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Research Paper

Determining the Appropriate Weights of Criteria in Multi-Criteria Decision-Making Using Cooperative Game: A Case Study of Bank

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ABSTRACT

Criteria weighting is a crucial step in the entire decision-making process. Determining the appropriate weights will lead to more reliable results. This study aims to use a coalitional game method for calculating proper criteria weights in multi-criteria decision-making (MCDM). In this paper, the Shapley value method is used to determine the weight of criteria. A numerical case study of 65 banks has been used to explain the efficiency of the proposed method. To this end, using the TOPSIS technique, the alternatives are ranked once in Shapley value and again in the Shannon entropy weighted matrix. Then the results are obtained applying Spearman rank correlation coefficient are compared to efficiency-based ranking using data envelopment analysis (DEA) as a powerful benchmarking method. In the proposed method, unlike many conventional weighting methods, the selection of criteria weights is made in a coalitional game with the participation of all criteria; the obtained weights are both intuitively and objectively fairer, and more reliable rankings are provided. According to the logical and fair calculation of weights, having a simple and understandable mathematical method, and no need for experts' judgment, the proposed method can be used in real problems. Especially where realistic ranking has a significant impact on the equitable allocation and absorption of resources.

1 Introduction

Determining the weights of criteria is one of the critical problems that occur in MCDM techniques. These values play a significant role in the ranking of alternatives. But the criteria weights are usually partially identified in real situations. Generally, the determination of weights is one of the essential complexities in MCDM methods. In the literature, various approaches to find the weights of attributes can be grouped into three categories: a) the subjective methods, b) the objective methods, and c) the integrated techniques. In subjective methods such as AHP (Analytic Hierarchy Process), ANP (Analytic Network Process), SWARA (Step-wise Weight Assessment Ratio Analysis) [42], Delphi, SMART (Simple Multi attribute Rating Technique), SIMOS, DEMATEL (Decision making trial and evaluation laboratory) and BWM (Best- Worst Method) [30], weights are determined only by decision-maker (DM), based on the previous practice, constraints, and DM's preferences. Several researchers employed subjective methods to determine the weights of decision factors. In contrast, Chou [6] utilized an ANP methodology for shipping registry

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selection by shipowners. Kilic et al. [17] proposed an integrated method that leverages DEMATEL and Elimination and Choice Expressing the Reality (ELECTRE) methods under an Intuitionistic Fuzzy (IF) environment. Yadegary and Avakh Darestani [38] used an integrated model based on DEMATAL and ANP to select suppliers in the mega-projects. Mousavi-Nasab and Sotoudeh-Anvari [25] provided a new weighting method, which integrates BWM and D numbers, called D-BWM. Kaviani et al. [16] developed a combined intuitionistic fuzzy analytic hierarchy process (IFAHP) and fuzzy multi-objective optimization approach to select suppliers and allocate the orders to them in the bottled water production context. Sadeghi et al. [32], Yazdani-Chamzini et al. [41], and San Cristobal [33] applied AHP or FAHP to determine the weights of decision criteria. But since these methods require a decision-maker to assign scores to reflect degrees of criteria importance subjectively, they suffer from some problems. Such as limited knowledge or information of decision-makers, feeling uncomfortable in assigning precise weights, creating a consensus of opinions in group decision making, the complexity of some methods, the interaction between attributes, a significant increase in the number of pairwise comparisons with the increasing number of criteria, drawbacks and requiring to particular software.

In the objective methods such as the Entropy method Shannon [35] and CRITIC (Criteria Importance through Intercriteria Correlations) Diakoulaki et al. [7] DMs have no role in specifying the weights, and they are determined based on a mathematical model and decision matrix. The objective weighting approaches are especially suitable for conditions where trustworthy subjective weights cannot be achieved. For instance, Şengul et al. [34] utilized interval Shannon's Entropy to determine the weight of attributes in a given problem. In the integrated methods such as AHP – Entropy technique, the weights of attributes are obtained using both previous groups. For example, Jing et al. [15] suggested a combination weighting approach, which integrates AHP with Shannon's Entropy. Lv et al. [20] combined weights determined by AHP and CRITIC, which considered the subjective and objective weights in a machining process plan.

