ensuring data stability and consistency. The variation coefficient for the expenses of insurance policies is equal to 1.200 and that for the investment index is 0.67. These values indicate that the investment index exhibited higher stability and consistency than did the practice expense indicator in the course of this research.

In assessing the normality of each of the indices, the histogram and a Kolmogorov–Smirnov test were used in SPSS software. The null and alternative hypotheses in the Kolmogorov–Smirnov test were as follows:

H0: Data on the indices follow the normal distribution.

H1: Data on the indices do not follow the normal distribution.

The results of the normalization of the indicators are shown in Table 6. The normalization test and the histogram of the indicators revealed that all the indices follow the normal distribution; thus, the null hypothesis cannot be rejected.

Table 5: Statistical Analysis of Data on Insurance Companies

Statistics	X1	X2	Z1	Z2	Y1	Y2
Mean	757766	7823810	8689008	1032878	741678	726935
Median	531931	3743915	5578126	600290	521480	566133
Maximize	3652253	28339909	27654591	4208193	2342862	1641675
Minimal	13	10	95	7	29	26354
Standard deviation	917307	7857235	7862301	1151705	714329	486579
Skewness	1.72	1.39	1.14	1.74	1.31	0.56
Stretching	3.03	1.16	0.428	2.34	0.8	-0.668
Number of observations	18	18	18	18	18	18

Table 6: One-Sample Kolmogorov–Smirnov Test

	X1	X2	Z1	Z2	Y1	Y2
No	18	18	18	18	18	18
Kolmogorov-Smirnov Z	0.957	0.935	0.738	0.997	1.062	0.824
Asymp. Sig.	0.318	0.346	0.647	0.296	0.209	0.504

The results of efficiency score of insurance companies using the above-mentioned models are listed in the Table 7. The efficiency of companies in the returns to scale variable in classical and two-stage networks has been investigated. The second column indicates the name of the insured companies studied at the given time interval. In the third column, the efficiency score of insurance companies with the classic method of data envelopment analysis (i.e. without regard to middleware) is provided only with the initial inputs and final outputs. The performance score obtained from the model (4) shows that Alborz, Karafarin, Mellat, Tejarateno, Khavarmianeh, Saman and Pasargad Insurance Companies are efficient, and other companies are inefficient. The fourth and fifth columns show the the efficiency of the DMUs without taking the next level into consideration. This amount was obtained from model (4) for each step. The first step (Marketing), refers to such insurance companies as Alborz, Asia, Khavarmianeh, Parsian and Saman. The other units are not efficient. The second step not only contains the insurance companies, such as Asia, Dana, Ma, Mellat, Khavarmianeh and Parsian which were active in both steps but also includes the other companies such as Dey, Tejarateno, Sarmad and Pasargad which are more successful in the process of investment and efficient, subsequently. The comparison of the second column with the third and fourth columns show that those companies which were efficient in the model (7) may be inefficient in one of the first or second steps or in both. For example, Tejaraeno , as an insurance company, is inefficient in the first step, but it has the efficiency of one in the classic model (4). It is not logical, and this incompatibility is caused by ignoring the middle products, not affecting the appropriate resources. The sixth and seventh columns list the efficiency obtained from the degree of the effect of each step on another one. The insurance companies such as Mihan, Albarz, Asia, Karafarin, Dana, Kosar, Khavarmianeh, Parsian and Saman are all efficient, but the insurance companies of Ma and Mellat had the efficiency of one, where they were not under the influence of the second level. In the second step, the only insurance companies which are still efficient are Mihan, Asia, Dana, Khavarmianeh and Saman. The eighth column indicates the overall efficiency score of the two-stage network of each unit which is the score of efficiency of the middle products, in contrast to the second column. The results show that Asia, Dana, Khavarmianeh and Saman companies are efficient units. It is obvious that those units which are efficient in this model, have the ability of marketing and investment. So, as a conclusion, the measured efficiency resulted from the twosteps method is logical, and it describes individuals' expectation about the physical relation of the whole process and its two sub-processes.

