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Case Study

Modelling Robust Optimization in DEA with Ratio Data: A Case Study of Commercial Banks

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ABSTRACT

In many practical problems, we face situations where the data ratio is important for the decision-maker (DM). Data envelopment analysis ratio-based (DEA-R) and ratio analysis models are presented to deal with the above issue in data envelopment analysis (DEA). If the data is uncertain, it is no longer possible to use the basic DEA-R and ratio analysis models to evaluate the efficiency of decision-making units (DMUs). In this paper, we will first discuss robust optimization modelling based on DEA-R models. In this regard, we consider a case where the inputs have an uncertain numerical value and the outputs have certain values. In the following, we present the ratio analysis model based on the set of common weights of all the ratios of input to output components and obtain this model for robust optimization. To show the validity of the proposed approach, we use it to evaluate the efficiency of 38 excellent banks that compete in the global market and compare the results of the proposed approach in this paper with the results of previous approaches.

1 Introduction

DEA is a non-parametric linear programming technique developed by Charnes et al. [14] to evaluate the efficiency of a set of homogeneous DMUs that use several inputs to produce several outputs. Classical DEA models evaluate the efficiency of units under evaluation in the form of envelopment and multiplier models. Envelopment DEA models depict the DMU under evaluation on the efficiency frontier of the production possibility set (PPS). These models obtain the target DMU corresponding to the inefficient DMUs. Inefficient DMUs must reach the level of their inputs and outputs at the level of inputs and outputs from their corresponding target DMUs in order to obtain the desired performance level. The multiplier models consider a weight corresponding to each of the input and output components. These models obtain the optimal weights in such a way that the efficiency score of the under evaluation DMU is maximized compared to other DMUs. Based on traditional multiplier models, the efficiency score of DMUs is not evaluated under the same conditions. (Dyson and Thanassoulis, [21]). To deal with the above issue, DEA models were presented based on a set of common weights. The main goal of DEA models with the set of common weights is to evaluate all DMUs under the same conditions. These models obtain the efficiency score of all DMUs with only a set of common weights corresponding to each of the input and output components, while the weight corresponding to each of the input and

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output components can be different for distinct DMUs based on multiplier models (Kao and Hung [37]). Omrani [49] presented an approach to measuring the efficiency score of DMUs in the presence of uncertain data based on robust optimization. In his approach, he used the set of common weights of input and output components in the DEA model when we evaluate DMUs in the same condition. Salahi et al. [60] presented DEA models in envelopment and multiplier forms in the presence of uncertain data. Their approach obtained a different rank than the approach presented by Omrani [49] for all DMUs. At first, they presented their model for the CCR model in an envelopment form based on robust optimization. In the following, they proposed a model based on the method presented by Kao and Hung [37] for generating a set of common weights. Their model evaluates DMUs in the same conditions. They showed the superiority of their models over Omrani's model [49] statistically. In classic DEA models, all inputs and outputs have definite and exact and certain values. However, in the real world, we face many cases where the input and output data do not have exact values. As we know, a small disturbance in the data may have a great impact on the optimality and feasibility of the solution of these models. Therefore, the development of these traditional DEA models is important to deal with uncertain data. Many studies have been done to deal with uncertain data in DEA, for example, we can mention the following studies:

DEA with fuzzy data (Hatami-Marbini et al. [29]; Zhou et al. [77]), DEA with random data or stochastic DEA (Olesen and Petersen [47]), DEA based on the bootstrap technique (Odeck [45]), Imprecise DEA in the presence of interval data (Zhu [78]), DEA based on the robust DEA (RDEA) optimization technique (Sadjadi and Omrani [57]; Tavana et al. [67]). Imprecise DEA received a flurry of interest from its introduction where the uncertain data are characterized by bounded intervals. The main drawback of imprecise DEA methods is that the upper and lower efficiency bounds are merely determined and no information within the efficiency interval is provided (Zhu [78]). The other approach is stochastic DEA approach. This approach is based upon stochastic optimization to deal with stochastic uncertainty. This approach requires the determination of a probability distribution function for noisy data, although in real-world problems this assumption may not work properly because there is insufficient empirical evidence to choose a specific distribution function. Furthermore, it is difficult to apply stochastic optimization in DEA when the sample size is small (Olesen and Petersen [47]). Bootstrap DEA investigates the statistical properties of the efficiency measures estimated by DEA models using bootstrap algorithms. This approach has two main difficulties: (1) the number of iterations required in this algorithm and (2) finding a suitable value of the smoothing parameter (Odeck [45]). The other approach is fuzzy DEA. It handles vague and ambiguous data such as linguistic variables. The existing methods can be classified into (1) α −cut-based approach, (2) tolerance approach, (3) fuzzy ranking approach, (4) possibility approach. Notably, the applicability of fuzzy DEA is often questionable because there is no clear way to define the membership functions of fuzzy inputs and outputs, as well as little theoretical and computational complexity in fuzzy mathematical programming problems (Hatami-Marbini et al. [29]). The robust DEA is a suitable approach to deal with some drawbacks of the aforesaid methods. In this paper, we consider uncertain data in the framework of robust optimization. The robust optimization technique was first presented by Soyster [65] and then developed by Mulvey et al. [43], Ben-Tal and Nemirovski [7–9], and Bertsimas and Sim [10-12]. Ben-Tal and Nemirovski [9] replaced an uncertainty linear programming problem with its robust counterpart (RC) programming problem and obtained robust solutions to an uncertain LP problem by solving a RC. This approach can estimate the robust solution probability when the uncertain coefficients follow some natural probability distributions. It was also more conservative than the Soyster [65] approach. The weakness of the Ben-Tal and Nemirovski [9] approach is that it turns LP models such as DEA into second-order conical programming (for more details, see Ben-Tal and Nemirovski [9]). Bertsimas and Sim (BS) [13] and Bertsimas, Pachamanova, and Sim [10] introduced a new approach where the DM can make a trade-off between the level of protection for the constraints and the degree of conservatism of the solution. Contrary to the approach of Ben-Tal and Nemirovski [9], the RC problem proposed by Bertsimas and Sim [13] is a linear programming (LP) problem. At first, Sadjadi and Omrani [57] introduced DEA model in the presence of uncertain data. In DEA literature, DEA models in the presence of uncertain data are called Robust DEA (RDEA) model. Subsequently, several studies were conducted in this field. Sajjadi and Omrani [58] applied a bootstrap model for estimating the efficiency score of communication companies. Salahi et al. [60, 62] proposed a robust optimization approach based on common weights in DEA. Omrani et al. [50] applied RDEA to evaluate supply chain performance in the presence of uncertain data. Toloo and Mensah [68] presented robust optimization with non-negative variables based on DEA. Hatami-Marbini and Arabmaldar [33] developed RDEA for cost and revenue concepts in DEA. Toloo et al. [69] proposed a general RDEA based on the duality concept. Dehnokhalaji et al. [20] proposed a box-uncertainty in DEA and a robust performance measurement framework. In classic DEA models, data are absolute numerical values. However, in the real world, we may face many cases where the data is a ratio. For example, in evaluating the efficiency of university units, the ratio of the number of graduating students in an academic course to the total number of students enrolled in that course can be considered ratio data. The studies conducted to deal with ratio data in DEA include three categories:

The first category includes articles that use ratio data as decimal numbers in the model. These article use of ratio and volume data in the model simultaneously. Among the studies done in this field, articles of Olesen et al. [46, 48] can be mentioned. The second category includes articles that use the simultaneous presence of ratio and volume data in the model but the ratio data are replaced in the model as fractional numbers that have a specific numerator and denominator. These models use the numerator and denominator of fractions in the evaluation model. Among the studies carried out, we can mention the articles of Emrouznejad and Amin [22] and Hatami-Marbini and Toloo [31]. In the third category, we can refer to articles where the data are not inherently ratio and we only use the ratio of input to output components or vice versa as ratio data. These articles were initially presented as ratio analysis models and later as DEA-R models. Thanassoulis et al. [66] used ratio analysis and DEA models as a tool to evaluate the performance of DMUs. They also compared the results of these models. Fernandez-Castro and Smith [23] proposed ratio analysis model as linear model for evaluating the performance of DMUs. In the discussion of ratio analysis and DEA-R models, the input and output parameters are not ratios, and all parameters are absolute, but their ratios are used to define efficiency and calculate the efficiency score. Also, Chen and McGinnis [16] proposed a mathematical relationship between the DEA technique and ratio analysis. They described the relationship between metric ratio efficiency and technical efficiency in detail. Liu et al. [38] first proved the relationship between ratio analysis models and output-oriented DEA models without explicit inputs in VRS technology. Then, they compared the efficiency scores in ratio analysis and DEA for 15 basic research institutes in the Chinese Academy of Sciences (CAS). On the other hand, Despic et al. [19] developed DEA-R models by combining DEA and ratio analysis. Then, Wei et al. [74] evaluated 21 medical centres in Taiwan and, in addition to ranking the DMUs, studied the relationship between weights in multiplier models in DEA-R. In DEA-R theory, the efficiency function is defined as a weighted sum of outputs to inputs components (the output-oriented) or vice versa. Gerami et al. [27] developed slack based measure (SBM) DEA models based on DEA-R models to gain efficiency and super efficiency scores. It can be said that the main contribution of this paper is as follows: In this paper, DEA-R models presented in the input orientation under uncertainty conditions based on the BS approach. In this regard, we obtain the RC of the DEA-R model in the envelopment form. The efficiency score obtained corresponding to each DMU under the conditions that inputs and outputs have uncertain and certain values, respectively. In the following, the ratio analysis models examined and developed based on the set of common weights of all the ratios of input to output components. Also, the ratio analysis model obtained with a set of common weights under uncertainty conditions based on the BS approach. Also, The RC model is obtained corresponding to this model. The robust ratio analysis model is provided to find a set of common weights from all the ratios of input to output components corresponding to all the DMUs when input components have uncertain values but output components have certain values, respectively. In the end, to show the validity of the proposed approach in the paper, it has been used to evaluate the efficiency of 38 branches of Premium Bank that operate in the global competitive market and compare the results of the proposed approach in this paper with the results of previous studies in this field.The rest of this paper is organized as follows: In the second section, we review the previous studies done in the following subjects: the set of common weights, robust DEA, and ratio data in DEA. In the third section, DEA-R models examined and presented under uncertainty based on the BS approach. In the fourth section, ratio analysis models are provided based on a set of common weights from all ratios of input to output components corresponding to all DMUs, and we obtain these models in uncertainty based on the BS approach. In the fifth section, the proposed approach used in this paper to evaluate the efficiency of 38 excellent banks that are active in the global competitive market, and at the end, the results of this paper are presented.

2 Literature Review

In this section, we review the studies conducted in the following subjects: the set of common weights, robust DEA, ratio data in DEA.

