فصلنامه مدیریت شهری
(ضمیمه لاتین)
Urban management
No.**46** Spring **2017**
105 - 114
**Received** *14 Aug 2015*; **Accepted** *11 Jan 2016*

# Nonlinear inelastic dynamic analysis of space steel frames with semi-rigid connections in urban buildings

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## Abstract

Applied studies addressing semi-rigid connections have been limited. Scant information exists in regulations except little brief information. Therefore, this research analyzes the behavior of three-dimensional steel frames and semi-rigid connections based on beam-column method and non-linear dynamic analysis. Stability functions and geometric stiffness matrix were used to study the non-linear geometric behavior of members. Then the dynamic behavior of rotational springs was simulated by stiffening, and the non-linear load-displacement responses and the final results were compared to prior studies that have taken place. It was finally concluded that the method used to analyze the semi-rigid connections was efficient, and if one element per member is used and a precise monitoring is delivered, end sections and nonlinear steel frames behavior with semi-rigid connections can be predicted.

**Key words:** *Dynamic analysis; nonlinear inelastic analysis; stiffener matrix; Stability function; Steel structures; Semi-rigid connections*

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## 1. Introduction

Rigid and pinned connections are used in designing and analyzing steel frames to simplify a structure. Despite the theoretical assumptions, precise experimental analyses have shown that pinned connections do not perform completely like a pin, and moves the same beam to the column, which is also true of rigid connection. Therefore, in elastic designing method, connections are installed in three major ways, namely pinned, semi-rigid and rigid [1]. Therefore, the behavior of connection should be taken into account while designing. There have been numerous studies with respect to this issue by a number of different researchers. In earthquakes, one major problem is the local failure of steel frame connections. Scientists have paid a great deal of attention to the use of steel frames with semi-rigid connections. Due to numerous displacements, frames with semi-rigid connections are rarely used [2]. Researchers have focused on different types of connections, and they have concluded that the improvement of nonlinear steel frames performance is dependent on the proper designing of semi-rigid connections [3].

Due to the modification of connection forces, semi-rigid connections show a better performance with respect to their structural and elasticity. They are also economically more feasible. Compared with other two types of connections, less research has been carried out on semi-rigid connections; hence our major point of emphasis is on these connections. Semi-rigid connections modify the anchors at the end and in the middle of the beam. They also play a significant role in an efficient designing of the members. The difference between semi rigid and other connection is the dependence of connection stiffness and structural analysis parameters [4].

Semi-rigid connections affect the elastic and plastic features of a structure and changes the responses beam anchor responses, stiffness matrix, mass matrix, period and the seismic behavior of structures [5]. Studies on semi-rigid connections show that the flexibility of a connection is significantly affected by the structure's performance. It should also be noted that many regulations assume that connections and steel frames are semi-rigid [6].

To design this type of connection, several finite element methods exist. They are used for analyzing frames with semi-rigid connections. For example in designing semi-rigid connections, Chen and Lui [7] used the stiffness of the end of the connections for forming the stiffness matrix. In another study Chan and Lui [8] used the concept of elements balance to introduce a new element. They claimed that the displacement and stiffness matrix will be found if the fifth-order polynomial function of the element is obtained. Xu [9] also used the first level analysis for steel frames. Besides, Salatic and Sekulovic [10] used stability and assumed an element in which a second level anchor affects axial force and the bending. Recently, Ihaddoudène et al [6] have proposed a model in which three springs are assumed for semi-rigid connections. Considering the above mentioned points, in this study, we will focus on the dynamic non-linear non-elastic analysis of semi-rigid connections, and two numerical models designed by the above relations were put forward.

