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Selecting The Optimal Multi-Period Stock Portfolio with Different Time Horizons in the Credibility Theory Framework

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ABSTRACT

The purpose of this study is to present a multi-period portfolio model in which assets have different time horizons for corrections or an asset may not be traded for the first few periods and then enter the correction stage. In this model, fuzzy variables defined in a credibility space are used to describe the return, and the credibility measure controls the risk. The model's objective function is to maximize the portfolio's ultimate wealth, and a constraint is used to control portfolio risk, in which the validity of the portfolio's ultimate wealth below a certain threshold is controlled at a certain level of confidence. A combination of particle swarm optimization and simulation is used to find the best solution. Finally, using a numerical example, the model is implemented on a portfolio with 6 assets and 4 monthly time steps on the Tehran Stock Exchange. Practical implementation shows that the optimal portfolio has the ability to provide the final desired wealth of the investor at a 95% confidence level and the portfolio return, including transaction costs, is higher than the return of a single-period portfolio.

1 Introduction

In the realm of capital and risk management, portfolio theory is one of the most important and practical fields. Various models have been proposed since the introduction of the Markowitz model for selecting the optimal stock portfolio, which differ in terms of the type of objective functions (maximizing returns, minimizing risk, etc.), their number (from single-objective models to multiple-objective models), and the type and number of constraints. These constraints can apply to the expected return, the quantity of assets, the risk of liquidity, and so on. Various techniques for optimizing stock portfolio selection models have also been proposed. The single-period model is one sort of stock portfolio selection model. In single-period models, the contents of the portfolio are expected to remain unaltered until the investment horizon. Multi-period portfolio models, unlike single-period models, allow the investor to evaluate and alter the contents of the portfolio at regular intervals. As a result, multi-period models are likely to be more realistic. The stock portfolio used in this study is multi-period and can contain assets with various time horizons. For example, one stock in a portfolio may be changed or corrected every month, while

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another is changed or corrected every two months. In addition, an asset may not be exchanged for a few periods before being revised. As a result, the current study's innovation is to develop a flexible model of a multi-period portfolio that can accommodate various asset time horizons. For this purpose, the research uses the credibility theory. The beliefs of investors or experts (typically articulated as linguistic variables) regarding future situations are described by fuzzy variables specified in the credibility space, which are used to estimate returns and credibility measure to evaluate and control the risk. Therefore, the main research question is how to design and optimize a multi-period portfolio by considering different time horizons for portfolio assets within the framework of credibility theory. The structure of the article includes the sections of theoretical foundations and research background, research method, research findings, conclusions, and suggestions.

2 Theoretical Foundations and Research Background

In this section, we first discuss about the importance of different time horizons in multi-period portfolio design. Then, the application of fuzzy theory in portfolio design and optimization is discussed and the necessity of studying the portfolio from the perspective of credibility theory is explained. Finally, the approach of the present research and its innovation in comparison with other researches are explained.
The capital market is always volatile, and the price of financial assets is dynamically increased or decreased by the flow of supply and demand. Because of these dynamic oscillations, investors' estimates of future returns on various assets are frequently based on a multi-period perspective [24]. The verbal description of the market's future by many investors supports this assumption. For example, statements like "I predict that these stocks will grow first, then go through a period of decline and then continue to grow again," or "I buy stocks in this industry first and then move on to another industry after growth," show that investors have a multi-period perspective and that the multi-period portfolio should be studied [6]. When reviewing the research literature, it can be noted that the majority of research in the field of optimal multi-period portfolio has assumed that all assets have the same time horizon, for example are [5]. This assumption necessitates the investor's forecasting of returns for all assets and time ranges in the future. In fact, investors may forecast assets across a variety of time frames [19]. They might, for example, forecast and assess A share for the next month (next few months) and B share for the next two months (next few two months), using technical, fundamental, or statistical analysis. Forecasting returns across various time horizons involves various risks, and the amount of calculated risk can encourage an investor to select a time horizon.