2.1 The set of Common Weights in DEA

Roll et al. [55-56] proposed one of the early studies in the field of assigning a set of common weights in DEA. They presented an approach to finding an appropriate bound on the weights to reduce the deviation in importance according to the same factor for different DMUs. Roll and Golany [55] presented a conceptual framework for the behaviour of weights in DEA. They put a bound on the weights and then limited the range of the weights. Kao and Hung [37] presented a nonlinear programming model to generate a common set of weights for all DMUs. Their method obtains a vector of efficiency scores closest to the corresponding efficiency scores generated from traditional DEA models. Makui et al. [39] presented an approach to obtaining the set of common weights based on multi-objective linear programming (MOLP) and obtaining the efficiency scores of the DMUs. Chen et al. [15] presented a model to generate a set of common weights to evaluate the efficiency of DMUs based on changes in inputs and outputs. Zohrehbandian et al. [79] improved the method of Kao and Hung [37] by using the multiple criteria decision-making (MCDM) model and presented a nonlinear model to find the set of common weights. They showed that their method has a high correlation with the method of Kao and Hung [37]. Hosseinzadeh Lotfi et al. [35] proposed a suitable method for finding the set of common weights in DEA and used it for allocating fixed cost. Hatami-Marbini et al. [35] developed common weight models for central resource reduction and target setting. Jahanshahloo et al. [36] proposed a suitable method for fixed cost allocation in DEA based on the principles of the set of common weights and efficiency deviation. Ghazi and Hosseinzadeh Lotfi [24] presented a model based on the set of common weights in DEA for evaluating and allocating budgets from natural gas distribution companies in Iran. Zarei Mahmoudabadi and Emrouznejad [76] presented balanced performance evaluation models under uncertainty based on models with common weight structures in the presence of fuzzy data. Soltanifar et al. [64] presented a new approach for resource allocation and target setting in DEA based on a set of common weights and multi-objective optimization. They calculate the efficiency of DMUs based on the ratio of output to input weights .They also used bargaining theory to evaluate DMUs. Afsharian et al. [1] presented a study and review of a collection of articles that used the set of common weights in DEA models to evaluate performance. They separated these models based on functional aspects and model form. Contreras et al. [17] and Contreras [18] reviewed the studies conducted in the DEA articles based on the set of common weights technique. Hatami-Marbini and Saati [32] presented network efficiency evaluation models based on the method of a set of common weights in DEA. Omrani et al. [51] presented different models to evaluate the efficiency of DMUs based on the strategy of the best and worst solution in DEA. They used the DM's superiority information in the common weights model.

2.2 Robust DEA

Sadjadi and Omrani [57] introduced a new DEA model for dealing with uncertain data. These models called robust DEA (RDEA) in the literature DEA. They applied their model for evaluation of electric distribution companies in presence of uncertain input and output data. Sadjadi and Omrani [58] used a bootstrap model to estimate the efficiency of communication companies in Iran. Their approach was used to deal with imprecise data in an uncertain environment. Wang and Wei [70] presented RDEA models in the presence of uncertain data based on the MCDM programming structure. Sadjadi et al. [59] presented a robust super efficiency DEA model for evaluating and ranking provincial gas companies. They showed that the model presented by them is superior to the chance-constrained programming models from the computational point of view, and all the DMUs with uncertain inputs and outputs can be ranked. Omrani [49] presented an ideal planning technique with robust optimization to study a set of common weights in DEA. He presented a DEA model with the structure of the set of common weights in the presence of uncertain data based on a robust optimization approach. He presented a RC to the DEA model. He obtained the ideal solution for each DMU and then obtained a set of common weights for all DMUs using the optimal programming technique. He used the provided approach to evaluate the efficiency of provincial gas companies in Iran. Salahi et al. [60, 62] presented a robust optimization approach based on the set of common weights in DEA. They presented the RC corresponding to the CCR model in envelopment and multiplier forms. They showed that these models are equivalent. They showed that by choosing the common weights strategy, we can have an evaluation under the same conditions for all DMUs. In this regard, they presented robust solutions corresponding to their RDEA model based on the set of common weights and compared the results of their approach with the results of previous approaches by considering the ideal solution. Omrani et al. [50] presented supply chain performance evaluation models in the presence of uncertain data based on robust optimization. Arabmaldar et al. [2] presented a robust super-efficiency DEA model. Salahi et al. [61] obtained Russell's non-radial measure in DEA to measure the performance of interval and ellipsoidal uncertainty sets in the best and worst cases. Toloo and Mensah [68] presented robust optimization with non-negative variables based on DEA. They presented a new framework for RDEA with the idea of reducing the computational aspect. They presented RDEA based on reduced RC. They used their approach to evaluate the performance of 250 banks operating in the European Union. They showed that the models provided by them reduce the number of calculations by 50% to solve DEA problems with non-negative variables. Their approach would reduce the number of calculations without changing the optimal solution. Shirazi and Mohammadi [63] developed a new RDEA approach in the presence of undesirable outputs. They calculated the efficiency of the airlines and presented a corrective plan to the top managers of these companies to improve their performance. Salahi et al. [62] developed RDEA models in the envelopment and multiplier forms. At first, they proposed a new DEA model in the constant returns to scale (CRS) technology in the presence of uncertain data. They obtain the RC model corresponding to the CCR model. In the following, using the Kao and Hung [37] method to determine a set of common weights, they obtained the CCR multiplier model in the presence of uncertain data. They showed that it is possible to obtain more correct ranks for the DMUs based on the provided approach than the method provided by Omrani [49]. They increased the discriminating power of the weights based on the provided models. Arabmaldar et al. [3] presented a robust worst-practice interval DEA model in the presence of non-discretionary factors. They reviewed the worst-practice frontier (WPF) DEA model fundamentally and presented a new robust WPF-DEA model in the presence of interval data and non-discretionary factors. Their approach was presented based on robust optimization and considering uncertain data. They used Monte-Carlo simulation to calculate the conformity of rankings in the efficiency interval as well as determine the price of robustness in order to select the worst-performing suppliers. Wu et al. [57] provided models for evaluating efficiency in presence of data uncertainty in DEA. Nasrabadi et al. [44] presented RDEA models based on interval data. Further they analyzed the robustness of the efficiency score. Hatami-Marbini and Arabmaldar [33] presented cost and revenue efficiency evaluation models in the presence of uncertain data based on robust optimization. They chose two different scenarios in this regard. In the first scenario, the input and output data are uncertain, and in the second scenario, the price corresponding to the input and output is also uncertain. They presented a general framework for measuring economic efficiency based on robust optimization. Toloo et al. [69] presented a general framework for RDEA based on the concept of duality in optimization problems. They presented robust optimization for the fractional form of DEA models, and these models were presented in input and output orientation and presented the relationships between the solutions to primal and dual problems in RDEA. They used two perspectives: pessimistic and optimistic. Omrani et al. [51, 53] presented a novel best-worst method for robust DEA. They provided information on the superiority of the DM in the process of evaluating the efficiency of the DMUs in an uncertain environment. The proposed model endeavors to provide a novel efficiency score that is more reliable and compatible with real problems by taking advantage of the best-worst method to use experts' opinions and RDEA to model uncertainty. Hatami-Marbini et al. [34] developed a new robust productivity growth and efficiency measurement in presence of undesirable outputs and applied it for the evidence from the oil industry. Omrani et al. [54] presented a robust credibility DEA model with a degree of fuzzy confusion. At the beginning, they presented a fuzzy credibility approach to building fuzzy data and a fuzzy set and used robust optimization to solve it. They considered the degree of turbulence in an exact and fuzzy manner. They used their approach to evaluate the efficiency of 28 hospitals in northwest Iran. Omrani et al. [52] proposed a robust credibility DEA model with Fuzzy Perturbation degree and applied it for evaluating hospitals performance. Dehnokhalaji et al. [20] presented a new RDEA model in the presence of uncertain data based on a robust optimization approach. They obtained the efficiency score of DMUs with interval inputs and outputs. They presented two different methods for ranking DMUs, which were different from the previous methods. They applied their approach to evaluate the performance of a set of hospitals. Arabmaldar et al. [4] proposed a generalized robust DEA model based on directional distance function. They obtained PPS in the robust environment. Arabmaldar et al. [5] proposed a new Robust DEA with variable budgeted uncertainty. They obtained a novel robust DEA model with variable budgeted uncertainty that is less conservative than extant robust DEA models. Also, they suggested a solution for specifying the probabilistic bounds for constraint violations of the uncertain parameters in robust DEA models.

2.3 Ratio data in DEA

In general, in DEA, we encounter three categories of ratio data. In the first category, the vector corresponding to inputs and outputs of the DMUs can be considered as $DMU_j = (X_j^v, X_j^R, Y_j^v, Y_j^R), j = 1, \dots, n$. X_j^v and Y_j^v are input and output components that have absolute numerical values and are non-ratio numbers. Also, X_j^R and Y_j^R are input and output components that have ratio values. In this way, we have volume (absolute) numbers in some input and output components and ratio numbers in others. The studies in the first category include those that have ratio data in the form of decimal numbers that are obtained from the result of dividing two numbers. The numerator and denominator of fractions corresponding to these decimal numbers may not be available. From this category, the studies conducted by Olesen et al. [46, 48] can be referred, as pointed out. Olesen et al. [46, 48] developed constant and variable returns to scale technology in the presence of ratio data. They showed that in the presence of ratio data, the convexity axiom in building the PPS is not satisfying to produce in the presence of absolute and ratio data at the same time. They presented the axioms for building the PPS in these conditions and obtained efficiency evaluation models in radial and non-radial forms.

The studies of the second category include studies that have ratio data in the form of fractions, and the numerator and denominator corresponding to these fractions are available. From the studies conducted to deal with the data in this category, the studies conducted by Emrouznejad and Amin [22], Hatami-Marbini, and Toloo [31] can be refereed. In this category, the ratio data can be expressed as $X_j^R = \frac{n_j}{d_j}$ and $Y_j^R = \frac{p_j}{q_j}$. We consider that the numbers n_j, d_j, p_j, and q_j corresponding to the numerator and denominator of these fractions are absolute and accessible numbers. For example, in evaluating the efficiency of hospital units, one of the input components can be considered as the ratio of the number of successful operations performed to the total number of operations performed in a period of time. This input component can be considered ratio data. In this case, n_j represents the number of successful operations and d_j represents the total number of operations. Emrouznejad and Amin [22] showed that the convexity axiom of the underlying assumptions for estimating the PPS is not established in the presence of ratio data in variable returns to scales (VRS) technology. They developed performance evaluation models in the presence of ratio data. They presented two different strategies as the first and second solutions. In the first solution, they put the numerator and denominator with ratio input components as numbers n_j, and d_j as new input and output components in the evaluation model, respectively. Also, they put the numerator and denominator corresponding to the ratio output component in the form of numbers p_j, and q_j as new output and input components in the evaluation model, respectively. In their second solution, they put the convex combination of n_j numbers, i.e., numerators, and the convex combination of d_j numbers, i.e., denominators, in the model separately. Hatami-Marbini and Toloo [31] showed that the efficiency evaluation models proposed by Emrouznejad and Amin [22] have problems. They developed these models to deal with the presence of ratio data and included the numerator and denominator with ratio data as discretionary and nondiscretionary data in the evaluation model. They solved the problems related to the models presented by Emrouznejad and Amin [22].

In the third category, the inputs and outputs of the DMUs can be considered as follows:

$DMU_j = (X_j^v, Y_j^v)$ where X_j^v and Y_j^v are input and output components that have absolute numerical values and are non-ratio numbers. In this category, the ratio numbers as $\frac{X_j^v}{Y_j^v}, \frac{Y_j^v}{X_j^v},$ numbers cab be considered. It should be noted that X_j^v and Y_j^v are available input and output components. The numerator and denominator corresponding to these fractions are the input and output components. In this category, we define ratio data as the ratio of all input components to output components, or vice versa. In our evaluation model, we use all these ratios to evaluate efficiency. The convexity axiom of the underlying assumptions for estimating the PPS is established in the presence of ratio data in the third category. Several papers have been presented evaluating the efficiency of DMUs for dealing with ratio data in this category. Despic et al. [19] presented DEA-R models in the input orientation to measure efficiency of DMUs based on all ratios of input to output components. They obtained the relationship between the efficiency score of the CCR and DEA-R models. Wei et al. [72–74] obtained DEA-R models in the input orientation based on the ratios of input to output components They showed that DEA-R models compared to traditional DEA models that have an intrinsic weight restriction. DEA-R models can avoid problems such as efficiency underestimation and pseudo-inefficiency in traditional DEA models such as CCR. They presented new relationships between the DEA and DEA-R models. They showed that the efficiency scores obtained from DEA-R models are greater than or equal to their corresponding scores obtained from DEA models in the input orientation.

