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Original Research

Application of Meta-Heuristic Algorithms in Portfolio Optimization with Capital Market Bubble Conditions

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ABSTRACT

The existence of bubbles in the market, especially the capital market, can be a factor in preventing the participation of investors in the capital market process and the correct allocation of financial resources for the economic development of the country. On the other hand, due to the goal of investors in achieving a portfolio of high returns with the least amount of risk, the need to pay attention to these markets increases. In this research, with the aim of maximizing return and minimizing investment risk, an attempt has been made to form an optimal portfolio in conditions where the capital market has a price bubble. According to the purpose, the research is of applied type, and in terms of data, quantitative and post-event, and in terms of type of analysis, it is of descriptive-correlation type. In order to identify the months with bubbles in the period from 2015 to 2021 in the Tehran Stock Exchange market, sequence tests and skewness and kurtosis tests were used. After identifying periods with bubbles, the meta-heuristic algorithms were used to optimize the portfolio. The results indicate the identification of 14 periods with price bubbles in the period under study. Also, in portfolio optimization, selected stock portfolios with maximum returns and minimum risk are formed. This research will be a guide for investors in identifying bubble courses and how to form an optimal portfolio in these conditions.

1 Introduction

One of the significant signs of development in today's world is related to high economic growth, which requires effective and sufficient investment. Also, the development and maintenance of financial power in any society is directly related to the reasonable and appropriate investment of that society. Therefore, organizations and people try to allocate part of their money or assets to investment. The capital market is considered as the main source of capital attraction and guidance, and therefore, efficient analysis of stocks in the capital market is necessary. Investors are always trying to find a suitable place to invest and choose different methods for investment [37]. One of the important features of industrialized and developed countries is the existence of an active and dynamic market of money and capital. In other

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words, if people's savings are directed to the production sector with the right mechanism, in addition to the return it brings to the owners of capital, they can also be useful as the most important factor of financing for launching the economic plans of the society and if they enter the unhealthy economic flows they will have adverse effects on the society [31]. The stock market, as part of the capital market, plays a very important role in directing savings to the economically productive sectors in all countries. But today in the economies of many developing countries, the situation of macroeconomic variables is not consistent with the rise of stock market indices. One of the causes of these problems is the fluctuations of asset prices and especially the formation of stock price bubbles. Whenever there is widespread disruption and diversion in the market, equipping and allocating financial resources will be a serious problem. One of the factors causing these problems is the price bubble [14].

The basic idea behind bubbles is the fact that traders and investors maintain the belief that, although the value of stocks has not been overestimated according to its principles, there are still additional specimens to buy. Therefore, a bubble in the stock price trend away from the equilibrium determined by its principles is important because market participants understand the existence of suitable opportunities. In fact, the movement of asset prices away from their principles indicates the occurrence of complete predictions of market participants, which are due to events outside the market. Basically, the existence of bubbles in asset markets indicates that market participants are not allocating their savings to the best possible investment. In addition, rational bubble analysis, based on rational expectations, is an indeterminate determinant, usually occurring when delegates' current decisions depend on both the current market price and their expectations of future prices [6]. The stock market in particular is a natural condition for studying bubbles, because stocks have a fundamental basis in terms of the present value of future dividends [42].

Studies on bubbles, on different types of assets in the economy over time, have occupied researchers in various fields. Understanding why changes in market asset prices relative to their intrinsic values are so important is that a methodology has not yet been developed that is able to predict such price fluctuations [13]. The existence of bubbles primarily affects those who invest in stocks and also leads to consequences that are almost always catastrophic for the whole economy. One striking example of the impact of the bubble on the economy in the United States is the dot-com crisis in 2000, and the Dutch tulip sector in the seventeenth century and more recently in the "lateral mortgage" catastrophe [13].

In addition to the issue of bubbles, investment managers also pay attention to the optimality of the portfolio of financial assets under their management [14]. Investment diversification is a convenient way to reduce the risk of total investment that has been used for years in the early years of portfolio theory. Usually, a diversification strategy is used to invest in different securities from different sectors, companies, businesses, places and governments [18].

The first mathematical model of portfolio selection was formulated in 1952 by Markowitz. In the Markowitz portfolio selection model, the return of a portfolio is measured as the expected value of the random variable of portfolio return, and portfolio risk is calculated by the variance of portfolio return. Markowitz showed that by assuming that the investor accepts a high level of risk (in other words, the investor is risk-averse) or want a lower efficiency threshold (Be a risk averse investor), the optimal portfolio can be obtained by solving the problem of second-degree planning [35].

In real conditions, the limitations of the problem are generally greater than the limitations of the quadratic optimization problem. For this reason, the second-order problem-solving method can no longer be used to solve such problems [19] and when the number of assets in the problem is large, it becomes a difficult nonlinear planning problem and efficient boundary search will not be possible by conventional methods [15]. For many years, advanced mathematics and computers have helped humans solve such complex problems to help them overcome environmental uncertainty and ambiguity as much as possible. Among the methods that in recent years in solving many optimization problems, have untied human ambiguities and have had a successful approach in response to complex problems, are methods and algorithms called innovative (meta-innovation) [30].

Meta-heuristic algorithms are pivotal search algorithms that approach the optimal solution slightly each time the algorithm rotates. These algorithms do not provide any guarantee for finding the optimal global, but experience has shown that they have a good performance in achieving the optimal global nonlinear programming [19]. In this regard, nature-inspired optimization algorithms, including evolutionary algorithms (EAs) and Swarm Intelligence (SI), are part of the computer information industry that has become very popular over the past few decades [27].

This research intends to form an optimal stock portfolio in a situation where the capital market is suffering from a price bubble. In this regard, first of all, the problem that exists, which is the formation of an optimal portfolio in the conditions of the price bubble, is discussed and then in the continuation of the research, the theoretical bases and background of the researches in this field will be presented. Then, explanations about the community and the sample and method of conducting the research will be provided. In the following, statistics and information obtained from the research process according to the methods used are presented. Finally, after performing the analysis on the obtained information, conclusions are made and the results of this research are compared with previous researches. This study intends to provide an effective and practical solution for investors in the field of stock portfolio optimization.

2 Theoretical Foundations and the Literature Review

2.1 Price Bubble

Whenever there is widespread disruption and diversion in the market, equipping and allocating financial resources will be a serious problem. One of the factors causing these problems is the price bubble [13]. The bubble originates from a kind of expectation so that this expectation creates the motivation for the bubble to occur and form [38]. In the bubble, rising prices lead to increased investor enthusiasm, increased demand and consequently rising prices again [14].

