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Research Paper

Investigating portfolio performance with higher moment considering entropy and rolling window in banking, insurance, and leasing industries

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ABSTRACT

According to modern portfolio theory, diversification should cover the risk. This theory is based on the normality of asset return. Experimental findings indicate that the assets return non-normality. Higher moments are sed to upgrade traditional models with the primary presumption of a normal distribution in recent years. This study uses a higher moment and the entropy for diversification and selects a portfolio given a non-normality assumption. It is essential to use up-to-date information to increase the model's efficiency, and accordingly, we used the rolling window for new price information. For the financial information method, we use the total index return in the last five working days and weigh the shares of the banking, insurance, and leasing industries on the next working day and evaluate this for three years. Python, math, and NumPy libraries were used to analyze the data. The comparison between models based on the portfolio evaluation indices indicates that, given using entropy for diversification, a much higher moment model can provide better portfolio selection results in most cases. The results showed that the mean-variance-skewness-entropy model, according to the performance evaluation criteria of ASR, MADR, SSR, OMEGA, and Jensen, and the modified Treynor show better performance than the other models and only in the SR evaluation model, which is somewhat traditional, it has shown poorer performance than other models. Therefore, the hypothesis of using entropy as a criterion to improve portfolio performance can be confirmed. Comparing the models based on portfolio evaluation indices indicates that the use of entropy for diversification does not significantly reduce the optimal values of other objective functions. As observed, higher efficiency was obtained when using entropy and higher moments than in other models.

1 Introduction

Investment is a two-dimensional process and involves risk and return. The two factors are two sides of the same coin, and both should be evaluated to make decisions in this regard. Therefore, if the information

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about the stock risk is available, its performance can not be discussed. It is impossible to examine different investment solutions only through returns without considering the risk. Although all investors prefer higher returns, investors are also risk-averse. To properly evaluate portfolio performance, whether the returns are large enough in terms of risk must be determined. For accurate performance evaluation, portfolio performance should be evaluated based on adjusted risk [1]. Optimal portfolio performance means selecting an asset portfolio to maximize the investor's utility. Markowitz presented his classical model based on the criteria of mean and variance. The most significant assumptions of this model were to consider the return on assets as a standard random variable, which various researchers in their research have shown that the return on assets does not follow the normal density function; Therefore, the classical model proposed by Markowitz loses its efficiency in choosing the optimal stock portfolio. [2] [3], [4], and [5] show that the utility of investors is not quadratic, and Higher moments such as kurtosis and skewness should also be considered, and due to the progress of processors, MVS and MVSK models have been proposed to evaluate the optimal performance of the portfolio. [4-5] On the other hand, models such as MVS in times of Financial crisis lose their effectiveness in portfolio diversification, especially when the number of assets used in the issue is limited. Another criterion introduced for this purpose is portfolio entropy, a suitable criterion for portfolio diversification. Studies indicate that response is more efficient when using entropy as a risk indicator, especially for out-of-sample data [6].

In 1992, The researchers showed that the utility is adjusted by higher moments such as kurtosis and skewness, and accordingly, models such as MVS and MVSK were proposed to optimize the portfolio. The MVS model was also questioned after the 2008 financial crisis because it failed to attract investors due to its limited assets. Accordingly, entropy, the measure of portfolio diversification, was considered one of the moments. Given this, models that take higher moments and entropy into account can be more efficient for investors. Therefore, the shortcomings of the previous models led us to examine a new model for selecting a portfolio from the banking, insurance, and leasing industries in the present study and to answer this question: whether the new model has a better performance in selecting portfolio compared to previous models? The resources allocation for each country and organization to improve the situation is one of the most vital pillars of sustainable development. Over the years, as the most critical center of asset allocation, financial markets have become increasingly important. Financial markets have developed with economic developments in recent years. One of the pillars of these markets is investing in financial industries such as banking, insurance, and leasing, which has increasingly attracted the attention of investors and has witnessed specialized companies for investment in this section. Meanwhile, due to the changes in these industries, there is a need for changes in line with these conditions for investment actors. Financial calculations have undertaken significant changes, and portfolio selection methods have been developed to increase investor utility. Markowitz proposed his classical optimization model based on the criteria of mean and variance, in which the return on assets was assumed to be expected, and then various researchers questioned the normality of assets return.

