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Multilevel Convergence and Cluster Fluctuations Based on Price Bubbles and Fractal Structure

Farzin Axon, Seyed Hossein Nasl Mousavi, Abbas Ali Pour Aghajan

Department Of Accounting, Qaemshahr Branch, Islamic Azad University, Qaemshahr, Iran

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ABSTRACT

Background & Objective: Cluster fluctuations and fractal structures are the essential features of space-time correlation in complex financial systems. However, the microscopic mechanism of creating and expanding these two features in financial markets remains challenging.

Materials and Method: In the present study, the process of forming cluster fluctuations according to the fractal structure of financial markets is investigated using a factor-based model design and considering a new interactive mechanism called multilevel convergence. Virtual agents trade in different groups is measured at three levels: stock, segment, and market, according to market performance and their mass behavior.

Results: The results show that multilevel convergence is one of the microscopic mechanisms of the microstructure of financial markets, along with providing new insights into space-time correlations of financial markets.

Conclusion: In other words, multilevel collective behavior is an essential factor in cluster fluctuations, price bubbles, and market fractals and should be considered in interpreting the concept of risk and defining risk management strategies from this perspective.

1 Introduction

Financial markets are one of the most complex social systems that have recently attracted the attention of many researchers and analysts in other sciences, such as physics and psychology, due to their ease of access and abundance of data. From physicists' point of view, the dynamic behavior and structure of complex financial systems can be characterized by space-time correlation functions. From a theoretical point of view, space-time correlations are significant in understanding price dynamics and portfolio optimization. This article focuses on the space-time correlation's fractal structure and clustering of oscillations. Cluster fluctuations indicate that the yield variance varies over time, large fluctuations tend to follow large fluctuations, and small fluctuations follow small fluctuations in the same direction [40,15,17]. On the other hand, the spatial structure of the market is described by examining the cross-sectional correlation of stocks [25,18,12,4]. While the return series is quantitatively non-correlated, between a few minutes and a few weeks, the absolute value of the return shows a positive and significant self-correlation

* Corresponding author. Tel.: +989123387221
E-mail address: *Nseyedhossein2@yahoo.com*

that slowly disappears [18]. The purpose of fractal theory is to study the behavior of nonlinear periods of system sensitivity to initial conditions. Accordingly, chaotic behavior is an integral part of a system, but this pattern is the reason for long-term memory in the market and the absence of chaotic behavior when a transparent and predictable pattern is created with a constant period of market behavior [40]. A random matrix[1] (RMT)[2] is one of the methods to analyze the market's fractal structure. A set of business segments can be defined using stochastic matrix theory, and a mechanism can be visualized as a portfolio [35]. Thus, the yield correlation matrix (matrix C) is analyzed to examine the relationship between the components [28]. The largest value of the C matrix significantly differs from the Wishart matrix distribution[3] [17]. This eigenvalue somehow reflects the market style, i.e., the correlation of the total market price and the corresponding eigenvector components is relatively the same for all stocks. Every vector corresponding to the most significant value represents a particular segment. The large eigenvalues stand for segments that can affect the fractal structure of the market.

In recent years, successful models have been proposed to study cluster fluctuations, including Lux and Marchesi [19], Ma et al. [13], Feng et al. [8], and Perona [21]. The effect of market players' behavior on this phenomenon is one of the aspects discussed in the occurrence of cluster fluctuations, and the results of previous studies in this regard are sometimes contradictory [35,28,19,5,41]. Although many studies have been devoted to the fractal structure of financial markets, its microscopic mechanism is still controversial [12]. Both fractal structure and cluster fluctuations are essential features of stock markets, and their mechanism in a model remains challenging. Therefore, the present study seeks to find the question of how interactive behavior at different market levels, according to its fractal structure, affects the occurrence of cluster fluctuations. One of the strongest simulation methods is factor-based modeling, which is widely used in various fields and is becoming a powerful validation tool for financial theories due to its strong controllability and reproducibility [15,23,31,15]. In the present study, a factor-based model [25] is combined with an interactive mechanism called multilevel massification to investigate the fractal structure of the market with cluster fluctuations. Apart from using the factor-based model, which is rarely used in financial field research, the study innovation is to apply mass production in different market layers and use actual market data at the share level, sector, market, and mass application. Considering the importance of analyzing and examining cluster returns of returns in issues related to risk management and share pricing, the results of the present study can be helpful in the development of theoretical and experimental literature in this field.