Theoretical foundations for understanding the phenomenon of speculative bubbles can be found to Keynes (1936), who compared the stock market to a beauty contest. Like a beauty contest, in the stock market, traders predict what the market will look like in the near future, and try to predict the average investor in order to make a profit by taking advantage of the sudden increase or decrease in value of securities, shares, securities [13]. When the price of a stock is different from its expected price in the next period, the bubble debate appears. In general, various reasons can be mentioned for the intensification of bubbles, the existence of bubbles can be related to high liquidity in the financial system. In general, fluctuations in the price of financial assets often consist of two main parts, one is the conventional part, which includes fundamental price changes, including the initial variables. The other is the unconventional part or false price changes known as speculative bubbles [38].

2.2 Stock Portfolio Optimization

It is well known that portfolio selection has always been one of the hot topics of research and early works can be found in the famous model of mean variance proposed by Markowitz in 1952 [45]. Markowitz reduced the correlation between assets, which determines the risk of securities under the influence of a certain level of return on the value of the investor's expected portfolio. The complexity of financial markets, the specialization of the topic of investment and the growth and development of financial instruments, made capital market participants need models and tools to help them achieve their ultimate goal of choosing the best portfolio. In fact, investing is a complex process involving deciding on the expected return. This complexity of the investment process led to the presentation and development of different methods and theories to optimize the portfolio day by day. Today, in global markets and very unstable markets such as stock exchanges, measuring efficiency and market risk management has become an important factor for competition and even the survival of financial institutions [22].

The issue of portfolio selection is how to allocate capital to a number of available assets in order to achieve maximum returns while minimizing risk. In the Markowitz portfolio selection model, the return of a portfolio is measured as the expected value of the random variable of return on the portfolio and portfolio risk is calculated by the variance of portfolio returns. Markowitz showed that by assuming that the investor accepts a high level of risk (in other words, the investor is risk-averse) Or if it wants a lower limit of return (investor is risk averse), the optimal portfolio can be obtained by solving the problem of convex quadratic planning [35]. In most portfolio selection issues, there must be an exchange between risk and return. Investors are also looking for the optimal weight of assets in the stock portfolio [7]. The well-known optimization problem of the financial literature has been solved using various methods and techniques such as parametric quadratic programming or linear programming. However, as the model adopts additional constraints, it becomes more difficult to optimize and traditional and definitive methods cannot achieve satisfactory results [40], [30].

2.3 Meta-Heuristic Algorithms

The main reason for using meta-heuristic algorithms in portfolio optimization is their ability to solve hard nonlinear problems. Because in some cases, when the limitations of the problem increase, it is no longer possible to solve problems with conventional solutions and because of this, these algorithms are used to solve such problems. Meta-heuristic algorithms are algorithms whose design is adopted from a kind of nature. These algorithms can be classified into different categories or, in other words, the classification of these algorithms can be done on different bases [19]. Since the introduction of the first meta-heuristic, major advances have been made, and numerous new algorithms are still being proposed every day. Meta-heuristic is chosen because of its efficient performance, large number of citations, specific evolutionary operators, interesting interaction mechanisms between members, conceptualization or management of parameters, and methods of stagnation prevention [16].

2.4 Artificial Bee Colony Algorithm (ABC)

The bee colony algorithm, first introduced by Karaboga in 2005, is a new branch of evolutionary algorithms (EA) inspired by the collective feeding behavior of real bee colonies [10]. This algorithm was introduced by Karaboga and developed in Karaboga and Basturk [25]. Compared to some popular evolutionary algorithms, such as particle swarm optimization, genetic algorithm, and Differential Evolution (DE), this algorithm achieves superior or comparable performance [11]. Bee colony is one of the effective and widespread optimization techniques based on intelligent swarming [25].

In the ABC algorithm, the arterial bee colony consists of three groups of bees: Working bees, Spectator bees, and scout bees. Working bees are responsible for exploiting the already discovered sources of nectar, and they inform other bees waiting in the hive about the quality of the food source being exploited. Spectator bees, depending on the information shared by the working bees, wait in the hive and create a food source for exploitation. Scouts search the environment to find new food sources. In the ABC algorithm, each food source is operated by only one recruited bee. In other words, the number of working bees is equal to the number of food sources around the hive. In addition, the location of a food source indicates a possible solution to the optimization problem, and the amount of nectar in a food

source corresponds to the quality of the solution. The number of working bees or Spectator bees is equal to the number of solutions available in the population [10].
Half of the bees are employed and the other half are observer bees. Scout bees are introduced as the trend evolves. For each nectar food source, there is a working bee. When a scout bee finds a new source of nectar, its location is maintained by the bee, and the new location replaces the previous location. After each worker bee completes its search cycle, it exchanges this information with Spectator bees. In a real beehive, this is done by a bee dance, but this formal dance can convey the direction, distance, and quality of the nectar source to the viewer [26]. The position of a food source, Xi = [xi1, xi2,…, x_{iD}] indicates a possible solution and the amount of nectar of a food source corresponds to the percentage of the associated solution. The ABC algorithm consists of four steps: initialization, working bee, spectator bee, and scout bee. In the early stages of ABC, SN food source positions are generated randomly with the search space. After producing food resources and allocating them to working bees, in the ABC working bee stage, each working bee tries to find a better quality food source based on i. The new food source, denoted as Ui = [ui1, ui2,…, u_{iD}], is calculated from Equation (1).

$$u_{ij} = x_{ij} + \emptyset(x_{ij} - x_{sj}) \tag{1}$$

Where i {1, 2,…, SN}, and SN represent the number of food sources. J is a random integer generated in the range [1, D], φ is a random number that is uniformly distributed in the range [-1, 1] and s is the index of a randomly selected solution. ABC changes each position in only one dimension in each iteration. If the new position has a better position in terms of stability, the position of the x_i source in the bee memory will be replaced by the candidate for the u_i position of the food source and will be replaced by the new food source. Each search bee selects a suggested food source based on the p_i probability value corresponding to the superior value, where:

$$p_i = fit_i / \sum_{j=1}^{SN} fit_j \tag{2}$$

If fit_i is a suitable food source i. After selecting a food source, a new candidate food source can be expressed by Eq. If a food source, i, cannot be improved for a predetermined number of cycles known as the allowable limit, that food source is discarded. Then, the scout bee randomly generates a new food source to replace i [34].