Thus, the classic model presented by Markowitz lost its efficiency in choosing the optimal portfolio. Capital market participants are concerned to reduce risk and, at the same time, increase returns. Accordingly, in addition to the mean, variance, and skewness, we decided to examine the use of entropy for a more suitable weight distribution for each stock. In this research, the effect of entropy on portfolio selection, including in banking, insurance, and leasing industries, has been addressed for the first time. One of the distinguishing features of this study is the study of the weight assigned to each share that has not been studied before. The study adds entropy as a diversification parameter to the mean-variance-skewness model (MVSM). In general, one of the essential concerns of investors and increasing returns and reducing risk is how to weigh each share in the portfolio. In this model, in addition to considering the traditional criteria for portfolio formation, we used entropy as a determining factor in the weight of each portfolio stock in order to increase the utility of investors. According to the above points, models that consider a much higher moment and entropy can have higher efficiency for the investor. The study aims to add entropy as a diversification parameter to the mean-variance-skewness model (MVSM). In general, one of the most critical concerns of investors and increasing returns and reducing risk is how to weigh each of the shares in the portfolio. In addition to considering the traditional criteria for portfolio formation in this model, we used entropy as a determining factor for the weight of each portfolio to increase the utility for investors. According to the above points, models that consider a combination of much higher moments and entropy can have higher efficiency for the investor. In this study, we try to examine a model that aims to maximize skewness (due to nonnormal return on assets) and, at the same time, entropy as a weighting criterion and, on the other hand, to minimize portfolio variance. The research background and a review of the basics are presented in the following. Finally, research findings and conclusions are presented.

Table 1: Metod Recherch

1. Literature review
↓
2. Portfolio selection models, including EWM, MinVM, MVM, and MVSEM model
↓
3. The portfolio performance evaluation models in this research
↓
4. Evaluation of the second stage models by the performance evaluation models of the fourth stage
↓
5. The results

1.1 Capital Market

The capital market is one of the fundamental pillars of the economic system of any country. This market is a place of accumulation of cheap and scattered resources towards different economic units. The symbol of the capital market is the stock exchange and its affiliated institutions. The proper functioning of the stock market can have practical consequences such as economic growth and development. In order to be able to direct savings to this market, investors' trust must be won. One of the most important issues is choosing the optimal portfolio. That is, what combination of assets should the investor choose to maximize the amount of utility. In modern portfolio theory, the return is assumed a random variable. One of the main issues in such a choice is the risk criterion. The investor must minimize the amount of risk and maximize the return as much as possible. The foundation of modern portfolio theory was laid by Markowitz. Markowitz drew variance as an indicator of investment risk [7]. Stocks are one of the components of investors' financial assets. Therefore, recognizing the factors affecting the value of this asset is of interest to investors [8]. Since the early 1960s, many researchers have sought to evaluate portfolio performance and have always sought to model and test existing models. In general, these models are based on two different theories: modern portfolio theory and Post-modern portfolio theory have been formed. In modern portfolio theory (MPT), the risk is defined as the variability of total returns around the average return and is calculated using the variance. In other words, modern portfolio theory considers equal weights for all positive and negative deviations given (favorable and unfavorable) uncertainty as a risk in terms of the distribution of deviations in the variance. This is why variance is recognized as a symmetric risk criterion and can be used when the distribution of returns is normal. This is even though research on emerging stock markets today has shown that the distribution of returns in these markets is not normal. Accordingly, the post-modern portfolio theory (PMPT) was proposed. The emergence of modern portfolio theory dates back to 1952 when Harry Markowitz published his article entitled Portfolio Selection [9]. Research on emerging stock markets has shown that the distribution of returns in these markets is not normal. This theory makes a clear distinction between desirable and undesirable fluctuations. In post-modern portfolio theory, only fluctuations below the rate of return of the investor's target are subject to risk. In contrast, all fluctuations above this target (given uncertainty) are considered investment opportunities to achieve the desired rate of return [10].

1.2 Multi-Criteria Decision Making and Higher Moments

Decision-making includes the correct goal setting, determining different and possible solutions, evaluating their feasibility, evaluating the consequences and results of the implementation of each solution, and finally selecting and implementing. The quality of management is a function of the quality of decision-making because the quality of plans and programs, the effectiveness and efficiency of strategies, and the quality of the results obtained from their implementation all depend on the quality of decisions made by the manager. In most cases, decisions are made when the decision-maker is satisfied with several criteria. Criteria may be quantitative or qualitative. In multi-criteria decision-making methods that the researcher has considered in recent decades, several criteria are used instead of one measure of optimality. If there are suitable tools for analysis, the investor can invest in the industry and company of choice by examining different stock market industries and selecting the desired portfolio. the optimal portfolio Selection requires an estimate of two factors: risk and return on securities [11].