The multi-period portfolio introduced in the present study considers the possibility of differences in time horizons for different assets and provides a framework for modeling these constraints. Therefore, each asset can have its own regular intervals for periodic adjustments until the final maturity. Also, an asset may not be traded before a certain period of time and may be traded after a certain time. In fact, the investor may have views on the minimum holding time of one or more assets and may not want to trade the asset for a certain period of time. After explaining the importance of different time horizons, we move on to fuzzy logic and credibility theory as the main research tool. Asset returns are frequently treated as random variables with a defined probability distribution in portfolio selection models, and historical data is utilized to estimate them. This method is founded on the notion that previous data may properly forecast future results. However, due to the ever-changing economic environment, it is difficult to estimate accurate probability distributions as well as sufficient data in practice [27]. In the real world, however, there are many other elements to consider when selecting a stock portfolio, such as social, political, psychological, and so on. Furthermore, investors frequently collect or express information in general terms such as "high risk," "poor profit," and so on. Fuzzy variables can be utilized to model in this instance [26]. With the introduction of fuzzy set theory, more researchers addressed the issue of choosing the optimal stock portfolio based on this theory. As an example, Peykani et al. [20] propose a novel Fuzzy Multi Period Multi-Objective Portfolio Optimization (FMPMOPO) model that is capable to be used under data ambiguity and practical constraints including budget constraint, cardinality constraint, and bound constraint. Rezaei et al. [23] used a combined method GJR-GARCH-EVT-copula to fuzzy multi-period portfolio selection, which takes into account the characteristics of financial data such as cluster fluctuations, wide sequence distribution and the structure of interdependence between assets. Farrokh [3] modeled stock portfolio selection by considering the limits of investment ratios to simultaneously optimize returns and risk in fuzzy uncertainty conditions. For this purpose, two new feasibility planning models were developed using the mean and possible adverse risk and fuzzy returns. Ansari et al. [1] considered the return as a fuzzy number and the absolute half value of deviation from the average as a risk criterion. In order to solve the mentioned models, two intelligent combined methods based on genetic algorithm and differential evolution algorithm have been used to optimize the portfolio and finally have been compared using economic performance criteria.

Shiri Ghahi et al. [24] made a comparative comparison of fuzzy stock portfolio optimization models. For this purpose, three stock portfolio optimization models were designed, and instead of considering a single-period stock portfolio model, a three-period model was used. Nouri and Mohammadi [18] in a study optimized fuzzy investment portfolios in which the mean return is a triangular fuzzy number and comes from historical data. Liu and Zhang [14] proposed the definitions of the lower and upper semi-variances. A general multi-period fuzzy portfolio optimization model with the objectives of maximizing both terminal wealth and the cumulative diversification degree of portfolios over the whole investment horizon is designed and a fuzzy multi objective nonlinear programming technique was applied to convert the proposed model into a single-objective model. Oprisor and Kwon [19] proposed a novel multi-period trading model that allows portfolio managers to perform optimal portfolio allocation while incorporating their interpretable investment views. This model's significant advantage is its intuitive and reactive design that incorporates the latest asset return regimes to quantitatively solve. Carlsson and Fullér [2] introduced the low and high probability averages of fuzzy numbers for use in the stock portfolio selection model. Zhang and Nie [30] defined low and high variance and covariance for fuzzy numbers and formulated the fuzzy mean-variance model. Zhang et al. [31] proposed a model with utility maximization based on the probabilistic mean of the value interval and the probabilistic variance. Karr and Bhattacharyya [8] proposed the mean-variance-skewness model using the concept of probabilistic weighted momentum of fuzzy numbers. Liu et al. [15] redefined the concepts of mean and variance for fuzzy numbers, proposed a concept of probabilistic skewness, and then formulated a fuzzy mean-variance-skewness stock portfolio selection model.

Yong et al. [27] proposed a fuzzy multi-period stock portfolio model based on the asymmetric and time-varying attitudes of investors towards profit and loss. Gupta et al. [6] developed a multi-period portfolio optimization model based on interval valued fuzzy numbers. Fuzzy theory began with membership functions, in which each fuzzy variable was introduced while simultaneously introducing its membership function. Then Zade [28], introduced the possibility of measuring a fuzzy event. Although the possibility theory is widely used in stock portfolio selection, there are still some limitations because the possibility measure is not self-dual. To overcome this shortcoming, credibility measure was proposed by Liu and Liu in 2002 and is accepted by more researchers. Huang [7] measured risk with entropy and proposed a mean-entropy portfolio selection model based on credibility theory. In the context of credibility theory, Qin et al. [22] proposed a portfolio selection model for minimizing cross-entropy. Qin and Kar [21] defined skewness for the fuzzy variable in the framework of credibility theory and in order to develop a fuzzy mean-variance-skewness model. In the framework of credibility theory, Vercher and

Bermudez [25] introduced a mean-half-absolute deviation portfolio selection model and used an evolutionary algorithm to find the approximate Pareto boundary. Guo et al. [5] in a study entitled "Fuzzy multi-period portfolio selection with different investment horizons," formed a multi-period portfolio selection model with a V-shaped transaction cost function, taking into account different time horizons. For this purpose, two mean-variance models were designed, one with the objective function of maximizing total income and the other with the goal of minimizing the variance of the final wealth of the portfolio. Finally, with a numerical example, the mentioned models were optimized using a fuzzy genetic algorithm on a 5-shares portfolio on the China Stock Exchange.