Mozaffari et al. [40] presented the relationship between DEA-R models and DEA models without obvious inputs. Mozaffari et al. [41] developed DEA-R models to evaluate cost and revenue efficiency. Mozaffari et al. [42] obtained the PPS in the presence of ratio data from two CRS and VRS technologies. They obtained efficient faces for these sets. Gerami et al. [25] obtained two-stage network efficiency evaluation models based on DEA-R models. They chose three strategies: black box, free link, and fix link. Gerami et al. [26] evaluated the efficiency of hospital supply chains based on DEA-R models. They presented a general model in the presence of all ratios of input to output components in a multi-stage network structure. Gerami et al. [27] presented non-radial DEA-R models as slack-based measure DEA-R (SBM-DEA-R) models in input and output orientations. They obtained the relationship between the DEA-R, SBM-DEA-R, and DEA models. They obtained super efficiency models corresponding to SBM-DEA-R models to rank efficient DMUs. Wanke et al. [71] developed DEA-R models to evaluate the efficiency of a two-stage network in the presence of random data. They also included undesirable outputs in the evaluation model. Ghiyasi et al. [28] proposed a novel inverse DEA- model with application in hospital efficiency evaluation.

3 Robust optimization DEA-R

In this section, the RC of DEA-R model in the input orientation based on the BS approach are presented. Consider n DMUs as $DMU_j = (X_j, Y_j) \in R^{m+s}$, $j = 1, \dots, n$. These DMUs use the input vector $X_j = (x_{1j}, \dots, x_{mj}) \in R^m$, $j = 1, \dots, n$, to generate the output vector

$Y_j = (y_{1j}, \dots, y_{sj}) \in R^s$, $j = 1, \dots, n$. Also suppose that $DMU_o = (X_o, Y_o)$ is the DMU under evaluation. The DEA-R model in the input orientation is as follows (Gerami et al. [27]).

$$\begin{aligned} \theta^{R^*} &= Min\ \theta^R \\ s.t.\ & \sum_{j=1}^{n} \lambda_j \left(\frac{x_{ij}}{y_{rj}}\right) \le \theta_R \left(\frac{x_{io}}{y_{ro}}\right), \quad i = 1, \dots, m,\ r = 1, \dots, s, \\ & \sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \ge 0,\ \ j = 1, \dots, n. \end{aligned} \tag{1}$$

Definition 3.1 $DMU_o = (X_o, Y_o)$ is called DEA-R efficient if $\theta^{R^*} = 1$ otherwise $DMU_o = (X_o, Y_o)$ is DEA-R inefficient.

In robust optimization which used in this paper, we consider only the input vector value as uncertain numbers and the output vector values as certain numbers. Also suppose that N_i^x are the set of indices related to the DMUs that have an uncertain value in the i-th component of their input. In this paper, the BS approach used to define input components that have uncertain values. Suppose that the scaled deviation of a nominal input value from the i-th input of DMU_j i.e. x_{ij} is as $\Gamma_{ij}^x = \frac{\tilde{x}_{ij} - x_{ij}}{\hat{x}_{ij}}$, $i = 1, \dots, m$, $j = 1, \dots, n$. $\tilde{x}_{ij}$ is the i-th uncertain input from DMU_j and $\hat{x}_{ij} = e^x x_{ij}$ is the estimation accuracy. e^x shows the uncertainty level (percentage of perturbation). We assume that scaled deviation means Γ_{ij}^x is a random variable that has an unknown value. However, it has a symmetrical distribution and its values are in the range [-1,1]. If the scaled deviation variable is equal to zero, that is, $\Gamma_{ij}^x = 0$, then $\tilde{x}_{ij}$ has a certain value. In this case, $j \notin N_i^x$. The aggregated scaled deviation for the i-th expression, i.e. $\sum_{j=1}^n \Gamma_{ij}^x$ can choose any value in the interval $[-|N_i^x|, |N_i^x|]$. But it bounded to $\sum_{j=1}^n \Gamma_{ij}^x < h_i^x$, where h_i^x is a parameter and is not necessarily an integer and any value in the interval $[0, |N_i^x|]$. In this paper, its value considered to be an integer for simplicity. The purpose of choosing the parameter h_i^x is to adjust the level of the robustness of the proposed approach against the level of conservatism of the solution and is called the budget of uncertainty of constraint i. The DEA-R model considered in the input orientation in the presence of input components from DMUs that have uncertain values. Also, the output components considered that to be certain numbers.

Theorem 3.1 The RC of DEA-R model in the input orientation based on BS approach is as follows.

$$\begin{aligned} {\theta_{Ro}^R}^* &= Min\ \theta_{Ro}^R \\ s.t.\ & \sum_{j=1}^n \lambda_j \left(\frac{x_{ij}}{y_{rj}}\right) - \theta_{Ro}^R \left(\frac{x_{io}}{y_{ro}}\right) + \sum_{j \in N_i^x} \alpha_{ij}^x + h_i^x \gamma_i^x \le 0, \quad i = 1, \dots, m,\ r = 1, \dots, s, \\ & \sum_{j=1}^n \lambda_j = 1, \quad \lambda_j \ge 0, \quad j = 1, \dots, n, \\ & \alpha_{ij}^x + \gamma_i^x \ge \lambda_j \left(\frac{\hat{x}_{ij}}{y_{rj}}\right), \quad i = 1, \dots, m,\ j \in N_i^x, j \ne o,\ r = 1, \dots, s, \\ & \alpha_{io}^x + \gamma_i^x \ge (\theta_{Ro}^R - \lambda_o) \left(\frac{\hat{x}_{io}}{y_{ro}}\right), i = 1, \dots, m, o \in N_i^x, \\ & \alpha_{ij}^x \ge 0, \gamma_i^x \ge 0, i = 1, \dots, m,\ j = 1, \dots, n. \end{aligned} \tag{2}$$

Proof: Consider the first constraint of model (1). By replacing uncertain input values as $\tilde{x}_{ij} = \Gamma_{ij}^x \hat{x}_{ij} + x_{ij}$ in this constraint , we will have:

$$\sum_{j=1}^n \lambda_j \left(\frac{\tilde{x}_{ij}}{y_{rj}}\right) - \theta_{Ro}^R \left(\frac{\tilde{x}_{io}}{y_{ro}}\right) = \sum_{j=1}^n \lambda_j \left(\frac{x_{ij}}{y_{rj}}\right) - \theta_{Ro}^R \left(\frac{x_{io}}{y_{ro}}\right) + \sum_{\substack{j \ne o \\ j \in N_i^x}} \lambda_j \Gamma_{ij}^x \left(\frac{\hat{x}_{ij}}{y_{rj}}\right) + (\lambda_o - \theta_{Ro}^R) \Gamma_{io}^x \left(\frac{\hat{x}_{io}}{y_{ro}}\right), i = 1, \dots, m,\ r = 1, \dots, s, \quad o \in N_i^x, \quad \text{and} \tag{3}$$

$$\sum_{j=1}^n \lambda_j \left(\frac{\tilde{x}_{ij}}{y_{rj}}\right) - \theta_{Ro}^R \left(\frac{\tilde{x}_{io}}{y_{ro}}\right) = \sum_{j=1}^n \lambda_j \left(\frac{x_{ij}}{y_{rj}}\right) - \theta_{Ro}^R \left(\frac{x_{io}}{y_{ro}}\right) + \sum_{\substack{j \ne o \\ j \in N_i^x}} \lambda_j \Gamma_{ij}^x \left(\frac{\hat{x}_{ij}}{y_{rj}}\right),\ i = 1, \dots, m,\ r = 1, \dots, s, \quad o \notin N_i^x,$$

Where $\sum_{j \in N_i^x} |\Gamma_{ij}^x| \le h_i^x$ and $-1 \le \Gamma_{ij}^x \le 1$.

Therefore, the RC corresponding to model (1) will be as follows:

$$
\begin{aligned}
& Min\ \theta_{Ro}^{R} \\
& \quad s.t.\ \ \sum_{j=1}^{n} \lambda_j \left(\frac{x_{ij}}{y_{rj}}\right) - \theta_{Ro}^{R}\left(\frac{x_{io}}{y_{ro}}\right) + \\
& \max_{\substack{\sum_{j\in N_i^x}\left|\Gamma_{ij}^x\right| \le h_i^x, \\ -1\le \Gamma_{ij}^x \le 1}} \left\{\sum_{\substack{j\neq o \\ j\in N_i^x}} \lambda_j \Gamma_{ij}^x \left(\frac{\hat{x}_{ij}}{y_{rj}}\right) + (\lambda_o - \theta_{Ro}^{R})\Gamma_{io}^x\left(\frac{\hat{x}_{io}}{y_{ro}}\right)\right\} \le 0,\ \ i=1,\dots,m,\ r=1,\dots,s,\ o\in N_i^x, \qquad (4) \\
& \quad \sum_{j=1}^{n} \lambda_j \left(\frac{x_{ij}}{y_{rj}}\right) - \theta_{Ro}^{R}\left(\frac{x_{io}}{y_{ro}}\right) + \\
& \max_{\substack{\sum_{j\in N_i^x}\left|\Gamma_{ij}^x\right| \le h_i^x, \\ -1\le \Gamma_{ij}^x \le 1}} \left\{\sum_{\substack{j\neq o \\ j\in N_i^x}} \lambda_j \Gamma_{ij}^x \left(\frac{\hat{x}_{ij}}{y_{rj}}\right)\right\} \le 0,\ \ i=1,\dots,m,\ r=1,\dots,s,\ o\notin N_i^x \\
& \quad \sum_{j=1}^{n} \lambda_j = 1,\quad \lambda_j \ge 0,\ \ j=1,\dots,n.
\end{aligned}
$$

As can be seen, model (4) includes inner optimizations problems. To solve these problems, some variables of the outer problem are considered as constant values in the inner problem. Also, the value of the optimal objective function of the inner problem is a part of the outer problem. To solve inner problems in model (4), the problem solved in such a way that model (4) becomes a linear programming (LP) model. In inner problems from model (4), the variables θ_{Ro}^{R} and λ_j are decision variables from model (4) and we put them as parameters in inner problems. In inner problems from model (4), Γ_{ij}^x considered as decision variables. In order to solve inner problems, we use the property of strong duality in optimization problems. (For more detail see Bazaraa, Jarvis, and Sherali [6]). The absolute function is removed in the inner problem of model (4). First, the following feasible region of inner problem substitutes as follows.

$\left\{\Gamma_{ij}^x \middle| \sum_{j\in N_i^x}\left|\Gamma_{ij}^x\right| \le h_i^x, -1 \le \Gamma_{ij}^x \le 1\right\}$ by $\left\{\Gamma_{ij}^x \middle| \sum_{j\in N_i^x}\Gamma_{ij}^x \le h_i^x, 0 \le \Gamma_{ij}^x \le 1\right\}$.