2. Beam–column element formulation

To consider the effects of the axial force derived from the side displacement of beam-column, we used stability functions proposed by Chen and Lui [11] to reduce the time of modeling and solution. Therefore, only one element was needed for each member to assume the effect of $p - \delta$. The force equation – spatial displacement of beam-column element as formulated by Kim and Thai are as follows:

$$\begin{Bmatrix} \Delta P \\ \Delta M_{yA} \\ \Delta M_{yB} \\ \Delta M_{zA} \\ \Delta M_{zB} \\ \Delta T \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & S_{1y}\frac{EI_y}{L} & S_{2y}\frac{EI_y}{L} & 0 & 0 & 0 \\ 0 & S_{2y}\frac{EI_y}{L} & S_{1y}\frac{EI_y}{L} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{1z}\frac{EI_z}{L} & S_{2z}\frac{EI_z}{L} & 0 \\ 0 & 0 & 0 & S_{2z}\frac{EI_z}{L} & S_{1z}\frac{EI_z}{L} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{GJ}{L} \end{bmatrix} \begin{Bmatrix} \Delta\delta \\ \Delta\theta_{yA} \\ \Delta\theta_{yB} \\ \Delta\theta_{zA} \\ \Delta\theta_{zB} \\ \Delta\emptyset \end{Bmatrix}$$

Where G and E are the modules of elasticity and shear modulus of material; A and L are the area and length of beam-column element; j is torsional constant; In is the moment of inertia in axis n (n=y, z); ΔP, $\Delta M_{yA}$, $\Delta M_{yB}$, $\Delta M_{zA}$, $\Delta M_{zB}$ and ΔT are the variation of axial forces; A and B are respectively the end moment of axes related to y, z and bending; Δδ, $\Delta\theta_{yA}$, $\Delta\theta_{yB}$, $\Delta\theta_{zA}$, $\Delta\theta_{z}$ and ΔΦ respectively show are the incremental axial displacement, joint rotations, and angle of twist, $S_{1n}$ and $S_{2n}$ of stability functions which are related to axis n (n=y, z). These are stated as follows:

$$S_{1n} = \frac{K_nLsin(K_nL) - (K_nL)^2cos(K_nL)}{2 - 2cos(K_nL) - K_nL - K_nLsin(K_nL)} \quad if \ \ P < 0$$

$$S_{1n} = \frac{(K_nL)^2cosh(K_nL) - K_nLsinh(K_nL)}{2 - 2cosh(K_nL) - K_nL - K_nLsinh(K_nL)} \quad if \ \ P > 0$$

$$S_{2n} = \frac{(K_nL)^2 - K_nLsin(K_nL)}{2 - 2cos(K_nL) - K_nLsin(K_nL)} \quad if \ \ P < 0$$

$$S_{2n} = \frac{K_nLsin(K_nL) - (K_nL)^2}{2 - 2cosh(K_nL) + K_nLsinh(K_nL)} \quad if \ \ P > 0$$

Where

$$K_n = \sqrt{|P|/EI_n}$$

Where EA, EIn and GJ are axial, bending and torsional stiffness of the beam–column element defined as follows:

$$EA = \sum_{j=1}^{h} W_j \left( \sum_{i=1}^{m} E_i A_i \right)_j$$

$$EI_y = \sum_{j=1}^{h} W_j \left( \sum_{i=1}^{m} E_i A_i Z_i^2 \right)_j$$

$$EI_y = \sum_{j=1}^{h} W_j \left( \sum_{i=1}^{m} E_i A_i y_i^2 \right)_j$$

$$GJ = \sum_{j=1}^{h} GW_j \left( \sum_{i=1}^{m} (y_i^2 + z_i^2) A_i \right)_j$$

Where h is the total number of sections along the element; m is the total number of axes divided by cross-sections; $W_j$ is weight factor of Ith point, and yi and $Z_i$ are the coordinates of ith axis in the cross-section.

To estimate the effect of the transformation of cross-section on the beam-column element, the equation of force-displacement is defined as follows:

$$\begin{Bmatrix} \Delta P \\ \Delta M_{yA} \\ \Delta M_{yB} \\ \Delta M_{zA} \\ \Delta M_{zB} \\ \Delta T \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & C_{1y} & C_{2y} & 0 & 0 & 0 \\ 0 & C_{2y} & C_{1y} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{1z} & C_{2z} & 0 \\ 0 & 0 & 0 & C_{2z} & C_{1z} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{GJ}{L} \end{bmatrix} \begin{Bmatrix} \Delta\delta \\ \Delta\theta_{yA} \\ \Delta\theta_{yB} \\ \Delta\theta_{zA} \\ \Delta\theta_{zB} \\ \Delta\emptyset \end{Bmatrix}$$