Credibility theory and fuzzy variables defined in the credibility space help to reflect the views of investors or experts on the future return and risk of assets [9]. The portfolio selection model of the present study uses credibility theory to model returns as well as to ensure that constraints related to the minimum of expected portfolio wealth as a risk measure. The approach of the present study in multi-period modeling with the possibility of different maturities is different from that of Guo et al. [5] in several respects. The most important difference is that the present research model is set up so that the objective function becomes a linear function of fuzzy variables, in which the expected value can be distributed linearly, taking the objective function out of the form of a fuzzy variable and linearizing it. Another important difference in risk modeling is that in the Guo et al. [5] study, variance was used for this purpose. The variance takes into account both positive and negative changes around the mean; it is not directly related to the amount of loss, and it is difficult to determine its controlling amount [7]. With respect to the criterion of value at risk, the amount of portfolio loss is controlled at a certain level of confidence with the help of the credibility amount.

In the research stock portfolio, transaction costs are also considered to be more in line with real conditions. Considering trading costs makes it possible to choose the optimal stock portfolio in more realistic conditions, especially when the number of trades or the value of the trade increases. In addition, the model of the present study is self-finance. The self-finance portfolio in stock portfolio theory refers to the stock portfolio that provides the necessary cost to purchase shares or pay the transaction fee itself. That is, the necessary budget is provided through the sale of the contents of the stock portfolio. The details of the research model are discussed in the next section.

3 Research Method

At the beginning of this section, we briefly introduce the credibility theory and the fuzzy variables defined in the credibility space. The following definitions and theorems about the credibility theory are taken from the credit space from the book "Theory of Uncertainty" written by Liu [12].

Suppose Ω be a nonempty set and P be its power set (the set of all subsets of Ω). In this case, the function $Cr:\mathrm{P}\rightarrow[0,1]$ is called a credibility measure if the following conditions are satisfied:

1.Normality: $Cr\{\Omega\}=1$;

2. Monotonicity: $Cr\{A\}\leq Cr\{B\}\ if\ \ A,B\in P, A\subseteq B$;

3. Self-dual: $Cr\{A\}+Cr\{A^c\}=1\ \forall A\in P$;

4. Maximiality: $Cr\{\bigcup_i A_i\}=\sup_i\ Cr\{A_i\}\ \ if\ \ \sup_i\ Cr\{A_i\}<0.5$.

If Cr is a credibility measure on Ω, then the (Ω,P,Cr) space is called a credibility space. For example, if μ is a real function with a positive value such that $\sup\mu(x)=1$, then

$$Cr\{A\}=\frac{1}{2}(\sup_{x\in A}\mu(x)+1-\sup_{x\in A^c}\mu(x)) \tag{1}$$

is a credibility measure on the power set of real numbers? It can be shown that the credibility measure also satisfies in subadditivity property $Cr\{\bigcup_i A_i\}\le\sum_i Cr\{A_i\}\quad A_1,A_2,...\in P$. The function $\xi:(\Omega,\mathrm{P},Cr)\to R$ is called a fuzzy or measurable variable. Fuzzy membership function or credibility function for ξ is defined as

$$\mu(x)=(2Cr\{\xi=x\})\wedge 1 \tag{2}$$

Where $\wedge$ is the minimum operator. With this definition, the creditability of a set can be calculated in terms of membership function or credibility function as:

$$Cr\{\xi\in B\}=\frac{1}{2}(\sup_{x\in B}\mu(x)+1-\sup_{x\in B^c}\mu(x)),\quad B\subseteq R \tag{3}$$

The three well-known and common credibility functions of linear, triangular and trapezoidal are defined by the following rule.

$$\mu_1(x)=1\quad a\le x\le b,\mu_2(x)=\begin{cases}\frac{x-a}{b-a} & a\le x\le b\\ \frac{x-c}{b-c} & b\le x\le c\end{cases},\mu_3(x)=\begin{cases}\frac{x-a}{b-a} & \frac{x-c}{b-c}\\ 1 & b\le x\le c\\ \frac{x-d}{c-d} & c\le x\le d\end{cases} \tag{4}$$

and their shapes is as follows.