2 Theoretical foundations and research background

In stock markets, temporal price changes and their relationships are complex. The price dynamics of a market naturally arise from individual stocks. Recent studies have shown that the price dynamics of a market can be broken down into different impulse modes, such as market and market segment [5]. Naturally, the market state includes all the stocks in a market, and the fractal state or part is affected by the interaction of stocks in a part of the market, making price dynamics a multilevel market [18]. In financial markets, collective behavior is one of the social behaviors in which investors become groups when making decisions, and these groups can be large. Price dynamics depend on the collective behavior of investors

[1] One of the new methods to study correlation and reduce the noise of correlation matrix in financial series
[2] Random Matrix Theory
[3] Correlation matrix of non-correlated time series

[21]. Investors may converge on the behavior of the masses or the market due to information asymmetry or distortion [43], observe the behavior of other investors, and make investment decisions based on these observations [45]. Then, a continuous investment trend emerges in the investor network, which can affect the occurrence of cluster fluctuations and price bubbles [36]. This phenomenon can be interpreted by the Multi Fractal Model of Asset Returns (MMAR) as a result of the dynamic structure of information transmission. In addition to being able to estimate changes in the yield series in a highly complex process involving various fractal Brownian motion modes, the MMAR explains many of the indisputable facts of capital markets, such as cluster fluctuations, broad tails, market memory, and price bubbles [29]. The dynamic structure of market information transfer can affect the homogeneity of investors, which can further affect market prices and determine their volatility. Different environments have different dynamic transfer structures [10]. According to research on transfer dynamics, the transfer structure influences the speed and power of information transfer and, therefore, the structure of investors in the capital markets [7]. For this purpose, multi-fractal structure analysis provides a more accurate understanding of fractal lines in time series and more or less changes in data structure [1].

In summary, the empirical background shows that collective behavior explains many statistical features of financial markets and significantly affects their occurrence [44]. In addition, the extent to which investors engage in collective behaviors may vary for various reasons [11]. Cheng et al. [7] examined the effects of changes in the trend of collective behavior in the network structure of investors. In this study, increasing the trend of collective behavior slows down the process of product sales in the network, and the time of outbreak Delays release. In addition, the tendency to collective behavior affects the size and scope of dissemination. Ducknageles and Rotundo [5] showed that the tendency for collective behavior decreases in stressful market conditions, and the network structure of investors and the market have a justifying role in mass production. However, the empirical evidence of Balkillar et al. [21] indicates that the tendency to collective behavior is more prevalent in turbulent periods, and the signals of speculators decrease in collective behavior. Wang and Wang [35] found that the intensity of collective behavior affects market fluctuations in addition to the fact that both the quality of confidential information and the number of mass leaders have a positive and significant relationship with their followers' tendency toward massism. Martinez et al. [26] evaluated the fractal structure and identified the transition time from efficient market random behavior to collective behavior and found that a higher amount of fractals in the financial series increases the probability of starting the transition to mass formation and bubble formation. Chen et al. [14] indicated that the multilevel mass mechanism explains a segmented structure in financial markets. Connovichs and Gantis [36] presented a three-state model of collective behavior in financial markets, stating that the observed statistical features (such as wide tails and cluster fluctuations) are consistent in high-frequency financial markets in the presence of collective behavior. Venice et al. [41] stated that both amateur and professional investors are prone to massism, and this tendency is more pronounced in professional investors. The collective behavior of market factors affects the statistical features such as cluster fluctuations, but its mechanism is ambiguous in combination with the multilevel mass mechanism to study the fractal structure of the market and cluster fluctuations. In the present study, it is assumed that the collective behavior of virtual agents consists of three different levels: share level, segment, and market, and then its effect on market structure and cluster fluctuations is examined.

1) Model design and determination of parameters
2) Multilevel convergence

The present research model is based on the daily transactions of virtual agents, i.e., buying, selling, and holding shares. The model is defined based on the number of N virtual agents, n shares, and nsec sections.