Each technique has its advantages and shortcomings and what is very significant is how the criteria weights are determined. Roberts and Goodwin [31] provided an overview containing the advantages and disadvantages of using different weighting methods. There is no consensus on the best method of determining criteria weights in the literature, either subjectively or objectively. However, there is an agreement that the weights calculated by applying specific methods (objective methods) are more accurate than the weights obtained by the methods of a direct criteria weight assignment based on the expert's understanding (subjective methods) [27]. This study aims to use a coalitional game method for calculating the proper criteria weights in the MCDM framework. Cooperative game theory is a well-known and widely used adopted approach in many different fields such as political and economic sciences, business, machine learning, online marketing, and in general for forecasting the coalition structure and the way players will negotiate to increase their utility. So far, a lot of researches have been done in this field. For example, Razmi et al. [29] modeled a restructured natural gas distribution network as a cooperative game to estimate the potential cost savings allocation for various collaboration scenarios. He et al. [10] introduced a framework for allocating river basin water in a cooperative way. Liang et al. [18] presented a new method called uncertain a-coordination value based on the uncertain cooperative game to solve public resource allocation among vulnerable groups. Mousavi-Nasab et al. [23] designed an approach using cooperative game and data envelopment analysis (DEA) to solve the resource allocation problem based on overall equipment effectiveness (OEE) among a set of comparable and uniform DMUs (decision-making units) in a fairway. Jiang et al. [15] presented a decentralized method in which unmanned aerial vehicles (UAVs) negotiate with each other for the best rendezvous positions by using the Nash bargain. Yang and Morita [39] utilized data envelopment analysis (DEA) and Nash bargaining game (NBG) theory to improve inefficient banks. Feizabadi and Alibakhshi [9] examined the interaction effect of customer integration (i.e., coordination) and shared relationship governance (i.e., cooperation) on supply chain adaptability and firm performance. Casajus and Huettner [4] provided new characterizations of the equal surplus division value based on the Shapley value, the equal surplus division value, and the equal division value. Jahangoshai Rezaee [13] applied the Shapley value as a cooperative game for determining the appropriate and fair weights regarding the importance of each objective. Memarpour et al. [21] studied the monetary policies of the central bank to determine the interest rate on deposits in the interaction with the Iranian banking system in the form of cooperative and non-cooperative games. The Cooperative game theory has great potential for determining the weight of decision matrix criteria in the field of multi-criteria decision-making. To the best of our knowledge, this capacity has not been considered so far. The absence of this field in the literature is clear. Most previous studies on the use of objective decision matrix weighting methods have focused on the use of Shannon entropy despite its weaknesses. Therefore, applying the Shapley value, a concept in the cooperative game theory, to achieve the purpose of the present study is largely novel. We aim to employ this method because of its fair advantages, very straightforward framework, and intuitive nature. We hope that this study will enhance the cooperative game theory as a valuable tool for this purpose. The specific objectives of this paper can be expressed as follows:

- To introduce a weighting method, without reliance on the judgment of experts and decision-makers
- Weighing decision matrix criteria by a simple and understandable mathematical procedure
- To Calculate the weights of the criteria in a logical, fair, and agreed manner with respect to all criteria
- Using the DEA as an impartial judgment in determining appropriate weights

The Shapley value, a solution concept in the cooperative game theory, is used to achieve these goals. In this way, first in the normalized decision matrix, each criterion is considered a player and each alternative a contributor. Thus, from different alternatives (contributors), in the criteria (players) dataset, estimating a fair allocation of the weights of the collaboration between the criteria can be derived. This can be considered as a case of horizontal cooperation and maybe studied using the cooperative game theory. Second, the decision matrix alternatives are ranked by the TOPSIS technique, once considering the weights calculated based on Shapley value and again regarding the weights obtained from the Shannon entropy weighting method, the most frequently used objective weighting technique. The research literature indicates that the TOPSIS technique is the most popular and widely accepted method compared to other MCDM techniques in the selection problems [24, 37]. Third, to evaluate the fairness of weights, the ranking of alternatives obtained from two weighting methods is compared to the ranking based on the efficiency of DEA using Spearman's rank correlation coefficient.

Data envelopment analysis (DEA) Charnes et al. [5] is a widely used non-parametric frontier analysis method implemented in linear programming for comparing the inputs and outputs of a set of comparable decision-making units (DMUs). It is evident that to make a meaningful comparison of ranking approaches; identical weights must be considered for decision criteria. Still, the standard DEA models enable the under evaluation DMUs free to determine the most favorable endogenous inputs and outputs weights. In the present paper, this feature of data envelopment analysis has been used as a strong point in determining the criteria weight of a decision matrix. Various DEA models have been successfully used in different problems. For example, the reader is referred to Lozano [19], Omrani [26], Jahanshahloo et al. [14], Izadikhah [12], and Moslemi et al. [22]. The structure of the paper is as the following. In Section 2, Shapley value, Shannon entropy, CCR model of DEA, TOPSIS method, and the concept of Spearman's rank correlation coefficient are reviewed. In Section 3, the research methodology is presented. In Section 4, the approach is illustrated with two examples. The last section summarizes and concludes the whole context.