Where

$$C_{1y} = \frac{K_{1y}^2 - K_{2y}^2 + K_{1y}A_{sz}GL}{2K_{1y} + 2K_{2y} + A_{sz}GL}$$

$$C_{2y} = \frac{-K_{1y}^2 + K_{2y}^2 + K_{2y}A_{sz}GL}{2K_{1y} + 2K_{2y} + A_{sz}GL}$$

$$C_{1z} = \frac{K_{1z}^2 - K_{2z}^2 + K_{1y}A_{sy}GL}{2K_{1z} + 2K_{2z} + A_{sy}GL}$$

$$C_{2z} = \frac{-K_{1z}^2 + K_{2z}^2 + K_{2z}A_{sy}GL}{2K_{1z} + 2K_{2z} + A_{sy}GL}$$

$$K_{1n} = S_{1n}(EI_n/L)$$

$$K_{2n} = S_{2n}(EI_n/L)$$

From the above equation the force-deformation element can be defined as:

{ΔF}=[K_e ]{Δd}

We also have the following

$$\{\Delta F\} = \begin{bmatrix} \Delta P & \Delta M_{yA} & \Delta M_{yB} & \Delta M_{zA} & \Delta M_{zB} & \Delta T \end{bmatrix}^T$$

$$\{\Delta d\} = \begin{bmatrix} \Delta\delta & \Delta\theta_{yA} & \Delta\theta_{yB} & \Delta\theta_{zA} & \Delta\theta_{zB} & \Delta\emptyset \end{bmatrix}^T$$

Where the effect of P–Δ is the impact of axial force p in relation to the last transverse displacement of the member. This effect could be assumed as the geometric stiffness matrix ($K_g$) which is as follows:

$$[K_g]_{12\times12} = \begin{bmatrix} [K_s] & -[K_s] \\ -[K_s]^T & [K_s] \end{bmatrix}$$

$$[K_s] = \begin{bmatrix} 0 & a & -b & 0 & 0 & 0 \\ a & c & 0 & 0 & 0 & 0 \\ -b & 0 & c & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

And we have

$$a = \frac{M_{zA} + M_{zB}}{L^2}, b\frac{M_{yA} + M_{yB}}{L^2}, c = \frac{P}{L}$$

The displacement of a beam-column element could be assumed in the following condition: the transformation of the element and the displacement of rigid and the increase in the transformation of {Δd} in the equation {ΔF}=[K_e ]{Δd} which is obtained via using displacement {ΔD} as suggested in the following relationship:

$$\{\Delta d\} = [T]_{6\times12}\{\Delta D\}$$

$$[T]_{6\times12} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1/L & 0 & 1 & 0 & 0 & 0 & 1/L & 0 & 0 & 0 \\ 0 & 0 & -1/L & 0 & 0 & 0 & 0 & 0 & 1/L & 0 & 1 & 0 \\ 0 & 1/L & 0 & 0 & 0 & 1 & 0 & -1/L & 0 & 0 & 0 & 0 \\ 0 & 1/L & 0 & 0 & 0 & 0 & 0 & -1/L & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

In this equation, tangent stiffness matrix of beam-column element could be defined as follows by assuming the effects of P–δ and P–Δ.

$$[K]_{12\times12} = [T]^{T}{}_{6\times12}[K_e]_{6\times6}[T]_{6\times12} + [K_g]_{12\times12}$$

## 3. Beam-to-column connection element

### 3.1. Spring element

An independent string element with zero length and three degrees of movement and rotation is used to simulate the beam-column connection. Figure 1 shows a multi-spring element with two nodes and same characteristics.

The transducer spring possesses linear stiffness while for rotational condition, stiffness is linear or non-linear. This analysis does not address the joint effects of the six elements of a spring attached.