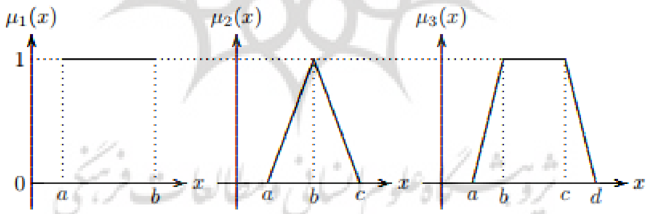

Fig. 1: Linear, triangular and trapezoidal fuzzy numbers from left to right, respectively

For a fuzzy variable, the cumulative distribution is defined as $\Phi(x)=Cr\{w\in\Omega|\xi(w)\le x\}$. For example, for a triangular fuzzy variable, the cumulative distribution would be

$$\Phi_2(x)=\begin{cases}0 & x\le a\\ \frac{x-a}{2(b-a)} & a\le x\le b\\ \frac{x+c-2b}{2(c-b)} & b\le x\le c\\ 1 & x\ge c\end{cases} \tag{5}$$

Fuzzy variables $\xi_1, \xi_2, ..., \xi_n$ with membership functions $\mu_1, \mu_2, ..., \mu_n$ are called independent whenever the membership function of the vector $\xi = (\xi_1, \xi_2, ..., \xi_n)$ is as $\mu(x_1, x_2, ..., x_n) = \min_i \mu_i(x_i)$. In the following, we express the Zadeh Extension Principle, which computes the membership function for a function of fuzzy variables. Zadeh Extension Principle: Suppose independent fuzzy variables $\xi_1, \xi_2, ..., \xi_n$ with membership functions $\mu_1, \mu_2, ..., \mu_n$ and $f : \mathbb{R}^n \to R$ are arbitrary functions. In this case, the membership function $\xi = f(\xi_1, \xi_2, ..., \xi_n)$ is in the form of relation (6).

$$\mu(x) = \sup_{x = f(x_1, x_2, ..., x_n)} \min_i \mu_i(x_i) \tag{6}$$

For a fuzzy variable, the expected value is defined as

$$E(\xi) = \int_0^{\infty} Cr\{\xi \geq r\}dr - \int_{-\infty}^{0} Cr\{\xi \leq r\}dr \tag{7}$$

For example, for the triangular fuzzy variable (a,b,c), the expected value is equal to $(a+2b+c)/4$. The defined mathematical expectation has a linear property, i.e. for two fuzzy variables ξ, η and $a \in R$ we have:

$$E(a\xi + \eta) = aE(\xi) + E(\eta) \tag{8}$$

In the following, we will introduce the research multi-period portfolio model with different time horizons based on creditability theory. Suppose the number of portfolio assets is equal to N with the index set $I = \{1, 2, ..., N\}$. The research multi-period portfolio is closed at the moment t = 0 and is sold in the maturity of t = T, and at times $t \in \{1, 2, ..., T-1\}$ after closure can be corrected or changed. The portfolio assets are divided into three categories in terms of time. The first category of assets is that its returns are available on all time horizons (on all time horizons, it is possible to change them through buying and selling) and this category is shown with S_1. The second category, which is shown with S_2 contains assets that are available at regular intervals with length $\delta > 1$ and therefore it is possible to change in a number of horizons. The third category that is shown with S_3, after a primary time period $\delta > 1$ is available for all the horizons after that. With these definitions, the series of time horizons for assets i shown with H_i are:

$$H_i = \begin{cases} \{0, 1, 2, ..., T-1, T\} & i \in S_1 \\ \{0, \delta_i, 2\delta_i, ..., [\frac{T}{\delta_i}]\delta_i, T\} & i \in S_2 \\ \{0, \delta_i, \delta_i + 1, ..., T-1, T\} & i \in S_3 \end{cases} \tag{9}$$

In addition, $H_i^- = H_i - \{T\}$ is the set of horizons corresponding to asset i with the possibility of modifying the asset and therefore the final maturity time has been removed. According to different time horizons, for $t \in \{1,2,...,T-1\}$, a number of assets have the ability to change in order to modify the stock portfolio. This set of assets is indicated by Π_t. Therefore

$$\Pi_t = \{i \in I,\ t \in H_i\} \quad t \in \{1,2,...,T-1\} \tag{10}$$

the function b or reverse shift is defined as follows, which for each time, calculates the time corresponding to the return before that time according to the type of asset.