2 Background

In this section, a brief explanation of Shapley value, Shannon entropy, CCR model of DEA, TOPSIS method, and the concept of Spearman's rank correlation coefficient are presented, respectively.

2.1. Shapley Value

The Shapley value was introduced by Shapley [36] as a particular function in the coalitional game. Shapley value is an allocation plan for payoffs based on the contribution of players. It reflects the relative contribution of players and is simple to be processed with mathematical methods. Hence, it is used widely in both economics and political sciences. The coalitional game is defined with characteristic function as follows:

Definition1. Coalitional game with characteristic function $\{N, v\}$ consists of a finite set N (the set of players) and characteristic function v that associates a real number $v(S)$, which is the worth of S with every nonempty subset S of N (a coalition). In the coalitional game, there is an assumption that:

$$v(S \cup T) \geq v(S) + v(T), \tag{1}$$

For all subsets S and T with $S \cap T = \emptyset$. That is to say, the payoff of a coalition must be more than the sum of the payoffs that each player could receive if he does not join the coalition. This character of the function v is called super-additive. Based on the characteristic function v, Shapley value can be defined as follows:

Definition2. Shapley value is defined by the formula (2):

$$\varphi_i(v) = \sum_{\substack{S \subset N \\ i \in S}} \frac{(s-1)!(n-s)!}{n!} \left[v(S) - v(S \backslash i)\right], \tag{2}$$

Where s and n are the numbers of players in S and N, respectively for any $i \in N$. $v(S)$ is called the worth of coalition S and $v(S \backslash i)$ is the worth of coalition S not containing player i. The value of formula (2) means the contribution or payoff of the player i in a coalitional game $\{N, v\}$. So, the Shapley value is a payoff profile presented as an n-dimensional vector for an n- person game [40].

2.2 Shannon Entropy Method

The entropy method is based on Shannon's entropy theory [35]. Entropy is a term that measures the uncertainty associated with random phenomena of the expected information content of a certain message. A discrete probability distribution represents this uncertainty. The entropy method estimates the weights of various criteria from the given payoff matrix and is independent of the views of the decision-maker. This method is particularly useful to explore contrasts between sets of data. These data sets can be mapped as a set of alternative solutions in the payoff matrix, where each alternative solution is evaluated in terms of its outcome. The philosophy of this method is based on the amount of information available and its relationship with the importance of a criterion. The basic idea is that the attribute having relatively less dispersion is assigned a lower weight because an index in which all alternatives have similar performance offers little information and is unimportant in the overall evaluation process [7]. The weights can be calculated according to the following procedures.

Step 1: Construct normalized performance rating matrix, using Equation (3)

$$r_{ij} = \frac{a_{ij}}{\sum_{i=1}^{m} a_{ij}}; \quad i = 1,2, \dots, m; \quad j = 1,2, \dots, n. \tag{3}$$

Step 2: Calculate the entropy E_j of the data of attribute j

$$E_j = -K\sum_{i=1}^{m} r_{ij}\ln r_{ij},\ K = 1/\ln m \tag{4}$$

Step 3: Calculate the degree of divergence (d_j) of average intrinsic information contained by each attribute c_j

$$d_j = 1 - E_j \tag{5}$$

Step 4: Calculate the weight for each attribute

$$W_j = d_j / \sum_{K=1}^{n} d_K \tag{6}$$

2.3 CCR Model

DEA is a non-parametric and non-statistical method that is widely used for estimating the relative efficiency of a homogeneous set of decision-making units (DMUs) that use multiple inputs (resources) to generate multiple outputs (products). The relative efficiency is calculated by maximizing the ratio of the weighted sum of outputs to the weighted sum of inputs. The first DEA model (so-called CCR) presented by Charnes et al. [5], was made based on the earlier work of Farrell [8]. To measure the efficiency of a DMU, the following model, which is known as the input-oriented CCR multiplier model, can be utilized:

$$\begin{aligned} &\theta_0 = Max \sum_{r=1}^{s} u_r\, y_{r0} \\ &s.t. \\ &\textstyle\sum_{i=1}^{m} v_i\, x_{i0} = 1, \\ &\sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} \le 0, \qquad j = 1,2,\dots,n \\ &u_r, v_i \ge 0 \qquad r = 1,2,\dots,s\ , \quad i = 1,2,\dots,m \end{aligned} \tag{7}$$

