Journal of Philosophical Investigations
Print ISSN: 2251-7960 Online ISSN: 2423-4419
Homepage: https://philosophy.tabrizu.ac.ir

On the Architectonic Idea of Mathematics

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Article Info

Article type:
Research Article

Article history:
Received 25 July 2024
Received in revised from 29 July 2024
Accepted 29 July 2024
Published online 07 August 2024

Keywords: architectonic, geometry, mathematics, arithmetic, construction.

ABSTRACT

The architectonic is key for situating Kant's understanding of science in the coming century. For Kant the faculty of reason turns to ideas to form a complete system. The coherence of the system rests on these ideas. In contrast to technical unity which can be abstracted a posteriori, architectonic ideas are the source of a priori unity for the system of reason because they connect our reasonable pursuit to essential human ends. Given Kant's focus on mathematics, in the architectonic and his critical philosophy more generally, we must have some sense of the architectonic idea of mathematics. In this paper, I argue for the key principles of the architectonic idea of mathematics: 1) because mathematics is grounded in a priori intuition, it is a peculiarly human activity; 2) the method of mathematics is one of a priori construction, a method only mathematics can employ and: 3) the objects of mathematics are extensive magnitudes. Given these principles, we can use the architectonic idea to have some clarity about how mathematics has dealt with historical development.

Cite this article: Valdez, E. (2024). On the Architectonic Idea of Mathematics. *Journal of Philosophical Investigations*, 18(47), 203-218. https://doi.org/10.22034/jpiut.2024.18293

 Publisher: University of Tabriz.
https://doi.org/10.22034/jpiut.2024.18293

Introduction

One way to consider the role and significance of Kant's philosophy for the future is to examine how Kant anticipated future developments in human investigation. This is best exhibited in Kant's account of an architectonic of reason. Kant's Doctrine of Method aims to determine "the formal conditions of a complete system of pure reason," where reason is the faculty of inferring and concluding from concepts (B, 735). The coherence of such a system rest on ideas that are grounded in an empirical consciousness but have no given or empirical object as referent. Among those formal conditions is systematicity, which Kant characterizes in the Architectonic. Given the importance of mathematics for the ends of reason and the many developments in mathematics since Kant, we should be able to express some of the principles of the architectonic idea of mathematics. In what follows I aim to give some exposition of what, for Kant, must be the architectonic idea of mathematics.

For Kant, mathematics concerns how we construct magnitudes a priori. First, I discuss architectonic unity, distinguishing it from technical unity and emphasizing its importance for the interests of reason. Reason requires that an architectonic idea is regulative. Second, I consider how mathematics is a peculiarly human science and why that peculiarity is tied to the essential ends of human reason. Mathematics takes as its substratum the a priori intuitions of space and time, the conditions for human sensibility, and they form the basis of nature. Third, I outline Kant's account of construction in mathematics. I then consider what for Kant are the objects of the mathematical domain, namely extensive magnitudes. Finally, I consider briefly how the architectonic idea helps Kant incorporate some developments in mathematics.

1. Architectonic Ideas

Reason is the faculty of inferring from principles. The understanding is constrained by rules, but reason proceeds through principles. Reason seeks a coherence that is not given by empirical objects. Whereas the understanding serves to understand perceptions, reason seeks to comprehend possible experience, of which actual experience is only a part (B, 367). To do this reason turns to ideas, a conjunction of concepts that complete a system. These ideas are still only possible in an empirical consciousness that brings coherence to the manifold of what is given in experience. Ideas are a composite of concepts that we infer in order to give coherence to possible experience.

Systematicity is the fundamental feature of the architectonic because "systematic unity is that which first makes ordinary cognition into science, i.e., makes a system out of a mere aggregate of it, architectonic is the doctrine of that which is scientific in our cognition in general, and therefore necessarily belongs to the doctrine of method" (B, 860). Only through this systematicity is more knowledge not merely an aggregate but an integral part of our science. Systematicity anticipates more knowledge to be incorporated. It expects growth and development from within over time. For

Kant, architectonic structure is the source of science's rigor. In architectonic unity, method and structure form the unity of knowledge a priori, while contingent purposes and experiences form a technical unity a posteriori. In investigating the bounds of metaphysics, we can grasp an architectonic idea of metaphysics whose structure precedes its content.