The relationship between the force vector $\{\Delta F_S\}$ and displacement vector of spring element {ΔUS} with six degrees of freedom could be as follows:

$$\{\Delta F_S\} = [K_S]\{\Delta U_S\}$$

$$[K_S] = \begin{bmatrix} K_x^{lin} & 0 & 0 & 0 & 0 & 0 \\ 0 & K_y^{lin} & 0 & 0 & 0 & 0 \\ 0 & 0 & K_z^{lin} & 0 & 0 & 0 \\ 0 & 0 & 0 & K_y^{non} & 0 & 0 \\ 0 & 0 & 0 & 0 & K_z^{non} & 0 \\ 0 & 0 & 0 & 0 & 0 & K_x^{non} \end{bmatrix}$$

Where [Ks] is the tangent stiffness matrix for every spring. Tangential stiffness for linear springs $K_n^{lin}$ or for non-linear springs $K_n^{non}$ is as follows:

$$K_n^{lin} = R_{k,n}^{lin}$$
$$K_n^{non} = R_{kt,n}^{non}$$

Where $R_{k,n}^{lin}$ is the mathematical constant of the linear spring, and $R_{kt,n}^{non}$ is the tangential stiffness of non-linear spring related to axis n (n=x, y, z).

### 3.2. Semi-rigid connection models

Numerous models have been proposed by different researchers to estimate the nonlinear responses of rigid connections. Kishi and Chen [13] propose the most popular model for the analysis of semi-rigid connections in which only three parameters are needed to be extracted from the curve of movement-rotation of connections, and stiffness is always a positive number. Movement-rotation connection equation proposed by Kishi and Chen could be obtained via the following equation:

$$M = \frac{R_{ki}|\theta_r|}{[1 + (|\theta_r|/\theta_0)^n]^{1/n}}$$

Where M and $\theta_r$ are the movement and the rotation of the connection; n is the figure parameter; $\theta_0$ is the rotation of the basic plastic, and $R_{ki}$ is the stiffness of the primary connection. The movement-rotation equation could be stated as follows:

$$M = \frac{(R_{ki} - R_{kp})|\theta_r|}{\left[1 + \left|\frac{(R_{ki} - R_{kp})|\theta_r|}{M_0}\right|^n\right]^{1/n}} + R_{kp}|\theta_r|$$

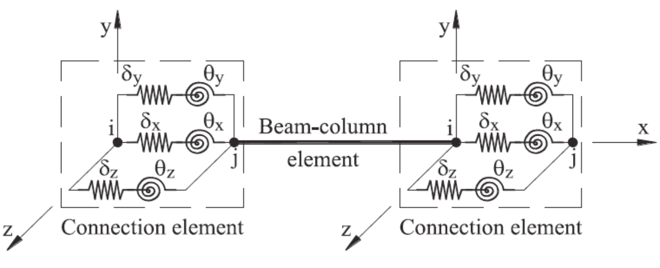


Fig 1. Spring element model


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Where $R_{kp}$ is the relative stiffness caused by the transformation, and $M_0$ is the basic anchor. Lui and Chen have proposed the following descriptive equation:

$$M = M_0 + \sum_{j=1}^{n} C_j \left(1 - exp^{-\frac{|\theta_r|}{2j\alpha}}\right) + R_{kf}|\theta_r|$$

Where α is a numerical factor; $R_{kf}$ is the relative transformation stiffness of the connection; $M_0$ is the primary movement; $C_j$ is bending modulus, and n is the number of phrases.

## 4. Cyclic Behavior of Semi-rigid Connections

Independent stiffness model is used for rigid connections in Figure 2 since it is simple to use to analyze the periodic behavior of these connections. Instantaneous tangential stiffness can be estimated by differentiating Chen's equations.

## 5. Nonlinear solution procedure

One method used in solving numerical problems is Newmark's method due its simplicity [14]. The remaining forces in each stage are removed by reiterating Newton-Raphson. The improved equation of the structure movement can be written as

$$[M]\{\Delta\ddot{D}\} + [C]\{\Delta\dot{D}\} + [K]\{\Delta D\} = \{\Delta F\}$$

Where $\{\Delta\ddot{D}\}$, $\{\Delta\dot{D}\}$ and $\{\Delta D\}$ are respectively acceleration, velocity and displacement vector; [M], [C] and [K] are respectively the matrix of mass, damping and tangential stiffness, and $\{\Delta F\}$ is the external force vector. Damping matrix is as

$$[C] = \alpha_M[M] + \beta_k[K]$$

Where $\alpha_M$ and $\beta_k$ are respectively mass factors and damping stiffness. If the mode has the equal ratio of ξ in this case the mass and stiffness could be defined as follows:

$$\alpha_M = \xi \frac{2\omega_1\omega_2}{\omega_1 + \omega_2}$$

$$\beta_k = \xi \frac{2}{\omega_1 + \omega_2}$$

Where $\omega_1$ and $\omega_2$ are the natural radian frequencies of the first and the second mode.