2.5 Particle Swarm Optimization Algorithm

Particle swarm optimization (PSO) technique, first introduced by Kennedy and Eberhart, is a metaphorical algorithm classified in intelligent swarm methodology that mimics and moves the coordinated behavior of birds and moves, and creating group information, to help them in the decision-making process in a coordinated manner [18, 7].

Particle swarm optimization is an evolutionary algorithm that aims to find an effective solution to optimization problems. Particles in the swarm work together to exchange information about places they have visited and what they have discovered. Each particle has a position and velocity in a search space and has a neighbor associated with it. The particle moves and recalls the best situation it has visited and the security of the neighborhood. Uses superior position to update speed [10].
Particle swarm optimization is primarily designed to mimic the aggressive behavior of birds and train fish. Each individual, called a particle, in the PSO population represents a potential solution to the optimization problem. The PSO population is referred to as a swarm composed of a number of particles. Particle i in iteration t with velocity vector $v_i^t = [v_{i1}^t, v_{i2}^t, \dots, v_{iiD}^t]$ and the position vector $x_i^t =$

$[x_{i1}^t, x_{i2}^t, \ldots, x_{iD}^t]$where I {1,2,…,NP} and Np are the population sizes. $x_{id} \in \{ l_d , u_d \}$ and d {1,2,…,D} Where D is the number of dimensions and $l_d$and $u_d$are the lower and upper squares of the dth dimension of the search space, respectively. Each particle passes through space at a speed. The new velocities and position of the particles are updated for subsequent iterations using the following two equations:

$$v_{id}^{t+1} = wv_{id}^t + c_1 r_1 \left(pbest_{id}^t - x_{id}^t\right) + c_2 r_2 \left(gbest_d^t - x_{id}^t\right) \tag{3}$$

$$x_{id}^{t+1} = x_{id}^t + v_{id}^{t+1} \tag{4}$$

$$w_t = w_{max} - \frac{(w\max - w\min) * t}{t\ max} \tag{5}$$

Where w is the inertial weight. Pbesti = [pbesti1, pbesti2,…, pbestiD] is the best position found by particle i, gbesti = [gbesti1, gbesti2,…, gbestiD] is the best historical situation found by the whole group; c1 and c1 are the acceleration coefficients. The weight of inertia w is used for trade in exploration and exploitation. r_1 and r_2 represent two random independent numbers that are uniformly distributed on [0,1]. [33, 22, 7]

In particle swarm optimization, the whole population and potential solutions are called swarm and particle, respectively. The PSO provides guidance for encouraging repetitive particles to promising locations. In the first step, the PSO randomly values a number of particles to fly through the multidimensional search space. In each iteration, particle i records two values: Position vector $x_i(t) = (x_{id}(t), \ldots, x_{id}(t))$ and velocity vector vector $v_i(t) = (v_{id}(t), \ldots, v_{id}(t))$. Each particle regulates its motion by recording its own experience (cognitive experience) as well as the experience of all the particles in the swarm (social experience). By repeating t, the dth dimension of the i-particle changes its coordinates using equations [3].

In PSO, the particle population moves through the search space by following the current optimal particles and changing positions to find the optima. The position of a particle indicates a possible performance solution for optimization. Performance evaluation according to the position of the particle provides the importance of that solution [23].

Across the group, each member, a particle, adjusts its position and velocity by evaluating both its own experience and the experience of other particles in the atmosphere to aid the decision-making process in a coordinated manner. In the PSO technique, a particle is transferred from one position to another in the space of a multidimensional solution to seek the convergence of an optimal solution. The position in the solution space is related to solving the problem under study. In general, the behavior of each particle is regulated by making a compromise between group memory and individual memory [18].

3 Literature Review

The concept of the bubble has entered the literature of economics since the early 17th century. Since then, several examples have been mentioned as the era of the price bubble. During the period 1990-85, Japanese assets as well as the American Internet market in 1998-2000 were in a bubble, the latter of which is known as Dot-com madness [17]. The research of Robert Schiller [39] with an article entitled

"Are stock price changes greater than those associated with changes in stock profits?" Can be considered as one of the first researches done on bubbles. In this paper, Schiller concludes by using data from the years 1871-1986 and using the boundary variance test, or that prices are too volatile, that price changes cannot be explained by changes in the present value of cash profits. In their research, Anderson and Brooks [1] developed and tested an empirical asset pricing model that allowed speculative bubbles to affect stock returns. They showed that stocks with larger bubbles resulted in higher returns. They also stated that there are many common changes in stock returns and can be attributed to market risk, due to the movement of bubbles instead of guiding principles.

Costa et al. [13], in a stock study in Brazil, identified a bubble in the Sao Paulo Stock Exchange index. They performed econometric tests based on the position of each in each period, for twenty-seven stocks, for the period between early 1990 and early 2010. In order to detect the presence of bubbles, they used the Johansen non-combination test or the Granger non-causal test between intrinsic value, dividends and stock returns and market value (closing price of stocks). Preliminary results showed the existence of bubbles in twenty of the twenty-seven stocks, at a significant level of 5%. Caspi and Graham [12] used data from the Israeli office-to-market ratio from July 1996 to August 2014. The results showed that no evidence of bubbles was observed. Also, the results showed the potential presence of strong non-fixed fluctuations in the market. Shu and Zhu [41], using the daily data of the Shanghai Shenzhen China Stock Market Index from January 2002 to April 2018, developed an advanced bubble detection method based on a unique periodic login confidence index to identify the primary causes of positive and negative bubbles in the Chinese stock market. They used the Phillips-Prone test and the Dickey-Fuller test to improve LPPLS index performance. The results showed the ability to detect positive and negative bubbles related to known historical events based on the unique periodic rule login index. Pavlidis et al. [32] proposed a new way to test speculative bubbles in divided capital markets, using a community of Chinese companies holding Hong Kong shares. They used two unit root and regression tests to predict the presence of bubbles in this market. The findings show the dynamism of trading in the Hong Kong stock market and the existence of a turbulent period with a trading bubble in 2007 and the market collapse in 2014 and 2015.