A science's unity must come from its a priori methodology, not from already experienced answers or affections. This architectonic structure is distinct from a mere technical or coincidental unity, "[t]he whole is therefore articulated; and not aggregated: it can, to be sure, grow internally, but not externally, like an animal body, whose growth does not add a limb but rather makes each limb stronger and fitter for its end without any alteration of proportion" (B, 861). Kant expects the sciences to add to the cognition of the articulated whole of sciences so that the system of truths that unifies them should grow to be a fuller system. This articulated whole is not to be understood as finished, but complete as in forming a system. The system serves as ground for completion from within.

Technical unity arises from the contingent coincidence of cognition. In this case, the unity is determined by the parts but also exhausted by it. There is nothing from within in furtherance of which the unity can grow. Its completion stems from a terminus rather than a purpose. Growth must come from the a posteriori addition of further coincidence. Technical unity looks back while architectonic unity looks forward in anticipation. Architectonic unity stems from "a single supreme and inner end, which first makes possible the whole" (B, 861). In this case the unity precedes any of the parts unified and growth occurs from within. The science only grows if that growth is a part of that same idea of reason.

On Kant's view, philosophy admits of the same architectonic structure, though it has been poorly expressed at times, leading to confusion about its method and aims. Kant is working to recognize the architectonic idea of philosophy and its reasonable ends. It is insufficient merely to historically study the work of previous philosophers or adopt their truths. One must philosophize in accord with the architectonic idea of philosophy. While Kant hopes to bring about a revolution in philosophy, the revolution is not one that changes the architectonic idea of philosophy but works to realize it and bring philosophy closer to scientific status. Philosophy, for Kant, admits of a systematic unity that is the basis of its scientific status. Thus far, philosophers have been unable to set philosophy on the path to science because they have misread the end of philosophy.

In an architectonic idea we find a systematic unity that understands cognitions in light of the same purpose or function. Science requires systematicity because it must yield knowledge of necessary laws. Isolated moments of cognition can only yield knowledge of laws when they are united under a larger explanatory function. The architectonic of reason structures reason and in turn all systematic reasonable pursuits. Thus, it is the location within this a priori structure that indicates the function of a science. As such, functions are embedded in other functions and unite other functions under them. Much like concepts, these functions have both extension and intension. The organic metaphor Kant uses to describe architectonic structure maps onto the intension and extension of functions within the architectonic. As an example, we think of the digestive system as having a purpose. This gives purpose to the disparate elements of the digestive system each with their own subordinate functions. The function of the digestive system itself is also an element of a larger function, i.e. the function of the organism. Particular sciences have explanatory functions. Those functions can be understood as being in service of a higher reasonable end. But also, within those sciences there are questions or issues that require a further determined idea. Kant uses this relationship to argue that the work of the Critique can be situated in a larger framework of science that alludes to a taxonomy of the sciences. In this larger framework there are subordinate sciences that require metaphysics but there are also broader interests of reason of which philosophy and metaphysics are only a part. As such, the architectonic serves its highest function, the essential ends of human reason. Kant uses the architectonic to argue that philosophers have thus far only dealt with the scholastic concept of philosophy, philosophy as concerned with rigor and precision. What he postulates as the ideal is philosophy in its cosmopolitan concept, as advancing the essential ends of humanity, calling on philosophers to see their work as tied to essentially practical ends.

Gabriele Gava argues that if we adhere strictly to the link to the essential ends of human reason, some of the disciplines we recognize as science today would fall short of the architectonic unity that Kant prescribes since humanity's essential ends are necessarily practical and physics and chemistry and the like are theoretical. Gava sees this tension due to a double function of the relationship between reason and its ends. One function concerns the immediate explanatory ends of a science and the other the mediate ends that point to the priority of practical philosophy.

> Kant also claims that architectonic unity is only possible by making reference to the essential ends of reason. Essential ends are practical and are so treated by practical philosophy. This would imply the quite implausible consequence that only philosophy, insofar as it considers the systematic relationship between our knowledge and these essential practical ends, can become a science (Gava, 2005, 373).

If architectonic unity requires a science to recognize the essential ends of humanity, many seemingly united or systematic realms of inquiry would fall short of scientific status given their agnosticism with regard to the essential ends of humanity. Kant grants that we may be mistaken about the architectonic unity of some investigations, but he also seems committed to the idea of physics and chemistry as sciences. Gava proposes eliminating the requirement that architectonic unity necessarily be linked to the essential ends of human reason, holding that an a priori internal unity is enough to ensure scientific status while unity in philosophy would still prioritize the practical ends of reason over the theoretical. Gava's point is that systematicity concerns a certain structural feature that allows for the unity of cognitions. We can keep this feature without having to turn to the essential ends of humanity. Such ends would seem to denote content of the function rather than a strictly structural feature. I doubt, however, that this tension rises to the level of problem or contradiction.