Table7: Efficiency Score of Insurance Companies under Models

DMU_p	Insurance Companies	θ^*_p Model (4)	θ^*_p Model(4)	θ^*_p Model(4)	θ^1_p Model(7)	θ^{*2}_p Model (7)	$\theta_p^{overal*}$ Model (7)
DMU01	Novin	0.7300	0.4456	0.9714	0.4456	0.3849	0.3849
DMU02	Mihan	0.0700	1.0000	0.2225	1.0000	0.0377	0.0377
DMU03	Alborz	1.0000	1.0000	0.8271	1.0000	0.8245	0.8245
DMU04	Asia	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DMU05	Dey	0.7400	0.9821	1.0000	0.9821	0.7413	0.7413
DMU06	Karafarin	1.0000	1.0000	0.7372	1.0000	0.7372	0.7372
DMU07	Taavon	0.4600	0.5430	0.5691	0.5430	0.2930	0.2930
DMU08	Dana	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DMU09	Kosar	0.5130	1.0000	0.7125	1.0000	0.4051	0.4051
DMU10	Ma	0.9500	1.0000	1.0000	0.8628	0.7298	0.7298
DMU11	Mellat	1.0000	1.0000	1.0000	0.8308	0.8531	0.8531
DMU12	Tejarateno	1.0000	0.3163	1.0000	0.3163	0.3163	0.3163
DMU13	Khavarmianeh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DMU14	Parsian	0.8400	1.0000	1.0000	1.0000	0.7125	0.7125
DMU15	Saman	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DMU16	Sarmad	0.9500	0.5892	1.0000	0.5892	0.5142	0.5142
DMU17	Razi	0.4100	0.8063	0.5483	0.8063	0.4306	0.4306
DMU18	Pasargad	1.0000	0.9126	1.0000	0.9126	0.9126	0.9126

In Table 8, both the maximum and minimum amount of $u_0^*+v_0^*$ is measured in efficient units and $(u^*, w^*, v^*, u_0^*, v_0^*)$ are the optimum responses of the two-step network DEA model (7). Therefore, based on the theorem 1, it is obvious that the efficient unit of Khavarmineh has constant returns to scale. It means that there is no difference between the efficiency of the production of the small or large units. Dana, Asia and Saman insurance companies are the units which have decreasing returns to scale. The choice of the larger amount means that the products will be produced more expensively

Table 8: Analysis of The Returns to Scale and The Sensitivity of Classification of Returns to The Scale of Efficient Units by The Classic Method**.**

DMUp	Min $u_0^*+v_0^*$	Max $u_0^*+v_0^*$	RTS	α^*	β^*	$\alpha^* + \beta^*$
DMU04	-1.0759	-1.0400	D	144533	65436	798869
DMU08	-M	-0.2948	D	81002	666210	747212
DMU13	-0.0023	1.0000	C	13.5	29	42.45
DMU15	-0.8446	-0.0916	D	45	23999	24044

The amounts of α^* and β^* for companies of Asia, Dana and Saman obtained from model (10) show the maximum amount of decreasing of the outputs and inputs of these units, so the classification of returns to scale do not change. Khavarmianeh is a company with a constant return to scale. To reach the maximum probable changes in the amount of inputs and outputs, models (11) and (12) are used to maintain the classification of returns to scale. It is shown in Table that $\alpha^* = \min\{\alpha^{1*}, \alpha^{2*}\}$ and $\beta^* = \min\{\beta^{1*}, \beta^{2*}\}$. In Table 9, the score of the efficiency of the insurance companies in constant two-step network model was listed whereas $(\theta_p{}^*, \lambda^*, \delta^*)$ is an optimal solution obtained from the model (6) for the unit under evaluation p th, $1\lambda^* + 1\delta^*$ are determined and the classification of the units is recognized based on the theorem 2.

Table 9: Analysis The Returns to Scale and The Sensitivity of Classification of Returns to The Scale of Efficient Units by The Classic Method**.**

DMU_p	Insurance Companies	$\theta_p{}^{overal*}$ model(6)	Min $1\lambda^* + 1\delta^*$	Max $1\lambda^* + 1\delta^*$	RTS	$\boldsymbol{\alpha^* + \beta^*}$	$\boldsymbol{\alpha^*}$	$\boldsymbol{\beta^*}$
DMU01	Novin	0.0621	15984/6210	15984/6210	D	883387	518163	365224
DMU02	Mihan	0.0102	590/7099	590/7099	D	1659201	1490421	168780
DMU03	Alborz	0.2287	0/9033	0/9033	I	2788191	624452	2163739
DMU04	Asia	0.1876	1/3948	1/3948	D	4969512	2237142	2323396
DMU05	Dey	0.3007	0/3533	0/3533	I	4276763	1326947	2949816
DMU06	Karafarin	0.1674	0/1674	0/1674	I	1633746	370564	1263182
DMU07	Taavon	0.1468	0/0694	0/0694	I	2820203	1545439	1266961
DMU08	Dana	0.2189	1/3863	1/3863	D	23428112	19483046	2342811
DMU09	Kosar	0.1471	0/2042	0/2042	I	2817903	1031535	1786368
DMU10	Ma	0.2342	0/2624	0/2624	I	2667125	680955	1986170
DMU11	Mellat	0.0680	0/0889	0/0889	I	316145	467579	269866
DMU12	Tejarateno	0.0570	8/3374	8/3374	D	641808	542824	98984
DMU13	Khavarmianeh	1/0000	1/0000	1/0000	C	23	10	13
DMU14	Parsian	0.2118	0/7263	0/7263	I	3553883	11012693	2541190
DMU15	Saman	0.4351	1/4351	1/4351	D	971984	365224	606760
DMU16	Sarmad	0.2617	0/1759	0/1759	D	644062	289441	354621
DMU17	Razi	0.1757	0/3818	0/3818	D	1254687	6129878	641709
DMU18	Pasargad	0.2106	0/5658	0/5658	D	2491223	857441	1633782