The study of semivariance in portfolio theory is as old as the variance. Semi-variance was introduced as a risk measure after two papers in 1952 by Markowitz and Ray. Ray tried to provide a practical way to determine the best interaction between risk and return. According to Ray, investors first seek to maintain the principal and then consider their capital's minimum return. In 1994, Sortino and Price used adverse risk to assess the performance of mutual funds. They used the term undesirable deviations instead of semivariance below the target rate. Using monthly data for the ten years to December 1992 for two mutual funds and six stock market indices, they demonstrated the usefulness of adverse risk to assess the performance of mutual funds [12]. In their study entitled "Combining Multi-Criteria Decision Making Techniques (MCDM) for stock selection based on the Gordon model, "Lee et al. identified the criteria that affect share prices. In this study, they extracted the criteria affecting the three critical elements of the Gordon model according to a review of the research literature. Criteria affecting the three main criteria of the Gordon model (projected dividends, discount rates, and growth rates) included industry outlook, revenues, operating cash flow, dividend payout ratio, market beta, risk-free returns, revenue growth rates, and dividend growth rates. According to the mean-variance model, only the primary and secondary moment related to the expected return and the variance-covariance of the return matrix are considered; however, this moment is primarily suitable for a portfolio that does not have a normal return distribution. Therefore, much research was done to consider more moments. Chunhachinda et al., Arditti, and Levy stated that more moments should not be ignored, as they influence investors' decisions [13]. Studies in 1971 and 1975 showed that investors prefer positive skewness. Since then, studies have been conducted in recent years that have tested skewness as the third moment in evaluating portfolio performance. Arditti in 1971 1975, Kraus and Litzenberger in 1976, and Harvey and Siddique showed that investors prefer positive skewness. Since then, studies have been conducted to test skewness as the third moment in evaluating portfolio performance [3]. In 1982, Price and Nantell examined the relationship between traditional beta and downside beta in a sample selected from US equity investment funds. When examining the relationship between traditional beta and downside beta, they found that the return on assets in the market has negative skewness [14]. In 1997, Hamid and Prakash studied the skewness in portfolio selection, and their experimental results showed that considering skewness in investor portfolio decisions causes fundamental changes in the structure of the optimal portfolio [15].

Friend and Westerfield in 1980 and Harvey and Siddique in 1999 argued that skewness plays a vital role in securities pricing. In 2008, Hutson et al. examined the relationship between trading volume and return skewness in 11 international stock markets using daily and monthly data from 1980-2004. Their results showed that high trading volume leads to negative skewness in returns [16]. Parkersh et al., Harvey et al., Houston also discussed more moments in the asset allocation system if the returns do not follow a possible symmetric distribution. In addition, they showed that the investor could achieve a high return when skewness is considered in the decision-making process. Therefore, the MVM was expanded to add skewness to the model. This model was called the mean-variance-skewness (MVS) model [17]. Christie et al. stated that the expected rate of return is related not only to systematic risk but also to skewness and kurtosis [18]. In 2009, Lee et al. proposed a mean-variance-skewness portfolio selection model with fuzzy parameters [19]. Chiao et al. conducted a study in Taiwan in 2003 that confirms such a relationship between stock returns and skewness. They concluded that in bullish periods, the effect of skewness and kurtosis in describing stock returns is more significant than in bearish periods [20]. On the other hand, Chunhachinda et al. Showed that the portfolio weight derived from MVM and MVSM usually focuses on low assets, while one of the asset allocation goals is diversity [21]. In another study, Parkash et al. confirmed the emphasis of the MV and MVS models on low assets. Bera and Park also showed that the above models are ineffective for portfolio diversification [22].

Entropy has been accepted as a measure of diversity in many studies to estimate diversity. As shown, the higher the entropy of the measured portfolio, the greater the diversity. Bera and Park made the first attempts to use entropy for the objective function in multiple objective functions to select a portfolio. To create a well-diversified portfolio, they proposed asset allocation models based on entropy and cross-entropy. The resulting weights are positive if entropy is used as the objective function to determine portfolio weights. This means that a model with entropy leads to the optimal situation in choosing the optimal portfolio depending on theoretical and practical reasons. In 2010, Usta and Mert Kantar examined the portfolio performance using entropy and concluded that entropy could improve portfolio performance. In addition, Gilmore et al. showed that portfolio variance decreases with increasing diversity [23]. On the other hand, the relationship between diversity and skewness has been addressed in many studies. Sears and Trennepohl have shown that if investors are trying to get positive skewness, it can reduce diversity. Simkowitz and Beedles also examined the skewness of portfolio returns when diversity increased and concluded that the increase in diversity results from a sharp decrease in portfolio skewness [24]. In addition, Hyung showed that when diversity reduces portfolio variance, skewness. For these reasons, skewness and diversity contradict each other for portfolio selection [25]. In 2018, Nabizadeh and Behzadi showed that higher moments in which entropy is taken into account help optimize the portfolio [26]. It is worth mentioning that in this research, the portfolio optimization approach of other researchers such as [27], [28], [29] and [30] were used. In 2020, Zhao et al. examined pairwise entropy models to measure dependence on stock markets, and the results showed that pairwise entropy could better show dependence on stock markets [31]. In 2020, Liu et al. used entropy-based metrics to identify various trading behaviors [32]. In 2020, Lu et al.proposed an index called polarity to measure trade imbalances in the Chinese stock market based on entropy, and the findings show that entropy-based trade imbalances are predictable [33].