where θ_0 is the efficiency measure of DMU under evaluation (DMU_0), u_r and v_i are the non-negative weights of the r-th output and the i-th input, and y_{rj} and x_{ij} are the r-th output and the i-th input of the j-th DMU, respectively. DMUs with $\theta_0 = 1$ are efficient and form boundary points, while DMUs with $\theta_0 < 1$ are inefficient. To evaluate all DMUs, the process is repeated for each DMU_j. The output-oriented version of CCR multiplier model is defined as follows:

$$\begin{aligned} &\varphi_0 = Min \sum_{i=1}^{m} v_i\, x_{i0} \\ &s.t. \\ &\textstyle\sum_{r=1}^{s} u_r\, y_{r0} = 1 \\ &\sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} \le 0, \qquad j = 1,2,\dots,n \\ &u_r, v_i \ge 0 \qquad r = 1,2,\dots,s, \qquad i = 1,2,\dots,m \end{aligned} \tag{8}$$

where $\varphi_0 \ge 1$. Since the efficiency measure is considered as a value which belongs to the interval (0, 1], the efficiency can be defined as $\theta_0 = {\varphi_0}^{-1}$.

2.4 TOPSIS Method

TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) is a multiple criteria decision making (MCDM) method to rank alternatives or select the best one. TOPSIS was first introduced by Hwang and Yoon [11]. The basic principle of the TOPSIS method is based on the fact that the chosen alternative should have the shortest Euclidean distance from Positive Ideal Solution (PIS) and the farthest Euclidean distance from Negative Ideal Solution (NIS). The alternatives on particular criteria are presented in a decision matrix as the following:

$$A = [a_{ij}]_{m\times n} = \begin{matrix} & c_1 & \cdots & c_n \\ A_1 & a_{11} & \cdots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ A_m & a_{m1} & \cdots & a_{mn} \end{matrix} \quad (9)$$

where i is the alternative index ($i = 1,2,\dots,m$), m is the number of alternatives, j is the criterion index ($j = 1,2,\dots,n$) and n is the number of criteria. Also, A_1, A_2, …, A_m and C_1, C_2, …, C_n refer to the alternatives and criteria, respectively. The elements of the matrix are related to alternative i with respect to criterion j.
The key steps in TOPSIS are as follows:

(1) Normalize the decision matrix $[a_{ij}]_{m\times n}$ by using equation (10):

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} {a_{ij}}^2}}, \quad i = 1,2,\dots,m; \; j = 1,2,\dots,n. \quad (10)$$

(2) Construct the weighted normalized decision matrix $[v_{ij}]_{m\times n}$ by utilizing the following equation:

$$[v_{ij}]_{m\times n} = [w_j]_{1\times n} \times [r_{ij}]_{m\times n}, \; i = 1,2,\dots,m; \; j = 1,2,\dots,n. \quad (11)$$

Where w_j is the weight of the j-th criterion and, $\sum_{j=1}^{n} w_j = 1$.

(3) Determine the best ideal solution and the anti-ideal solution:

$$V_j^+ = \{v_1^+, v_2^+, \dots, v_n^+\} = \left\{ \operatorname*{Max}_i v_{ij} \middle| j \in I, \; \operatorname*{Min}_i v_{ij} \middle| j \in I' \right\} \quad (12)$$
$$i = 1,2,\dots,m; \; j = 1,2,\dots,n.$$

$$V_j^- = \{v_1^-, v_2^-, \dots, v_n^-\} = \left\{ \operatorname*{Min}_i v_{ij} \middle| j \in I, \; \operatorname*{Max}_i v_{ij} \middle| j \in I' \right\} \quad (13)$$
$$i = 1,2,\dots,m; \; j = 1,2,\dots,n.$$

Where I is related to the benefit criteria and I' is related to the cost criteria.

(4) Calculate the separation of each alternative from the positive ideal (d^+) and negative ideal (d^-) solution measures by employing equations (17) and (18):

$$d_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - V_j^+ \right)^2}, \; \forall i \quad (14)$$

$$d_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - V_j^- \right)^2}, \forall i \quad (15)$$

(5) Compute the relative closeness to the ideal solution for all alternatives:

$$CL_i^* = \frac{d_i^-}{d_i^- + d_i^+} \quad (16)$$

(6) Create the priority ranking for all alternatives in a descending order.