Each section includes $N/_{n_{sec}}$ is a share. Each virtual agent holds only one share randomly selected from n shares. The rational shareholder trades his stock, considering real records and stock performance at different time scales, which is included in the model to match the real market and better describe the behavior of virtual agents. Considering the investment horizon as described above, the weighted average share return, $\acute{R}_\iota$ which is the basis of factor decision-making to maintain the share, is calculated as described in Model 1 [25]:

$$\acute{R}_\iota(t) = K\sum_{l=1}^{L}\left[\xi_l \sum_{m=0}^{i-1} R_i(t-m)\right] \tag{1}$$

L is the maximum investment horizon; provided that $\sum_{l=1}^{L}\xi_L = 1$ is considered, the value of ξ_l will be as follows:

$$\xi_l = {l^{-1.12}}\Big/{\sum_{l=1}^{L} l^{-1.12}} \tag{2}$$

As mentioned, the agent's trading decisions are based on past returns. Therefore, on day t + 1, the virtual factor that holds the share i with the investment horizon condition absolute $\sum_{m=0}^{L-1} R_i(t-m)$ is the basis for estimating the previous stock performance. To ensure the compatibility of the oscillation value $\acute{R}_\iota(t)$ with $R_i(t)$ and to guarantee the condition$\left|\acute{R}_\iota(t)\right|_{max} = R_i(t)_{max}$, the value of the coefficient K is:

$$K = {1}\Big/{(\sum_{l=1} L \sum_{m=l} L\ \xi_m)} \tag{3}$$

Obviously, if L = 1 (investment horizon is one day), then $\acute{R}_\iota(t)$ will be equal to $R_i(t)$.

As mentioned, the collective behavior of factors is analyzed at the share, segment, and market level. Therefore, the factor with share i with "factor i share", the factor with share i belonging to section s, with "factor of section S" and the group consisting of "share factor i" or "factor S" with "group I share", respectively. "And" Section Group S "is defined. Factors with a share of i are first classified in the "Share I group"; Collective behavior at the i-share level is similar to mass behavior in other models that simulate only one share. These groups form larger groups in each section, representing collective behavior at the S-level level. Since the sum of all segments constitutes the market, groups at the S-level level become larger groups that constitute collective behavior at the market level. Figure (1) shows the mechanism of multilevel collective behavior.

- Degree of collective behavior at the share level:

The value of D is defined as follows to measure the degree of collective behavior at the "Share I Group" level on day t:

$$D_i^I(t) = {\overline{n_\iota}(t)}\Big/{N_i} \tag{4}$$

Where $D_i^I(t)$ indicates the degree of collective behavior at the level of "share group I", $\overline{n_\iota}(t)$ shows the average of virtual agents in "share group I", N_i presents the number of virtual agents that have share i have. Since the collective behavior of the factors based on their estimation is from the performance records of the share, therefore $\overline{n_\iota}(t)$ is:

$$\overline{n_\iota}(t) = \left|R_\iota(t\acute{} - 1)\right| \tag{5}$$

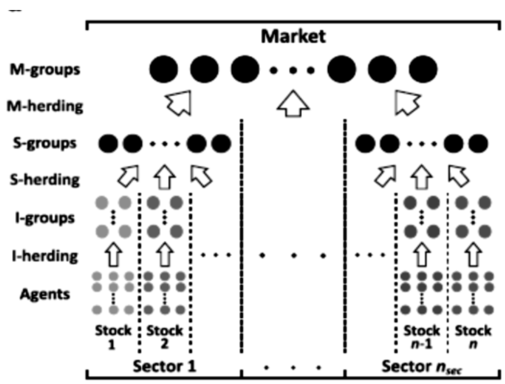

Fig. 1: Mechanisms of multilevel collective behavior [13]

- Degree of collective behavior at the ward level:

The agents randomly join each of the "Share I Group", after the collective behavior at the share level for all n available shares, the number of "Share I Group" in the S segment and the M market are:

$$N_S^I(t) = \sum_{i \epsilon s} \left[{}^{1}\!\big/_{D_i^I(t)} \right] \tag{6}$$

$$N_M^I(t) = \sum_{i} \left[{}^{1}\!\big/_{D_i^I(t)} \right] \tag{7}$$

Where $N_S^I(t)$ and $N_M^I(t)$ represent the number of "share group I" in segment S and market M, respectively. Each part of the stock comprises similar characteristics at the sector level. Therefore, the degree of collective behavior of the factors is influenced by the simultaneous movement of stock prices in the sector, i.e., the stock price in a sector simultaneously increases and decreases. The share group I in a section forms larger groups called the "section group" S. Thus, the degree of collective behavior at the level of section S is:

$$D_S^S(t) = {}^{n \times (H_S - H_M)}\!\Big/_{N_S^I(t)} \tag{8}$$

Where $D_S^S(t)$ is the degree of collective behavior of factors in S, H_S and H_M are, respectively, the stock price correlation in S, and in the market, $n \times (H_S - H_M)$ also represents the net price correlation in Section S. Other variables are the same as before.