1.3 Rolling Window Approach

In this study, in order to calculate the performance evaluation index, the rolling window approach has been used. In this approach, WL = 70, the 70-day estimation window is usually considered. The optimal weights are calculated using the window data, and the optimal values calculated for each objective function in each step. This process will be repeated by rolling the window, and new data and the optimal weights are recalculated. Finally, the T-WL portfolio (weight vector) is obtained, where T is the desired period, and WL is the window length. At each stage, using the calculated weights, the value of Rp.t+1 is calculated as the out-of-sample portfolio return for the period t + 1.

Rp.t+1=WTt rt+1

Where rt + 1 is the efficiency vector at t + 1. Therefore, the number of T-WL out-of-sample returns for each optimally calculated portfolio can be obtained to determine SR and MSR and compare models. Also, in Otsa and Kantar (2011), the ZJK statistic, with zero mean and variance, compares SR. the variance is calculated according to the following equation [34].

2 Structures

This research aims to use the higher moment to select a portfolio of banking, insurance, and leasing industries in the Tehran Stock Exchange. The present study is applied in terms of purpose. It is also descriptive in terms of collection method and is retrospective due to historical data. The statistical population of the present study is all companies in the banking, insurance, and leasing industries listed on the stock exchange. companies operated between 2018 and 2020 and have met the following criteria are selected as a research sample:

1. Do not change the fiscal year in the mentioned period.
2. Active in the stock exchange in 2018-2020.
3. Have trades in more than 90% of the trading days of the period in question.

Accordingly, they had 26 shares given these criteria. The Iran Financial Information Processing Center, stock exchange, and also source Arena websites have been used to collect information. After collecting the data, the data were analyzed using Python, math, and NumPy libraries. Several studies are conducted in the field of stocks in our country, while in the specialized field in the industries of this research, the research has not been done as it should be. Also, many of the researches that have been done have focused more on the mean-variance model and have paid limited attention to a higher moment such as skewness in the mean-variance model. Given these conditions and shortcomings, such as paying close attention to the positive skewness and other issues discussed below, we decided to provide a model that has both the advantages of the previous models and can help diversify. To this end, we added entropy to the previous models for better diversification. note that to improve the evaluation and the updated stock update information, we changed our evaluation to 5-day intervals. In the following, we explain this process. First, we start from the mean-variance models to reach the models with the higher moment, i.e., the mean entropy skewness variance, then we examine the performance according to the available indices.

2.1 Traditional portfolio selection models

This section presents the most famous traditional portfolio selection model and definitions. The portfolio vectors are the weight of the i-th portfolio risk asset. The weights of the portfolio are as follows:

$$\sum_{i=1}^{n} xi = x^T 1 = 1 \tag{1}$$

Where 1 is a n × 1 vector, and T denotes the transposition of the vector. In addition, portfolio weights are in the range of [0,1], i.e., short sell is not allowed, because if short sell is to be allowed, this number can be more than one, and the weight of some stocks can be even more than one.

The excess return vector is as follows:

$$R = (R_1, R_2, R_3)^T \left(\underset{1}{\approx}{R} - r_f, \underset{2}{\approx}{R} - r_f, \ldots \underset{n}{\approx}{R} - r_f \right)^T \tag{2}$$

R_i = yield premium of the i-th fund

r_f = The total return index is considered a criterion for evaluating the overall performance.

The mean excess return vector is as follows:

$$\text{E[R]=M=}(m_1, m_{2,}, \ldots, m_n)^T \tag{3}$$

Where $m_i = E(R_i)$ and E is the expected operator.

The n × n variance-covariance matrix of the excess return is as follows:

$$E[R - E[R]\ ]^2 = V \tag{4}$$

Where V contains the following terms:

$$\sigma_{ij} = E[(R_j - E[R_j])(R_j - E[R_j])] \tag{5}$$

Which shows the variance between the returns on assets i and j for ∀ (i, j) ϵ [1,… n].

The skewness is as follows:

$$E[R - E[R]]^3 = S \tag{6}$$

Which includes the following terms:

$$S_{ijk} = E[(R_i - E[R_i])(R_j - E[R_J])(R_k - E[R_k])] \tag{7}$$

Which shows the skewness between the returns of assets i, j and k for ∀ (i, j, k) ϵ [1,… n].