$$b_i(t) = \begin{cases} t-1 & i \in S_1 \\ (n-1)\delta_i & i \in S_2, t = n\delta_i \\ 0 & i \in S_3, t \le \delta_i \\ t-1 & i \in S_3, t > \delta \end{cases} \tag{11}$$

The return of the portfolio for different time horizons can be estimated and predicted by the investors or experts, for which fuzzy variables are used. In the following, it is assumed that fuzzy variables $\xi_{it}\ \ i \in I, t \in H_i^-$ are used to describe the returns. ξ_{it} is the return on the stock i in the moving step from t to t + 1. The share of each asset in the stock portfolio in terms of currency (nine percent) in different time horizons is also indicated by the variable $x_{it}\ \ i \in I, t \in H_i$. At any time and before moving to the next time horizon, the investor can modify the contents of his portfolio for assets that can be adjusted at that time. And for this purpose $\Delta x_{it}\ \ i \in I, t \in H_i^-$ indicates the amount of correction or change in each asset that can take a positive value in the sense of purchase or negative in the sense of sale. Obviously, in the first step to move from time zero to one, $\Delta x_{i0} = 0\ \ i \in I$ In the following, the dynamics governing the value of each asset at different times is considered as

$$x_{i,t} = (x_{i,b(t)} + \Delta x_{i,b(t)})(1 + E(\xi_{i,b(t)}))\quad i \in I, t \in H_i, t \ge 1 \tag{12}$$

The objective function of the research portfolio is to maximize the wealth of the portfolio at the final maturity by considering the transaction cost. Equivalently, the objective function can be considered as maximizing the change in wealth from closing to final maturity. So far, the main assumption that has been made is to place a dynamic based on the expected value of returns for stock price movements. However, it is possible to deviate from expected value at any time; because in principle the change in value of an asset from step t-1 to t is calculated based on Equation (13).

$$x_{i,t} = (x_{i,b(t)} + \Delta x_{i,b(t)})(1 + \xi_{i,b(t)})\quad i \in I, t \in H_i, t \ge 1 \tag{13}$$

Note that in Equation (13) it is assumed that $x_{i,b(t)}$ is a positive fixed number but $x_{i,t}$ is a function of the fuzzy variable $\xi_{i,b(t)}$ and therefore is considered fuzzy variable itself. The research portfolio is self-finance and the necessary cost to buy assets is provided in different time horizons in order to improve the portfolio by selling other assets at the same time. In addition, the research portfolio also considers transaction costs, for which to model them, a factor of C is used (a percentage that represents the transaction cost for total sales). Thus, the change in the value of the portfolio in the moving from t-1 to t, denoted by ΔW_t, is a fuzzy variable that is calculated as Equation (14).

$$\Delta W_t = \sum_{i\in\Pi_t}(x_{i,b(t)} + \Delta x_{i,b(t)})(1+\xi_{i,b(t)}) - \sum_{i\in\Pi_t} x_{i,b(t)} - c\sum_{i\in\Pi_t}|\Delta x_{i,b(t)}| = \sum_{i\in\Pi_t}(x_{i,b(t)} + \Delta x_{i,b(t)})\xi_{i,b(t)} + \sum_{i\in\Pi_t}\Delta x_{i,b(t)} - c\sum_{i\in\Pi_t}|\Delta x_{i,b(t)}| \quad (14)$$

By specifying the amount of change in the value of the portfolio at each time step, the change in the value of the stock portfolio from zero to the final value at the final maturity, which is indicated by W_T is equal to:

$$W_T = \sum_{t=1}^{T}\sum_{i\in\Pi_t}(x_{i,b(t)} + \Delta x_{i,b(t)})\xi_{i,b(t)} + \sum_{t=1}^{T}\sum_{i\in\Pi_t}\Delta x_{i,b(t)} - c\sum_{t=1}^{T}\sum_{i\in\Pi_t}|\Delta x_{i,b(t)}| \quad (15)$$

As mentioned, the research portfolio selection model seeks to maximize the ultimate wealth of the stock portfolio, while considering a number of limitations. One of these limitations is the consideration of risk. Credibility measure is used to model it. The investor wants that the stock portfolio changes at maturity over zero be greater than a threshold value at a certain confidence level (e.g. 95%). For this purpose, the uncertainty caused by the event related to the value of the portfolio being below a certain value is controlled by $Cr\{W_T < d\} < \alpha$;where d is the threshold of change of wealth that must be controlled at the error confidence level of α (which for example can be 0.05). Therefore, the model for selecting the research optimal stock portfolio is as follows.

