Adv. Math. Fin. App., 2024, **9**(3), P.840-855

Advances in Mathematical Finance & Applications

www.amfa.iau-arak.ac.ir
Print ISSN: 2538-5569
Online ISSN: 2645-4610
Doi: 10.22034/amfa.2023.1973569.1830

Original Research

Designing a Model to Investigate the Process of Forming Cluster Fluctuations according to the Fractal Structure in Financial Markets

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ARTICLE INFO

Article history:
Received 2022-11-26
Accepted 2023-05-02

Keywords:
Factor-Based Modeling
Multi-level Multivariate Clustering
Cluster Volatility
Fractal Structure of the Market(FSM)

ABSTRACT

Cluster fluctuations and fractal structures are important features of space-time correlation in complex financial systems. However, the microscopic mechanism of creation and expansion of these two features in financial markets remains challenging. In the current research, by using factor-based model design and considering a new interactive mechanism called multi-level clustering, the formation process of cluster fluctuations was investigated with regard to the fractal structure of financial markets. For this purpose, the daily information of the final price of 150 shares that were accepted in the Tehran Stock Exchange, after the final screening, was entered in 5 sections with 30 shares in each section, in the desired model, and they were aggregated in three stock levels., sector and market were measured. Due to the fact that some investors have a longer investment horizon in the stock market and due to the limitation of the investigated time period, the maximum investment horizon of 1000 days has been determined in the model.In addition, the data studied in the research model are from August 2012 to September 2018. The findings of the research showed that the intensity of the tendency of collective behavior at the sector level is much stronger than at the market level. In addition, based on the findings of the research, it was determined that the distribution of simulation eigenvalues in three levels is significantly similar to the distribution of real data. Also, according to the investor's time horizon, the studied series always has a long-term memory for fluctuations. In addition, it was found that long-term memory is directly related to fractal dimensions. The findings of this research, in addition to providing a new insight into the space-time correlations of financial markets, show that multi-level conglomeration is one of the mechanisms for creating the microscopic microstructure of such markets. In other words, multi-level collective behavior is an important factor in the occurrence of cluster and fractal fluctuations in the market, and therefore, it should be considered from this point of view in the interpretation of the concept of risk and the definition of risk management strategies.

1 Introduction

Financial markets are one of the most complex social systems that, due to the ease of access and abundance of data, have recently attracted the attention of many researchers and analysts of other sciences

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such as physics and psychology. From the point of view of physicists, the dynamic behavior and structure of complex financial systems can be characterized by space-time correlation functions. From a theoretical perspective, space-time correlations are very important in understanding price dynamics and optimizing investment portfolios. Fractal structure and clustering of fluctuations are important features of space-time correlation, which are focused on in this article. Clustered fluctuations indicate that the variance of returns varies over time, and large fluctuations tend to follow large fluctuations and small fluctuations tend to follow small fluctuations in the same direction [1- 4]. On the other hand, the spatial structure of the market is described by examining the cross-sectional correlation of stocks [5-7], which quantitatively means While the yield series itself is uncorrelated, over a time span of several minutes to several weeks, the absolute value of the yield shows significant positive autocorrelation that slowly decays [8]. The purpose of the fractal theory is to study the behavior of the nonlinear periods of the sensitivity of the system to the initial conditions. Based on this, chaotic behavior is an inseparable part of a system, but if there is a specific and predictable pattern with a fixed periodicity in the behavior of the market; This pattern is the reason for the existence of long-term memory in the market and the absence of chaotic behavior [9]. One of the ways to investigate and analyze the fractal structure of the market is to use random matrix (RMT). Using random matrix theory, it is possible to determine a set of business sectors and visualize a mechanism like a portfolio for it [8]; In this way, the efficiency correlation matrix (C matrix) is analyzed to check the relationship between the components [10,11] and the largest value of the C matrix has a significant deviation from the distribution of the Wishart matrix [12]. This eigenvalue somehow represents the style of the market, that is, the correlation of the total market price and the corresponding eigenvector components is relatively the same for all stocks. In developed markets, each eigenvector corresponding to the largest value represents the stock of a particular sector. These large eigenvalues stand for sectors that can affect the fractal structure of the market.

In recent years, successful models have been proposed to study cluster fluctuations, including the studies of Ma et al. [10], Feng et al. [11] and Schmitt [12]. did One of the discussed aspects in the occurrence of cluster fluctuations is the effect of the behavior of market players on the occurrence of this phenomenon, in such a way that the results of previous studies in this regard are sometimes contradictory [5,9,11,13]. On the other hand, although many studies have been devoted to investigating the fractal structure of financial markets, its microscopic mechanism is still a matter of debate [4]. Both fractal structure and cluster volatility are important characteristics of stock markets, and the mechanism of these two characteristics in a model is still challenging; Therefore, the current research seeks to find out the question that how does interactive behavior at different levels of the market, according to its fractal structure, affect the occurrence of cluster fluctuations? One of the powerful simulation methods is agent-based modeling, which is widely used in various fields and is becoming a powerful validation tool for financial theories due to its controllability and strong repeatability [14-16]. In the current research, an agent-based model [11] combined with an interactive mechanism, called multi-level clustering, is used to investigate the fractal structure of the market with cluster fluctuations. The innovation of the current research, apart from using the factor-based model, which is far less used in the researches of the financial field, is the application of stratification in different layers of the market and using real market data, so that at the share level, Sector and market are applied in bulk. Considering the importance of analyzing and investigating cluster fluctuations of yield in the topics related to risk management and share pricing, it is obvious that the results of the present research can help researchers in expanding the theoretical and empirical literature in this field.