Mean entropy variance skewness equations for portfolio. The mean, variance, and third central moment in portfolio returns and portfolio entropy weights (probabilities) are as follows:

$$E[R_P] = E[X^T R] = \sum_{i=1}^{n} X_i m_i = X^T M \tag{8}$$

where

$$R_P = \sum_{i=1}^{n} X_i R_i \tag{9}$$

is Return on portfolio.

The variance of the portfolio is as follows:

$$\sigma^2[R_P] = E[X^T R - E[X^T R]]^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \sigma_{ij} = X^T V X \tag{10}$$

and

$$S_3 = E[X^T R - E[X^T R]]^3 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} X_i X_j X_K \sigma_{ijk} = X^T S(X \otimes X) \ldots s_3 R_P \ldots \otimes \tag{11}$$

$$SK[R_p] = \frac{S_s[R_P]}{\sigma^3{}_p[R_p]} \tag{12}$$

Shanon's entropy method is a multi-criteria decision-making method that aims to weight and prioritize criteria and indicators. This method is usually used as an auxiliary method for methods such as TOPSIS or ELECTRE. This method was proposed by Shannon and Weaver in 1974 and is used in the criteria matrix to calculate the weight of criteria. This method is used when there is also an alternative in addition to the criteria. The alternative can be, for example, to rank several companies or the number of strategies we want to choose.

Table 2: Shannon entropy calculation method

	X1	X2	. . .	Xn
A1	R11	R12	...	X1n
A2	R21	R22	...	X2n
. . .	. . .	. . .		. . .
Am	Rm1	Rm2	...	Xmn

$$p_{ij} = \frac{r_{ij}}{\sum_{i=1}^{m} r_{ij}}$$

The entropy Ej is calculated as follows, and k is a constant value that holds the Ej between 0 and 1:

$$E_j = -K \sum_{i=1}^{m} p_{ij} \; Ln(p_{ij}) \qquad K = \frac{1}{Ln\ m}$$

Next, the dj value (degree of deviation) is calculated, which states how much the relevant index (dj) provides the decision-maker with helpful information. The closer the measured values to each other indicates that the alternatives are not much different in terms of that index.

Therefore, the role of that indicator in decision-making should be reduced equally.

$d_j = 1 - E_j$

Then the weight, Wj, is calculated, and the best weight is selected

If the decision-maker has already considered a certain weight (λj) for the index, the new weight W^0j is calculated as follows:

$$W^0{}_j = \frac{\lambda_j \; W_j}{\sum_{j=1}^{n} \lambda_j W_j} \qquad\qquad W_j = \frac{d_j}{\sum_{i=1}^{n} d_j}$$

Our portfolio certainly includes many stocks, and the weights are not always equal in the portfolio. In such cases, we must assign a weight to each stock because it can affect our returns and other criteria according to the criteria. In general, weights help a lot in achieving the goals.

In this study, we also briefly use entropy as follows:

$$H(x) = -\sum_{i=1}^{n} X_i \, InX_i = -X^T (Inx) \tag{13}$$

Where Inx is as follows:

$$InX = \left(InX_1, InX_2, ..., InX_n\right) \tag{14}$$

H (x), the Shannon entropy, is a concave function $X_1, ...X_n$ for portfolio weights. The maximum value and i ≠ j, for j = of InX is when X_i=1 / n for i = 1,2,. N. H (X) reaches its minimum, 0, when X_i=1 1,… n. Thus, the properties of the entropy scale, H (X), provide a good measure of the probability distribution of a diverse portfolio, which can be used to measure portfolio diversity.

2.2 Portfolio Selection Models

2.2.1 Equal Weights Model

In the EWM model, the important thing is that the portfolio weights are equal, X_i=1/n for i = 1,2,… .n, and does not include optimization and estimation. EWM completely ignores the mean and variance of returns. Some investors use this simple model to allocate assets, and some complex models are derived from this model.

2.2.2 Minimal Variance Model

In the MinVM model, asset weights are obtained by minimizing the variance-covariance of the portfolio return matrix. MinVM is as follows:

$$\text{Min } X^T VX \tag{15}$$

Subject to $X^T 1 = 1$

$X_i \geq 0$ for i=1,2,…,n

According to studies on MinVM, empirical studies show that MinVM performs better than the MVM model. Even when we use the Sharpe ratio and other measurement criteria, we see that the MinVM performance is better than the MVM model (when we calculate the mean and variance).

2.2.3 Mean-Variance Model

Markowitz's MVM model is based on higher expected returns are possible by accepting more risk. MVM is as follows:

$$\text{Min } X^T V X \tag{16}$$

$$\text{Subject to } X^T M = \mu, X^T 1 = 1$$

$$X_i \geq 0 \text{ for i=1,2,…,n}$$

Where μ is the predetermined expected return for the portfolio, the expected return is equal to the total index.