The architectonic idea is not part of the psychology of the scientist, or even philosophers or historians of science. That is, someone inquiring need not always be conscious of the mediate purpose of a science for that science to work toward that purpose or function.

> Nobody attempts to establish a science without grounding it on an idea. But in its elaboration the schema, indeed even the definition of the science which is given right at the outset, seldom corresponds to the idea; for this lies in reason like a seed, all of whose parts still lie very involuted and are hardly recognizable even under microscopic observation. For this reason, sciences, since they have all been thought out from the viewpoint of a certain general interest, must not be explained and determined in accordance with the description given by their founder, but rather in accordance with the idea, grounded in reason itself, of the natural unity of the parts that have been brought together. For the founder and even his most recent successors often fumble around with an idea that they have not even made distinct to themselves and that therefore cannot determine the special content, the articulation (systematic unity) and boundaries of the science (B, 862).

Those of us working in service of scientific inquiry are likely to have an operative definition of a science that does not give full expression to the idea that situates it within the architectonic of reason. The biologist may not comprehend how biology serves the essential ends of reason even though biology as a science maybe be necessary for a coherent view of such ends. That might give us cause to question our definition or anticipate development but not to abandon the presumption of the scientific status of our idea. The cogs in a machine, the bees in a swarm, the organs in an organism need not be aware of the whole of the system for the system to be its intelligible organizing principle.[1] Though the explanations within a science may be ultimately in service of the essential ends of humanity, the ends of humanity need not be expressed in every cognition of that science. The duality of function that Gava points to is not one that sees these functions as in conflict with one another. For Kant, we cannot simply talk about the structure of reason in the abstract. We must talk of the structure of human reason; the structure then cannot take its form without situating the essential ends of humanity as that in service of which all science ultimately gets its function. To acknowledge the essential practical ends of humanity is not to abandon the ends of our science. Rather these functions should be understood as ordered.

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[1] I would also hold that the collection or cohesion of scientific investigation is not enough to meet the standard for architectonic unity absent a connection to the ends of humanity but such disagreement is not central to the concern I am addressing concerning mathematics.

Kant rejects a Platonist notion of idea but uses the term because of its metaphysical baggage. As Onora O'neill, (1992, 285) points out these ideas are to be thought of as precepts not given realities,

> He defends his appropriation of this mis- leading Platonic term, not because but *in spite* of its metaphysical resonance. The term suits not because Kant too wants to endorse a classical, theoretical conception of reason, as correspondence of thought to its real archetypes, but because Plato's Ideas are potent symbols of striving for the most encompassing unity. The Platonic Ideas are an image of the unity of the highest principles that guide a quest for the Good and the Beautiful as well as the True. Kant allows himself this borrowing, which parallels his own three fundamental questions, but rejects the entire Platonic account of the metaphysical basis of unity and success in these quests. He firmly rejects all thought that his Ideas of Reason correspond to any real archetypes, and adopts a position that is irreconcilable with any form of the Platonic vision of Ideas as patterns for knowledge and mathematics (Oneill, 1992, 285).

Ideas for Kant are inferred for the sake of coherence so they do not depend on any one agent thinking them, but they are not the foundations of any metaphysical claims. There is a distinction between an idea within the architectonic framework and our contingent recognition of it. The scientific status of particular fields of inquiry is conditioned by an architectonic idea but not by our knowledge of the architectonic idea. Our recognition of an architectonic idea may be muddled or even absent. Mathematicians and physicists need not be thinking of the essential practical ends of human reason when engaged in mathematics in order for mathematics to be a part of the framework of human reason. In this sense, the unity Gava prescribes is the one we most concern ourselves with in identifying sciences.

> Thus, the metaphysics of nature as well as morals, but above all the preparatory (propaedeutic) critique of reason that dares to fly with its own wings, alone constitute that which we can call philosophy in a general sense. This relates everything to wisdom, but through the path of science, the only one which, once cleared, is never overgrown, and never leads to error. Mathematics, natural science, even the empirical knowledge of humankind, have a high value as means, for the most part to contingent but yet ultimately to necessary and essential ends of humanity, but only through the mediation of a rational cognition from mere concepts, which, call it what one will, is really nothing but metaphysics (B, 878).

The value that these sciences have must be examined through rigorous metaphysics, but this need not be a project for the physicist or mathematician according to Kant.