## 6. Numerical verification and discussion

In the analysis of such connections, Vogel [15] assumed two factors, namely nonlinear geometry and transformation development. Six-storey frame analyzed by his is shown in Figure 3.

To overcome the geometric defects of all columns and to simulate the non-vertical condition, coefficient ψ=1.45 was used. Yung modulus and yield stress of steel were assumed to be E=20500 N/mm$^2$ and $\sigma_y$= 235N/mm$^2$ respectively. For carrying out such analysis, five elements was assumed for each column;

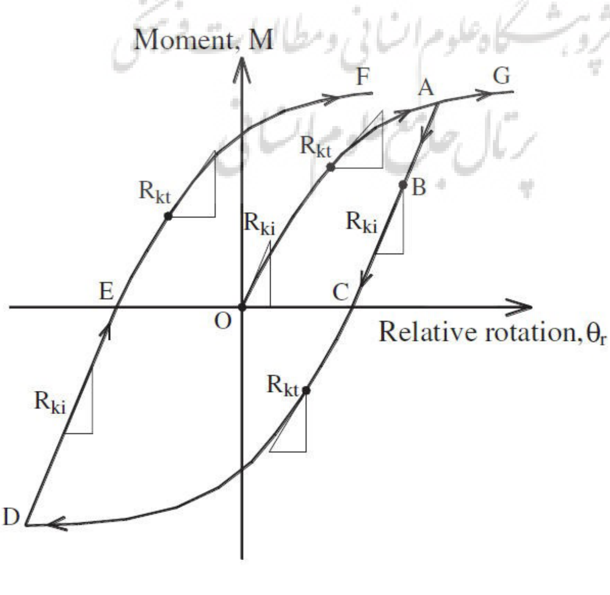


Fig 2. Independent hardening model

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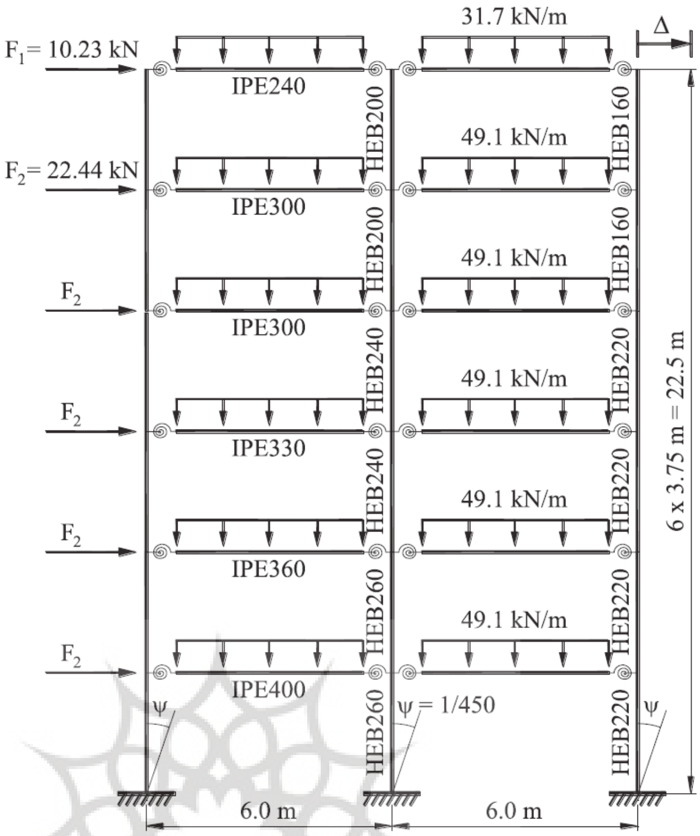


Fig. 3. Vogel six-story frame.