Rasekhi et al. [36], used the general unit root tests of the generalized Dickey Fuller Supremium sequence and the generalized Dickey Fuller General Supremium to detect and determine bubble periods in the Tehran Stock Exchange during the period April 2002 to January 2015. The results of these tests determined the explosive behavior and the existence of multiple bubbles in the Iranian stock market. In addition, all three evaluated indicators, including total price, price-to-earnings, and total real price indices, jointly showed the existence of bubbles in the periods of June 2003 to August 2003 and September 2009 to November 2009 and March 2010 to May 2011. Daryabour et al. [14], by determining the time periods in the period 2001 to the end of 2016 in Tehran Stock Exchange, and using variables such as price, monthly stock return, Monthly return of the whole market, Variance, Value at risk, adverse risk criteria in the financial period, sought to find a model to optimize their financial asset portfolio. After identifying the bubble periods in the studied period and optimizing the portfolio, they compared the performance of the formed portfolio with other portfolios in both bubble and non-bubble modes. The results showed that the bubble portfolio is in a much better position than the non-bubble portfolio (market portfolio), both in ascending and descending positions.

Saberi et al. [38], used the 10-year data of companies listed on the Tehran Stock Exchange from 2006 to 2015. The results show that the average return of a portfolio in the trading bubble space at a certain level of risk is greater than the return of a non-bubble portfolio based on subjective accounting. The results show that the average return of a portfolio in the trading bubble space at a certain level of risk is greater than the return of a non-bubble portfolio based on subjective accounting. In a study, Karaboga and Akay [23], compared the performance of a bee colony algorithm (ABC) with genetic optimization (GA), particle swarm optimization (PSO), differential evolution (DE) and ES. In their paper, the aim was to compare the performance of the standard version of the bee colony algorithm (ABC) with other known population-based algorithms. The results showed that the performance of the bee colony algorithm (ABC) is better or similar to these algorithms. However, it uses fewer control parameters and can be used effectively to solve multimodal and multidimensional optimization problems. Hong Mei et al. [21], were the first to apply the bee colony algorithm (ABC) to the portfolio optimization problem and compare it with the genetic algorithm. The experimental results showed that the bee colony algorithm (ABC) is higher than the genetic algorithm in terms of convergence speed and solution quality. Wang et al. [43] compared the bee colony algorithm (ABC) with the genetic algorithm (GA), tabu search (TS), simulated repayment (SA) and particle swarm optimization (PSO) algorithm And reported that the bee clone algorithm performed well to solve the portfolio optimization problem and obtained better solutions than other heuristic algorithms.

Zhu et al. [46] proposed a meta-heuristic approach to portfolio optimization using the Particle Swarm Optimization (PSO) technique. They compared particle swarm optimization technique with genetic algorithm (GA) and in their research results, recommended the use of hybrid techniques to improve particle swarm optimization performance. Haj-Nouri et al. [20] calculated the return and risk of each asset and prepared a portfolio optimization model based on the cardinality limit and investment income per share. In their research, they used the weed optimization algorithm (IWO) to solve the concluded model. The results showed that the algorithm used has a significant and better performance than other algorithms. Bacanin et al. [4] used the bee colony algorithm (ABC) to limit the problem of portfolio optimization with an efficient method with efficient constraint. They tested and compared the bee colony algorithm (ABC) with the genetic algorithm (GA) and the firefly algorithm (FA) on a set of five stock portfolios and pointed to the potential of the bee colony algorithm to effectively solve portfolio optimization problems. Rezaei Pouya et al. [37] used a proposed multi-purpose portfolio selection model using fuzzy normalization and uniform design method with a single-purpose programming model, which used the monthly data of the top 50 companies in the Tehran Stock Exchange in 2013. They solved their proposed model with three methods: weed algorithm, particle swarm algorithm, and gradient reduction method.

Kumar and Mishra [25] combined the bee colony algorithm (ABC) with statistical correlation concepts to accelerate its convergence. The resulting algorithm is named as Multi - Co-variancance based ABC (M-CABC) and is used to optimize the portfolio. The validation results confirmed the correct performance of the proposed algorithm in obtaining various optimal trading solutions simultaneously with realistic constraints. Yang et al. [45] presented a precise process to solve the problem of generalized limited portfolio optimization using the Irregular Random Flight Bird Swarm Algorithm (AI-BSA) and provided a numerical example to compare with both bird swarm and particle optimization algorithms. Numerical test of this paper showed that the selection of the initial period of real behavior (FQ) in the Irregular Random Flight Bird Swarm Algorithm (AI-BSA) does not have a significant effect on the performance of the algorithm. Therefore, it will be important to develop appropriate methods for selecting FQ to ensure the effectiveness of AI-BSA. On the other hand, in many practical applications, such as vehicle navigation, statistical clustering, etc., the AI-BSA proposed in this paper may provide a better current solution.

Kalayci et al. [24] used the three meta-heuristic algorithms of ant colony, bee colony, and genetic algorithm to solve the problem of securities optimization with cardinality constraints. They proposed their method based on seven general criteria in library data based on weekly prices between March 1992 and September 1997 from the Hang Seng, DAX 100, FTSE, S&P 100 and Nikkei 225 indices extracted by Cheng et al., As well as daily prices between May 2013 to April 2016, as described by Kalayci et al., Were used to evaluate the effectiveness of the proposed algorithm and compare it with the results of other research. Tekin Tezel et al. [43] tried to solve the problem of choosing an appropriate and efficient algorithm. They used fuzzy logic and fuzzy systems to create a collaboration scheme to automatically select appropriate Meta-heuristic algorithms and control the search process dynamically. Their study attempted to combine the benefits of different meta-heuristic algorithms to achieve a better search approach. Feshari and Mazaherifar [19], studied and compared the performance of genetic algorithms and weeds in obtaining the efficient boundary of the mean model of the mean variance bound to the Tehran Stock Exchange between 2014 and 2016. The results showed that the weed algorithm, despite using more time, was able to perform better.

Peymani Foroushani et al. [33] investigated the random mastery method in portfolio optimization and compared the performance of this method with portfolio optimization by Markowitz method, using performance evaluation criteria in Tehran Stock Exchange. The results show the superiority of the performance of the second-order random mastery method over the Markowitz method in the extra-sample and in-sample approach. Also, the second-order random mastery method has a higher cumulative efficiency than the Markowitz method. Amiri et al. [2] used a meta-heuristic algorithm called GRASP greedy stochastic matching search to solve the problem of portfolio optimization with cardinality constraints. In this study, in order to be more in line with the real world, two sets of constraints, including floor and ceiling constraints and cardinality constraints, have been added to the Markowitz model. Mansourian et al. [28], in order to implement an intelligent financial portfolio, upgraded the existing optimization methods based on Sharp ratio performance and provided intelligent methods for trading based on various algorithms.