2.5 Spearman's Rank Correlation Coefficient (ρ)

Spearman's rank correlation coefficient, which was first presented by Raju and Kumar [28], is defined as follow:

$$\rho = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}, \quad (17)$$

Where i, is the number of alternatives, n is the total number of alternatives and d_i is the difference between the ranks obtained through two different methods. ρ = 1 defines a perfect association between the ranks, ρ = 0 characterizes no association between the ranks and $\rho = -1$ defines perfect disagreement between the ranks.

3 Research Methodology

In this section, the steps of determining the fair criteria weights of a decision matrix using the proposed method are explained.
Step1: Calculate the criteria weights using the Shapley value.
In this step, first, the decision matrix $[a_{ij}]_{m\times n}$ is normalized by using equation (10), then, using equation (2), Shapley value (the average expected marginal contribution) of each player is calculated. This means that the total value of all possible marginal contribution is calculated by each player and then multiplied by the probability of the player entering the coalition. To calculate the weight of each criterion (player), the calculated Shapley value for each criterion is divided by the total Shapley value of all players.
Step 2: Calculate the criteria weights using the Shannon entropy.
At this stage, the weight of the criteria is calculated using equations (3-5).
Step 3: Rank the alternatives using the TOPSIS technique.
In this step, the alternatives are ranked once based on Shapley value weights and once again based on Shannon entropy weights implementing equations (10-16).
Step 4: Calculate the efficiency of DMs (alternatives) by the CCR model (7).
Step 5: Compute the Spearman's rank correlation coefficient.
Finally, to identify more reasonable and fair weights of the criteria, the rankings obtained from step 3 are compared to the rankings obtained from step 4 using Spearman rank correlation coefficient, equation (17). The flowchart of the proposed procedure can be seen in Fig. 1.

4 Numerical Example

In the following, the presented approach is illustrated with datasets from a real case study. In this problem, 65 banks are evaluated. Determining inputs and outputs can be done from the viewpoints of four perspectives; shareholder, customer, management, and employee. As it can be observed from Table 1, five selected typical attributes of the banking system used in numerous studies such as [1, 2, 3, 39] are as Table 1.

(1) Capital adequacy ratio (CAR), which belongs to the category of soundness, is a measurement of a bank's available capital expressed as a percentage of a bank's risk-weighted credit exposures. This criterion is used to protect depositors and promote the stability and efficiency of financial systems around the world.
(2) Net impaired assets per Shareholders' equity (NIA/SE), indicates the credit quality of a bank. An impaired asset is an asset that has a market value less than the value listed on the company's balance sheet. Shareholders' equity (or business net worth) shows how much the owners of a company have invested in the business, either by investing money in it or by retaining earnings over time.