As shown in the third column of Table 9, the Khavarvianeh Insurance Company is the only efficient unit under Model (6). As shown in the third column of Table 9, the Middle East Insurance Company is the only efficient unit under Model 6. The fourth and fifth columns show the minimum and maximum $1\lambda^* + 1\delta^*$ values in the optimal answer obtained from Model (6), which if both of them have a value greater than 1, ie the unit under evaluation has the decreasing returns to scale. If both values are less than 1, the unit under evaluation has the increasing returns to scale. And if, like the Khavarmianeh Insurance Company, the amount of time is equal to 1, or one of them is greater and the other is less than 1, then the unit under evaluation has a constant return to scale. The sixth column of the Table shows that companies such as Novin, Mihan, Asia, Dana, Tejarateno, Saman, Sarmad and Razi have decreasing returns to scale, and khavarmianeh is the only unit with a constant return to scale. The other units have increasing returns to scale. The α^* and β^* values shown in the eighth and ninth columns of Table 9 are obtained from models (28), (33), (35) and (36) according to the classification of returns to unit scale, which are a sTable interval for changes in the initial inputs and final outputs of the units. So that the classification of returns to the scale of the units under evaluation does not change. For example,Tavon insurance company has decreasing returns to scale, if all initial inputs reduce by $\alpha;\ 0 \leq \alpha \leq \alpha^* = 5181630$ and all final outputs reduce by $\beta;\ 0 \leq \beta \leq \beta^* = 16870$, it still has decreasing return to scale. Asia insurance company has increasing returns to scale, if all intial inputs increase by $\alpha;\ 0 \leq \alpha \leq \alpha^* = 2337142$ and all final outputs increase by $\beta;\ 0 \leq \beta \leq \beta^* = 2323396$, it still has decreasing returns to scale The same can be said about the Khavarianeh company, if all initial inputs reduce or increase by $\alpha;\ 0 \leq \alpha \leq \alpha^* = 10$ and all final outputs reduce or increase by $\beta;\ 0 \leq \beta \leq \beta^* = 13$, it still has constant returns to scale.

5 Conclusion

In economics, returns to scale describe what happens to long-run returns as the scale of production increases, when all input levels including physical capital usage are variable (able to be set by the firm). The concept of returns to scale arises in the context of a firm's production function. It explains the long-run linkage of the rate of increase in output (production) relative to associated increases in the inputs (factors of production). In the long run, all factors of production are variable and subject to change in response to a given increase in production scale. While economies of scale show the effect of an increased output level on unit costs, returns to scale focus only on the relation between input and output quantities. There are three possible types of returns to scale: increasing returns to scale, constant returns to scale, and diminishing (or decreasing) returns to scale. If output increases by the same proportional change as all inputs change then there are constant returns to scale (CRS). If output increases by less than the proportional change in all inputs, there are decreasing returns to scale (DRS). If output increases by more than the proportional change in all inputs, there are increasing returns to scale (IRS). A firm's production function could exhibit different types of returns to scale in different ranges of output. Typically, there could be increasing returns at relatively low output levels, decreasing returns at relatively high output levels, and constant returns at some range of output levels between those extremes. In mainstream microeconomics, the returns to scale faced by a firm are purely technologically imposed and are not influenced by economic decisions or by market conditions (i.e., conclusions about returns to scale are derived from the specific mathematical structure of the production function in isolation). Most of real-life production technologies are multi-stage in nature. In this article, we focused on returns to scale in two-phase networks, proposed a new method for sensitivity analysis of returns to scale, and presented a new model based on Kao and Hwang's [8] model, and concepts presented by Sarparast et al. [28]. We presented the basic DEA model and the generic two-stage process and reviewedstudies conducted by Kao and Hwang [8] and Sarparast et al. [28] We then proceeded to introduce a new method for returns to scale classification and provided a new model to sensitivity analysis of returns to scale classification for two-stage network systems. we used datafrom Sarparast et al's article [28] and compared two methods. The new method, in addition to examining the classification of returns to scale of efficient units, also examines the classification of returns to scale of inefficient units. Also the method of Sarparast et al. [28] and the new method were examined in 18 insurance company in Iran. Each insurance company considered as a two-stage network. The first stage marketing activity witch utilized operating expense and insurance expense in order to earn direct written insurance and reinsurance premiums, and then is the second stage insurance company which use direct written insurance and reinsurance premiums for the purpose of underwriting profits and investment profit. The result of the two methods were compared. In classic method, only efficient DMUs have been considered for sensitivity analysis of returns to scale, but, in new method, returns to scale of all DMUs including efficient and inefficient DMUs can be analyzed. And this topic helps managers make decisions about how much confusion can be created in the values of inputs and outputs in a way that does not change the classification of returns to the scale of units.

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