- The degree of collective behavior at the market level

The simultaneous movement of the total market price influences the collective behavior of factors at this level. The S groups in different segments represent the characteristics of the whole market in the larger M groups. Thus, the degree of collective behavior at the market level is:

$$D_S^M(t) = {}^{n \times (H_M)}\!\Big/_{N_S^M(t)} \tag{9}$$

Wher e: $N_S^M = {}^{\overline{H} \times N_M^I(t)}\!\Big/_{H_S}$ (10)

$$\overline{H} = \left({}^{\sum_S H_s}\!\big/_{n_{sec}} \right) \tag{11}$$

Where $D_S^M(t)$ is the degree of collective behavior at the market level. Other variables are defined as before. All factors are in group M after the occurrence of collective behavior at all three levels. Each agent makes only one trading decision per day, and the trading decision of the agents in an M group has an equal probability. Given that each factor has a share of i, the decision of factor α on day t is defined as follows:

$$\theta_\alpha(t) = \begin{cases} \mathbf{1} & \textbf{Purchase} \\ \mathbf{-1} & \boldsymbol{sell} \\ \mathbf{0} & \boldsymbol{hold} \end{cases} \tag{12}$$

The probability of buying and selling in group M is considered equal according to Chen et al. (2016) and Feng et al. (2013) $P_{sell} = P_{buy} = P$, so the probability of keeping the share (P_{hold}) is equal to 1-2P. In addition, the return on share i is defined as the difference between supply and demand, so the return on factor α for share i is equal to:

$$R_i(t) = \sum_{\alpha \in i} \theta_\alpha(t) \tag{13}$$

2.3 Estimation of market and segment collective behavior parameters

As mentioned, H_S and H_M parameters are defined at the sector and market level, to calculate the degree of collective behavior (convergence). Simultaneous movement of stocks can be determined by the similarity of the signal and the amplitude of the return volatility [19]. Thus, on the day (t) and according to the sign $r_i(t)$, the range of fluctuations of ascending and descending trends on day t is defined as $V_+(t)$ and $V_-(t)$, respectively:

$$\begin{cases} V_+(t) = \sum_{i, r_i(t)>0} {r_i}^2(t)/n_s \\ V_-(t) = \sum_{i, r_i(t)<0} {r_i}^2(t)/n_s \end{cases}, \quad n_s = n/n_{sec} \tag{14}$$

In addition, the normalized values of efficiency according to model (15) are entered in the calculations;

$$r_i(t) = [R_i(t) - \langle R_i(t) \rangle]/\sigma, \sigma = \sqrt{\langle R_i(t) \rangle} - \langle R_i(t) \rangle^2 \tag{15}$$

to compare time series:. Where $\langle R_i(t) \rangle$ the mean return on I is in period t and (σ) is the standard deviation of the return. Since these two trends are usually out of balance, and one of the two regimes mentioned dominates the stocks in each segment (n_s) in each period, so the range of dominant fluctuations $V^d(t)$ and non-dominant $V^n(t)$ is defined as follows:

$$\begin{cases} V^d(t) = max[V_+(t), V_-(t)] \\ V^n(t) = min[V_+(t), V_-(t)] \end{cases} \tag{16}$$

The total amplitude of the oscillation is calculated by the difference $V^d(t)$ and $V^n(t)$. According to the mentioned cases, finally, the degree of convergence of H_S and H_M is:

$$\begin{cases} H_M = \langle \zeta(t) \rangle . \langle V^d(t) - V^n(t) \rangle \,|market \\ H_S = \langle \zeta(t) \rangle . \langle V^d(t) - V^n(t) \rangle \,|s - sector \end{cases} \tag{17}$$

$$\zeta(t) = n^d(t)/n_s \tag{18}$$

Where H_S and H_M, are respectively the degree of convergence (collective behavior) at the sector and market level, $\zeta(t)$ and $n^d(t)$ present respectively the percentage and number of shares in the dominant regime4 in period t and other variables according to the previous definitions.