Therefore, the first constraint of model (4) rewrite in an equivalent form as follows:

$$
\sum_{j=1}^{n} \lambda_j \left(\frac{x_{ij}}{y_{rj}}\right) - \theta_{Ro}^{R}\left(\frac{x_{io}}{y_{ro}}\right) + \max_{\substack{\sum_{j\in N_i^x}\Gamma_{ij}^x \le h_i^x, \\ 0\le \Gamma_{ij}^x \le 1}} \left\{\sum_{\substack{j\neq o \\ j\in N_i^x}} \lambda_j \Gamma_{ij}^x \left(\frac{\hat{x}_{ij}}{y_{rj}}\right) + (\lambda_o - \theta_{Ro}^{R})\Gamma_{io}^x\left(\frac{\hat{x}_{io}}{y_{ro}}\right)\right\} \le 0,
$$
$$
i=1,\dots,m,\ r=1,\dots,s,\ o\in N_i^x, \qquad (5)
$$

The inner problem in relation (5) can be written as the following LP.

$$
\begin{aligned}
\max\ & \sum_{\substack{j\neq o \\ j\in N_i^x}} \lambda_j \Gamma_{ij}^x \left(\frac{\hat{x}_{ij}}{y_{rj}}\right) + (\lambda_o - \theta_{Ro}^{R})\Gamma_{io}^x\left(\frac{\hat{x}_{io}}{y_{ro}}\right) \\
& \sum_{j\in N_i^x}\Gamma_{ij}^x \le h_i^x, \\
& \Gamma_{ij}^x \le 1,\quad j\in N_i^x, \qquad (6) \\
& 0 \le \Gamma_{ij}^x,\ j\in N_i^x.
\end{aligned}
$$

In model (6), λ_j and θ_{Ro}^{R} are parameters and Γ_{ij}^x, $j\in N_i^x$ are decision variables. The dual model (6) is written as follows.

$$
\begin{aligned}
& \min \ \sum_{j\in N_i^x} \alpha_{ij}^x + h_i^x \gamma_i^x \\
& S.t. \quad \alpha_{ij}^x + \gamma_i^x \geq \lambda_j \left(\frac{\hat{x}_{ij}}{y_{rj}}\right), i = 1, \dots, m, \ r = 1, \dots, s, \ j \in N_i^x, j \neq o \\
& \qquad \alpha_{io}^x + \gamma_i^x \geq (\theta_{Ro}^R - \lambda_o) \left(\frac{\hat{x}_{io}}{y_{ro}}\right), i = 1, \dots, m, \ r = 1, \dots, s, \ o \in N_i^x, \\
& \qquad \alpha_{ij}^x \geq 0, h_i^x \geq 0, i = 1, \dots, m, j \in N_i^x.
\end{aligned} \tag{7}
$$

α_{ij}^x and γ_i^x are dual variables corresponding to the first and second constraints of model (6). Therefore, constraint (5) will be as follows.

$$
\begin{aligned}
& \sum_{j=1}^n \lambda_j \left(\frac{x_{ij}}{y_{rj}}\right) - \ \theta_{Ro}^R \left(\frac{x_{io}}{y_{ro}}\right) + \sum_{j\in N_i^x} \alpha_{ij}^x + h_i^x \gamma_i^x \leq 0, \quad i = 1, \dots, m, \ r = 1, \dots, s, \\
& \qquad \alpha_{ij}^x + \gamma_i^x \geq \lambda_j \left(\frac{\hat{x}_{ij}}{y_{rj}}\right), \quad i = 1, \dots, m, \ j \in N_i^x, j \neq o, \ r = 1, \dots, s, \\
& \qquad \alpha_{io}^x + \gamma_i^x \geq (\theta_{Ro}^R - \lambda_o) \left(\frac{\hat{x}_{io}}{y_{ro}}\right), = 1, \dots, m, o \in N_i^x, \\
& \qquad \alpha_{ij}^x \geq 0, \ \gamma_i^x \geq 0, \ i = 1, \dots, m, \ j \in N_i^x, \lambda_j \geq 0, \ j = 1, \dots, n.
\end{aligned} \tag{8}
$$

Similarly, the second constraint of model (4) will be as follows:

$$
\begin{aligned}
& \sum_{j=1}^n \lambda_j \left(\frac{x_{ij}}{y_{rj}}\right) - \ \theta_{Ro}^R \left(\frac{x_{io}}{y_{ro}}\right) + \sum_{j\in N_i^x} \alpha_{ij}^x + h_i^x \gamma_i^x \leq 0, \quad i = 1, \dots, m, \ r = 1, \dots, s, \\
& \qquad \alpha_{ij}^x + \gamma_i^x \geq \lambda_j \left(\frac{\hat{x}_{ij}}{y_{rj}}\right), \quad i = 1, \dots, m, \ j \in N_i^x, \ r = 1, \dots, s, \\
& \qquad \alpha_{ij}^x \geq 0, \ \gamma_i^x \geq 0, \ i = 1, \dots, m, \ j \in N_i^x, \ \lambda_j \geq 0, \ j = 1, \dots, n.
\end{aligned} \tag{9}
$$

By placing relations (8) and (9) in model (4), model (2) is obtained, and the proof is complete.■

4 Robust ratio analysis based on the set of common weight

In this section, at first, the ratio analysis model presented to evaluate the efficiency of the DMU under evaluation in the presence of ratio data. We consider the weight corresponding to the ratio of the i-th input component to the r-th output component as w_{ir}. The ratio analysis model to evaluate the efficiency of $DMU_o = (X_o, Y_o)$ as the DMU under evaluation is as follows: (Chen and McGinnis [16]).

$$
\begin{aligned}
& \min \ \sum_{r=1}^s \sum_{i=1}^m w_{ir} \ \left(\frac{x_{io}}{y_{ro}}\right) \\
& s.t. \ \sum_{r=1}^s \sum_{i=1}^m w_{ir} \ \left(\frac{x_{ij}}{y_{rj}}\right) \geq 1, \\
& \qquad\qquad w_{ir} \geq \epsilon, i = 1, \dots, m, \ r = 1, \dots, s.
\end{aligned} \tag{10}
$$

Definition 4.1 DMU_o is called ratio-efficient based on model (10) if an optimal solution of model (10) is $w_{ir}^* > 0, = 1, \dots, m, \ r = 1, \dots, s$, so that the score of the optimal objective function of model (10) is equal to one.

Now, we present multiple objective linear programming (MOLP) for calculating the efficiency score of all DMUs based on the ratio analysis model. This model is based on the set of common weights of all ratios of input components to all output components as follows:

$$\begin{aligned} &\min \left\{\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir} \left(\frac{x_{ij}}{y_{rj}}\right), j=1,\dots,n\right\} \\ &s.t.\ \sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir} \left(\frac{x_{ij}}{y_{rj}}\right) \geq 1, \\ &\qquad w_{ir} \geq \epsilon,\ i=1,\dots,m,\ r=1,\dots,s. \end{aligned} \tag{11}$$

Considering that the model (11) is a MOLP model, the weighted sum method can be used to solve this model. Therefore, the model (11) will be in the form of a single-objective model as follows:

$$\begin{aligned} &\min \sum_{j=1}^{n}\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir} \left(\frac{x_{ij}}{y_{rj}}\right) \\ &s.t.\ \sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir} \left(\frac{x_{ij}}{y_{rj}}\right) \geq 1, \\ &\qquad w_{ir} \geq \epsilon,\ i=1,\dots,m,\ r=1,\dots,s. \end{aligned} \tag{12}$$

Model (12) determines the set of common weights of all ratios of input components to all output components in such a way that the efficiency of all DMUs is maximized based on these weights. According to the strategy of a set of common weights, all DMUs are evaluated based on a set of common weights. In other words, all DMUs are evaluated in the same condition. Let $(w_{ir}^*: i=1,\dots,m,\ r=1,\dots,s)$ is an optimal solution of model (12). The efficiency score DMU_j resulting from model (4.3) defined based on a set of common weights as follows.

$$\frac{1}{\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}^* \left(\frac{x_{ij}}{y_{rj}}\right)} \tag{13}$$

Definition 4.2 DMU_o is called ratio-efficient based on model (12) if $\frac{1}{\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}^* \left(\frac{x_{io}}{y_{ro}}\right)} = 1$.

In this section, the method of Kao and Hung [33] used to find the set of common weights from all ratios of input components to all output components from all DMUs based on the robust optimization approach under conditions where input components are uncertain and output components are certain. The norm 1 applied to measure the distance between the robust DEA-R efficiency scores obtained from model (2). The efficiency score obtained from the ratio analysis model based on the set of common weights, namely model (12). The model (14) presented with the aim of minimizing the distance between the efficiency scores obtained from models (2) and (12) based on the set of common weights of all ratios of input components to all output components as follows:

$$\min \sum_{j=1}^{n} \left| \theta_{Rj}^{R} - \frac{1}{\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{x_{ij}}{y_{rj}}\right)} \right|$$
$$S.t. \quad \sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{x_{ij}}{y_{rj}}\right) \geq 1, \qquad (14)$$
$$w_{ir} \geq \epsilon,\ i = 1, \ldots, m,\ r = 1, \ldots, s.$$

In model (14), w_{ir} is the weight corresponding to the ratio of the i-th input component to the j-th output component and θ_{Rj}^{R} is the robust DEA-R efficiency score obtained from model (2) in the evaluation of DMU_j. ϵ is a positive parameter that prevents the weights from becoming zero? This number is non-Archimedean. The objective function score of the model (14) is the same for all values of ϵ. Now, in order to transform the model (14) into a linear model, we use the fact that

$$\theta_{Rj}^{R} \geq \frac{1}{\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{x_{ij}}{y_{rj}}\right)}$$

or equivalently

$$\left(\theta_{Rj}^{R}\left(\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{x_{ij}}{y_{rj}}\right)\right) - 1\right) \geq 0.$$

The above relationship is established because

$$\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{x_{ij}}{y_{rj}}\right) \geq 0.$$

Therefore, we have the following linear programming which minimizes sums the deviations namely $\left(\theta_{Rj}^{R}\left(\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{x_{ij}}{y_{rj}}\right)\right) - 1\right)$ from zero for all DMUs.

$$\min \sum_{j=1}^{n} \left(\theta_{Rj}^{R}\left(\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{x_{ij}}{y_{rj}}\right)\right) - 1\right)$$
$$S.t. \quad \sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{x_{ij}}{y_{rj}}\right) \geq 1, \qquad (15)$$
$$w_{ir} \geq \epsilon,\ i = 1, \ldots, m,\ r = 1, \ldots, s.$$

In the models presented in the third section in the envelopment form, the slack corresponding to the ratios of input to output components are not considered for the convenience of presenting the formulation of the robust optimization model based on the DEA-R model. However, we suppose that the weight corresponding to the ratio of the i-th input component to the r-th output component is greater than or equal to ϵ in the ratio analysis model in the multiplier form.

Now we obtain the formulation of the robust optimization problem corresponding to model (15). Assume that IN_j^x represents the set of input indices from DMU_j that have uncertain values. Also assume that Γ_{ij}^{x} is also defined as in the second section.

Also, assume that $\sum_{i \in IN_j^x} \left|\Gamma_{ij}^{x}\right| \leq \beta_j^x,\ j = 1, .., n,$ where the parameter $\beta_j^x \in \left[0, \left|IN_j^x\right|\right],\ j = 1, .., n,$ is not necessarily an integer, but for simplicity, we assume that it is an integer. Now, in Theorem (4.1), an RC of model (15) are provided.