Philosophy, however, must at some point address the essential ends of human reason directly and systematically because the internal a priori relation science has to the essential ends of humanity are to be understood by the philosopher. Part of the task of philosophy is to understand such ends and situate them in relation to the essential ends of humanity, something the ideal philosopher is tasked with investigating. The question of the essential ends of humanity falls within the immediate ends of philosophy. Gava cites[1] Kant's remark of the logician and mathematician being only artists of reason—in contrast to the philosopher as legislator—as evidence that the possibility of only philosophy meeting the standard for science is one Kant entertains. But though this is perhaps a careless remark on Kant's part it is important to note the context in which he makes it. When Kant distinguishes between the scholastic concept and the cosmopolitan concept of philosophy, he claims that only the latter relates all science to the essential ends of humanity. It is only in this respect that the mathematician and logician would not be up to the task because the task is a philosophical one. As such it is one for the ideal philosopher,

> From this point of view [that of the ideal philosopher] philosophy is the science of the relation of all cognition to the essential ends of human reason (*teleologia rationis humanae*), and the philosopher is not an artist of reason but the legislator of human reason. It would be very boastful to call oneself a philosopher in this sense and to pretend to have equaled the archetype, which lies only in the idea. (B, 867).

By this Kant means that we have not yet come to realize the relation between all our cognition and the essential ends of humanity. The mathematician may be versed in the rational cognition of geometry but that does not equip her to relate even mathematical cognition to the essential ends of humanity. As Guyer notes only an architectonic can hope to bridge the seeming divide between the determinism of a system of nature and the fundamental freedom that must be postulated for practical human ends,

> Kant's basic idea is that to think rationally is to think systematically: to think about nature rationally is to think about it systematically: to think about our own conduct rationally is to think about it systematically; and ultimately, we must think about how mankind could collectively achieve a systematic union of ends within the system of nature (Guyer, 2005, 3).

This occurs only when philosophy turns its attention to these essential ends. For Kant, any attempt to offer an assessment of reason requires making sense of mathematics. This has been a

[1] The scholastic philosopher cannot thus be considered a real philosopher, but only an "artist of reason" [Vernunftkünstler], along with the mathematician, the logician and the naturalist. By contrast, the true philosopher is a "legislator of reason" [Gesetzgeber der menschlichen Vernunft; KrV, A 839/B 867], even though this idea of the philosopher as a "knower of wisdom" [Kenner der Weisheit; V-Met-L2/Pölitz; AA 28, 534] remains only an archetype. P.378

source of confusion for philosophers because metaphysicians have continually *misused* mathematics in hopes of advancing philosophy,

> But what obscured the fundamental idea of metaphysics from yet another side was that, as a priori cognition, it shows a certain homogeneity with mathematics, to which, as far as a priori origin is concerned, it is no doubt related; but the comparison between the kind of cognition from concepts in the former with the manner of judging a priori through the mere construction of concepts in the latter requires a difference between philosophical and mathematical cognition—thus a decided heterogeneity is revealed, which was always felt, as it were, but was never able to be brought to distinct criteria. Thus, it has been the case until now that since philosophers themselves erred in the development of the idea of their science, its elaboration could have no determinate end and no secure guideline…. (B, 872).

Kant distinguishes between the cognition from concepts of the philosophical method and the cognition from the construction of concepts of the mathematical method. The method of mathematics is not available to philosophy, but too many philosophers have attempted to use it. In distinguishing these methods Kant offers some principles of the architectonic idea of mathematics.

2. Mathematics as Human Endeavor

The critical standpoint shifts metaphysics from a concern of how cognition conforms to objects to a concern for how objects conform to cognition, a reversal of the priority of matter over form. Kant views the rationalists before him as investigating the matter of the world but his Copernican Revolution in philosophy turns the attention to the form of the objects of cognition. That is, the form of our experience of the world is embedded in our epistemic condition and not in things in themselves. Thus, an investigation of nature must turn to an investigation of the a priori structure of our experience. Rather than consider what the world is like from a perspective outside of it, we can ask only from a perspective within the world. It follows then that mathematics is also a distinctly human investigation. Mathematics draws on the formal conditions of space and time but space and time are a priori intuitions *for us*. Thus, mathematics is wholly a priori but only an investigation of what we have ourselves put into the objects of investigation. Other rational beings may be subject to the same logical constraints but nothing requires that such beings have the same intuitive frameworks for experiencing the world. Rational beings with different intuitive frameworks—including computers that can adhere to logical constraints and make large computations very quickly—would mathematize differently, "all these principles, and the representation of the object with which [mathematics] occupies itself, are generated in the mind completely *a priori*…." (B, 299) This is not then an idiosyncratic or solipcistic activity. It remains wholly a priori but its intelligibility has its seat in the structure of the human subject. The coherence of our empirical consciousness requires an a priori spatiotemporal manifold and mathematics is the science through which we can treat this manifold as object.