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| Connections parameters | A- Single web angle | C – Flush endplate |
|---|---|---|
| $M_0$ (KN. m) | 0.0 | 0.000 |
| $R_{kf}$(KN. m/rad) | 5.332 | 108.925 |
| α | 0.0005117 | 0.000318 |
| $C_1$ | -4.892 | -28.287 |
| $C_2$ | 137.140 | 573.189 |
| $C_3$ | -661.841 | -3433.98 |
| $C_4$ | 1465.397 | 8511.3 |
| $C_5$ | -1510.926 | -9362.57 |
| $C_6$ | 590.0 | 3832.899 |
| $R_{ki}$(KN. m/rad) | 5440.592 | 12340.198 |

Table1. Parameters of connections for the Chen–Lui model

one element for each column, and five integer point for each elements. Chen and Chui also assumed pins at the end of each beam to analyze rigid connections as could be observed from the figure.

Parameter adaptation curve in the descriptive model proposed by Chen- Lui is put in Table 1 [8]. In this model, in both conditions, the end page has the same angle and level of connections with single web.

In the following figure, the anchor-rotation nonlinear relation for these connection proposed by Vogel could be observed.

The results of the above analysis are in agreement with the results achieved by Chui and Chen as could be observed in the following figure.

The comparison of the two figures shows

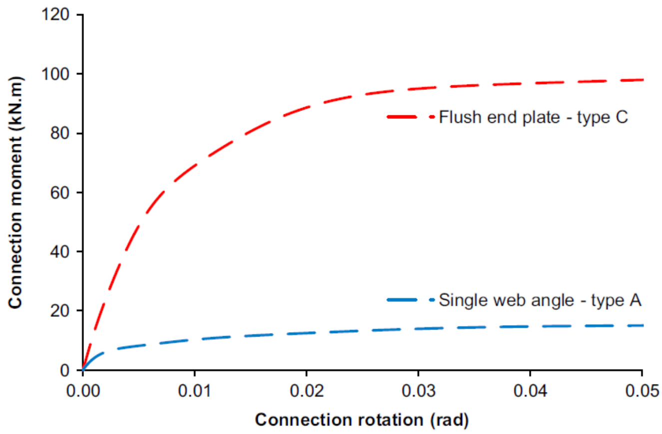


Fig 4. Moment–rotation curve of semi-rigid connections for Vogel ix-storey frame

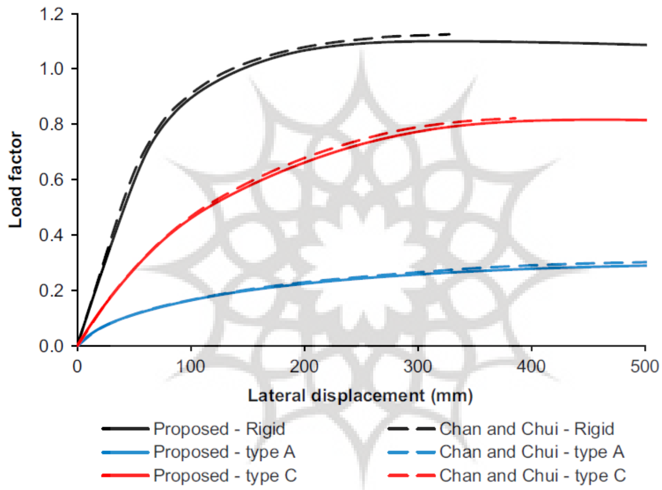


Fig 5. Load–displacement curve of Vogel six-storey frame



that load-displacement curve is a little lower than that of Chen and Chui. This difference is due to using approximate figure function in geometric effect estimation in Chen and Chui's research. However, in the previous method, stability function was used to find the effects of the second level nonlinear geometry.

In another model by Oribson [16], a six-storey frame with rigid connection was studies. The geometric characteristics of the model are depicted in the following figure. The steel yield stress was 250 Mpa; elasticity modulus was 206850 Mpa, and cross-sectional modulus was 79293 Mpa. In beam-column connections frame with screw connection was in on top with the bevel of the living room. The parameters of the semi-rigid model of Kishi-Chen, which are presented in the following table, were used in their connections. In this analysis, two weak and strong conditions were assumed for beam-column connections.