Theorem 4.1 Assume that $i \in IN_j^x$ and the uncertain input components are defined as $\tilde{x}_{ij} = \Gamma_{ij}^{x}\hat{x}_{ij} + x_{ij}$ as in the second section, and the output components also have definite and certain values. The RC model corresponding to model (15) based on BS approach will be as follows.

$$\begin{aligned}
&\min \sum_{j=1}^{n}\left(\theta_{Ro-j}^{R}\left(\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{x_{ij}}{y_{rj}}\right)\right)-1\right)+\sum_{i\in IN_j^x}\varphi_{ij}^{x}+\psi_j^x\beta_j^x\\
&S.t.\quad \sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{x_{ij}}{y_{rj}}\right)-\sum_{i\in IN_j^x} t_{ij}^{x}-f_j^x\beta_j^x \geq 1,\ j=1,..,n, \qquad (16)\\
&\qquad t_{ij}^{x}+f_j^x \geq -\left(\sum_{r=1}^{s} w_{ir}\left(\frac{\hat{x}_{ij}}{y_{rj}}\right)\right),\ \ i\in IN_j^x,\ \ j=1,..,n,\\
&\qquad \varphi_{ij}^{x}+\psi_j^x \geq \left(\sum_{r=1}^{s} w_{ir}\left(\frac{\hat{x}_{io}}{y_{ro}}\right)\right)\theta_{Rj}^{R},\ \ i\in IN_j^x,\ \ j=1,..,n,\\
&\qquad w_{ir}\geq \epsilon,\ i=1,\dots,m,\ r=1,\dots,s,\\
&\qquad \varphi_{ij}^{x},\ \psi_j^x,\ t_{ij}^{x},\ f_j^x \geq 0.
\end{aligned}$$

Proof: Assume that the input components are uncertain as $\tilde{x}_{ij}=\Gamma_{ij}^{x}\hat{x}_{ij}+x_{ij}$, $i=1,\dots,m,\ r=1,\dots,s,$ and the output components have certain values. These values will replace in model (15). The objective function and constraints of the model (15) will be as follows.

$$\left(\theta_{Rj}^{R}\left(\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{x_{ij}+\Gamma_{ij}^{x}\hat{x}_{ij}}{y_{rj}}\right)\right)-1\right)=\left[\theta_{Rj}^{R}\left(\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{x_{ij}}{y_{rj}}\right)-\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{\hat{x}_{ij}}{y_{rj}}\right)\Gamma_{ij}^{x}\right)-1\right], \qquad (17)$$

And

$$\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{x_{ij}+\Gamma_{ij}^{x}\hat{x}_{ij}}{y_{rj}}\right)=\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{x_{ij}}{y_{rj}}\right)+\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{\hat{x}_{ij}}{y_{rj}}\right)\Gamma_{ij}^{x} \geq 1, \text{ where } \sum_{i\in IN_j^x}\left|\Gamma_{ij}^{x}\right|\leq \beta_j^x,\ j=1,..,n. \qquad (18)$$

According to relations (17) and (18), model (15) can be presented as follows.

$$\begin{aligned}
&\min \sum_{j=1}^{n}\left[\theta_{Rj}^{R}\left(\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{x_{ij}}{y_{rj}}\right)\right)-1+\max_{\substack{\sum_{i\in IN_j^x}\left|\Gamma_{ij}^{x}\right|\leq\beta_j^x,\\ -1\leq\Gamma_{ij}^{x}\leq 1}}\left\{\theta_{Rj}^{R}\left(\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{\hat{x}_{ij}}{y_{rj}}\right)\right)\Gamma_{ij}^{x}\right\}\right],\\
&S.t.\ \sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{x_{ij}}{y_{rj}}\right)+\min_{\substack{\sum_{i\in IN_j^x}\left|\Gamma_{ij}^{x}\right|\leq\beta_j^x,\\ -1\leq\Gamma_{ij}^{x}\leq 1}}\left\{\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{\hat{x}_{ij}}{y_{rj}}\right)\Gamma_{ij}^{x}\right\}\geq 1, \qquad (19)\\
&\qquad w_{ir}\geq\epsilon,\ i=1,\dots,m,\ r=1,\dots,s.
\end{aligned}$$

According to the explanations in the proof of Theorem (3.1) about solving inner problems, we consider the inner problem in the objective function of model (19). According to the discussion in solving the inner problem in the proof of Theorem (3.1), the constraints can be replaced as follows.

$\sum_{i\in IN_j^x}\left|\Gamma_{ij}^x\right| \leq \beta_j^x, -1 \leq \Gamma_{ij}^x \leq 1$, by $\sum_{i\in IN_j^x}\Gamma_{ij}^x \leq \beta_j^x\ ,\ 0 \leq \Gamma_{ij}^x \leq 1$, respectively.

First, consider the inner problem of the objective function of model (19) as follows.

$$\begin{aligned} &\max \left\{\theta_{Rj}^R\left(\sum_{r=1}^s\sum_{i=1}^m w_{ir}\left(\frac{\hat{x}_{ij}}{y_{rj}}\right)\right)\Gamma_{ij}^x\right\} \\ &S.t.\quad \sum_{i\in IN_j^x}\Gamma_{ij}^x \leq \beta_j^x,\quad j=1,..,n, \\ &\qquad\qquad \Gamma_{ij}^x \leq 1,\quad i\in IN_j^x,\quad j=1,..,n, \\ &\qquad\qquad 0 \leq \Gamma_{ij}^x,\ i\in IN_j^x,\quad j=1,..,n, \end{aligned} \tag{20}$$

The dual model (20) is as follows.

$$\begin{aligned} &\min \sum_{i\in IN_j^x}\varphi_{ij}^x + \psi_j^x\beta_j^x \\ &S.t.\qquad \varphi_{ij}^x + \psi_j^x \geq \left(\sum_{r=1}^s w_{ir}\left(\frac{\hat{x}_{io}}{y_{ro}}\right)\right)\theta_{Rj}^R,\quad i\in IN_j^x,\quad j=1,..,n, \\ &\qquad\qquad w_{ir}\geq\epsilon,\ \varphi_{ij}^x,\ \psi_j^x,\ t_{ij}^x,\ f_j^x \geq 0, i\in IN_j^x, r=1,\dots,s,,\quad j=1,..,n. \end{aligned} \tag{21}$$

Similarly, the inner problem of the constraints of model (19) can be written as follows:

$$\begin{aligned} &\min \sum_{r=1}^s\sum_{i=1}^m w_{ir}\left(\frac{\hat{x}_{ij}}{y_{rj}}\right)\Gamma_{ij}^x \\ &S.t.\quad \sum_{i\in IN_j^x}\Gamma_{ij}^x \leq \beta_j^x,\quad j=1,..,n, \\ &\qquad\qquad \Gamma_{ij}^x \leq 1,\quad i\in IN_j^x,\quad j=1,..,n, \\ &\qquad\qquad 0 \leq \Gamma_{ij}^x,\ i\in IN_j^x,\quad j=1,..,n. \end{aligned} \tag{22}$$

The dual model (22) is as follows.

$$\begin{aligned} &\max\ -\sum_{i\in IN_j^x}t_{ij}^x - f_j^x\beta_j^x \\ &S.t.\qquad t_{ij}^x + f_j^x \geq -\left(\sum_{r=1}^s w_{ir}\left(\frac{\hat{x}_{ij}}{y_{rj}}\right)\right),\quad i\in IN_j^x,\quad r=1,\dots,s,\ j=1,..,n, \\ &\qquad\quad w_{ir}\geq\epsilon,\ i=1,\dots,m,\ r=1,\dots,s, \\ &\qquad\qquad t_{ij}^x,\ f_j^x \geq 0,\ i\in IN_j^x,\quad r=1,\dots,s,\ j=1,..,n. \end{aligned} \tag{23}$$

By placing models (21) and (23) instead of the inner problems of model (19), the model (16) is obtained, and the proof is complete.■

5 An Application for Commercial Banks in The Global Competitive Market

In this section, an application of the proposed approach in this paper has been shown. For this purpose, we evaluated 38 commercial banks that operate in a globally competitive market in 2020. These banks belong to 38 different countries. These banks have the same activity in the global market. The dataset mainly covers the variables from the balance sheet and income statement. The data for commercial banks is taken from the Fitch Solutions database (https://www.fitchsolutions.com/fitch-connect). In many cases, the data may not be available in the real world as exact numbers, and the data may have a small fluctuation over a period of time. Therefore, in order to evaluate the correct efficiency, we must use DEA models in the presence of uncertain data. In evaluating the efficiency of commercial banks, considering that the input data can have a small fluctuation during a period of time, the robust DEA and robust DEA-R models used to evaluate the efficiency of banks. To evaluate the banks, we assume that the input and output data have uncertain and certain values, respectively. The input and output variables in the evaluation of banks are as follows:

Inputs: personnel expenses, total interest expenses, and non-interest expenses.

Outputs: net interest income, non-interest income.

The **inputs** are defined as follows:

Personnel Expenses: This cost includes salaries and benefits related to bank employees.

Total interest expenses: These expenses include the expenses that the bank pays to customers for bank interest.

Non-interest expenses: These expenses include expenses that the bank spends on administrative, general, depreciation, and doubtful claims.

Also, the **outputs** are defined as follows:

Net interest income: The income from the difference between interest paid and interest received is the main source of bank income. The income from the interest difference means interest income minus interest expense. According to another definition, a certain amount of money that the borrower must pay to the lender is called interest. This money is separate from the principal of the amount exchanged between the two parties as a loan. In other words, the interest must be paid in excess of the principal. In other words, the interest rate is the rate that is charged by the borrower to prevent a decrease in the value of money. This is due to the difference in the value of money at two different times. It means that the purchasing power of the money exchanged is greater at the time of receiving the loan than at the time of paying it back. Also, in normal market conditions, in order to compensate for the lender's investment opportunities, an amount may be added to this rate as the lender's expected minimum profit. Therefore, the net interest income includes the bank's income through the bank's interest that it receives from its customers.

Non-interest income: These incomes include the bank's income from fees such as deposit fees, transaction fees, insufficient funds fees, annual account fees, monthly account service fees, credit card issuance fees, and the cost of late loans. Interest-free income is one of the solutions to increase income and delay the liquidity of the bank in the event of an increase in these rates.

The data sets and characteristics of commercial banks are given in Tables 1 and 2.

To evaluate the efficiency of banks, at first the results of the DEA model in the input orientation is obtained. The results are in the fourth column of Table 2. As can be seen, banks B1, B6, B10, B13, B16, B20, B22, B29, B30, and B37 are efficient banks, and other banks are inefficient. Also, to evaluate the efficiency of banks, we obtain the results of the DEA-R model in the input orientation. The results are in the fifth column of Table 2. As can be seen, similar to the results of the DEA model, banks B1, B6, B10, B13, B16, B20, B22, B29, B30, and B37 are efficient banks, and other banks are inefficient. As can be seen, all the efficiency scores related to all the banks based on the DEA-R model are greater or equal to their corresponding scores obtained from the DEA model in the input orientation. As previously stated, DEA models have an inherent weight restriction and may not truly calculate efficiency scores. To face the problems created, such as underestimation of efficiency and pseudo-inefficiency (Wei et al. [72–74]), we can use DEA-R models instead of DEA models, and the results show this well. As can be seen in the fourth and fifth columns of Table 2, the ranking of each bank based on the efficiency scores obtained from the DEA and DEA-R models is shown in parentheses next to their efficiency scores. The rank corresponding to each bank based on the DEA and DEA-R models is not the same. For example, the rank corresponding to Bank B36 obtained from the DEA and DEA-R models is equal to 7 and 11, respectively. The scatter plot used to check the possible relationship between two sets of numbers. In Figure 1, a relationship between the ranking scores of the DEA and DEA-R models for different banks is provided. There is a positive and linear relationship between the ranking scores.