Markowitz's MVM model is widely used for portfolio selection, although there are still some drawbacks to MVM. For example, MVM leads to poor portfolio performance.

2.2.4 Mean Skewness Variance Model

In the context of MVSM, it has been shown that investors' preferences for positive skewness in the distribution of returns are consistent with the concept of absolute risk aversion reduction. Also, a greater tendency for positive skewness emphasizes a precautionary motive. MVSM is as follows:

$$\text{Minimize } X^T V X \tag{17}$$

$$\text{Maximize } X^T S(X \otimes X)$$

$$\text{Subject to } X^T M = \mu, X^T 1 = 1$$

$$X_i \geq 0 \quad \text{for i=1,2,…,n}$$

According to studies on MVSM performance, introducing skewness in MVM can lead to much better portfolios for non-normal return distributions.

3 Conclusions

Multi-objective portfolio selection based on mean-variance-skewness-entropy

The first attempts to use entropy in the objective function for portfolio analysis are seen for Park, Samantha, and Zinc. Jana and Roy added the entropy function to MVSM for better portfolio distribution and introduced the mean-variance-skewness-entropy model. They used deviation instead of variance under normal conditions and used linear skewness. They also used the fuzzy programming technique to solve the multi-objective model, which included entropy but ignored the experimental evaluations of the model or the comparison of the model with the well-known portfolio model. In this study, we introduce MVSEM and evaluate its actual performance compared to well-known portfolio selection models and various performance comparisons. The higher moment model based on the mean, variance, skewness, and entropy is as follows:

$$\begin{aligned}
&\text{Minimize } \mathbf{x}^{\mathbf{T}}\mathbf{v}\,\mathbf{x} \\
&\text{Maximize } \mathbf{x}^{\mathbf{T}}\mathbf{s}(\mathbf{x}\otimes\mathbf{x}) \\
&\text{Maximize } -\mathbf{x}^{\mathbf{T}}\,\mathbf{ln}(\mathbf{x}) \\
&\text{Subject to} \\
&\mathbf{x}^{\mathbf{T}}\mathbf{M} = \mu,\ \mathbf{x}^{\mathbf{T}}\mathbf{1} = 1 \\
&x_i \geq 0 \text{ for i=1,2,…,n}
\end{aligned} \tag{18}$$

There is a multi-objective optimization problem to obtain portfolio weights from MVSEM. To solve this problem, we use the Weighted sum model. If the weighted sum method is used to optimize the multiple objectives given in the above equations, the optimization is obtained as follows:

$$\begin{aligned}
&\text{Minimize } X^TVX - X^TS(X\otimes X) + X^TIn(X) \\
&\text{Subject to } X^TM = \mu, X^T1 = 1 \\
&X_i \geq 0 \text{ for i=1,2,…,n}
\end{aligned} \tag{19}$$

So by solving the above relationship, the optimal weights for the portfolio can be obtained.

If we consider the risk aversion or investor risk preferences in terms of portfolio variance, skewness, and entropy, each of the above equations can be weighted, and it depends on the investors' preferences. So the above equation can be written as follows:

$$\begin{aligned}
&\text{Minimize } \lambda_1 X^TVX - \lambda_2 X^TS(X\otimes X) + \lambda_3 X^TIn(X) \\
&\text{Subject t to } X^TM = \mu, X^T1 = 1 \\
&X_i \geq 0 \text{ for i=1,2,…,n}
\end{aligned} \tag{20}$$

Where $X_i \geq 0$ and i = 1,2,3. To calculate the optimal point, our weights should be as follows:

$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$

Thus, the different combinations of λ_i values represent the combination of the portfolio components. For example, MVSEM is the same as MVM when $\lambda_1 = 1$ and $\lambda_2 = \lambda_3 = 0$.

Evaluation of portfolio performance

This section evaluates various portfolio performance metrics to evaluate MVSEM performance compared to EWM, Min VM, MVM, and MVSM.

Some performance measurement strategies are presented in the research background to evaluate portfolio performance models. In this research, we look at some ways to measure performance. One of these models is the Sharpe ratio. The Sharpe ratio formula is given below:

$$SR=\frac{E\lfloor R_p \rfloor}{\sqrt{\sigma^2\lfloor R_p \rfloor}} \tag{21}$$

Where R_P is the return on the portfolio.