Simulation model: According to Table 1, the model parameters for simulation are:

Table 1: Simulation Model Parameters

Number of shares (N)	**150**
Number of sections (SEC)	**5**
Number of agents (AGENTS)	**1000**
Maximum investment horizon (L)	**1000**
Probability of buying or selling (P)	**0.363**

Daily data on the final price of 150 shares listed on the Tehran Stock Exchange is imported into five sections after the last screening, with 30 shares in each section from August 2013 to September 2017. According to Feng et al. [17] and Chen et al. [16], the investment horizon of 94% of the factors is less than 500 days. The number of factors should not be too small to adapt the simulator as much as possible to the characteristics of a real market. The values of N do not significantly affect the section's structure, the distribution of the C matrix's specific value, and the oscillation correlation's amplitude. The investment horizon is from one day to more than one year [12]. The maximum investment horizon of 1000 days is set in the model because some investors have a longer horizon in the stock market due to the period limit [25].

First, the probability of buying, selling, and daily maintenance of a single investor in the real market determines the parameter P. As mentioned, the probability of buying and selling is assumed to be equal, i.e., Pbuy = Psell. According to the sample companies and in the period under review, the institutional investor's average retention percentage is 60.3%, and the actual shareholder is 39.7%. The average exchange rate between the number of shares held by an actual shareholder is assumed to be 1.64 [23]. Considering 250 trading days per year, the probability of daily trading is equal to:

$$\frac{1.64}{0.397\times 250} = p_{buy}(t) + p_{sell}(t) \Rightarrow 2p = 0.0165 \Rightarrow p = 0.00826 \qquad (19)$$

The factors in group M are related to simulating the phenomenon of collective behavior, and it is assumed that if factor a in group M decides to buy or sell shares, the whole group will make the same decision. Given that the average number of agents in a group is $n \times H_M$, the probability of buying or selling a group is equal to:

$$P = 1 - (1-p)^{n\times H_M} \qquad (20)$$

Thus, the probability of buying (selling) is equal to 0.363. All n shares in the first level of the investment horizon L have zero returns, so the first 500 points of the return data are omitted. The simulator evaluates the time series $\mathrm{R}_i(t)$ for each share by estimating H_M and H_S,. For each share i on day t, the value of $\acute{R_\iota}(t)$ is calculated according to Equation (1). Then the value of $D_i^I(t)$ is calculated according to Equation (4) and (5). Virtual agents at the stock level randomly join one of the $1/_{D_i^I(t)}$ "Share I Group"; Then the "share group I" in section S joins one of the $1/_{D_S^S(t)}$ "group section S" and finally the group section S, joins

[4] To measure the similarity in the return sign

$^{1}/_{D_S^M(t)}$ group M5. The factors in an "M group" make a similar decision (buy/sell and hold) with the probability of P_{buy} ,P_{sell} ,P_{hold} after simulating convergence at these three levels. Thus, the return per share on day t is counted according to Equation (13). The groups are dissolved after the decision, and a return time series is obtained for all stocks in the market by repeating this process. Figure (2) shows the simulation process.

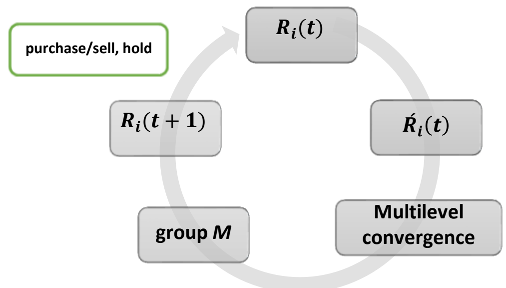

Fig. 2: Simulation Process

Simulation results: Table 2 shows the amount of mass production in each segment and the whole market. As mentioned, H_S and H_M parameters indicate the intensity of the trend of simultaneous movement at the sector and market levels, respectively. These values, which are based on the actual data of the sample companies, show that the intensity of the collective behavior tendency at the sector level is much stronger than at the market level.

Table 2: Values of H_S and H_M parameters (degree of bulk density) in segments 1 to 5 and the whole market.