Based on the objectives of this research and according to the theoretical foundations presented above, the following hypothesis can be expressed:

Hypothesis 1: There is a significant difference between the returns obtained from the meta-heuristic algorithms used to optimize the portfolio in market bubble conditions.

Hypothesis 2: There is a significant difference between the risks obtained from the meta-heuristic algorithms used to optimize the portfolio in market bubble conditions.

4 Research Type, Population and Statistical Sample

According to the purpose, this research is applied, and in terms of data, quantitative and post-event, and in terms of analysis, it is a descriptive-correlational research. In order to obtain the research data, considering that it is directly related to the Stock Exchange and Securities Organization, the data available on the website of the Tehran Stock Exchange and related sites and related sites, as well as the information available in the Rah-Avard-Nouvin software have been used. In terms of data collection and analysis of the results obtained from this, this research is descriptive and since MATLAB software is used to select the optimal portfolio, it is analytical. The statistical population of this research is all public joint stock companies with shares that can be traded on the Tehran Stock Exchange, which are active in the period considered in this study and their shares are traded, provided that they have price bubble conditions in the period under study. The reason for choosing this statistical community is due to the availability of relatively comprehensive information about the status of companies and their financial and economic performance. Considering the closest period to the current situation in terms of the novelty of statistical data for better analysis and also because to accurately perform the process of review and application of algorithms, it is necessary to have complete information accurately and correctly, the time period for reviewing the total number of transactions performed in the Tehran Stock Exchange during the period 2015-2021 was considered. To select the sample of this research, from all companies active in the stock market, the shares of companies that have all the following conditions are selected:

1. Due to the period of study and the need for the information needed to conduct research, companies that have been active in the period from 2015 to 2021 and have been listed on the stock exchange until the end of 2014 and have not been expelled from the stock exchange during the desired period.
2. Their financial information is available.
3. Do not prohibit the transaction and interrupt the transaction for more than 3 months in a year during the desired time period.
4. Not be part of banks, financial and credit institutions and investment companies.
5. In order to observe the comparability, the financial year of the company should end at the end of March of each year.
6. The company does not have a change in the financial year during the period under review.
7. Considering the cardinal limit on the number of shares and selected companies to 20 companies.

According to these cases, from the remaining 153 companies, 20 companies that have the above conditions to form a portfolio will be selected.

5 Data Analysis Method

In this research, in order to identify the courses with price bubbles, the sequence test and the skewness and kurtosis test were used, and based on the time intervals that were common in both tests, periods with a price bubble were identified. Then, to optimize the portfolio using the artificial bee clone meta-heuristic algorithm and the Particle swarm optimization, MATLAB software version 2015 was used.

6 Research Model and How to Implement It

There is a process for implementing and testing the algorithm, which is as follows:

1. Data selection: The first step is data selection. The financial data of the top companies in the Tehran Stock Exchange, according to the considered restrictions, for the years 2015 to 2021, which are collected using the Rah-Avard-Nouvin software and related sites of the Tehran Stock Exchange, are collected.
2. Clearing and preparing data: In this step, data whose independent variables do not exist due to incomplete information or that could not be calculated are deleted. In continuation, the shares of 153 companies that meet the conditions in this study remain.
3. Data normalization for use in the model: If we enter the data raw in the software, it will reduce the speed and accuracy of the algorithm. Therefore, in order not to face such a problem and also in order to standardize the value of the data before the test, the input data should be normalized (standardized). In other words, all data should be equated between 1- and 1. Formula (6) is used to normalize the data:

$$\text{Yi} = \frac{yi - y\,min}{y\max - y\,min}\,(hi - Li) + Li \tag{6}$$

Where: Yi = input values normalized by the equation, y_i = principal input values, y_{min} = smallest input value, y_{max} = largest input value, hi = high value at normalization interval (here equal to +1), Li = lower value at normalization distance (Here -1).

1. Determining the objective function: In this research, the original Markowitz model is used. Markowitz model has data or inputs which are:
2. Expected return per share
3. Deviation of the expected return criterion as a criterion for determining the risk per share
4. Covariance, as a measure that shows the alignment between the returns of different stocks.

The Markowitz model was based on expected return and risk characteristics of securities, which is essentially a theoretical framework for analyzing risk and return options. According to her theory, an efficient investment portfolio is a portfolio that has the highest return or the lowest risk at a certain level of risk for a certain level of return [5]. The Markowitz model is expressed as Equation (7):

$$\begin{aligned} &\text{Min } \sum_{i=1}^{n} \sum_{j=1}^{n} wiwj\sigma ij \\ &\text{Max } \sum_{i=1}^{n} wi\mu i \\ &\text{s.t: } \sum_{i=1}^{n} wi = 1 \\ &W_i \geq 0 \quad i=0,1,\dots,N \end{aligned} \tag{7}$$

In order to choose a safer portfolio, by entering the coefficient λ in the objective function, an attempt has been made to include both risk and return criteria in the objective function, while minimizing the risk and maximizing the return. In fact, λ is only a weighting parameter whose value varies in the range [0,1] and by which the investor's valuation is applied to risk or return. As a result of this change, the model presented in Equation (7) will be as follows:

$$\begin{aligned} &\text{Max } z = \lambda\ \lambda \sum_{i=1}^{n} wi\mu i - (1-) \sum_{i=1}^{n} \sum_{j=1}^{n} wiwj\sigma ij \\ &\text{s.t: } \sum_{i=1}^{n} wi = 1 \\ &W_i \geq 0 \quad i=0,1,\dots,N \end{aligned} \tag{8}$$

The above model is incapable of optimizing the selection of a restricted stock portfolio under integer constraints. To solve this problem, the integer constraint as equation (9) is added to the model:

$$\sum_{i=1}^{n} Zi = K \tag{9}$$

According to this limitation, if invested in share i, the value of Zi is equal to one, and if not invested in this share, the amount of Zi is equal to zero. In this formula, K is the number of shares that the investor wants to have in her portfolio and invest in them. Accordingly, the model will change as equation (10):

$$\begin{aligned} &\text{Max } z = \lambda\ \lambda \sum_{i=1}^{n} wi\mu i - (1-) \sum_{i=1}^{n} \sum_{j=1}^{n} wiwj\sigma ij \\ &\text{s.t. } \sum_{i=1}^{n} wi = 1 \\ &\sum_{i=1}^{n} Zi = K \\ &W_i \geq 0 \quad i=0,1,\dots,N \ \text{و}\ Zi \ \{0,1\} \end{aligned} \tag{10}$$

To consider the amount of weight of each asset in the portfolio, a minimum amount and a maximum amount are considered. In other words, if xi takes a value other than zero, relation (11) must be established:

$$\alpha \leq x_i \leq \beta, \square x_i \neq 0 \tag{11}$$

In this regard, α and β are the lower and upper limits for the weight of i's share in the portfolio, respectively.