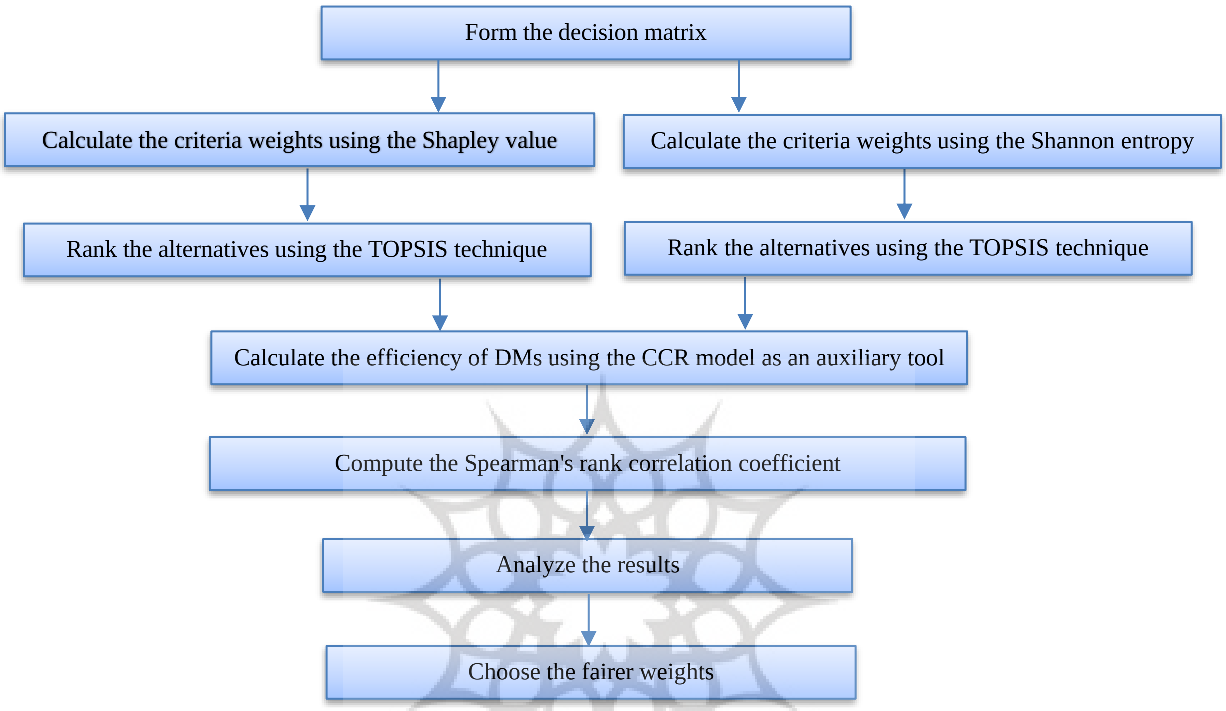

Fig1: Proposed General Framework

(3) Return on average equity (ROAE), which is an indication of profitability, is a financial ratio that measures the performance of a company based on its average shareholders' equity outstanding.
(4) Cost per income (C/I), which is an indication of efficiency, is a key financial measure used to aid in the valuation of a bank. It is calculated by dividing operating costs by operating income, and as such a lower ratio indicates a more profitable bank.
(5) Dividends per share (DPS) computed as the ratio of dividend paid to the number of outstanding shares, is the sum of declared dividends issued by a company for every ordinary share outstanding.
Variables for each bank are categorized in Table 1 as inputs (C/I and DPS) and outputs (CAR, NIA/SE, and ROAE) based on management perspective. The criteria weights obtained by the Shannon entropy (W_{Shan}) and, the Shapley value (W_{Shap}), along with the Shapley value (V_{Shap}), are also displayed in this table. The value of coalitions on the normalized data of the criteria is shown in Table 2. In this table, Inputs and outputs are shown from A to E, respectively. In Table 3, because of text limitation and preventing duplication, the Shapley value of the first criterion is only calculated using Formula (2) as an example.
By observing the weights obtained from the two studied methods, the inadequacy of Shannon entropy weights is quite understandable. Input 1, which indicates the bank's efficiency, weighs about one percent, and Output 1, which means the performance and financial stability of the bank, weighs approximately zero. On the other hand, input 2, which represents earnings per share, is about 78%. While, the importance of

these weights by the Shapley value method is about 12, 13, and 39%, respectively, which indicates a more proportionate and more logical fit of the weights.