The Sharpe ratio was based on the mean-variance theory and is only valid when the return distribution is normal. The Sharpe ratio gives us a misleading answer when the return distribution is skewed. There are several ways to choose the optimal portfolio for the Sharpe ratio. The adjusted for skewness Sharpe ratio, which is used for portfolio skew, is as follows:

$$ASR = SR\sqrt{1+\frac{SK(R_p)}{3}SR} \tag{22}$$

Another way is the mean absolute deviation ratio, which considers risk as to the mean absolute deviation, as follows:

$$MADR = \frac{E[R_P]}{E([R_P - E[R_P]])} \tag{23}$$

The Sortino-Satchell Ratio is a measure of performance based on the partial moment as follows:

$$SSR = \frac{E\lfloor R_p \rfloor}{\sqrt{E\lfloor \max(-R_{P''}O)^2 \rfloor}} \tag{24}$$

Keating and Shadwick used the omega ratio in 2002 to evaluate portfolio performance. This criterion pays attention to all elements of the return distribution and divides the returns higher than the target return into the returns lower than the target return. Since the omega ratio combines all the distribution moments and considers the investor's preferences for profit and loss, it can be calculated through the investment profit zone to loss zone. The following formula describes this phenomenon:

$$\Omega(r) = \int_r^b [1-F(x)]dx \Big/ \int_a^r F(x)dx$$

Where r is the target rate of return, F (x) is the cumulative distribution function of the returns, and [a, b] is the distance of the returns. The Jensen ratio adjusted by Momquli and Dabusi by replacing beta-Estrada with the traditional beta was introduced in the Jensen index, and it was introduced as Mamogli and Dabusi's alpha:

$$\alpha_P^{MD} = R_P - [R_F + \beta_P^D(E(R_M) - R_F)]$$

α_P^{MD} Momquli and Dabusi Alpha, R_P: portfolio return, R_F: risk-free return, β_P^D:: downside beta-Estrada.

$$\beta_P^D = \frac{E\{Min[(R_P-\mu_P),0].Min[(R_M-\mu_M),0]\}}{E\{Min[(R_M-\mu_M),0]^2\}}$$

Adjusted Momquli and Dabusi Treynor ratios are also based on downside beta and are calculated as follows.

$$\text{MDP} = \frac{R_{P-MAR}}{\beta_P^D}$$

The above criteria were the several performance evaluation criteria selected because of their relevance. We also use the Jarque test to show that the data distribution is not normal and symmetric. The statistics of this test is calculated based on the following equation:

$$\text{JB} = \frac{n}{6}\left(s^2 + \frac{(K-3)^2}{4}\right)$$

Where n represents the number of samples, s the skewness, and k the kurtosis of the data, note that this statistic follows the chi-square distribution with two degrees of freedom. The Jarque test showed that we have a non-normal distribution in all stocks.

3.1 Results and Discussion

Using the available information, we calculated mean, variance, skewness in 2018-2020. Data were analyzed using Python and the math and NumPy libraries. Now that stock weights in the portfolio have been obtained using 5-day periods, we need to compare this information with the portfolio valuation metrics and examine which one is the more appropriate model for portfolio selection. Below is the mean value obtained for each of the evaluation criteria.

Table 3: Results of performance evaluation of different models

Models	SR	ASR	MADR	SSR	OMEGA	α_P^{MD}	MD_p
EWM	0.0148	0/0121	0.0083	1174/765	0.5235	0.544	-0.249
MinVM	0.0216	0/0164	0.0085	1134/995	0.3491	-0.0423	-0.540
MVM	0.0201	0/0170	0.0089	1134/433	0.4328	0.864	0.767
MVSEM (0/5,0/5)	0.0167	0/0164	0.0164	1143/551	0.5635	1.064	0.486
MVSEM (0/5,0/5,0)	0.0172	0/0185	0.0168	1180/951	0.5498	0.892	2.443
MVSEM (1/3,1/3,1/3)	0.0152	0/0157	0.0148	1262/119	0.5611	0.944	1.873

Source: Researcher Findings

We used SR, ASR, MADR, SSR, OMEGA, adjusted Jensen ratio, and adjusted Treynor ratio and examined the following results. We allocated λ as desired. For example in
MVSEM (0.5.0, .05)

We have assigned 0.5 to the first term of the MVSEM model , 0 to and 0.5 λ_3. It should be noted that in the model

MVSEM (0.5, 0, 0.5)

Most of the first and third terms are essential to us, that is, the term that includes variance and entropy.

in this model

MVSEM (0, 0.5, 0.5)

The term includes skewness, entropy is essential, and the term containing variance is not essential.

and in the model

MVSEM (1/3, 1/3, 1/3)

All terms are equally important to us.