	H_M	H_1	H_2	H_3	H_4	H_5
The amount of mass	0.636	0.491	0.414	0.438	0.431	0.546

Source: Research Calculations

Descriptive statistics and econometrics of the simulated yield distribution series related to each sector and the market are presented before examining cluster fluctuations in each series. Table (3) shows the descriptive statistics of each segment and the total market. The estimated price returns are calculated using the following equation:

$$r_t = \ln(p_t) - \ln(p_{t-1}) \tag{21}$$

In which, rt indicates the return on assets at time t, pt indicates the price of assets at time t, and pt-1 suggests the price of assets at time t-1. As shown in Table (3), the distribution of the efficiency series is not normal, as confirmed by the Jark statistic. According to the elongation statistics, the series of each sector and course of the whole market has a wide tail, which is a proven feature in financial markets, including Iran [12,19,25]. Given the value of the Advanced Dickey-Fuller (ADF) statistic and the critical limit of McKinnon for the statistic[6], the whole market-wide return series is at the 5% error level. BDS tests measure nonlinear dependence by rejecting null hypothesis, indicating the existence of a nonlinear series. According to the value of the Z statistic of the mentioned test and the rejection of the null hypothesis with significance at the 5% line level, the simulated yield series has a nonlinear dependence and is not independent and

[5] Larger group or market group
[6] 2.84

similar (IID). In addition, Lagrangian incremental test (LM) results indicate the presence of the arch effect at a significance level of 5%, indicating cluster fluctuations. Hurst statistics are estimated to explore market fluctuations further, and Hurst power is obtained by calculating the slope of the curve $\log(R/S)/\log(N)$ and using the regression method regarding N changes. The highest value represents the average period of the model rotation [36]. The value of this statistic indicates the measurement of long-term memory and the failure of a time series (Hurst, 1951). According to Hurst's output, the value of the Hurst statistic is 0.5, indicating the existence of a completely random series. As shown in Table (3), the value of the Hurst market series statistic is 0.512424, which is slightly higher than 0.5 and indicates long-term market memory. Hurst power is calculated as a moving average for every 1000 recent data (according to the investor time horizon) [18]. In addition, the amplitude of this statistic is always higher than 0.5, which indicates that the study series has always had long-term memory for fluctuations (Figure (3)).

Table 3: Distribution the series of eleven econometric statistics

	SEC1	***SEC2***	***SEC3***	***SEC4***	***SEC5***	***MARKET***
N	37726	37652	37826	37587	37741	188532
MEAN	0.000873	-0.00542	0.00077	0.00127	-0.00167	0.00085
STD	0.0307	0.0175	0.0201	0.0224	0.0156	0.0193
SKEWNESS	0.821	-0.342	0.153	-1.720	-0.591	-0.742
KURTOSIS	11.184	6.179	12.147	10.151	8.472	9.572
JARQ-BERA	3.2551	4.1685	2.4498	7.2162	5.1789	2.1935
ADF						*-32.15******
BDS						*42.88******
LM						$R^2 = 95/87$ *115.66******
HURST						*0.56122******
FRACTAL DIMENSION						*1.43877*

*****: Significance at the level of 5% error
Source: Research Calculations

Fig. 3 shows the average Hurst power trend over the period under review;

Fig.3: Hurst power of the simulated efficiency series
Source: Research Calculations

Long-term memory is directly related to fractal dimensions. The fractal dimension of a time series shows the number of fluctuations and instability. The relationship between the fractal dimension and Hurst power is a time series equal to: [13].

$$CapD = 2 - H \tag{22}$$

The 1.5 fractal dimension represents the IID series and has a random step. Therefore, if the fractal dimension is 1 to 1.5, the series will have long-term memory. According to Hurst power, the fractal dimension of the simulated series is 1.438776, indicating that the series has a long-term memory. Finally, the cluster fluctuations of the share i according to the autocorrelation function of the fluctuations are calculated as follows: [24].

$$A_i(t) = {\left[\langle |r_i(\acute{t})||r_i(t \acute{+} t)|\rangle - \langle |r_i(\acute{t})|\rangle^2\right]}/{A_i^o} \tag{23}$$

$$open - angleA_i^o = \langle |r_i(\acute{t})|^2 \rangle = \langle |r_i(\acute{t})|\rangle^2 \tag{24}$$

Where $A_i(t)$ the self-correlation is the value of the share fluctuations i in period t and $\langle |r_i(\acute{t})|\rangle$ indicates the average rate of return correlation during period t. Therefore, the autocorrelation function of the oscillations for the estimated series is $(t) = {\sum_i A_i(t)}/{n}$. Figure (4) shows the mean autocorrelation functions for the simulated efficiency series. AS shown, the values for the simulation series correspond to the actual data.