Table 1: The Inputs and Outputs Data of 65 Banks

Bank code	Inputs		Outputs			Bank code	Inputs		Outputs		
	C/I	DPS	CAR	NIA/SE	ROAE		C/I	DPS	CAR	NIA/SE	ROAE
1	64.29	3.96	13.14	0.3959	2.87	35	59.75	3.48	10.55	0.4019	5.47
2	62.98	44.78	10.84	0.2453	4.44	36	62.09	2.84	10.06	0.6272	5.28
3	66.21	2.94	12.41	0.4713	3.52	37	59.09	3.49	13.72	0.2523	4.09
4	47.17	2.82	13.20	0.7858	5.55	38	55.50	3.98	11.05	0.4018	8.80
5	61.98	3.48	11.80	0.3004	5.55	39	60.66	37.36	10.17	0.6170	4.96
6	46.01	66.58	11.90	0.3905	5.71	40	65.34	4.47	12.67	1.0417	4.60
7	71.52	28.78	13.74	0.2345	3.64	41	54.59	3.53	10.07	0.2568	6.25
8	57.49	4.99	11.71	0.2477	4.65	42	65.83	2.48	8.55	0.6661	3.48
9	66.19	3.49	9.70	0.2337	4.06	43	58.19	4.03	9.46	0.6557	7.57
10	58.49	24.66	11.32	0.3910	6.68	44	70.15	3.50	10.94	0.3445	5.01
11	58.69	12.52	9.49	0.6520	7.22	45	65.93	2.49	8.33	0.5041	3.87
12	42.21	6.41	11.20	0.2699	9.26	46	51.60	29.93	10.64	0.4313	7.47
13	43.69	5.49	11.63	0.3438	9.61	47	67.55	2.43	9.44	0.7297	2.46
14	50.41	2.8	9.47	0.5087	8.12	48	67.97	2.95	11.15	0.5293	2.74
15	68.49	2.48	9.83	0.6241	1.82	49	42.05	3.64	10.71	0.4715	4.73
16	53.28	7.96	13.68	0.2949	5.03	50	64.51	2.96	10.15	0.3563	5.08
17	71.36	2.49	10.13	0.5165	6.74	51	60.34	3.41	14.24	0.3455	4.76
18	57.40	2.49	10.78	0.4771	4.41	52	69.07	2.99	12.20	0.2156	3.48
19	50.77	2.99	12.00	0.3529	4.18	53	64.66	2.98	9.85	0.5973	4.63
20	65.91	1.53	9.15	1.0342	15.47	54	73.16	34.84	10.67	0.6219	4.56
21	63.50	2.49	10.86	0.6231	3.97	55	48.82	7.87	14.55	0.2572	5.26
22	74.62	2.48	9.21	0.5772	2.81	56	47.58	5.96	10.49	0.5170	12.32
23	45.16	1.56	10.21	1.0815	6.70	57	66.29	2.39	9.84	0.5045	3.74
24	49.06	4.37	12.14	0.4689	4.96	58	51.14	3.48	11.26	0.4191	5.54
25	45.05	5.39	13.58	0.3798	4.84	59	74.98	3.24	10.59	0.6360	5.00
26	84.88	4.99	10.68	0.5409	7.46	60	68.97	2.76	9.86	0.9196	3.83
27	63.24	3.48	12.26	0.2305	3.79	61	54.59	3.98	10.17	0.6488	3.12
28	52.28	2.99	10.39	0.3467	6.98	62	62.13	24.97	10.25	0.7833	9.44
29	60.60	2.88	13.45	0.5057	1.28	63	59.79	2.47	10.64	0.557	2.64
30	65.00	2.99	10.77	0.5988	7.06	64	65.43	2.92	13.07	0.2518	4.22
31	65.65	3.49	10.89	0.2932	3.95	65	58.63	3.36	12.03	0.4384	4.39
32	67.91	2.95	10.58	0.3690	6.23	W_{Shan}	0.0137	0.7775	0.0094	0.0981	0.1013
33	49.86	3.49	12.55	0.2617	4.47	V_{Shap}	0.1188	0.3897	0.1313	0.1672	0.2130
34	47.03	3.23	11.98	0.2866	5.37	W_{Shap}	0.1165	0.3820	0.1288	0.1639	0.2088

The relative closeness to the ideal solution ($CL^*_{i\ Shan}$ and $CL^*_{i\ Shap}$) for all alternatives is calculated by using the TOPSIS technique, the efficiency of DMUs (alternatives) is calculated by CCR model (7) (θ_{CCR}) are illustrated in Table 4. Also, the rank of each alternative is shown in parentheses. By comparing the ranking of alternatives resulting from the two different weighting methods, the obvious and significant difference in rankings is evident in some cases. Except efficient Bank 20, which is ranked first based on the CCR model and the TOPSIS technique, considering both the Shannon entropy and Shapley value weighting methods. This indicates the dominant superiority of this bank over other banks. By referring to Table 3 and considering the input and output values of this bank, its superiority is somewhat intuitively recognizable. This bank has the lowest value in the DPS input attribute and the highest value in the ROAE output attribute, So that the significant weight dispersion of the two used methods could not change its position. Also, Bank 23 has almost the same conditions as Bank 20. This efficient bank, based on the DPS input attribute, has the second-lowest value and has the highest output among the banks in the NIA/SE attribute. But in the case of other efficient banks, the situation is not like this. Bank 13 is ranked 50th with Shannon weighting and 7th with Shapley weighting.

Table 2: The Worth of Coalitions

Coalitions	V(S)	Coalitions	V(S)	Coalitions	V(S)
A	0.1737	BD	0.6778	ADE	0.7087
B	0.5852	BE	0.7065	BCD	0.8092
C	0.1607	CD	0.3868	BCE	0.8379
D	0.2563	CE	0.4298	BDE	0.7991
E	0.3287	DE	0.5738	CDE	0.6749
AB	0.6794	ABC	0.8108	ABCD	0.9034
AC	0.2982	ABD	0.7720	ABCE	0.9321
AD	0.3806	ABE	0.8007	ABDE	0.8933
AE	0.4636	ACD	0.5205	ACDE	0.8098
BC	0.7166	ACE	0.5647	BCDE	0.9305
				ABCDE	1.0247