5. Selection of stock portfolio based on artificial bee colony meta-heuristic algorithm and invasive weed algorithm: In the continuation of the above steps, with the help of MATLAB software and using the tools available in it, the optimal stock portfolio is selected with the artificial bee colony algorithm and the invasive weeds algorithm.

6.1 Sequence Test

One way to test for the existence of a bubble is to test the serial dependence of successive returns over time. One of the tools to identify serial dependence is the sequence test. This is done based on a test in non-parametric statistics called pairs test, to show a meaningful and logical difference between the counted pairs and the expected pairs.

Table 1: Last 12 Months Return for The Period Under Review in Excel Software

Last 12 months return	2015	2016	2017	2018	2019	2020
3/21 To 4/20	-0.00918	0.0153	0.0036	0.0146	0.07005	0.1082
4/21 To 5/21	-0.015	0.017	0.0028	0.0126	0.0966	0.1364
5/22 To 6/21	-0.0123	0.0125	0.0045	0.027	0.0949	0.1558
6/22 To 7/22	-0.0079	0.01015	0.0047	0.024	0.0916	0.1883
7/23 To 8/22	-0.009	0.016	0.00275	0.0445	0.0851	0.1771
8/23 To 9/22	-0.011	0.0184	0.0078	0.056	0.0907	0.1608
9/23 To 10/22	-0.00117	0.0177	0.0072	0.067	0.0763	0.1528
10/23 To11/21	-0.0128	0.0185	0.0086	0.1125	0.0664	0.1513
11/22 To12/21	-0.0086	0.0212	0.0135	0.047	0.0766	0.1425
12/22 To 1/20	0.0038	0.013	0.0161	0.0512	0.0838	0.1135
1/21 To 2/19	0.024	-0.0019	0.018	0.049	0.0925	0.1024
2/20 To 3/20	0.024	-0.00525	0.0166	0.0605	0.0902	0.1021

Table 2 shows the values related to the average monthly return of each month for the period under review.

Table 2: Monthly Returns for The Period Under Review in Excel Software

Monthly returns	2015	2016	2017	2018	2019	2020
3/21 To 4/20	0.0657	-0.0341	0.0150	-0.0093	0.1405	0.3565
4/21 To 5/21	-0.0467	-0.0237	0.0238	-0.0004	0.0527	0.3917
5/22 To 6/21	0.0060	-0.0501	-0.0294	0.1432	0.0916	0.3237
6/22 To 7/22	0.0523	0.0235	0.0256	-0.0052	0.0819	0.473
7/23 To 8/22	-0.0270	0.0420	0.0186	0.26	0.059	-0.0757
8/23 To 9/22	-0.0509	-0.0209	0.0398	0.1774	0.1189	-0.0764
9/23 To 10/22	0.0241	0.0154	0.0079	0.1375	-0.0101	-0.1067
10/23 To 11/21	0.0013	0.0113	0.0281	-0.0518	-0.0026	-0.0209
11/22 To 12/21	-0.0223	0.0101	0.0690	-0.0885	0.1582	0.0531
12/22 To 1/20	0.0838	-0.0108	0.0206	0.0713	0.1488	-0.1988
1/21 To 2/19	0.1631	-0.0202	-0.0007	-0.0280	0.1791	0.0459
2/20 To 3/20	0.0342	-0.0055	-0.0190	0.1201	0.0645	0.0608

If this difference is seen in the market, which is represented in the total index or in companies, then there is a bubble, and if this difference is meaningless, there is no bubble [14]. To perform this test, first, the monthly return of the examined period is calculated separately for each month. Then, the return of each month is compared with the average return of the previous 12 months, and if this return is higher than the average of the last 12 months, it is shown with a positive sign, otherwise, it is shown with a negative sign. Finally, if the number of consecutive positive signs is five or more, there is a possibility of a bubble. In Table 1, the average return of the last 12 months for each month in the period under review is shown in summary using Excel software.

6.2 Skewness and Kurtosis Test

In this test, if the skewness is negative, there is a possibility of a price bubble, because after the growth of prices, its decrease will be more than the increase due to the psychological atmosphere that is created, as a result, if a share is skewed to the left and is not normal, there is a possibility of a bubble.

Table 3: The Results of Doubling the Average Return of the Past One Year for Each Month of the Desired Period Using Excel Software

Double the average return of the past 12 months	2015	2016	2017	2018	2019	2020
3/21 To 4/20	0.01836	0.0306	0.0072	0.0292	0.1401	0.2164
4/21 To 5/21	0.03	0.034	0.0056	0.0252	0.1932	0.2728
5/22 To 6/21	0.0246	0.025	0.009	0.054	0.1898	0.3116
6/22 To 7/22	0.0158	0.0203	0.0094	0.048	0.1832	0.3766
7/23 To 8/22	0.018	0.032	0.0055	0.089	0.1702	0.3542
8/23 To 9/22	0.022	0.0368	0.0156	0.0112	0.1814	0.3216
9/23 To 10/22	0.00234	0.0354	0.0144	0.134	0.1526	0.3056
10/23 To11/21	0.0256	0.037	0.0172	0.225	0.1328	0.3026
11/22 To12/21	0.0172	0.0424	0.027	0.094	0.1532	0.285
12/22 To 1/20	0.0076	0.026	0.0322	0.1024	0.1676	0.227
1/21 To 2/19	0.048	0.0038	0.036	0.098	0.185	0.4048
2/20 To 3/20	0.048	0.0105	0.0332	0.121	0.1804	0.2042