The unique load of the floor was 9.6 KN/m2 changed into the integrated loads in the upper section of the column. Wind loads and point loads along the direction y were assigned to each pin of beam-column. In beam-column element, for each member, there were two points with round numerical length. For the cross-section of all members, 48 axes were assumed for the wings, and 18 axes for web.

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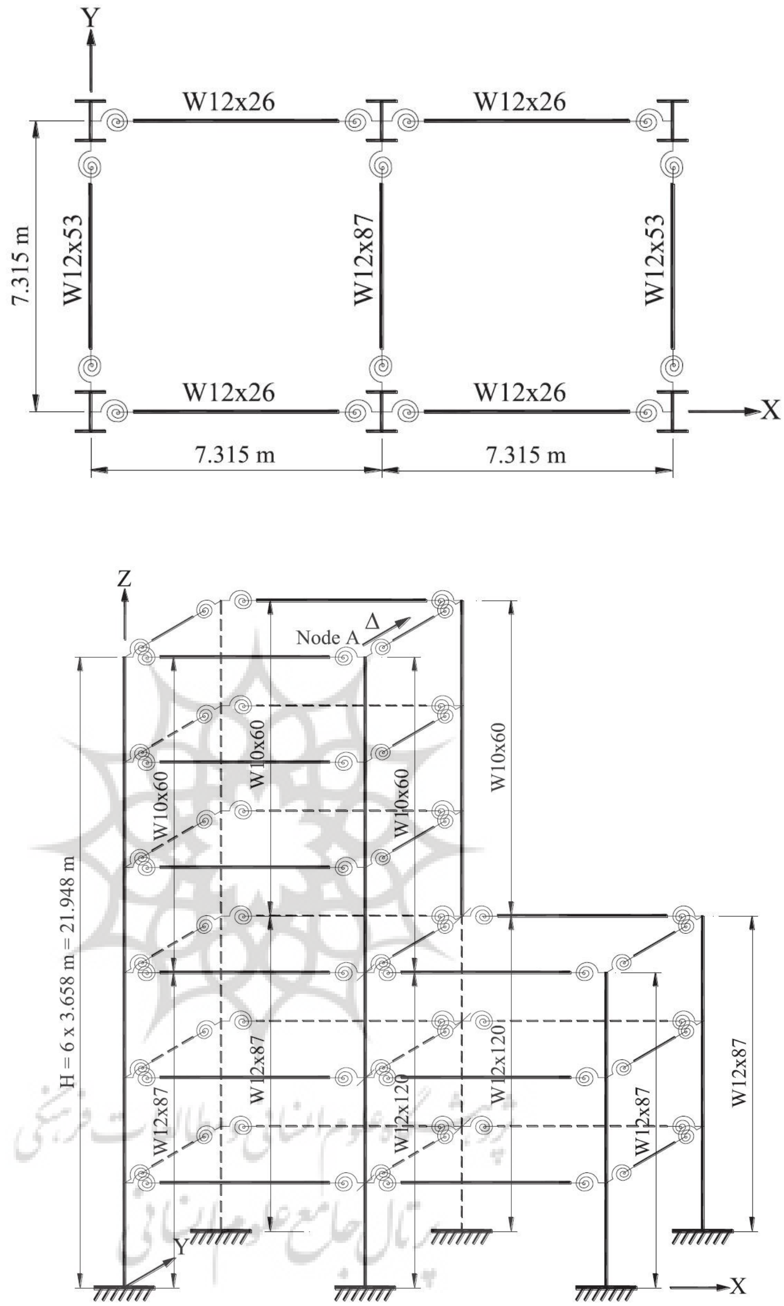


Fig. 6. Plan view and Perspective view of Orbison six-storey space frame with semi-rigid connections

| Beam section | Bending-axis | $M_u(KN.m)$ | $R_{ki}(KN.m/rad)$ | n |
|---|---|---|---|---|
| $W12 \times 87$ | Strong-axis | 300 | 160503.2 | 1.57 |
| | Weak-axis | 300 | 52267.75 | 1.57 |
| $W12 \times 53$ | Strong-axis | 300 | 92185.09 | 1.57 |
| | Weak-axis | 300 | 20776.82 | 1.57 |
| $W12 \times 26$ | Strong-axis | 200 | 44247.8 | 0.86 |
| | Weak-axis | 200 | 3752.54 | 0.86 |