2 Theoretical Foundations and Research Background

In stock markets, time-oriented changes are also complex in stock prices and relationships between them. The price dynamics of a market naturally arise from individual stocks. Recent studies show that the price dynamics of a market can be divided into different impulse states such as the market and part of the market [14]. Naturally, the market state includes all stocks in a market and the fractal state or part is affected by the interaction of stocks in a part of the market. Therefore, the price dynamics of a multi-level market [15]. In financial markets, crowding is one of the collective behaviors in which investors become groups when making decisions, and these groups can be many or large. Since collective behavior of investors is necessary for price dynamics, it may also be multi-level [16]. For reasons such as information asymmetry or information distortion [17], investors may converge to the behavior of the masses or the market and apply their investment decisions by observing the behavior of other investors [18,19]. Then, a continuous trend of investment among the network of investors emerges, which can affect the occurrence of cluster fluctuations [18]. In fact, the dynamic structure of market information transmission can affect the homogeneity of investors, and this can further affect market prices and determine their volatility. It is obvious that different environments have different dynamic transmission structures [20]. Research on transmission dynamics shows that the structure of transmission can determine the speed and power of information transmission and therefore strongly influence the structure of investors in the capital market [21,22]. Therefore, multi-fractal structure analysis is used to provide a more accurate understanding of fractal lines in time series and small and large changes in the data structure [23].Because identifying the process governing stock market returns in order to make optimal decisions and reduce risk costs is very important for investors and financial policymakers, as well as the importance of analyzing markets and trying to better understand them after challenging the assumptions of an efficient market and The discovery of the universal truths of wide sequences, the clustering of fluctuations in financial time series, the tendency of analysts from models with completely random properties with a normal distribution towards modified Levy and multifractal models [17]. Multifractal has been used in different parts of the market.In addition, one of the most important tasks of financial economics is modeling and forecasting price fluctuations of risky assets. Because according to analysts and policymakers, price volatility is a key variable that helps to understand market fluctuations [25]. Therefore, analysts need to have a correct forecast of price volatility as a necessary input to perform tasks such as risk management, portfolio allocation, value at risk assessment and pricing of options and futures contracts. Because the volatility of the asset market plays an important role in monetary policies and the consequences of the recent financial crisis on the world economy showed how important the volatility of financial markets is in the implementation of effective monetary policy [20].Therefore, access to high-frequency time series and dealing with the two characteristics of "broad sequence" and "volatility clustering" for many financial markets is a large part of the literature in this subject area, and in fact, the main motivation of using multifractal models is to A lot is caused by these two features [23]. Also, since in financial markets, price volatility is a standard for measuring price fluctuations of a financial instrument over time, and volatility cannot be observed directly, and it must be measured using appropriate criteria or as part of a The asset pricing model estimated stochastically, therefore, multifractal models are suitable to overcome the full range of behavior of absolute moments [24].However, the studies show that the works that have been done in the field of multifractal models so far and the people who work in this field are few. The reason for this avoidance can be due to the fact that the first generation of multifractal models was untested and unfamiliar to financial economists. However, their non-causal principles regarding the construction of the hierarchical structure of dependencies seem to be very different from the well-known recurring time series models that have been used so far. In addition, multifractal includes scaling functions and distribution of Hölder components, which are not known in finance and economics, and the use of standard statistical models to deduce multifractal processes seems tiresome and impossible [22]. However, all these obstacles have disappeared with the emergence of the second generation of multifractal models (MSM and MRW), which behave better statistically and have a causal and repetitive nature. On the other hand, besides the better performance of these models in experimental applications, they have advantages such as having a clear definition of continuous asymptotes so that their continuous and discrete applications can be embedded in a consistent framework [24]. Therefore, the scientific approach to this approach, while increasing the scientific richness of this subject area, provides the basis for extensive and more professional research for accounting and financial science researchers.In short, the examination of the empirical background indicates that collective behavior, in addition to explaining many statistical characteristics of financial markets, also has a significant effect on their occurrence [21]. In addition, the extent of investors' tendency to collective behavior may be different for various reasons [24]. while investigating the effects of changes in the tendency to collective behavior, in the network structure of investors and in an epidemic manner, concluded that the increase in the tendency to collective behavior slows down the process of spreading the sale of products in the network and time postpones the spread of the outbreak; In addition, the tendency to collective behavior has an effect on the size and scope of the publication. The results of the experimental investigation by Duckangels and Rotando [23] also show that in stressful conditions of the market, the tendency to collective behavior decreases and the network structure of investors and the market plays a role in justifying the occurrence of crowding; Meanwhile, the empirical evidence of Balkillar et al. [25] indicates that the tendency to collective behavior exists more in chaotic and turbulent periods, and speculators' signals lead to a reduction in collective behavior. Also, Meng and Feng [25] found that in addition to the fact that both the quality of confidential information and the number of Tudeh leaders have a positive and significant relationship with the tendency of their followers to turn to Tudeh Wari, the intensity of collective behavior has an effect on the occurrence of market fluctuations. Ravindranath [23] by examining the fractal structure and identifying the transition time from the random efficient behavior of the market to the collective behavior, found that the higher the amount of fractal in the financial series, the more likely it is to start the process of transition to mass and bubble formation. The research results of Chen et al. [4] show that the multi-level clustering mechanism explains the creation of sectoral structure in financial markets. While presenting the three-state model of collective behavior in financial markets, Lux [28] state that in the presence of collective behavior, the observed statistical characteristics (such as wide tails and cluster fluctuations) in financial markets coincide with high frequency. Is. Venezia et al. [25], state that both amateur and professional investors tend to behave in a mass manner, and this tendency is more intense in professional investors.According to the mentioned cases, the collective behavior of the market factors has an effect on the statistical characteristics, including cluster fluctuations, but its mechanism in combination with the multi-level clustering mechanism to investigate the fractal structure of the market and cluster fluctuations is unclear[26]. In the current research, it is assumed that the collective behavior of virtual agents consists of three different levels: share, sector and market level, and then its effect on the market structure and cluster fluctuations is investigated.