$$\begin{aligned}
&\max \;\; E(W_T)\\
&st:\\
&Cr\{W_T < d\} < \alpha\\
&x_{i,t} = (x_{i,b(t)} + \Delta x_{i,b(t)})(1+E(\xi_{i,b(t)})) \quad i\in I, t\in H_i, t\geq 1\\
&\sum_{i\in I} x_{i,0} = V_0\\
&\sum_{i\in\Pi_t}\Delta x_{i,t} = 0 \quad t = 0,1,\dots,T-1\\
&x_{i,t} + \Delta x_{i,t} \geq 0 \quad i\in I, t\in H_i^-\\
&\Delta x_{i0} = 0 \quad i\in I_i\\
&x_{i,t}\geq 0 \quad i\in I, t\in H_i
\end{aligned} \quad (16)$$

Where the limit $\sum_{i\in I} x_{i,0} = V_0$ indicates the amount of initial investment and $\sum_{i\in\Pi_t}\Delta x_{i,t} = 0 \quad t = 0,1,\dots,T-1$ indicates the stock portfolio self-financing. Limit $x_{i,t} + \Delta x_{i,t} \geq 0$ also shows that short selling is not possible. In this section, an algorithm for solving the model (16) is presented. According to Equation (8), giving to the linear property of expected value about fuzzy variables, the objective function can be rewritten as (17).

$$E(W_T) = \sum_{t=1}^{T}\sum_{i\in\Pi_t}(x_{i,b(t)} + \Delta x_{i,b(t)})E(\xi_{i,b(t)}) + \sum_{t=1}^{T}\sum_{i\in\Pi_t}\Delta x_{i,b(t)} - c\sum_{t=1}^{T}\sum_{i\in\Pi_t}|\Delta x_{i,b(t)}| \quad (17)$$

In Equation (17), the objective function exits the form of a fuzzy variable and becomes linear. To calculate the constraint of changes in the value of the portfolio from closing to the final maturity, i.e. $Cr\{W_T < d\} < \alpha$ at a desired point from the vector of decision variables (not necessarily possible) simulation is used, because the analytical form is very complex and untraceable.. For this purpose, suppose that fuzzy variables representing returns (for all assets and time horizons) are of triangular type and independent of each other. For each fuzzy variable, a number of m samples are extracted. Accordingly, if Φ_2^{-1} is the inverse function of the distribution of a triangular fuzzy variable (5) and the samples of $\varepsilon_1, \varepsilon_2, ..., \varepsilon_m$ are extracted from the uniform distribution in the range of zero to one(For example, with the help of MATLAB software), $\Phi_2^{-1}(\varepsilon_1), \Phi_2^{-1}(\varepsilon_2), ..., \Phi_2^{-1}(\varepsilon_m)$ samples extracted are from the fuzzy variable. Next, the set Σ_{it} represents the samples extracted from the triangular fuzzy variable ξ_{it} i.e.

$$\Sigma_{it} = \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_M \,|\varepsilon_i \; comes \;\; from \; \xi_{it}\} \quad i \in I, t \in H_i \tag{18}$$

The set $S = \otimes \Sigma_{it} \quad i \in I, t \in H_i$ means the set of all scenarios, is defined equal to the Cartesian product of the sets Σ_{it}. Now, for each scenario $s \in S$, the wealth changes corresponding to the scenario, i.e. $W_T(s)$ are calculated based on Equation (15). Thus, for all scenarios, a set $\{W_T(s), s \in S\}$ is formed, which shows the changes in the portfolio wealth for all sample scenarios. If μ is a vector membership function consisting of ξ_{it} s then according to Equation (3):

$$Cr\{W_T < d\} = Cr\{s : W_T(s) < d\} = \frac{1}{2}(\sup_{\{s:W_T(s)<d\}} \mu(s) + 1 - \sup_{\{s:W_T(s)\geq d\}} \mu(s)) \tag{19}$$

and since ξ_{it} s are assumed to be independent, $\mu(s) = \min_{i\in I, t\in H} \mu_{i,b(t)}(s)$, therefore according to the Zadeh expansion principle in relation (6):

$$Cr\{W_T < d\} = \frac{1}{2}(\sup_{\{s:W_T(s)<d\}} \min_{i\in I, t\in H} \mu_{i,b(t)}(s) + 1 - \sup_{\{s:W_T(s)\geq d\}} \min_{i\in I, t\in H} \mu(s)) \tag{20}$$

By calculating $Cr\{W_T < d\}$, the amount of deviation from the considered error level α is measured by the following equation.

$$e = \max\{Cr\{W_T < d\} - \alpha, 0\} \tag{21}$$

By calculating the portfolio risk based on simulation, the meta-heuristic algorithm particle swarm optimization or PSO in MATLAB programming environment is used to optimize the nonlinear model (16), where to cover the constraint $Cr\{W_T < d\} < \alpha$, the amount of error calculated in Equation (21) is added to the objective function as a penalty.