Table 4: Courses with participation in sequence tests, and skewness and kurtosis tests

Year	Joint exam courses	Year	Joint exam courses
2015	2015/3/21 To 2015/4/21	2018	2018/5/22 To 2018/6/22
	2015/6/22 To 2015/7/23		2018/7/23 To 2018/10/23
	2015/9/23 To 2015/10/23		
2016	2015/12/22 To 2016/2/20	2019	2019/3/21 To 2019/4/21
	2016/6/21 To 2016/8/22		2019/11/22 To 2019/12/22
2017	2017/4/21 To 2017/5/22	2020	2020/1/21 To 2020/2/20
	2017/6/22 To 2017/9/23		2020/3/20 To 2020/7/22
	2017/10/23 To 2017/12/22		

And if the elongation is less than normal, the dispersion of the variance will be more, and this factor, along with the skewness, indicates the occurrence of bubbles [17]. To perform this test, first calculate the monthly returns of the examined period and if the monthly return is more than double the average return of the last 12 months, there is a possibility of a bubble. To perform this test, a comparison was made by using Excel software and calculating the monthly return of each month separately, and then calculating double the average return of the last one year for each month. In Table 3, the information related to the calculation of doubling the average return of the last year is shown every month for comparison with the monthly return. After performing the sequence tests and the skewness and kurtosis tests, the time intervals that are common to both tests are determined as bubble periods. The results of this study are shown in Table 4.

7 Optimization Using Meta-Heuristic Algorithms

After determining the periods with price bubbles, with MATLAB software version 2015 and using the tools available in it, considering that this research intends to use two meta-heuristic algorithms, the optimal stock portfolio is selected. For this purpose, according to the considered objective function and the limitation in choosing the number of shares in the portfolio to the number of 20 shares, separate coding to measure the return of each share, the risk of each share, the limitation of choosing the number of shares in the portfolio, the weight of each share in the selected portfolio, is done. Next, coding was done for the main objective function of this research and finally transferring its result to the main algorithm code (bee colony algorithm, Particle swarm optimization).

Considering that the objective function is the same in all the meta-heuristic algorithms used in this research, the codes considered for calculating the return, risk, weight of the portfolio, and the number of stocks in the portfolio have been written once and used for each algorithm.

Also, according to the basic code structure of the algorithms, and based on the objective function of this research, only the objective function part of the algorithm coding and the number of repetitions until reaching the desired answer were changed. The number of repetitions performed in each of the two meta-heuristic algorithms in order to achieve a better result, in this research, 8000 repetitions for each

Table 5: Yield and risk of selected portfolio in ABC algorithm and PSO

Meta-heuristic algorithm	PSO		ABC	
Period	Risk	Returns	Risk	Returns
2015/3/21 To 2015/4/21	0.01547	0.222629	0.0089	0.066897
2015/6/22 To 2015/7/23	0.1537	0.143485	0.0471	0.014625
2015/9/23 To 2015/10/23	0.01277	0.448379	0.0035	0.029041
2015/12/22 To 2016/2/20	0.00193	0.027721	0.004	0.019579
2016/6/21 To 2016/8/22	-0.000321	0.157585	0.1739	0.01208
2017/4/21 To 2017/5/22	0.01557	0.201555	0.00067009	0.006555
2017/6/22 To 2017/9/23	0.00252	0.046097	-0.00000051634	0.009634
2017/10/23 To 2017/12/22	0.01269	0.21915	0.0013	0.024126
2018/5/22 To 2018/6/22	0.00261	0.192242	0.00042279	0.041858
2018/7/23 To 2018/10/23	0.00174	0.145917	0.00054542	0.11678
2019/3/21 To 2019/4/21	0.00646	0.621087	0.0091	0.391309
2019/11/22 To 2019/12/22	0.00149	0.418694	0.0018	0.470745
2020/1/21 To 2020/2/20	0.0022	0.360783	0.00080658	0.134555
2020/3/20 To 2020/7/22	0.0016	0.151426	0.00094829	0.080605

desired period of time. Outputs of code execution in MATLAB software, including:

1- Selected shares in the portfolio,

2- The weight of each of the selected shares in the order of selection in the stock portfolio,

3- The return of each of the selected shares in the stock portfolio in the order of selection,
4- The risk of each of the selected shares in the stock portfolio in the order of selection.

In these algorithms, according to the average value and variance of each share and according to the intended objective function, to select the optimal portfolio of shares including 20 shares that maximize the return and minimize the risk, is attempted. Based on the periods with price bubbles obtained, the optimal portfolio of each period was determined using meta-heuristic algorithm of artificial bee colony and Particle swarm optimization.

Also, based on the information obtained from the portfolios and the weight of each in the studied periods, the amount of return and risk of each period is shown in Table 5.

8 Research Hypotheses Test

After obtaining each of the values mentioned in the previous step, according to the return and risk of each of the portfolios in the used algorithms, comparing the return and risk obtained from each algorithm with each other, in order to confirm or reject the research hypotheses done. In this step, first, the covariance test of each algorithm is checked. For this purpose, the investment risk in each of the methods is examined and compared. This case was investigated by the covariance test of the paired sample T-test in SPSS software version 26. In this test, the following hypotheses were used:

$$\begin{cases} H0\text{: The variance of the two groups is not equal} \\ H1\text{: The variance of the two groups is equal} \end{cases}$$

In the above test, whenever the value of the maximum level of significance is less than 0.05, the assumption of inequality of variance will be rejected with 95% confidence [5].

9 The covariance test of the paired sample T test

In Table 6, the results of the covariance test of the T-test of two paired samples, which was performed by SPSS software version 26, are presented.

Table 6: The results of the T-test of two paired samples

algorithms	**Paired Differences**					**t**	**df**	**Sig. (2-tailed)**
	Mean	**Std. Deviation**	**Std. Error Mean**	**Confidence ۵۵% Interval of the Difference**				
				Lower	**Upper**			
ABC - PSO	**0.00162**	**0.05696**	**0.01522**	**-0.03127**	**0.03451**	**0.107**	**13**	**0.917**

According to the result obtained from the covariance test and the rejection of the H0 hypothesis, the Wilcoxon test at a confidence level of 0.99 is used in SPSS software version 26 to check the research hypotheses. In the Wilcoxon test, a two-by-two comparison was made, once between the returns and once between the risks of the portfolios obtained from each algorithm, in order to investigate the research hypotheses. In this test, two hypotheses for return and two hypotheses for risk were set as follows.