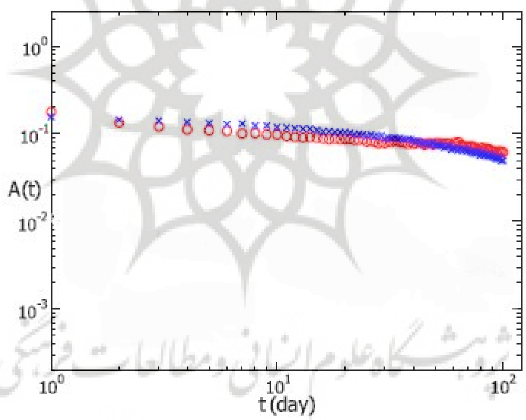

Fig.4: Mean autocorrelation function of real data fluctuations and simulated efficiency series.
Source: Research Calculations

the C-matrix of two simultaneous7 correlations8 is first calculated to determine the spatial structure of a series:

$$C_{ij} = \langle r_i(t)\, r_j(t)\rangle\,. \tag{25}$$

Where 〈r_i (t) r_j (t)〉 is the average correlation during period t and C_ij represents the correlation of stock returns i and j. In addition, C is a matrix symmetric with the condition C_ii = 1, and the values of the

[7] A mathematical quantity is defined as the product of two-time functions. The degree of cross-correlation of measuring the similarity of two series is a function of the displacement of one to the other. Thus, self-correlation is the mutual correlation of a signal with itself
[8] Equal-time cross-correlation matrix C

other C_ij statements are in the range [-1,1]. The first, second, and third eigenvalues of the matrix C are λ0, λ1, and λ2, respectively, which are determined by the components of the special vector ui (λ). The simulation results, in comparison with the actual data, are shown in Figure (5). According to historical data, for λ0, the eigenvector components are almost uniform in all segments, but the eigenvector λ1 is significantly affected by section (5), and the eigenvector λ2 is significantly affected by section (1). It is worth noting that these features are also observed in the simulated time series.

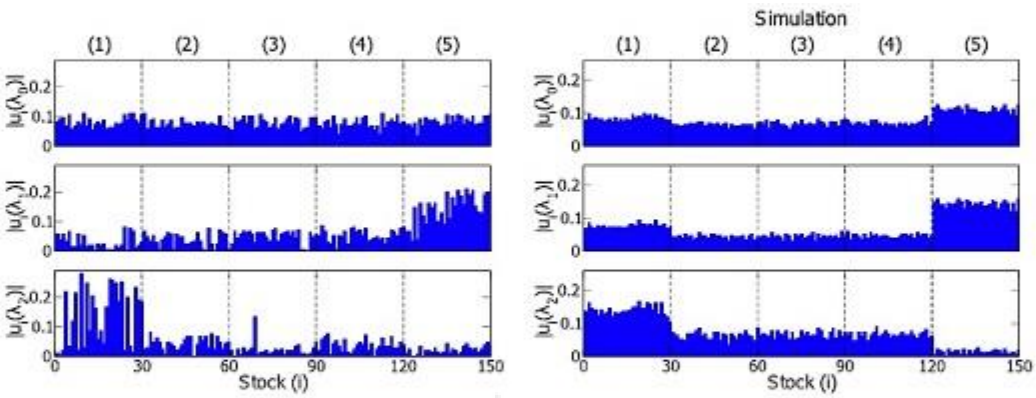

Fig. 5: Absolute value of specific vector components ui (λ) Triple values of double-correlation matrix C calculated based on historical data and simulated returns.
Source: Research Calculations

Figure (6) shows how to distribute the triple eigenvalues of the C correlation matrix and the simulated values. As can be observed, the eigenvalues (λ0, λ1, λ2) of the C matrix are (5.13, 7.45, 26.01) and the simulation values are (3.82, 7.93, 24.62), respectively, indicating how the eigenvalues are distributed. The simulation is significantly similar to the actual data distribution.