4 Model Implementation

In this section, the proposed multi-period portfolio model is implemented on a sample stock portfolio of the Tehran Stock Exchange. The portfolio consists of six shares, including Saderat Bank, Alborz Insurance, Iran Khodro Investment Company, Shargh Cement, and Tehran Petrochemical, which are indicated by numbers one to six. Research by Zhang et al. [29], Guo et al. [5], Li et al. [11] and Zhou et al. [32] also used six, five, five and six assets, respectively. The stock portfolio has four periods. Each period is equal to one month. Stocks with numbers 1 and 2 will be traded monthly, and stocks 3 and 4 will be traded once every two months until the final maturity. Shares 5 and 6 cannot be traded before the first two months, after which they can be traded on a monthly basis. Therefore, according to relations (9) and (10),

$$H_i = \begin{cases} \{0,1,2,3,4\} & i \in S_1 = \{1,2\} \\ \{0,2,4\} & i \in S_2 = \{3,4\} \\ \{0,2,3,4\} & i \in S_3 = \{5,6\} \end{cases} \quad (22)$$

$$\Pi_0, \Pi_2 = \{1,2,3,4,5,6\} \qquad \Pi_1 = \{1,2\}$$
$$\Pi_3 = \{1,2,5,6\} \qquad \Pi_4 = \{1,2,3,4,5,6\}$$

The data in Table 1are related to an investor's forecast of six shares in the research portfolio, for each forecast a triangular fuzzy variable is presented. Note that the time horizon of each forecast varies according to the type of share. For example, for member stocks of set S_2, forecasts are bi-monthly.

Table 1: Prediction of six portfolio assets for 4 time horizons based on fuzzy variable

Time Horizon / Assets	**1**	**2**	**3**	**4**
1	(0/033,0/05,0/067)	(-0/024,-0/01,0/004)	(-0/013,0/02,0/053)	(0,0/02,0/04)
2	(0/042,0/06,0/078)	(-0/03,-0/02,-0/01)	(0/035,0/05,0/065)	(-0/005,0/02,0/045)
3	(0/07,0/1,0/13)	-	(-0/045,-0/01,0/025)	-
4	(0/036,0/05,0/064)	-	(0/035,0/06,0/085)	-
5	(0/124,0/15,0/176)	-	(-0/052,-0/03,-0/008)	(0/012,0/03,0/048)
6	(0/045,0/08,0/115)	-	(-0/042,-0/04,-0/038)	(0/038,0/05,0/062)

The initial capital of the portfolio is equal to 1000 million Rials and the transaction cost of the total purchase and sale is equal to $c = 0.015$. In addition, the investor wants to maximize the wealth of the stock portfolio at the final maturity; and at the 95% confidence level, he expects the final wealth to be no less than 108 million. Thus

$$\alpha = 0.05, \; P = 100M, \; Cr\{W_T < 8\} < 0.05 \quad (23)$$

In order to optimize the model (16), 1000 samples were extracted for each triangular fuzzy variable representing return. Then, the optimal solution was calculated and optimized using the PSO algorithm in MATLAB software with 200 particles and 1000 repetitions. The optimal amount of each asset in terms of currency $x_{i,t}$, for different time horizons, is presented in Table 2. Mark - indicates that the asset is not tradable on this time horizon. The numbers in Table 2 are on the scale of 10 million Rials.

Table 2: The Optimal Value of Assets (in Terms of 10 million Rials) in the Optimal Portfolio

Time Horizon / Assets	0	1	2	3	4
1	32/04	33/642	51/521	0	0
2	21/24	22/514	4/326	54/811	0
3	0	-	0	-	0
4	25/02	-	26/271	-	58/205
5	21/7	-	24/955	0	0
6	0	-	0	0	57/551

The optimal changes or modifications of the stock portfolio in time horizons or $\Delta x_{i,t}$ are presented in Table 3, the positive sign means the purchase and the negative sign means the sale. The sum of each column is equal to zero, which indicates the self-financing of the stock portfolio. Mark - indicates that the asset is not tradable on this time horizon.