9.1 Hypotheses used to compare the efficiency of meta-heuristic algorithms in the Wilcoxon test:

H0: There is no significant difference between investment returns in stock portfolios obtained from meta-heuristic algorithms.
H1: There is a significant difference between investment returns in stock portfolios obtained from meta-heuristic algorithms.

The results of the Wilcoxon test to compare the performance of meta-heuristic algorithms are shown in Table 7.

Table 7: The Results of the Wilcoxon Test to Compare the Returns of the Stock Portfolios Obtained in the Algorithms

	PSO - ABC
Z	-3.045
Asymp. Sig. (2-tailed)	0.002

9.2 Hypotheses Used to Compare the Risk of Meta-Heuristic Algorithms in the Wilcoxon Test:

H0: There is no significant difference between the investment risks in stock portfolios obtained from meta-heuristic algorithms.
H1: There is a significant difference between the investment risks in stock portfolios obtained from meta-heuristic algorithms.

The results of the Wilcoxon test to compare the risk of meta-heuristic algorithms are shown in Table 8.

Table 8: Wilcoxon Test Results to Compare the Risk of Stock Portfolios Obtained in the Algorithms

	PSO - ABC
Z	-1.538
Asymp. Sig. (2-tailed)	0.124

In the following, according to the results of the Wilcoxon test, the hypotheses of this research are examined.
Hypothesis 1: There is a significant difference between the returns obtained from the meta-heuristic algorithms used to optimize a portfolio under market bubble conditions.

In order to answer this hypothesis, first, time periods with price bubbles in the investigated period were identified by using sequence test, and skewness and kurtosis test. Then, according to these time periods with price bubbles, based on the objective function considered to obtain returns, and based on the shares and weights of each of them in the selected optimal portfolio, the returns were calculated for the periods with price bubbles. After that, the Wilcoxon test was used to check the difference between the returns of meta-heuristic algorithms in the selected stock portfolios.
Based on the results of the SPSS software output for the Wilcoxon test, at a significance level of 0.002 between the bee colony algorithm and the Particle swarm optimization, it can be stated that there is no

significant difference between the returns obtained from the meta-heuristic algorithm and this hypothesis is rejected at a significance level of 5%. In other words, it can be said that the superiority of algorithms in the field of return is relative to each other and there is no sign of complete superiority.

Hypothesis 2: There is a significant difference between the risks obtained from the meta-heuristic algorithms used to optimize a portfolio under market bubble conditions.

In order to answer this hypothesis, first, time periods with price bubbles in the investigated period were identified by using sequence test, and skewness and kurtosis test. Then, according to these time periods with price bubbles, based on the objective function considered to obtain risk, and based on the shares and weights of each of them in the selected optimal portfolio, risk was calculated for periods with price bubbles. After that, the Wilcoxon test was used to check the difference between the risks of meta-heuristic algorithms in the selected stock portfolios.
Based on the results of the SPSS software output for the Wilcoxon test, at a significance level of 0.124 between the bee colony algorithm and the Particle swarm optimization, it can be stated that there is no significant difference between the risks obtained from the meta-heuristic algorithm and this hypothesis is rejected at a significance level of 5%. In other words, it can be said that the superiority of algorithms in the field of risk is relative to each other and there is no sign of complete superiority.

10 Conclusion and Discussion

Choosing a stock portfolio in investment matters is a difficult and difficult task to decide which stock is in a better position compared to other stocks and has the merit of being selected and placed in one's investment portfolio and how to allocate one's capital between these Stocks are complicated. These conditions become more difficult when there is a price bubble in the capital market and it increases the amount of risk for the investor compared to normal conditions without a bubble. In this research, two meta-heuristic algorithms were used in order to select an optimal portfolio in the conditions of the existence of a bubble in the capital market, taking into account the two goals of maximizing returns and minimizing risk. Based on the results of tests related to the identification of the price bubble, the months with the price bubble were determined, which indicates the turbulent and risky capital market in Iran for investment. In this case, investors must carefully follow the trend of prices so as not to risk losing their capital in the process of investing and making a profit, at the time of price peak and approaching price fall. According to the purpose of research on portfolio optimization in capital market bubble conditions, and based on the specified periods with bubbles in the market, the artificial bee colony algorithm and PSO algorithm were used for optimization. According to the results of using meta-heuristic algorithms to optimize the portfolio, in relation to the obtained returns, no significant difference was observed between the returns of periods with price bubbles in each of the two meta-heuristic algorithms. Accordingly, the first hypothesis of this study was rejected regarding the significant difference between the efficiencies of the meta-heuristic algorithms used. Also, by comparing the risk obtained between the optimal portfolios of each algorithm, there is no significant difference between the risk values in the two algorithms. Accordingly, the second hypothesis of the study about the existence of a significant difference between the risks of portfolios obtained from the two meta-heuristic algorithms was rejected.
Based on the findings mentioned above and the observation of the researchers, it can be stated that the above algorithms, due to the speed of the program execution and the logical phases considered in each algorithm, different results and different yields have been obtained. Among the algorithms implemented in this research, the bee colony algorithm has a higher accuracy and a longer execution time, while the

particle swarm algorithm with an average execution time has shown better results for each of the examined periods. In terms of the amount of risk mentioned for each of the two algorithms in the studied periods, no significant difference is observed. In general, it can be concluded that according to the results presented in each algorithm, the particle swarm algorithm has a higher speed and accuracy than the artificial bee colony algorithm. One of the limitations of this research is the non-generalizability of the findings of this research to stock exchanges of other countries and other time periods due to the condition of the price bubble, also mentioned other companies in the stock market due to the restrictions considered for selecting the statistical sample. This study is for all risk-averse and risk-averse investors, considering that price bubble courses were first identified. And then the portfolio was optimized, providing a suitable solution to achieve the desired return against the least amount of risk. Based on the objectives and results of this study, to identify courses with price bubbles, it is suggested to use other methods or a combination of methods to identify courses with price bubbles in future research, Or the methods used to identify the periods with price bubbles used in this study, be used for other markets and compare the results with the present study. Also, to optimize the stock portfolio, other algorithms should be used in other markets of Tehran Stock Exchange, including OTC market and commodity exchange or use the methods used in this study to identify price bubbles in the stock market of other countries.

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