Imagination is the faculty of representing an object in intuition that is not present. This faculty is divided in two. The reproductive imagination offers reproductions and distortions of objects that were previously given while the productive imagination presents objects that were not previously given. The productive imagination is a figurative synthesis. It is spontaneous and determines the sense a priori in accordance with the unity of apperception (B, 151). It yields only a priori determinations of space and time. An object is constructed when an intuitive manifold is united with a concept. That is, we only have a geometrical object when we have brought the spatial manifold under a conceptual unity. To cognize a line, one must draw it, but the drawing that is required is the a priori construction of it, regardless of whether or not it is accompanied by an empirical pencil and paper drawing. The unity of this act is the unity of consciousness in the concept of a line. Space as the form of outer intuition guides the construction. While the reproductive imagination is passive and governed by sense and empirical laws, the productive imagination yields the manifold of sense in accordance with a concept.

> Imagination is the faculty of representing an object even without its presence in intuition. Now since all of our intuition is sensible, the imagination, because of the subjective condition under which alone it can give a corresponding intuition to the concepts of understanding, belongs to sensibility; but insofar as its synthesis is still an exercise of spontaneity, which is determinative and not, like sense, merely determinable, and can thus determine the form of sense a priori in accordance with the unity of apperception, the imagination is to that extent a faculty for determining the sensibility a priori, and its synthesis of intuitions, in accordance with the categories, must be the transcendental synthesis of the imagination, which is an effect of the understanding on sensibility and its first application (at the same time the ground of all other applications) to objects of the intuition that is possible for us (B, 151-2).

Since it is the figurative synthesis that serves as the mathematical object and not the empirical image left behind, mathematics occurs only in the subject. It concerns the outside world, but the activity is in the subject.

Mathematics is connected to essential human ends because it is necessary for the various natural sciences that allow us to understand nature. As Kant puts it in the Metaphysical Foundations, “in any special doctrine of nature there can be only as much proper science as there is mathematics therein.” (4, 470) A robust understanding of nature is necessary for the essential ends of human reason because though such ends may be moral, they take place against the backdrop of the natural world. Thus, the full synthesis that brings together the determinism of nature and the freedom necessary for morality is only possible with a robust mathematical device, the same subjective condition for reasoning about the essential ends of humanity. We need good mathematical knowledge of nature to make good moral decisions about peace, climate change, agricultural policy etc. Devices that can compute quickly may produce data but data alone is not cognizable. More importantly, although being precise about those connections may prove elusive, we need not see it as a necessary step in identifying the human element of the architectonic idea of mathematics.

3. Construction

The mathematical method is integral for the architectonic idea of mathematics. Kant distinguishes between the philosophical and mathematical method.

> Mathematics proceeds through the construction of concepts. This method always treats its objects in concreto. [Philosophy] confine itself solely to general concepts, [mathematics] cannot do anything with the mere concepts but hurries at once to intuition, in which it considers the concept *in concreto*, although not empirically, but rather only as one in which it has exhibited a priori, i.e., constructed, and in which that which follows from the general conditions of the construction must also hold generally of the object of the constructed concept (B, 743-4).

In mathematical judgments, there is always an object before us. But mathematics is not an a posteriori endeavor in which we keep mathematical objects before us by relying on empirical objects or sensory affection. Rather we construct the objects of investigation, we produce an object in intuition that exhibits the formal conditions or features we are investigating.

For Kant, mathematics proceeds by definitions, axioms and demonstrations. A definition presents completely the concept of something while delineating its limits and contexts. The only concepts that allow for real definitions are those with an arbitrary synthesis with a priori construction because other attempts at definitions fall short of an exhaustive delineation of constitutive properties. Axioms are synthetic in that they appeal immediately to the intuition of space and are a priori in that they are necessarily true. Mathematics can synthetically combine two constructed concepts to yield another that can be likewise constructed in intuition and this is what Kant calls a demonstration. More precisely a demonstration that proceeds through the construction of concepts he calls a *Mathema* (B, 764). This constructed intuition is a priori. Given their immediacy and construction in intuition, axioms are intuitive principles to be distinguished from discursive principles. Mathematical construction also requires postulates, immediately certain intuitive principles that are necessary for construction,

> Now a postulate in mathematics is the practical proposition that contains nothing but the synthesis through which we first give ourselves an object and generate its concept, e.g. to describe a circle with a given line from a point on a plane; and such

> a proposition cannot be proved, since the procedure it demands is precisely the procedure through which we first generate the concept of such a figure (B, 287).