Table 4. Parameters of semi-rigid connections follow the Kishi–Chen model

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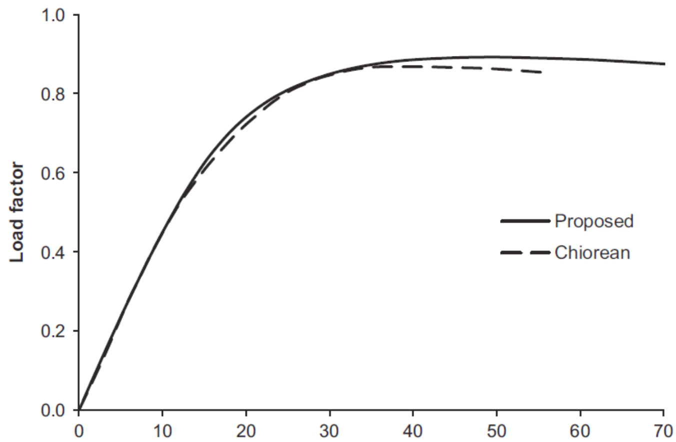


Fig 7. Load–displacement curve at Y-direction node A of Orbison six-storey frame.

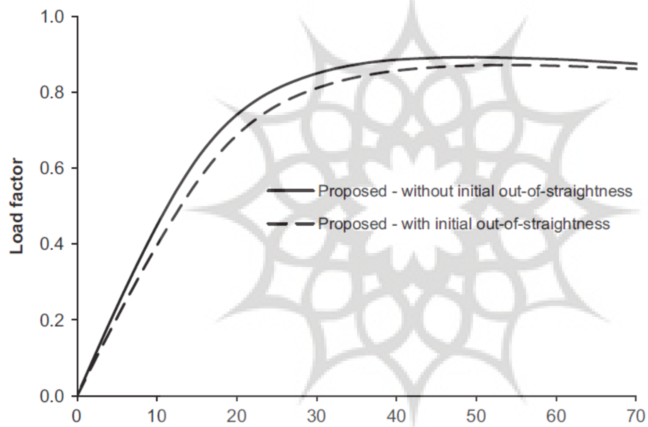


Fig 8. Load–displacement curve at Y-direction node A of Orbison six-storey frame with and without geometry imperfection

In the following figure, load-displacement curve on the roof for node A was estimated by computer, and it was compared with Chiorean results.

Kim and Chen proposed the tangential modulus reduction method to consider the initial curve using only one element of each member. In their plan, Yung modulus 0.85 ×E was used for all steel members to consider the initial curve of the members. In the following figure, load-displacement curves of semi-rigid connection frames are shown with and without considering geometric errors.Frame final load factors for conditions with and without initial curves of the members were respectively 0.872 and 0.892. Given the initial curve, the frame final load was reduced for 2.2 %. Therefore, it can be concluded that the plan and method used in the analysis of non-elastic, nonlinear response of steel frames with semi-rigid connection could be trusted.

## 7. Conclusion

Given the complicated behavior and performance of semi-rigid connections, the performance of connections is most often assumed to be rigid or pinned, however this assumption is not correct in practice, thus more research is needed with regard to these connection and the analysis of their nonlinear response. Therefore, in this study, we proposed an efficient numerical method to provide a non-elastic and dynamic analysis of steel frame by considering the nonlinearity of geometric analysis and the nonlinear response of the materials. To analyze the nonlinear condition of geometrical characteristics, stability functions were used, and to consider the nonlinear response of the materials, uniaxial stress - strain relation was drawn for each axis in the cross-section. To solve the numerical samples, efficient computer programs were used, and considering the outputs of numerical models, it was evident that if synthetic nonlinear effects of geometry, materials and connections are taken into account, the behavior of the semi-rigid steel frame connections can be predicted.

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