Table 3: Optimal Amount of Changes of Asset Value (in terms of 10 million Rials) in the Optimal Portfolio

Time Horizon / Assets	0	1	2	3
1	0	+18/4	-51/521	0
2	0	-18/4	+47/875	-54/811
3	0	-	-	-
4	0	-	+28/64	-
5	0	-	-24/995	0
6	0	-	0	54/811

The final wealth of the stock portfolio is 1157/56 million Rials, Therefore, the return on the portfolio is equal to 15.756%. The amount of transaction costs is equal to 44/917 million Rials, which, taking into account transaction costs, the return on the portfolio at the confidence level of 0/95 is equal to 11.2644%. In the following, we consider the previous portfolio selection as a single-period portfolio. This means that the portfolio will not be modified until the end of the fourth month after closing. For this purpose, the expected return from the beginning of the investment to the last investment horizon (4 months later) is according to Table 4.

Table 4: Prediction of Six Portfolio Assets for Final Horizon, Based on Fuzzy Variable

Assets	Return
1	(-0/02,0/06,0/09)
2	(-0/02,0/07,0/10)
3	(-0/04,0/08,0/12)
4	(-0/03,0/09,0/12)
5	(-0/04,0/11,0/13)
6	(-0/05,0/09,0/12)

In order to optimize the single-period version of model (16), 1000 samples were extracted for each triangular fuzzy variable representing return. Then, the optimal solution was calculated and optimized using the PSO algorithm in MATLAB software with 200 particles and 1000 repetitions. The optimal amount of each asset in terms of currency $x_{i,t}$, for different time horizons, is presented in Table 5. The numbers in Table 5 are on the scale of 10 million Rials.

Table 5: The Optimal Value of Assets (in terms of 10 million Rials) in the Optimal Single-Period Portfolio

Assets	Initial wealth	Final wealth
1	0	0
2	23/80	25/466
3	0	0
4	10/61	11/554
5	65/59	72/804
6	0	0

The final wealth of the stock portfolio is 1098/36 million Rials. Therefore, the return on the portfolio is equal to 9.836%. The amount of transaction costs is equal to 15 million Rials, which, taking into account transaction costs, the return on the portfolio at the confidence level of 0/95 is equal to 8.336%. Therefore, the multi-period approach with different time horizons has a better return at the same confidence level of 0.95 with respect to single period portfolio.

5 Conclusions and Suggestions

Operating in the financial markets necessitates the selection of the appropriate and optimal portfolio. Many models have been developed as a result of this, each of which focuses on a different component of the stock portfolio optimization. The capacity to differentiate the time horizons of assets in the construction of a multi-period portfolio is the main focus of the stock portfolio model provided in this study. The current study used fuzzy variables defined on a credibility space to provide a flexible approach to modeling such a portfolio. Credibility measure was utilized to control the stock portfolio loss, and fuzzy variables made it possible to model returns based on the opinions of investors and experts. Finally, the proposed model was optimized using a mix of simulation and PSO algorithm. What the present study has developed compared to other researches in the field of multi-period portfolio modeling is considering different time horizons for portfolio assets so that even one asset cannot be traded before a certain time. In addition, for the first time, the value at risk was used as a measure of risk in the structure of a multi-horizontal multi-period portfolio, which is consistent with the concept of risk as a loss amount. In addition, transaction costs were added to make it more realistic. Also, an important difference is that the present research model is set up so that the objective function becomes a linear function of fuzzy variables, in which the expected value can be distributed linearly, taking the objective function out of the form of a fuzzy variable and linearizing it.

Practical implementation and optimization of the research portfolio on a stock portfolio with six indices of the Tehran Stock Exchange for four monthly time horizons show that the optimal portfolio has the ability to provide the final desired wealth of the investor at a 95% confidence level and the portfolio return, including transaction costs, is higher than the return of a single-period portfolio. Investors and stock portfolio managers interested in financial modeling are encouraged to use the multi-period research portfolio model for practical purposes in the presence of assets with different time horizons. For this purpose, it is necessary to have a corresponding time horizon for each asset, and it is desirable that the forecast risk in this time horizon is low. It is therefore recommended to estimate the time horizon with the least risk for each asset with the available data set. Risk-avoiding investors with a multi-period approach are also advised to use this model for practical purposes, given the possibility of controlling portfolio changes.

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