These postulates, much like the postulates for God, freedom, and immortality are practical in that they are necessary for activity. The metaphysical objects are necessary for moral behavior and the mathematical postulates are necessary for geometrical construction. Unlike the postulates of the metaphysical objects, mathematical postulates concern a procedure or activity through which we construct something necessary for a mathematical demonstration[1].

For Kant, space does not give us geometrical objects, rather it conditions their construction. These objects are constructed through a conceptual synthesis. But the formal conditions of space give us the content that we synthesize to construct geometrical objects. This highlights the distinction between space and geometry. As Kant writes in his response to Kästner, metaphysical space "is considered in the way it is given, before all determination of it in conformity with a certain concept of object" (20, 419), Geometrical space is constructed through a determinate synthesis which distinguishes it from the a priori intuition of space. Space conditions the geometrical representations that are possible but not because it already contains such representations. Rather it does so by conditioning the synthetic construction of geometrical spaces and objects. Space provides the manifold that is in turn limited for our construction.

The activity of the productive imagination seems to presume or presuppose a perceptual analog to the precise mathematical properties of continuity and homogeneity. The condition for phenomenological or perceptual activities in intuitive spatial representation, prior to any conceptual synthesis, is distinct from the mathematical properties that require synthesis. The latter are not merely read off from our perception of our spatial representation. Quite the contrary, from the mere a priori representation of space, we cannot establish or discover properties beyond those considered in the Metaphysical Exposition. The space of geometrical activities is itself the product of an a priori synthesis.

In geometry, the activities of drawing a line, extending a line, circumscribing a circle—often referred to as straight-edge and compass—are foundational in that they make all other constructions possible. These elementary activities correspond to a priori phenomenological activities that have been united with a concept to produce an a priori synthesis. For Kant, the possibility of such constructions is uniquely conditioned by the a priori intuition of space. As he puts it in *On Kästner's Treatises*:

> That, however, the possibility of a straight line and a circle cannot be demonstrated mediately through inferences, but only immediately through the construction of

[1] In "Kant on Parallel Lines: Definitions, Postulates, and Axioms," Jeremy Heis offers a detailed account of Kant's characterization of mathematical postulates and the difficulty of considering the parallel postulate to be a genuine postulate. In this paper, I am taking Kant at his word about the role postulates play in pure mathematics, without such a detailed examination.

> these concepts (which is not at all empirical), is due to the fact that among all constructions (exhibition which are determined according to a rule in intuition *a priori*) some must be the first, such as the drawing (*Ziehen*) or the describing (in thought) of a straight line and the rotation thereof around a fixed point, where neither the latter can be derived from the former, nor these from any other construction of the concept of magnitude (20, 411).

The geometrical activities in question are inseparable from the magnitudes provided by intuition. To construct mathematically is to remain in immediate relation to intuition. Onof and Schulting (2014) show that this distinction between space as form of intuition and as formal intuition is present throughout the Critical period as evidenced in the Transcendental Deduction

> Space represented as object (as actually required in geometry), contains more than the mere form of intuition, it also contains the composition of the manifold given in accordance with the form of sensibility in an intuitive representation, so that the form of intuition gives the manifold, but the formal intuition gives the unity of the representation. I ascribed this unity in the Aesthetic merely to sensibility, only to note that it precedes all concepts, although it does presuppose a synthesis, which does not belong to the senses but through which all concepts of space and time first become possible. For since through it (as the understanding determines the sensibility) space or time are first given as intuitions, the unity of this a priori intuition belongs to space and time, and not to the concept of the understanding (∫24) (B160).[1]

Cognition of objects requires a synthesis. Space as the form of intuition is given, but it cannot be cognized. While singular, it merely forms human receptivity to the world and cannot be made a unity with any concepts. The formal intuition of space has been subjected to a kind of pre-conceptual unity so that rather than perceive a mere manifold there can be a representation of space as object. But once geometrical activity has begun, the manifold has been made a unity and synthesized with concepts. It has been subjected to a series of conceptual syntheses, among which might be continuity and homogeneity.[2]

[1] This footnote is key to the debate in Kant scholarship between conceptualism and non-conceptualism. In the former, all unity, even the form of sensibility, is owed to concepts. In the latter, sensibility to concepts requires a unity that is not owed to the understanding. I side with non-conceptualists in this regard although I think a conceptualist can recognize the same elements of the architectonic idea of mathematics. Where the conceptualist might take issue is in my characterization of construction.