Table 1: European Banks Data

Bank Name	Country	Bank	Inputs		
			Personnel Expenses	**Total Interest Expenses**	**Non-Interest Expenses**
Macquarie Group Limited	Australia	B01	3286.95	2035.9	4954.2
Erste Group Bank AG	Austria	B02	3066.68	2183.55	5402.92
KBC Group NV	Belgium	B03	2833.46	2186.23	5186.36
Banco Safra S.A.	Brazil	B04	588.55	1074.57	921.6
National Bank of Canada	Canada	B05	2037.09	2495.12	3460.73
Bank of China Limited	China	B06	13670.29	52663.7	25457.93
Banco Nacional de Costa Rica	Costa Rica	B07	221.71	357.06	338.12
Ceskoslovenska Obchodni Banka a.s. (CSOB)	Czech Republic	B08	426.83	441.77	906.19
Jyske Bank A/S	Denmark	B09	487.59	644.77	887.17
AS SEB Pank	Estonia	B10	53.04	12.41	76.04
Nordea Bank Abp	Finland	B11	3348.08	2204.48	5648.67
Deutsche Bank AG	Germany	B12	12739.02	7640.25	25259.04
Alpha Services and Holdings S.A.	Greece	B13	590.9	646.99	1408.82
OTP Bank Plc.	Hungary	B14	1037.94	656.5	2383.59
Arion Banki hf	Iceland	B15	96.94	161.72	192.13
ICICI Bank Limited	India	B16	1097.68	5509.15	3026.66
PT Bank Central Asia Tbk	Indonesia	B17	946.46	797.01	2124.25
Bank of Ireland	Ireland	B18	1239.72	605.87	2699.63
Banca Monte dei Paschi di Siena S.p.A.	Italy	B19	1835.48	856.12	4291.31
The Howa Bank, Ltd.	Japan	B20	24.57	1.92	59.7
JSC Rietumu Banka	Latvia	B21	25.79	18.98	47.69
AB SEB Bankas	Lithuania	B22	59.86	29.44	97.45
BGL BNP Paribas	Luxembourg	B23	582.26	318.63	953.45
Cooperatieve Rabobank U.A.	Netherlands	B24	5698.55	6803.23	8884.83
ANZ Bank New Zealand Limited	New Zealand	B25	652.38	1522.65	1138.36
DNB Bank ASA	Norway	B26	1401.45	1916.31	2585.54
Bank Pekao S.A.	Poland	B27	552.53	175.36	1221.43
Caixa Geral de Depositos, S.A.	Portugal	B28	538.71	555.26	955.4
The Saudi National Bank	Saudi Arabia	B29	946.59	734.53	1861.31
Slovenska sporitelna, a.s.	Slovakia	B30	192.59	36.01	411.94
Nova Ljubljanska banka d.d.	Slovenia	B31	200.74	67.64	393.08
FirstRand Limited	South Africa	B32	1851.92	3677.82	3266.35
Woori Bank	South Korea	B33	1957.44	2944.24	3205.31
Banco Santander, S.A.	Spain	B34	13118.6	16724.6	41211.1
Skandinaviska Enskilda Banken AB (publ)	Sweden	B35	1803.25	1497.29	2846.6
Credit Suisse Group AG	Switzerland	B36	11114.23	8957.69	18444.68
Turkiye Halk Bankasi A.S.	Turkey	B37	528.57	4739.71	1008.21
NatWest Group plc	United Kingdom	B38	5264.67	3116.13	10608.52

Figure 2 compares the efficiency scores of the DEA and DEA-R models. As can be seen, the dispersion between the ranking scores of the DEA and DEA-R models is less. Now, the results of the robust DEA-R model (model 2) have been analysed. As mentioned earlier, we assume that the input data are uncertain numbers and the outputs are certain numbers. In order to include uncertainty in the input data, the BS approach used. We assume that there is 5% disturbance in this data. According to the BS approach, we put $h_i^x = 1 + \varphi^{-1}(1 - \tau_i)\sqrt{n_i}$ to solve the robust DEA-R model and choose the correct h_i^x.

In this regard, φ^{-1} is cumulative distribution of standard Gaussian variable. τ_i shows the amount of confusion in the data. Also, in the first constraint of model (2), we have $n_i = 38$. If we put $\tau_i = 0.05$, the value of $h_i^x = 11$ is obtained. By solving the robust DEA-R model for $h_i^x = 11$, the results are shown in the second column of Table 3. As can be seen, banks B1, B4, B6, B10, B13, B16, B 17, B20, B21, B22, B25, B27, B29, B30, B35, and B37 are efficient banks and other banks are inefficient.

Table 2: European Banks Data and Efficiency Scores

Bank	Outputs		Efficiency (CRS Technology)	
	Net Interest Income	**Non-Interest Income**	**DEA**	**DEA-R**
B01	1147.93	6260.22	1 (1)	1 (1)
B02	5833.23	2879.45	0.5572 (23)	0.5948 (23)
B03	5499.03	3241.02	0.623 (19)	0.6584 (18)
B04	1299.36	106.02	0.5721 (22)	0.5721 (25)
B05	3194.92	2757.17	0.6887 (15)	0.6887 (17)
B06	64502.75	17437.3	1 (1)	1 (1)
B07	391.37	143.67	0.5179 (26)	0.5324 (28)
B08	1204.6	420.74	0.6291 (17)	0.6416 (19)
B09	835	555.3	0.5985 (21)	0.6232 (21)
B10	142.46	70.2	1 (1)	1 (1)
B11	5492.95	4870.05	0.7413 (12)	0.7413 (13)
B12	14022.53	15211.15	0.6025 (20)	0.6025 (22)
B13	1879.16	1265.87	1 (1)	1 (1)
B14	2633.84	1453.13	0.785 (8)	0.8382 (7)
B15	244.93	154.12	0.8212 (6)	0.8504 (6)
B16	4581.77	2012.86	1 (1)	1 (1)
B17	3867.18	1461.25	0.9496 (2)	0.9834 (2)
B18	2522.01	670.35	0.5249 (24)	0.5398 (27)
B19	1559.32	1981.23	0.5724 (21)	0.5724 (24)
B20	69.85	1.38	1 (1)	1 (1)
B21	41.36	46.96	0.9085 (2)	0.9085 (4)
B22	138.69	116.55	1 (1)	1 (1)
B23	1530	425.57	0.7081 (14)	0.7081 (15)
B24	9956.65	3137.61	0.4705 (27)	0.4731 (29)
B25	2153.9	553.99	0.7637 (11)	0.7637 (12)
B26	4498.76	1669.42	0.7828 (9)	0.8131 (10)
B27	1416.15	702.59	0.9019 (3)	0.9184 (3)
B28	1270.5	754.05	0.77 (10)	0.8177(9)
B29	4476.56	1374.35	1 (1)	1 (1)
B30	528.25	204.02	1 (1)	1 (1)
B31	364.61	261.93	0.8297 (5)	0.8297 (8)
B32	3414.83	2593.51	0.7169 (13)	0.7304 (14)
B33	4890.29	522.59	0.6239 (18)	0.6239 (20)
B34	39399.58	14262.2	0.6787 (16)	0.694 (16)
B35	3027.45	2958.94	0.8614 (4)	0.8677 (5)
B36	6684.27	17064.67	0.7853 (7)	0.7853 (11)
B37	2630.84	252.98	1 (1)	1 (1)
B38	10399.16	4125.31	0.5199 (25)	0.5432 (26)

The rank corresponding to each of the banks is given in parentheses next to their efficiency scores. If we consider the rank of efficient banks as one, the rank of inefficient banks is based on their efficiency scores. Among the inefficient banks, banks B23, B26, and B15 have a higher rank than other banks. According to the results, all the scores related to the efficiency scores obtained from the robust DEA-R model are greater than or equal to their corresponding values obtained from the DEA-R model. As can be seen, the corresponding ranking of banks based on robust DEA-R (model 2) and DEA-R models (model 1) is not the same. For example, bank B36 has a rank of 6 based on the robust DEA-R model, while it has a rank of 11 based on the DEA-R model. In Figure 3, a relationship between the ranking scores of DEA-R and robust DEA-R models for different banks is proposed. There is a positive and almost linear relationship between the ranking scores. As can be seen, the dispersion between the ranking scores of DEA-R and robust DEA-R models is less. Figure 4 compares the efficiency scores of the DEA-R (model 1) and robust DEA-R (model 2) models.

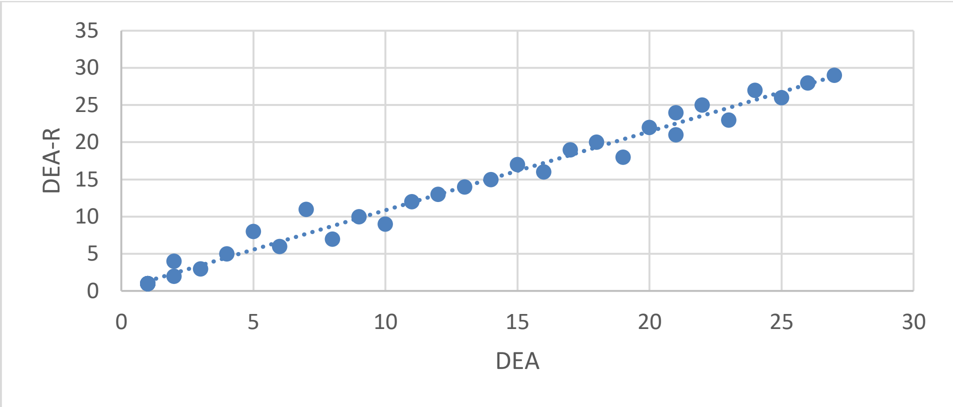

Fig. 1: Scatter Plots for Various Ranking Scores of Dea and Dea-R Models

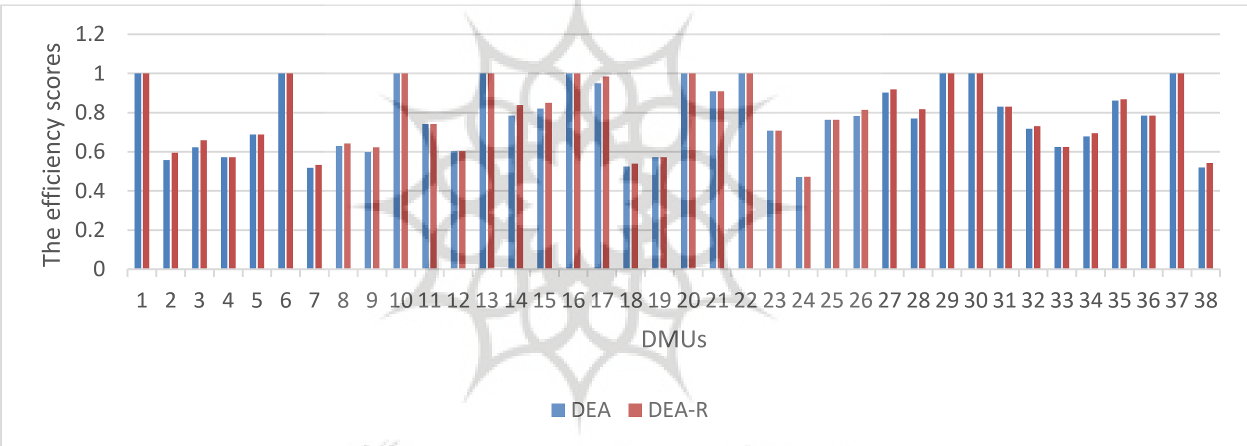

Fig. 2: Columns Plots for Efficiency Scores of DEA and DEA-R Models

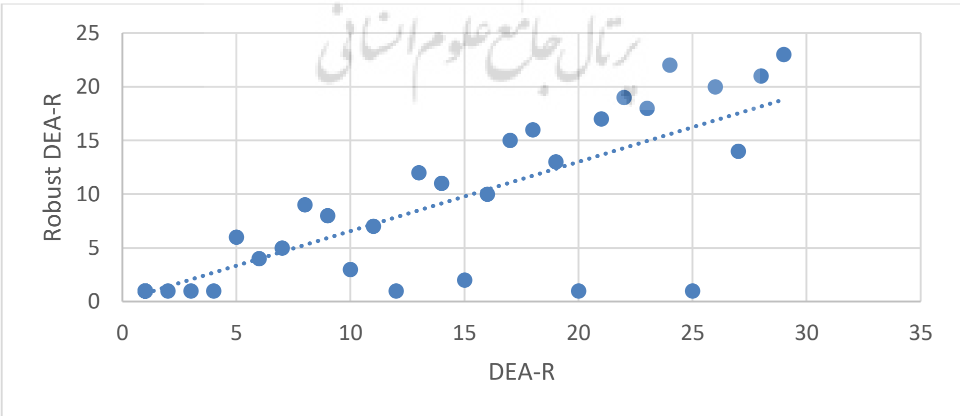

Fig. 3: Scatter Plots for Various Ranking Scores of DEA-R and Robust DEA-R Models