3 Model Design and Parameter Determination

3.1. Multi-Level Collective Behavior

The current research model is based on daily transactions of virtual agents, i.e. buying, selling and holding shares. The model is on the number defined based of N virtual agents, n shares and n_{sec} sectors. Each section contains N/n_{sec} shares. Each virtual agent keeps only one share that is randomly selected from among n shares. A rational shareholder, taking into account the real records and share performance in different time scales, trades his shares, which is included in the model to match the real market and better describe the behavior of virtual agents. Considering the investment horizon, the weighted average return of the share, $\acute{R_\iota}$, which is the basis for the agent's decision to hold the share, is calculated as described in Model 1: [4]

$$\acute{R_\iota}(t) = K \sum_{l=1}^{L} \left[\xi_l \ \sum_{m=0}^{i-1} R_i(t-m) \right] \quad (1)$$

L is the maximum investment horizon; With the condition that $\sum_{l=1}^{L} \xi_L = 1$ is considered, the value of ξ_l will be as follows:

$$\xi_l = l^{-1.12} \Big/ \sum_{l=1}^{L} l^{-1.12} \quad (2)$$

As mentioned, the agent's trading decisions are based on past performance; Therefore, on day t+1, the virtual agent who holds stock i with an investment horizon of l days, $\sum_{m=0}^{L-1} R_i(t-m)$ is the basis for estimating the stock's previous performance. In order to ensure the consistency of the fluctuation value $\acute{R_\iota}(t)$with $R_i(t)$ and in other words to guarantee the condition $\left|\acute{R_\iota}(t)\right|_{max} = R_i(t)_{max}$, the value of the coefficient K is:

$$K = 1 \Big/ \left(\sum_{l=1} L \ \sum_{m=l} L \ \xi_m\right) \quad (3)$$

Obviously, if L=1 (the investment horizon is one day), then $\acute{R_\iota}(t)$ will be equal to $R_i(t)$.
As mentioned, the collective behavior of factors is measured and analyzed at the share, sector and market level. Therefore, the agent with share i with "agent of share i", the agent with share i belonging to section s, with "agent of section S" and the group consisting of "agent of share i" or "agent of section S" respectively with "group of share I" " and "section group S" are defined. Factors that own share i are first classified in "share group I"; The collective behavior at the level of share i is similar to that of the mass in other models that simulate only one share. Then these groups form larger groups in each section that represent collective behavior at the level of section S. Finally, since the sum of all sectors constitutes the market, groups at the level of sector S become larger groups that constitute collective behavior at the market level. Figure (1) shows the mechanism of multi-level collective behavior.

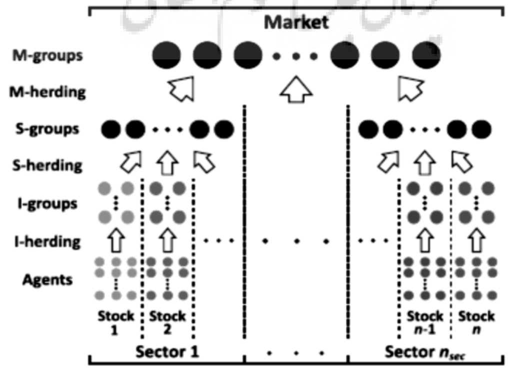

Fig. 1: The Mechanism of Multi-Level Collective Behavior [4]

≠ The degree of collective behavior at the share level:

In order to measure the degree of collective behavior at the level of "share group I" on day t, the value of D is defined as follows:

$$D_i^I(t) = {}^{\overline{n_\iota}(t)}\!/_{N_i} \tag{4}$$

where $D_i^I(t)$ represents the degree of collective behavior at the level of "share group I", $\overline{n_\iota}(t)$ represents the average of virtual agents in "share group I", $N_i$represents the number of virtual agents who own share i have. Because the collective behavior of the factors is based on their estimation from the performance records of the share, therefore $\overline{n_\iota}(t)$ is:

$$\overline{n_\iota}(t) = \left|R_\iota(t^{\prime} - 1)\right| \tag{5}$$

≠ Degree of collective behavior at the department level:

Agents randomly join each of the "share group I", after the collective behavior at the share level for all n available shares, the number of "share group I" available in the section S and the market M is:

$$N_S^I(t) = \sum_{i\epsilon s}\left[{}^{1}\!/_{D_i^I(t)}\right] \tag{6}$$

$$N_M^I(t) = \sum_{i}\left[{}^{1}\!/_{D_i^I(t)}\right] \tag{7}$$

where $N_S^I(t)$ and $N_M^I(t)$ represent the number of "share group I" in section S and market M, respectively. Because each sector is composed of shares with similar characteristics, at the sector level, the degree of collective behavior of factors is affected by the simultaneous movement of stock prices in the sector, that is, the share price in a sector rises and falls at the same time. The share group I in a sector forms larger groups that are called "Sector group". Thus, the degree of collective behavior at the level of S sector is:

$$D_S^S(t) = {}^{n \times (H_S - H_M)}\!\Big/_{N_S^I(t)} \tag{8}$$

where $D_S^S(t)$ is the degree of collective behavior of factors in the S, H_S and H_M sectors, respectively, is the correlation of stock prices in the S sector and in the M market, $n \times (H_S - H_M)$ also represents the net price correlation in is the S section. Other variables are also according to the previous definition.
The degree of collective behavior at the market level
The collective behavior of factors at this level is influenced by the simultaneous movement of the overall market price. S groups in different sectors represent the characteristics of the whole market and therefore they are placed in larger M groups. Thus, the degree of collective behavior at the market level is:

$$D_S^M(t) = {}^{n \times (H_M)}\!\Big/_{N_S^M(t)} \tag{9}$$

$$N_S^M = \overline{H} \times N_M^I(t) \Big/ H_S \quad (10)$$

$$\overline{H} = \left(\sum_S H_s \Big/ n_{sec}\right) \quad (11)$$

where $D_S^M(t)$ is the degree of collective behavior at the market level; Other variables are defined as before. After the occurrence of collective behavior in all three levels, all agents are placed in M groups; Each agent makes only one trading decision every day, and the trading decisions of agents in a group M have equal probability. Considering that each agent has a share i, therefore, the decision of agent α on day t is defined as follows:

$$\theta_\alpha(t) = \begin{cases} 1 & \text{buy} \\ -1 & sale \\ 0 & keep \end{cases} \quad (12)$$

According to the research of Chen et al. [4] and Feng et al. [11], the probability of buying and selling in group M is considered equal ($P_{sell} = P_{buy} = P$), so the probability of holding the share (P_{hold}) is equal to $1 - 2$P. In addition, the yield of share i is defined as the difference between demand and supply, so the yield of factor α for share i is equal to:

$$R_i(t) = \sum_{\alpha \in i} \theta_\alpha(t) \quad (13)$$

3.2. Estimation of Market and Sector Collective Behavior Parameters

As mentioned, H_S and H_M parameters are defined at the sector and market level, respectively, to calculate the degree of collective behavior (convergence). The simultaneous movement of stocks can be defined by the similarity of the sign and the range of the return volatility [22]; Thus, on day (t), in accordance with the symbol $r_i(t)$, the range of fluctuation of upward and downward trends on day t is defined as $V_-(t)$and$V_+(t)$:

$$\begin{cases} V_+(t) = \sum_{i, r_i(t)>0} {r_i}^2(t) \Big/ n_s \\ V_-(t) = \sum_{i, r_i(t)<0} {r_i}^2(t) \Big/ n_s \end{cases}, \quad n_s = n \Big/ n_{sec} \quad (14)$$

In addition, in order to compare the time series, the normalized efficiency values according to model (15) are included in the calculations;

$$r_i(t) = [R_i(t) - \langle R_i(t)\rangle] \Big/ \sigma \qquad , \sigma = \sqrt{\langle R_i(t)\rangle} - \langle R_i(t)\rangle^2 \quad (15)$$

where $\langle R_i(t)\rangle$is the average yield of share i in period t and (σ) is the standard deviation of yield. Since these two trends are usually not in balance and in each period, one of the two mentioned regimes dominates the stocks in each sector (n_s), so the dominant fluctuation range is $V^d(t)$and non-dominant $V^n(t)$ is defined as follows:

$$\begin{cases} V^d(t) = max[V_+(t), V_-(t)\,] \\ V^n(t) = min[V_+(t), V_-(t)\,] \end{cases} \quad (16)$$

It is obvious that the total oscillation amplitude is calculated through the difference of $V^d(t)$ and $V^n(t)$.