[2] Part of the case that I elsewhere make in considering Kant's views on geometry is that geometrical construction requires many syntheses and many of these we implicitly assume are necessary for all geometrical activity although they are merely a subset of all the geometrical syntheses that are possible.

4. Magnitudes

Mathematics constructs magnitudes. For Kant, a magnitude is given as continuous and thus extensive. Magnitudes draw on the formal properties of the sensible condition—space and time—and are not the empirical images left behind. As Daniel Sutherland points out, Kant's view of magnitude draws on the Greek theory of proportions which focuses on homogenous continuous magnitudes, irrespective of numbers.

> Kant thinks of mathematical cognitions as cognitions of magnitudes. Since his view of magnitudes derives from the Euclidean tradition, his account of mathematical cognition turns on the cognitions that make the theory of proportions possible. Those cognitions include the cognition of comparative size relations by means of the cognition of equality and part-whole relations. They also include the cognition of the part-whole composition relations of magnitudes (Sutherland, 2006, 539).

Only extensive magnitudes can be constructed and put into relations with one another in this way. Because all magnitudes are limitations of space and time, they are all situated within the same whole.

> Since the mere intuition in all appearances is either space or time, every appearance as intuition is an extensive magnitude, as it can only be cognized through successive synthesis (from part to part) in apprehension. All appearances are accordingly already intuited as aggregates (multitudes of antecedently given parts), which is not the case with every kind of magnitude, but rather only with those that are represented and apprehended by us as extensive (B, 203-4).

With extensive magnitudes, the representation of parts requires and presupposes the representation of the whole.[1] Mathematical construction is a matter of constructing the parts by delimiting the whole.

For Kant investigations are only properly scientific insofar as they admit of the application of pure mathematics. But such application is not possible if all we can do is enumerate the elements of a domain rather than construct extensive magnitudes. It is not simply that mathematics is concerned with *quanta*, but rather that it constructs magnitudes. These magnitudes are continuous and intuitive. Intuition conditions mathematical construction by providing intuitive content but that content alone does not make mathematics, it must be made a synthetic unity to form concepts and judgments,

[1] In "Infinity and Kant's Conception of the "Possibility of Experience,"" *Philosophical Review* Volume 73. No. 2, Charles Parsons argues that there is a tension in Kant holding there to be certain infinite features to our intuitive representations and characterizing intuitions as singular and immediate. Parsons notes that Kant does not think we can have an intuitive grasp of an actual infinity. So, the infinite features of intuition require a going over and processing of our experience. A kind of zooming in and out but such processing then is not singular and immediate.

> In fact, it is not images of objects but schemata that ground our pure sensible concepts. No image of a triangle would ever be adequate to the concept of it. For it would not attain the generality of the concept, which makes this valid for all triangles, right or acute, etc., but would always be limited to one part of this sphere. The schema of the triangle can never exist anywhere except in thought, and signifies a rule of the synthesis of the imagination with regard to pure shapes in space (B, 180).

Mathematical knowledge is always expressed in concepts. The *in concreto* method allows mathematics to attain universal conceptual knowledge while still being tethered to an *a priori* intuitive representation,

> Philosophical cognition thus considers the particular only in the universal, but mathematical cognition considers the universal in the particular, indeed even in the individual, yet nonetheless *a priori* and by means of reason, so that just as this individual is determined under certain general conditions of construction, the object of the concept, to which this individual corresponds only as its schema, must likewise be thought as universally determined (B, 742).

Mathematics is not merely a matter of turning to intuition. For Kant, all genuine cognition must turn to intuition. But in the case of mathematics there is a particular mode of tending to intuition that denotes the particular content that intuition provides. To be clear, I have distinguished three features of Kant's view concerning what denotes mathematics—who constructs, how construction occurs, and what construction produces—but these features are not separable. The way I have characterized construction requires that we view construction as a peculiarly human activity and as necessarily concerned with extensive magnitudes.