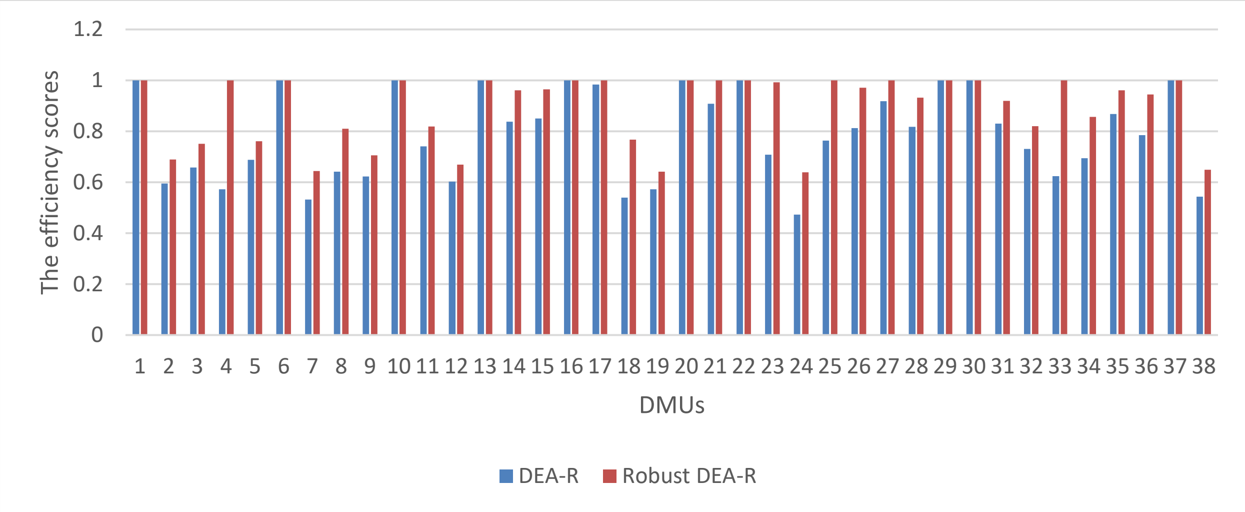

Fig. 4: Columns Plots for Efficiency Scores of DEA-R and Robust DEA-R Models

Now, comparing the results of the robust DEA model presented by Salahi et al. [62] by choosing $h_i^x = 11$. The results of this approach are shown in the third column of Table 3. Salahi et al. [62] assumed that all input and output components are uncertain numbers. Banks B1, B6, B10, B13, B17, B16, B15, B20, B22, B21, B27, B29, B31, B30, B35, and B37 are efficient banks, and other banks are inefficient, based on the approach results of Salahi et al. [62]. The rank corresponding to each of the banks is given in parentheses next to their efficiency scores. By comparing the results of the DEA model and the approach provided by Salahi et al. [62], we see that the results of robust DEA and DEA models are not the same. The efficiency scores obtained from the robust DEA model (Salahi et al. [62]) are larger than the corresponding scores obtained from the DEA model.

Now, in order to compare the results of robust DEA and robust DEA-R models under the condition that the input and output components are uncertain and certain numbers, respectively, we solve the robust DEA model of Salahi et al. [62] by putting $\Gamma_i^x = 11$. The results related to the model of Salahi et al. [62] are given in the last column of Table 3. By comparing the robust DEA (Salahi et al. [62]) and robust DEA-R (model (2)) models, it can be seen that all the efficiency scores obtained from the robust DEA-R model are greater or equal to their corresponding efficiency scores obtained from the robust DEA model. This result shows that the robust DEA-R model avoids the problem of underestimation of efficiency available in the robust DEA model. The robust DEA-R model introduces banks B4, B25, B27, and B33 as efficient banks, while these banks are inefficient according to the robust DEA model of Salahi et al. [62]. This result shows that the robust DEA-R model prevents pseudo-inefficiency in contrast to the robust DEA model. As can be seen, the corresponding ranking of banks based on robust DEA (Salahi et al. [62]) and robust DEA-R (model (2)) models are not the same. For example, bank B23 has a rank of 2 based on the robust DEA-R model, while it has a rank of 12 based on the robust DEA model. In Figure 5, a relation between the ranking scores of robust DEA (Salahi et al. [62]) and robust DEA-R (model (2)) models for different banks is provided. There is a positive and almost linear relationship between the ranking scores. The dispersion between the ranking scores of robust DEA and robust DEA-R models is almost zero. Figure 6 compares the efficiency scores of robust DEA and robust DEA-R models.

Table 3: The Efficiency Scores of Robust DEA and DEA-R

Bank	**Efficiency (CRS Technology)**		
	Robust DEA-R approach (model (2)): $\Gamma = 11.$	**Robust DEA approach (Salahi et al. [62]) : $\Gamma = 11$**	**Robust DEA approach by only input uncertain (Salahi et al. [62]) : $\Gamma = 11.$**
B01	1 (1)	1 (1)	1 (1)
B02	0.6892 (18)	0.6807 (19)	0.6159 (22)
B03	0.7507 (16)	0.7610 (14)	0.6885 (17)
B04	1 (1)	0.6989 (18)	0.6323 (21)
B05	0.7612 (15)	0.8413 (10)	0.7612 (13)
B06	1 (1)	1 (1)	1 (1)
B07	0.6444 (21)	0.6327 (22)	0.5725 (25)
B08	0.8099 (13)	0.7685 (12)	0.6953 (15)
B09	0.7055 (17)	0.7311 (17)	0.6615 (19)
B10	1 (1)	1 (1)	1 (1)
B11	0.8193 (12)	0.9055 (7)	0.8193 (10)
B12	0.6689 (19)	0.7360 (15)	0.6659 (18)
B13	1 (1)	1 (1)	1 (1)
B14	0.9614(5)	0.9590 (3)	0.8676 (7)
B15	0.9652(4)	1 (1)	0.9076 (5)
B16	1 (1)	1 (1)	1 (1)
B17	1 (1)	1 (1)	1 (1)
B18	0.7668 (14)	0.6412 (20)	0.5801 (23)
B19	0.6418 (22)	0.6993 (16)	0.6327 (20)
B20	1 (1)	1 (1)	1 (1)
B21	1 (1)	1 (1)	1 (1)
B22	1 (1)	1 (1)	1 (1)
B23	0.9919 (2)	0.8650 (9)	0.7826 (12)
B24	0.6392 (23)	0.5747 (23)	0.5200 (26)
B25	1 (1)	0.9330 (6)	0.8441 (9)
B26	0.9706 (3)	0.9563 (4)	0.8652 (8)
B27	1 (1)	1 (1)	0.9969 (2)
B28	0.9322 (8)	0.9406 (5)	0.8510 (9)
B29	1 (1)	1 (1)	1 (1)
B30	1 (1)	1 (1)	1 (1)
B31	0.9197 (9)	1 (1)	0.9171 (4)
B32	0.8205 (11)	0.8758 (8)	0.7924 (11)
B33	1 (1)	0.7622 (13)	0.6896 (16)
B34	0.8569 (10)	0.8292 (11)	0.7502 (14)
B35	0.9606 (6)	1 (1)	0.9521 (3)
B36	0.9452 (7)	0.9593 (2)	0.8679 (6)
B37	1 (1)	1 (1)	1 (1)
B38	0.6496 (20)	0.6351 (21)	0.5746 (24)

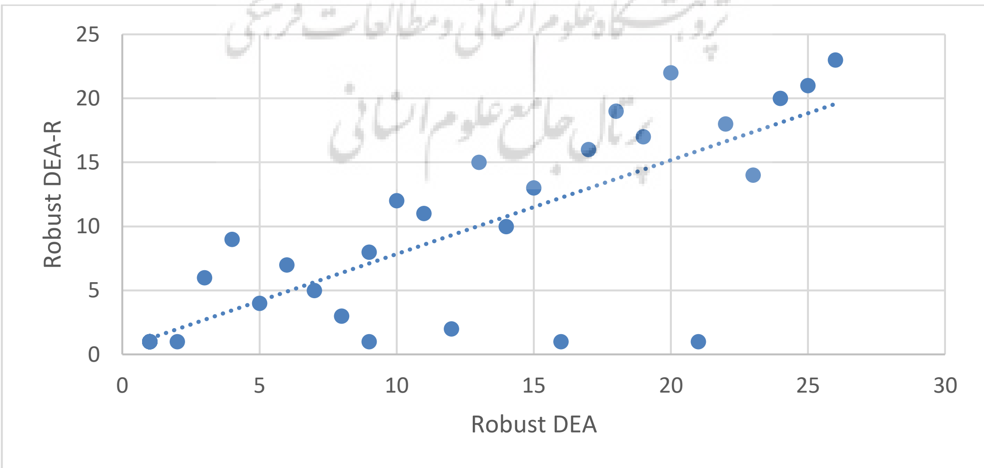

Fig. 5: Scatter Plots for Various Ranking Scores of Robust DEA and Robust DEA-R Models