According to the mentioned cases, finally the degree of convergence of H_M and H_S is:

$$\begin{cases} H_M = \langle \zeta(t) \rangle . \langle V^d(t) - V^n(t) \rangle \,|market \\ H_S = \langle \zeta(t) \rangle . \langle V^d(t) - V^n(t) \rangle \,|s - sector \end{cases} \tag{17}$$

,

$$\zeta(t) = {n^d(t)}/{n_s} \tag{18}$$

where H_S and H_M are respectively the degree of convergence (collective behavior) at the sector level and market level, $\zeta(t)$ and $n^d(t)$ are respectively the percentage and number of shares in the dominant regime in period t and other variables according to the previous definitions Is. Simulation model The parameters of the model for simulation are according to Table (1):

Table 1: Parameters of the Simulation Model

Number of Shares (N)	150
Number of Sectors (SEC)	5
Number of Agents	1000
Maximum Investment Horizon (L)	1000
Probability of Buying or Selling (P)	0/363

The daily information of the final price of 150 shares accepted in the Tehran Stock Exchange after the final screening is entered into the model in 5 sections with 30 shares in each section. The data is from August 2012 to September 2018. According to the research of Feng et al. [13] and Chen et al. [4], the investment horizon of 94% of factors is less than 500 days. In order to adapt the simulator as much as possible to the characteristics of a real market, the number of factors should not be too small, considering that the number of virtual factors is considered to be 1000 factors, taking into account the number of 150 investigated shares, the values of N=150000 which provides the desired characteristics of the model, the values of N do not have a significant effect on the structure of the section and the distribution of the eigenvalue of the C matrix, and it has a partial effect on the range of autocorrelation of fluctuations. The investment horizon is from one day to more than one year [9]. Due to the fact that some investors have a longer investment horizon in the stock market and due to the limitation of the investigated time period, the maximum investment horizon of 1000 days has been determined in the model [10]. In order to determine the P parameter, first, the daily buying, selling and holding probability of a single investor in the real market is determined. As mentioned, it is assumed that the probability of buying and selling is equal, that is, $P_{buy} = P_{sell} = P$; According to the sample companies and in the investigated time period, the average holding percentage of institutional investors is equal to 60.3% and real shareholders is equal to 39.7%. If the average exchange rate between the number of shares held and traded by a real shareholder is assumed to be 1.64 [4], considering 250 trading days per year, the probability of daily trading is equal to:

$$\frac{1.64}{0.397 \times 250} = p_{buy}(t) + p_{sell}(t) \Rightarrow 2p = 0.0165 \Rightarrow p = 0.00826 \tag{19}$$

The agents in group M are related to each other, and in order to simulate the phenomenon of collective behavior, it is assumed that if agent a in group M decides to buy or sell shares, the whole group makes the same decision; Considering that the average number of factors in a group is $n \times H_M$, therefore, the probability of buying or selling a group is equal to:

$$P = 1 - (1 - p)^{n \times H_M} \quad (20)$$

In this way, the probability of buying (selling) equal to 0.363 is calculated. In order to balance the model, the yield of the first level of the investment horizon L, for all n shares, is zero, and therefore the number of the first 500 data points of the yield has been removed. By estimating H_M and H_S, the simulation model estimates the time series $R_i(t)$ for each stock. For each share i on day t, the value of $\acute{R_i}(t)$ is calculated according to equation (1) and then the value of $D_i^I(t)$ is calculated according to equations (4) and (5). Virtual agents at the stock level randomly join one of $1/_{D_i^I(t)}$ "stock group I"; Then "Partition group I" in section S to one of $1/_{D_s^S(t)}$ "Part group S" and finally part group S, joins $1/_{D_S^M(t)}$ group M. After simulating the convergence in the three mentioned levels, the agents in a "group M" make a similar decision (buy/sell and hold) with the probability of P_{buy}. P_{sell} . P_{hold} hold are adopted, so the yield of each share on day t is calculated according to equation (13). Groups are dissolved after a decision is made; By repeating this process, the time series of returns for all stocks in the market is obtained.

≠ Simulation Results

Table (2) shows the amount of mass in each sector and the whole market. As mentioned, the H_S and H_M parameters respectively express the intensity of the trend of simultaneous movement at the level of the sector and the market, which at the level of 5 sectors is between 0.414 and 0.546 and for the whole market it is equal to 0.363. The mentioned values, which are based on the real data of the sample companies, show that the intensity of collective behavior at the sector level is much stronger than at the market level.

Table 2: Values of parameters H_S and H_M (degree of mass) in sections 1 to 5 and the whole market.

	H_1	H_2	H_3	H_4	H_5	H_M
Amount of mass	0.491	0.414	0.438	0.431	0.546	0.363

Source: Research calculations

Also, before examining the cluster fluctuations in each of the series, descriptive and econometric statistics of the distribution of the simulated yield series related to each of the sectors and the market as a whole are presented. Table (3) shows the descriptive statistics of each of the sectors and the whole market; It is worth mentioning that the efficiency of the estimated prices has been estimated using the following equation:

$$r_t = \ln(p_t) - \ln(p_{t-1}) \quad (21)$$

t represents the yield of the asset price at time t, pt represents the asset price at time t, and p_{t-1} represents the asset price at time t-1. As shown in table (3), in terms of normality, the distribution of the efficiency series of any of the sectors is not normal, this is confirmed by the Jarek statistic; In addition, according to the coefficient of curvature, the series of each sector and of course the whole market has a wide tail, which is a proven feature in financial markets including Iran [13,16]. According to the value of Advanced Dickey-Fuller (ADF) statistic and McKinnon's critical limit for the said statistic, the entire return series at the level of the whole market is at the 5% error level. Also, based on the findings of table (3), in the BDS test, in order to measure the non-linear dependence of the series, the rejection of the null hypothesis indicates the existence of a non-linear time series; Considering the value of the Z statistic of the mentioned test and rejecting the null hypothesis with significance at the 5% error level, therefore, the simulated efficiency series has a non-linear dependence and is not independent and identical (IID). In addition, the results of the increasing Lagrange (LM) test indicate the presence of the arch effect at a significance level of 5% and therefore indicate the presence of cluster fluctuations. In order to further investigate market fluctuations, Hurst's statistic is estimated; Hurst's power is obtained by calculating the slope of the $\log(R/S)/\log(N)$ curve and using the regression method in the domain of N changes. The highest value obtained represents the average period of alternating circulation of the pattern [9]. The value of the mentioned statistic shows the measurement of long-term memory and fragility of a time series [14]. According to Hurst's output, the value of the Hurst statistic is equal to 0.5, indicating the existence of a completely random series, the value of the statistic, in the range of 0.5 to 1, indicates the existence of long-term memory and the positive statistic values, but less than 0.5, indicate the unstable and unsustainable series. As shown in table (3), the value of the Hearst statistic of the market series is 0.561224, which is slightly higher than 0.5, and therefore indicates the existence of long-term memory of the market. It is worth noting that Hurst's power is calculated as a moving average for every 1000 recent data (according to the investor's time horizon) [19]. In addition, as shown in Figure (3), the range of the mentioned statistic is always higher than 0.5, which shows that the investigated series has always had a long-term memory for fluctuations.

Table 3: How to Distribute the Eleventh Series and Econometric Statistics

	Sec1	Sec2	Sec3	Sec4	Sec5	Market
N	37726	-0.00542	0.00077	0.00127	-0.00167	0.00085
Mean	0.000873	0.0175	0.0201	0.0224	0.0156	0.0193
Std	0.0307	-0.342	0.153	-1.720	-0.591	-0.742
Skewness	0.821	6.179	12.147	10.151	8.472	9.572
Kurtosis	11.184	1685.4	4498.2	2162.7	1789.5	1935.2
Jarq-Bera	2551.3	0.00542	0.00077	0.00127	-0.00167	0.00085
ADF						***-32.15
BDS						***42.88
LM						$(R^2 = 95/87)$ ***115.66
Hurst						***0.56122
Fractal dimension						1.43877

***Significance at the 5% error level

Source: Research calculations

Figure (3) shows the trend of Hurst's average power during the period under review;

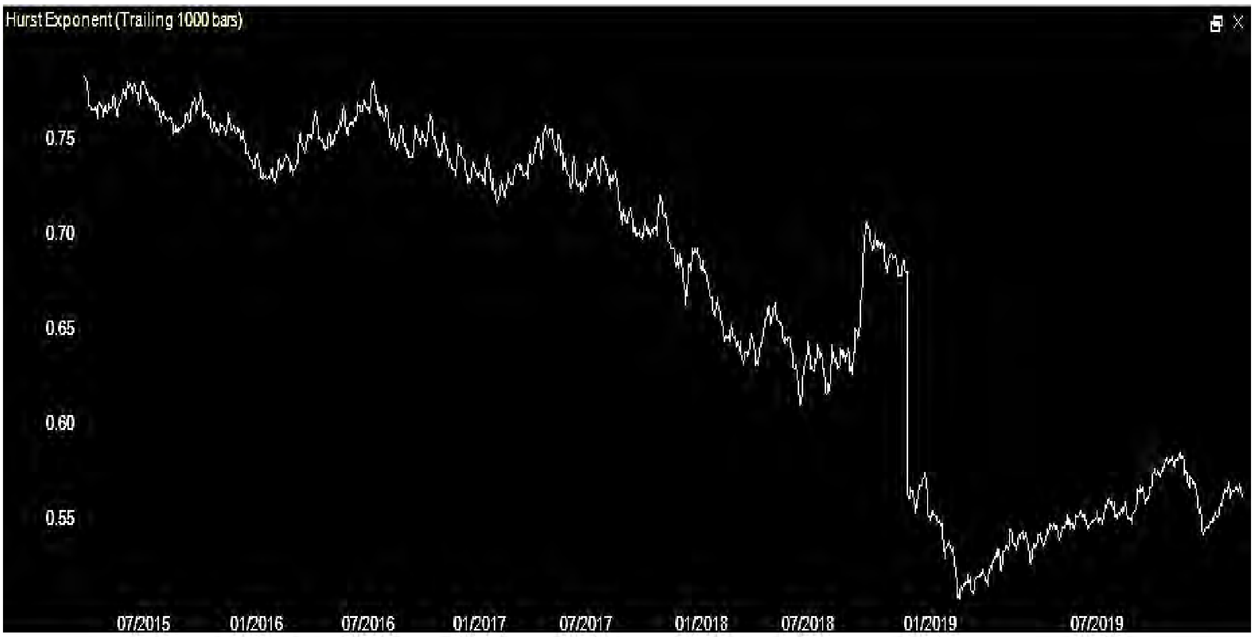

Fig 3: Hurst power series of simulated efficiency
Source: Research calculations

Based on the research findings reflected in Figure (3), long-term memory is directly related to fractal dimensions. In fact, the fractal dimension of a time series shows its fluctuation and instability. The relationship between the fractal dimension and the Hurst exponent of a time series is equal to [20]:

$$D = 2 - H \qquad (22)$$

Obviously, the fractal dimension of 1.5 represents the IID series and has a random step; Therefore, if the fractal dimension is in the range of 1 to 1.5, the series will have long-term memory. According to Hurst's power, the fractal dimension of the simulated series is calculated as 1/438776 and therefore the series has a long-term memory. Finally, the calculation of the cluster fluctuations of share i is done according to the autocorrelation function of the fluctuations as follows: [11]

$$A_i(t) = {\left[\langle |r_i(\acute{t})| |r_i(t + \acute{t})| \rangle - \langle |r_i(\acute{t})| \rangle^2\right]}\Big/{A_i^o} \qquad (23)$$

$$A_i^o = \langle |r_i(\acute{t})|^2 \rangle = \langle |r_i(\acute{t})| \rangle^2 \qquad (24)$$

where $A_i(t)$is the autocorrelation value of the fluctuations of share i in period t and $\langle |r_i(\acute{t})| \rangle$ represents the average correlation of returns during period $\acute{t}$. Therefore, the autocorrelation function of fluctuations for the estimated series is: $A(t) = {\sum_i A_i(t)}\big/{n}$.

. Figure (4) shows the average autocorrelation functions for the simulated yield series; As can be seen, the values for the simulation series match the real data.

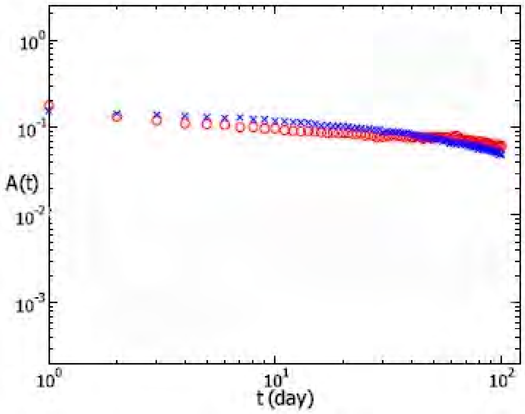

Fig.4 :Average autocorrelation function of real data volatility and simulated yield series.
Source: Research calculations

In order to determine the spatial structure of the series, first, the simultaneous bi-correlation matrix C is calculated, each row of which consists of:

$$C_{ij} = \langle r_i(t)\, r_j(t) \rangle \ . \tag{25}$$

where $\langle r_i(t)\, r_j(t) \rangle$ is the average correlation during the period t and C_{ij}represents the correlation of returns of stocks i and j. According to the definition, C is a symmetric matrix with the condition $C_{ii} =$ 1and the values of other C_{ij}domains are in the range [-1.1]. The first, second, and third eigenvalues of matrix C are respectively $\lambda_0.\lambda_1.\lambda_2$, and u_i(λ), which are determined by the components of the eigen-vector ui(λ). The simulation results in comparison with the real data are shown in Figure (5). According to the historical data, for λ_0, the eigenvector components are almost uniform in all sections, but the eigenvector λ_1 is strongly affected by section (5) and the eigenvector λ_2 is affected by section (1); It is worth mentioning that these features are also observed in the simulated time series.

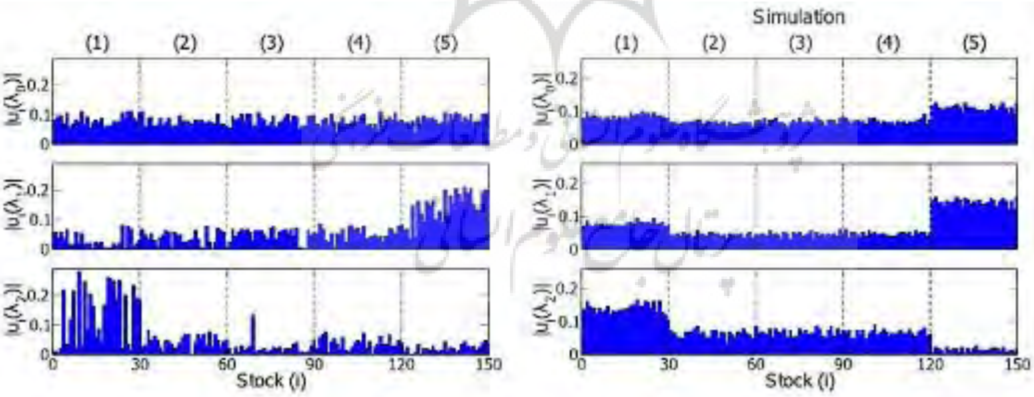

Fig .5: The absolute value of the components of the eigenvector u_i *(λ)* of the triple values of the bi-correlation matrix C calculated based on historical data and simulated returns
Source: Research calculations

Figure (6) shows the distribution of the three eigenvalues of the correlation matrix C and the simulated values. As can be seen, the eigenvalues of $(\lambda_0.\lambda_1.\lambda_2)$ of matrix C are respectively (5.13, 7.45, 26.01) and the simulation values are (3.82, 7.93, 24.62) and therefore it indicates that the distribution of eigenvalues The simulation is remarkably similar to the real data distribution.