Table 4: Comparison of results using performance evaluation criteria

Row	model	r_p^o	σ_p^o	SR	PT	P-value
1	EWM	0.0235	0.0875	0.2691	-	-
2	MVM	0.0274	0.0585	0.4688	0.0312	0.1946
3	MVSM	0.283	0.0586	0.04828	0.01612	0.1708
4	MVSKM	0.0321	0.0564	0.5699	0.0194	0.0318

The MVSKM model performs better than the other three models. Finally, the MVSKM model has the best performance among other models in terms of efficiency and standard deviation.

Evaluation of SR Results

Results indicate that the value of the Sharpe index for the MinVM model is higher than other models and shows that portfolio selection without considering criteria such as skewness and entropy can bring better performance for our portfolio in the Sharpe index. The lowest value is for the EWM model.

Evaluation of ASR Results

The value of the ASR index for the MVSEM model (0.0 / 5,0 / 5) is higher than other models and shows that using the MVSEM model, which considers skewness and entropy, can have better performance, and the lowest value is related to EWM. It has the weakest performance in the ASR ratio.

Evaluation of MADR Results

The value of the MADR index for the MVSEM model (0.0 / 5,0 / 5) is higher than other models, and the lowest value is related to EWM

Evaluation of SSR Results

The value of SSR index for MVSEM model (1 / 3,1 / 3,1 / 3) is higher than other models and shows that in SSR index, MVSEM model (1 / 3,1 / 3,1 / 3) will perform better. The weakest performance is related to the MinVM model.

Evaluation of OMEGA Results

The value of the OMEGA index for the MVSEM model (0 / 5.0,0 / 5) is higher than other models, and the lowest value is related to MinVM.

Evaluation of the Results of the Adjusted Jensen Ratio

The adjusted Jensen ratio value in the MVSEM model (0.5,0, .05) shows good performance, and the lowest value is related to MinVM.

Evaluation of the Results of the Adjusted Treynor Ratio

The adjusted Treynor ratio index value for the MVSEM model (0, 0.5, 0.5) is higher than the other models, and the lowest value is related to MinVM.

4 Conclusion

Portfolio variance is an indicator of risk measurement and has inadequacies since it is calculated based on historical data. Entropy is a measure of diversification in portfolio optimization. This paper evaluated the portfolio performance resulting from higher moments by considering entropy and rolling window in banking, insurance, and leasing industries and proposed a multi-criteria optimization approach to optimize the model with mean, variance, skewness, kurtosis, and entropy.Diversification is one of the critical dimensions in portfolio optimization. Portfolio variance has long been considered one of the risk measurement indicators with significant shortcomings. Entropy is one of the criteria for diversity. In this study, we evaluated the portfolio's performance resulting from higher moments by considering entropy, and the method was based on higher moments such as skewness and entropy. We weighted the portfolio in 5-day periods for 2018-2020. using different portfolio evaluation criteria, we examined the portfolio, and the results showed that using higher moments could provide better performance for investors. According to the performance evaluation criteria of ASR, MADR, SSR, OMEGA, and Jensen, and the adjusted Treynor, The mean-variance-skewness-entropy model shows better performance than the other models and only in the SR evaluation model, which is somewhat a traditional model, has weaker performance than other models. Therefore, the hypothesis of using entropy as a criterion to improve portfolio performance is confirmed. Portfolio performance evaluation is essential in the capital market and stock investment management. The ASR index for the MVSEM model (0.0 / 5,0 / 5) is higher than other models and shows that using the MVSEM model, which considers skewness and entropy, can have better performance lowest value is related to EWM. It has the weakest performance in the ASR ratio. The adjusted Treynor ratio index value for the MVSEM model (0, 0.5, 0.5) is higher than other models, and the lowest value is related to MinVM. Diversification is one of the critical dimensions in portfolio optimization. Portfolio variance is one of the leading indicators of risk measurement, and since it is calculated based on historical data, it also has shortcomings in this regard. Entropy is a measure of diversity in portfolio optimization. This paper includes entropy and higher moments in portfolio optimization models and has proposed a multi-criteria optimization approach to optimize a model that considers mean-variance, skewness, kurtosis, and entropy. Comparing the models based on portfolio evaluation indices indicates that the use of entropy for diversification does not significantly reduce the optimal values of other objective functions. As observed, when using Gini-Simpson entropy and higher moments, more efficiency was obtained than other models, and on the other hand, Shannon entropy resulted in more variation than Gini -Simpson entropy. Future studies recommend developing problem-solving methods in this regard, various heuristic algorithms, including evolutionary algorithms, and model frameworks from other entropy measurement functions. It is also suggested that these models are based on fuzzy logic to evaluate the performance of models. Therefore, the use of variance, skewness, and entropy in selecting a portfolio in the banking, insurance, and stock exchange industries can be suggested to investment companies, investors, and individuals who have some relationship with the capital market. Future studies may use other diversity criteria and study this model based on fuzzy logic.

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