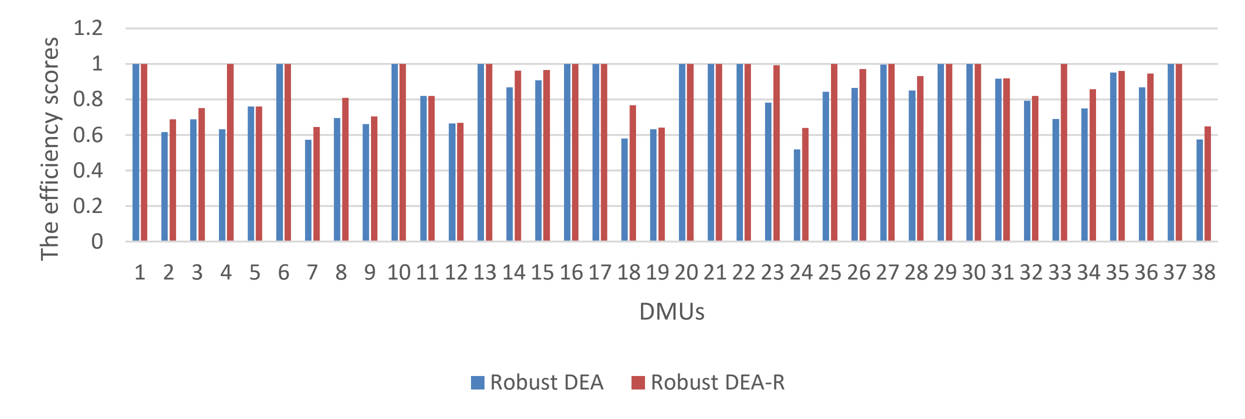

Fig. 6: Columns Plots for Efficiency Scores of Robust DEA and Robust DEA-R Models

Table 4: The Efficiency Scores of Models with The Set of Common Weight

Bank	Efficiency (CRS Technology)			
	DEA-R Common weight (model (12))	**DEA-R (model (1))**	**Robust DEA-R (model (2)): $\Gamma = 11$**	**Robust- DEA-R Common weight (model (16)): $\Gamma = 11$**
B01	0.2844 (34)	1 (1)	1 (1)	0.2853 (34)
B02	0.5742 (24)	0.5948 (23)	0.6892 (18)	0.5749 (24)
B03	0.6429 (19)	0.6584 (18)	0.7507 (16)	0.6437 (19)
B04	0.1602 (36)	0.5721 (25)	1 (1)	0.1595 (36)
B05	0.6616 (17)	0.6887 (17)	0.7612 (15)	0.6613 (17)
B06	0.9224 (5)	1 (1)	1 (1)	0.9096 (5)
B07	0.4558 (28)	0.5324 (28)	0.6444 (21)	0.4544 (28)
B08	0.6373 (20)	0.6416 (19)	0.8099 (13)	0.6365 (20)
B09	0.6092 (22)	0.6232 (21)	0.7055 (17)	0.6085 (22)
B10	0.8692 (6)	1 (1)	1 (1)	0.8726 (6)
B11	0.7039 (15)	0.7413 (13)	0.8193 (12)	0.7055 (15)
B12	0.4743 (27)	0.6025 (22)	0.6689 (19)	0.4757 (27)
B13	1 (1)	1 (1)	1 (1)	1 (1)
B14	0.7335 (11)	0.8382 (7)	0.9614(5)	0.7345 (11)
B15	0.8389 (7)	0.8504 (6)	0.9652(4)	0.8367 (7)
B16	0.9804 (3)	1 (1)	1 (1)	0.9645 (3)
B17	0.9529 (4)	0.9834 (2)	1 (1)	0.9526 (4)
B18	0.3769 (32)	0.5398 (27)	0.7668 (14)	0.3771 (32)
B19	0.3502 (33)	0.5724 (24)	0.6418 (22)	0.3516 (33)
B20	0.0525 (37)	1 (1)	1 (1)	0.0525 (37)
B21	0.7336 (12)	0.9085 (4)	1 (1)	0.7353 (12)
B22	0.9842 (2)	1 (1)	1 (1)	0.987 (2)
B23	0.5415 (25)	0.7081 (15)	0.9919 (2)	0.5424 (25)
B24	0.4006 (29)	0.4731 (29)	0.6392 (23)	0.4001 (29)
B25	0.6313 (21)	0.7637 (12)	1 (1)	0.6268 (21)
B26	0.7901 (9)	0.8131 (10)	0.9706 (3)	0.7884 (9)
B27	0.7098 (14)	0.9184 (3)	1 (1)	0.7116 (14)
B28	0.7948 (8)	0.8177(9)	0.9322 (8)	0.7948 (8)
B29	1 (1)	1 (1)	1 (1)	1 (1)
B30	0.66 (16)	1 (1)	1 (1)	0.6618 (16)
B31	0.6508 (18)	0.8297 (8)	0.9197 (9)	0.6529 (18)
B32	0.715 (13)	0.7304 (14)	0.8205 (11)	0.7122 (13)
B33	0.2308 (35)	0.6239 (20)	1 (1)	0.23 (35)
B34	0.5976 (23)	0.694 (16)	0.8569 (10)	0.5958 (23)
B35	0.7818 (10)	0.8677 (5)	0.9606 (6)	0.783 (10)
B36	0.3883 (31)	0.7853 (11)	0.9452 (7)	0.3893 (31)
B37	0.4117 (30)	1 (1)	1 (1)	0.396 (30)
B38	0.4939 (26)	0.5432 (26)	0.6496 (20)	0.4944 (26)

Now we will examine the results of the robust ratio analysis model based on the set of common weights, namely model (16). The results are in the last column of Table 4.

As can be seen, a robust ratio analysis model based on the set of common weights, namely model (16), evaluates all banks by considering only input components in the form of uncertain numbers and output components in the form of certain numbers. The model identifies banks B13 and B29 as efficient and other banks as inefficient. The rank corresponding to each of the banks is given in parentheses next to their efficiency scores. The rank of efficient banks is considered as one. The ranking of other banks is based on their efficiency scores obtained from the model (16). Banks B22, B16, and B17 have the second, third, and fourth ranks in terms of rank, while the rank of these units is equal to one based on the robust DEA-R model (model 2). This result shows that by using model (16), which uses a set of common weights from all ratios of input components to output components, we can rank all commercial banks based on efficiency scores.

The model (16) has a higher weight discriminating power than the robust DEA-R model (2), and all the banks under evaluation can be ranked. The robust ratio analysis model based on the set of common weights, namely model (16), evaluates all banks under the same conditions. A relation between the ranking scores of robust DEA-R (model 2) and robust ratio analysis models (model 16) for different banks proposed in Figure 7. There is a positive and non-linear relationship between the ranking scores. Figure 8 compares the efficiency scores of robust DEA-R and robust ratio analysis models.

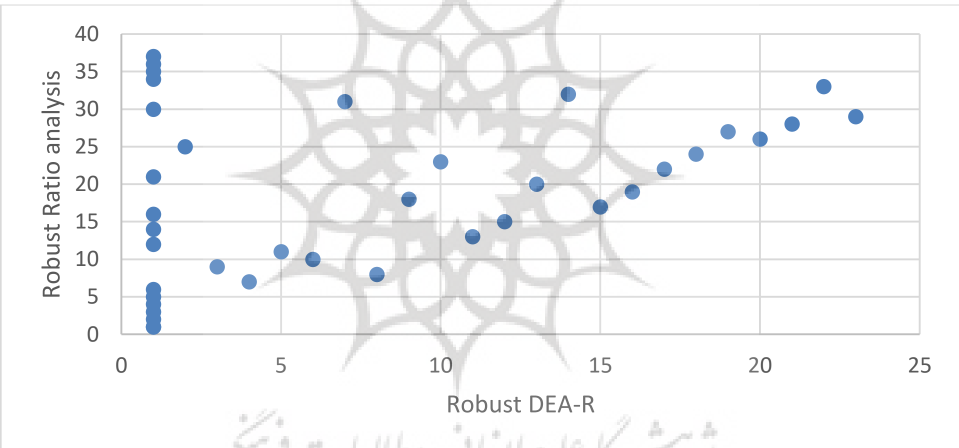

Fig.7: Scatter Plots for Various Ranking Scores of Robust Dea-R and Robust Ratio Analysis a Models

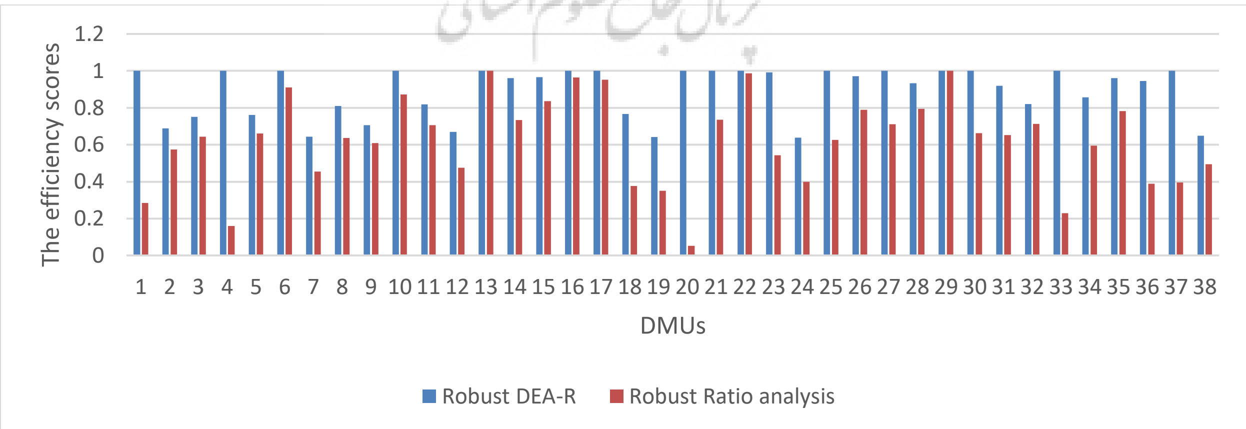

Fig. 8: Columns Plots for Efficiency Scores of Robust DEA-R and Robust Ratio Analysis Models

6 Conclusions

One of the techniques for evaluating the efficiency of a set of DMUs is DEA. In traditional DEA models, input and output data have absolute values. However, in many applications of the real world, such as evaluating the performance of banks and universities, we face many cases in which the ratio of input components to output components (and vice versa) of the DMUs is important for the DM. Therefore, in recent years, efficiency evaluation models have been developed based on the ratio of input components to output components, which can be referred to as ratio analysis and DEA-R models. Also, in the real world, we may face many cases where the input and output data have uncertain values. One of the proposed approaches for dealing with uncertain data is robust optimization. In this paper, DEA-R models presented to evaluate the efficiency of DMUs based on the ratio of input and output components under conditions of uncertainty. We assumed that the input components have uncertain values and the output components have certain values. The robust optimization used based on the BS approach and obtained the RC problem corresponding to the robust DEA-R model as a linear programming model. In order to increase the discriminating power of the weights, we presented the ratio analysis model based on a set of common weights of all the ratios of input to output components and presented this model under uncertainty conditions. To solve this model, we used the method of Kao and Hung [37] to solve the presented common weight model. The robust optimization applied based on the BS approach and obtained the RC problem corresponding to the robust DEA-R model as a linear programming model. According to the results obtained in the case study, it can be stated that the efficiency scores obtained from the robust DEA-R models are greater than or equal to the corresponding scores obtained from the robust DEA models, and this shows that the robust DEA-R models avoids the problem of underestimation of efficiency in contrast to the robust DEA models. Also, some DMUs are introduced as inefficient in the evaluation with the robust DEA model due to the zeroing of the weight corresponding to the input or output components, but in the evaluation with the robust DEA-R model, these DMUs are introduced as efficient. These results show that robust DEA-R models avoid the problem of pseudo-inefficiency compared to robust DEA models. In this paper we have shown that different and correct rankings obtained for DMUs based on robust ratio analysis models with a set of common weights, because these models use all the ratios of input to output components, and this issue increases the power of distinguishing weights. As future work, robust DEA-R and robust ratio analysis models can be developed in conditions where all input and output data are uncertain. Also, the models presented in this paper can be developed for the two-stage network structure. As another work, the proposed model in this paper can be developed for non-radial models, such as the SBM model.

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