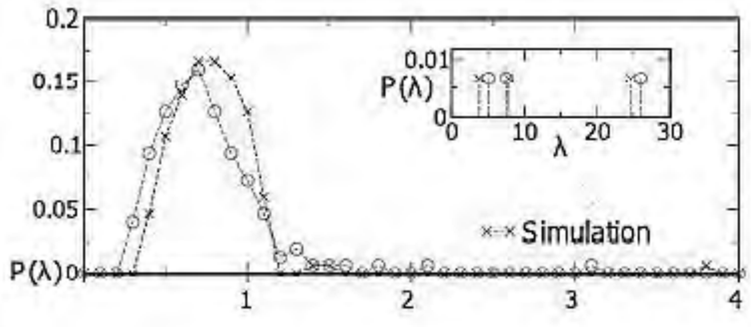

Fig.6: Probability distribution of eigenvalues of correlation matrix C and simulated values.
Source: Research calculations

4 Discussion and Conclusion

Identifying the process governing stock market returns in order to make optimal decisions and reduce risk costs is very important for investors and financial policy makers. The importance of analyzing the markets and trying to understand them better caused that after challenging the assumptions of the efficient market and discovering the universal truths of wide sequences, clustering of fluctuations in financial time series, analysts from models with completely random properties with The normal distribution tends towards Levy and multifractal models. This led to the expansion of the use of multifractal models in different parts of the market. In financial markets, fractal structure and cluster fluctuations are important characteristics of space-time correlation; But the mechanism of creating such cases still has many ambiguous dimensions, and especially how to combine these two features in one model is still challenging. Therefore, the current research, based on the aforementioned necessity and with the aim of microscopically investigating the mechanism of price dynamics in financial markets, including the Iranian capital market, and using factor-based modeling, the interactive mechanism of multi-level collective behavior of investors and its effect on cluster fluctuations and simulated and studied the fractal structure of the market. For this purpose, the daily information of the final price of 150 shares accepted in the Tehran Stock Exchange after the final screening, in 5 sections with 30 shares in each section, was entered into the desired model. Due to the fact that some investors have a longer investment horizon in the stock market and due to the limitation of the time period under investigation, the maximum investment horizon of 1000 days is determined in the model and their mass is measured in three levels of stock, sector and market. and checked. The findings of the research show that according to the appropriate matching of simulation results with experimental data, the mechanism of multi-level collective behavior, while requiring the fractal (segmental) structure of the market, also requires a new dimension from the point of view of space-time correlation. , provides the microstructure of financial markets.In other words, based on the findings of the research, it was determined that multi-level collective behavior is an important factor in the occurrence of cluster and fractal fluctuations in the market, and special attention should be paid to this type of behavior in the interpretation of the concept of risk and risk management strategies. In addition, based on the findings of the research, it was determined that in the context of price movement, the dynamic structure of information transmission in a market full of mass investors is of fundamental importance. Also, due to the mass investor's sensitivity, the transmission of market information can increase the homogeneity of investors, and in fact, the positive deviation of the

Hurst power from 0.5 and the presence of the fractal dimension contrary to 1.5 is due to the homogeneity of the investor structure. The results of the current research are consistent with the studies of of Feng et al [11], Chen et al. [4], Christoffersen et al. [19] and Cheng [21]. In any case, the use of the proposed model in this research, in which the factor-based approach is used, and this approach is less used in researches in the financial field, is one of the special features of this article. Another advantage of this research is the use of real data and the application of stratification in different layers of the market at the levels of share, sector and market, while in this article, considering the importance of analyzing and investigating cluster fluctuations of returns in related topics In terms of risk management and share pricing, we tried to present an operational model and simultaneously improve the theoretical literature of this field.In any case, despite the fact that the multilevel behavior of investors in financial markets has been common as a theoretical concept since the 1990s. But the research that examines the causes or factors that stimulate multi-level behavior, so far, very little has been done. In addition, most of the topics of the research conducted so far are relatively undeveloped and most of the empirical researches are still trying to answer the question of whether multilevel behavior can be found in the capital market or not. And this issue has hindered understanding the development of a comprehensive theoretical framework of multi-level behavior.At the same time, it should also be noted that the model proposed in this research, from an empirical point of view, should also be noticed by capital market activists and observers; Also, managers should study the behavior of fractal organizations carefully and try to implement and implement those features in the financial market. Of course, in this path, the shortcomings and obstacles must also be identified and addressed. On the other hand, researchers are suggested to study and investigate multi-level behaviors and the effects of price fluctuations in all kinds of complex financial systems in order to finally achieve a certain pattern in this field. It is also suggested to researchers to investigate the tendency towards multi-level collective behavior among both amateur and professional investors and compare their results with each other. According to the obtained results, it is suggested to the managers of the financial market that measures should be taken to reduce the emotional and irrational behavior of investors in crisis conditions, in order to help the stability of the market by establishing rules and increasing the diversity of financial instruments. Investors are also suggested to base their investment strategies on an adaptive approach with the country's economic conditions. Also, according to the predictability in the market, especially during economic development periods, consider long-term investment horizons in addition to short-term horizons in your choices.The main limitation of this research was the small number of researches that had been done in this field so far in Iran and other countries. Therefore, the generalization of the results of this research should be done with caution.

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