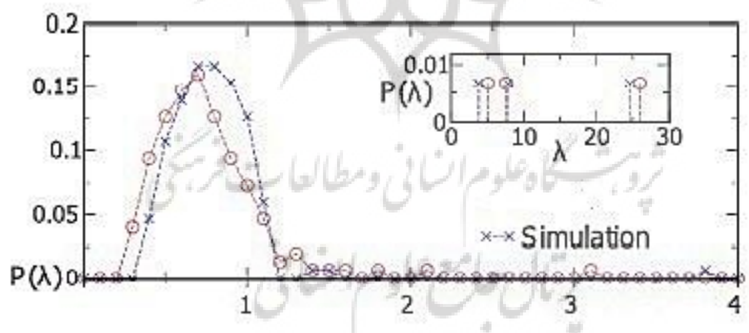

Fig 6: Probability distribution of specific values of C correlation matrix and simulated values.
Source: Research Calculations

3 Conclusion

In financial markets, fractal structure and cluster fluctuations are essential features of space-time correlation. However, the mechanism for creating such cases remains ambiguous, and combining these two features is challenging. Therefore, as necessary, the present study microscopically examined the mechanism of price dynamics in financial markets, including the Iranian capital market, using factor-based modeling, the interactive mechanism of multilevel collective behavior of investors, and its effect on fluctuations. The rationale for modeling is investors' individual and collective behaviors in real markets. The designed virtual agents trade in groups based on their previous performance and the market's historical data, and their collective behavior is examined. The fractal structure, cluster oscillations, and the distribution of specific correlation matrix values were calculated by determining the parameters and the simulated time series, and their compliance with the experimental data was investigated. The results indicate that the multilevel collective behavior mechanism requires the fractal structure of the market and provides a new dimension regarding space-time correlation at the microstructural levels of financial markets. In other words, multilevel collective behavior is an essential factor in cluster and fractal market fluctuations and should be considered in interpreting the concept of risk and defining risk management strategies from this perspective. In addition, the dynamic structure of information transfer is of fundamental importance for price movements in a market full of mass investors. The transfer of market information can increase the homogeneity of investors due to the sensitivity of the mass investor. The positive deviation of Hurst's power from 0.5 and the existence of a fractal dimension opposite to 1.5 is due to the homogeneity of the investor structure. The results of the present study are consistent with the investigations of Meng et al. [19], Chen et al. [12], Pilar et al. [22] and Kent [26].

Stock market investors come in various categories, the largest of which are individual investors. Participants in this market use their knowledge, experience, and interest in the stock market. Another group of participants are brokers who, in addition to their main duties, provide services to investors to buy and sell shares for themselves and their customers according to their history of continuous presence in the market. On the other hand, irrational increases that lead to a stock price bubble affect the behavior of investors and their analysis of market trends and cause the balance between risk and return to fluctuate in the pattern of investors. An investor's first goal is to earn more profit than he invests in the activity. These benefits can be an increase in the asset's price (here referred to as the securities) or the gain from holding the asset until its due date. In the meantime, increasing price levels based on logical analysis in any market is the basis of investors' work. Identifying the factors that affect these increases is essential because they may lead to a price bubble in the financial markets. From the investors' point of view, this price trend recognition is debatable from two aspects. First, identifying stocks that will enter the price bubble soon and experience market speculation increases can give higher risk-return returns to investors who sell stocks before the price rises. Buy-have leads to increased wealth. Second, buying the stock of companies with a price bubble that risks bursting the bubble can cause stock prices to fall, resulting in a sharp decline in the value of these stocks, huge losses for investors, and reduced wealth creation. Therefore, the price bubble's effect on investors' behavior is an inevitable part of the capital markets, which almost always happens and divides investors into beneficial and disadvantaged categories. Beneficial investors are those who sell stocks at previous prices. Affected shareholders are those who bought stock during the bubble and sold it with a price increase after the bubble burst.

The results show that multilevel convergence is one of the microscopic mechanisms of the microstructure of financial markets, along with providing new insights into space-time correlations of financial markets. Collective behavior at the macro level has been considered, and the empirical proof of multilevel collective behavior (the last level of the market) is essential. Controlling the network topology of market orders and the relationship of the network structure by the regulator is one of the proposed strategies to reduce the allowable number of similar orders. A developed and mature market also exhibits a relatively weak dynamic structure of the transfer because of the large number of heterogeneous investors, so expanding the market size is one of the solutions. Finally, the multilevel collective behavior mechanism, described by simultaneous price movements, can be applied to other complex systems with similar collective structures.

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