5. Consequences

The architectonic idea of mathematics brings coherence to the mathematical domain for interpreting Kant in light of developments in mathematics. This is useful both with respect to developments since Kant's day as well as those developments that are still to come. The architectonic idea of mathematics allows us to identify something as mathematical that Kant originally excluded and something as not mathematical that Kant originally included. Kant denies the intuitive possibility of certain non-Euclidean objects. NonEuclidean manifolds were not considered a part of geometrical activity. Kant may have viewed geometry unconventionally, but he did not include non-Euclidean geometry in the mathematical domain. The architectonic idea calls for constructing magnitudes in space with certain delimiting concepts. Coherent with that idea are various kinds of spaces, geometrical in virtue of the same thing, their reliance on the a priori intuition of space. Kant's error in rejecting objects of non-Euclidean geometry stems either from letting a founder like Euclid or Newton denote the idea of mathematics or from considering the empirical image associated with mathematical construction to be indicative of the a priori form of intuition. The methods and objects of non-Euclidean geometries arise from the same architectonic idea and are necessarily a part of maintaining the unity of the idea of geometry. While Kant was committed to certain strictly Euclidean principles of geometry, the architectonic idea allows us to recognize and rectify this error. What for contingent historical reasons seemed outside the scope of geometrical activity can through the architectonic idea of mathematics be understood as grounded in the same internal relations. Non-Ecudlidean geometry is a process of taking precise conceptual analogs to phenomenological activity and synthesizing them with the content provided by the a priori intuition of space. Non-Euclidean construction requires space for its concepts to have an intuitive referent and not be a matter of mere play.

Kant also holds that arithmetic is constructive in the same manner as geometry, but arithmetical procedures do not construct magnitudes in the way geometry does. This is not a concern about syntheticity[1] but about mathematical construction. Kant acknowledges a difference between geometry and arithmetic in a letter to Schulz when he says that arithmetic 'has no axioms, because it actually does not have a quantum, i.e., an object of intuition as magnitude, for its object, but merely quantity, i.e., a concept of a thing in general by determination of magnitude' (10:555-6). Kant grants that while particular numbers when applied to intuition have spatiotemporal referents, the mere consideration of the relationships of the concepts of magnitudes is itself prior to any application to objects. Thus, the consideration of arithmetical magnitudes would seem to require a mediating concept for the application to objects.

Arithmetic's reliance on intuition is quite different from the turn to intuition in geometry. Whereas geometry requires the formal conditions of space to provide content in order to construct any of its objects, arithmetic relies on intuition for the one after another (*hintereinander*) succession that is a part of the arithmetical process. As such, arithmetic does not construct mathematically or deal with extensive magnitudes. In this way, we can also see that it does not have the feature of being peculiarly human in the same way since it does not draw on the form of human subjectivity in the same way. Again, the technical unity or merely contingent contiguity of arithmetic regularly being employed in conjunction with mathematical activity contributes to seeing it as united with the mathematical. Within the architectonic idea, we must recognize arithmetic as having a different internal relation to human reason.

[1] There are accounts of analysis that require considering the activity of arithmetic to be synthetic. I have in mind something like R Lanier Anderson's, "It Adds Up After All: Kant's Philosophy of Arithmetic in Light of the Traditional Logic," *Philosophy and Phenomenological Research*, 69 (3), in which he argues that operator functions cannot be expressed through containment relations. But though certainly related the question of mathematical construction is distinct from the question of syntheticity and my point is that arithmetic does not construct magnitudes.

These errors in Kant are often used to argue that he either grossly misunderstood mathematics or that mathematics has changed so fundamentally since Kant that we cannot reconcile the claims he makes about mathematics with contemporary perspectives. Architectonic ideas should give us some pause in taking either of those two positions to an extreme. Even in laying out the framework of the architectonic Kant leaves room for progress and development, not simply so that others can continue what he began but also as a way of recognizing that he may have only a limited or contingent understanding of the a priori internal relations of scientific inquiry. As Paula Manchester argues, in the doctrine of method the architectonic precedes the history of reason precisely because Kant is leaving open a place in the schema for development, a place to be filled in by other rational investigators (Manchester, 2008, 147). If architectonic ideas are regulative with regard to functions and the essential ends of humanity, it follows that we ought to work toward understanding what serves as some of the content of the internal ideas of our scientific investigations. These ideas serve to direct the sciences and discovering or expressing them is a difficult task for us but a necessary one. Given that mathematics is devoid of empirical content and that it is the first evidence we have of the possibility of synthetic a priori principles, it should be among the first sciences whose architectonic idea we work to discover and express clearly. I have focused on questions of pure mathematics but for Kant this architectonic framework is for making sense of all scientific inquiry. We do not have to take Kant's word that chemistry must be understood a certain way. Rather, we must take developments in the sciences as opportunities to identify and further refine our understanding of the internal systematic unity of the sciences.

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