Table 3: Calculating the Shapley Value of the First Criterion

i= C/I (A)	ROAE (B)	NIA/SE (C)	CAR (D)	DPS (E)	(S-1)! × (n-S)!	V(S)	V(S\i)	` [V(S)-V(S\i)]× [(S-1)! × (n-S)!]
1	1	1	1	1	24	1.0247	0.9305	2.2608
1	0	1	1	1	6	0.8098	0.6749	0.8094
1	1	0	1	1	6	0.8933	0.7991	0.5652
1	1	1	0	1	6	0.9321	0.8379	0.5652
1	1	1	1	0	6	0.9034	0.8092	0.5652
1	0	0	1	1	4	0.7087	0.5738	0.5396
1	0	1	0	1	4	0.5647	0.4298	0.5396
1	0	1	1	0	4	0.5205	0.3868	0.5348
1	1	0	0	1	4	0.8007	0.7065	0.3768
1	1	0	1	0	4	0.7720	0.6778	0.3768
1	1	1	0	0	4	0.8108	0.7166	0.3768
1	0	0	0	1	6	0.4636	0.3287	0.8094
1	0	0	1	0	6	0.3806	0.2563	0.7458
1	0	1	0	0	6	0.2982	0.1607	0.8250
1	1	0	0	0	6	0.6794	0.5852	0.5652
1	0	0	0	0	24	0.1737	0.0000	4.1688
							Sum	14.26
							Shapley	0.1188

Or that bank 56 is ranked 49th with Shannon weighting and 2nd with Shapley weighting. In both cases, the difference in rankings is very significant. Here, the considerable effect of weights on the shift of ranks is evident. Therefore, to evaluate the fairness of the obtained weights and ranks, in addition to intuitive comparison, objective comparison of the rankings using data envelopment analysis has been used as a benchmark judge. However, in most cases, this technique cannot fully rank the DMUs, as in the recent case, the efficiency of six banks is equal to one (100%). So, naturally, it will not be possible to identify the best bank and also complete ranking. Since there is no need for subjective or objective weighting, the inputs and outputs choose their favorite weights. So, this method can be used as a suitable benchmark in determining the fairness of rankings obtained by various weighting methods. To make this comparison, using Spearman rank correlation coefficient (ρ), equation (17), the rankings obtained from The TOPSIS technique based on two weighting methods have been compared to the rankings obtained from the CCR method.

The results show that the ranks obtained by the CCR method with the ranks of TOPSIS technique based on Shannon entropy weighting have an inverse correlation of -0.045 and with Shapley weighting, a direct correlation of 0.238 indicates that the CCR method is more in agreement with Shapley weights. Fig. 2 shows a graphical representation of the ranking of the top six efficient banks in terms of the CCR method and the rankings assigned by the TOPSIS technique based on the two different weighting methods.

Table 4: CL_i^*, θ_{CCR}, W and ρ

Bank code	$CL_{i\ Shan}^*$	θ_{CCR}	$CL_{i\ Shap}^*$	Bank code	$CL_{i\ Shan}^*$	θ_{CCR}	$CL_{i\ Shap}^*$
1	0.92324 (51)	0.72867 (32)	0.77056 (53)	34	0.93357 (27)	0.90986 (09)	0.79576 (29)
2	0.33417 (64)	0.57954 (58)	0.32296 (64)	35	0.93527 (23)	0.67251 (42)	0.80211 (21)
3	0.93362 (26)	0.75858 (27)	0.78512 (38)	36	0.94379 (07)	0.64827 (45)	0.81350 (14)
4	0.94761 (06)	1.00000 (01)	0.82543 (10)	37	0.92704 (44)	0.83430 (20)	0.77860 (45)
5	0.93286 (32)	0.70898 (36)	0.79701 (27)	38	0.94039 (13)	0.84900 (19)	0.83884 (06)
6	0.02289 (65)	0.89501 (10)	0.09421 (65)	39	0.44837 (63)	0.61613 (53)	0.43596 (63)
7	0.57755 (60)	0.63732 (48)	0.53544 (61)	40	0.93229 (33)	0.77349 (24)	0.81001 (16)
8	0.91632 (52)	0.70475 (38)	0.77766 (48)	41	0.93330 (30)	0.73100 (31)	0.80114 (22)
9	0.92644 (46)	0.55341 (62)	0.77557 (49)	42	0.93928 (16)	0.55803 (61)	0.79245 (32)
10	0.64184 (58)	0.69132 (40)	0.61095 (59)	43	0.94256 (08)	0.69945 (39)	0.84002 (05)
11	0.82614 (57)	0.66810 (44)	0.76951 (54)	44	0.93222 (34)	0.60265 (56)	0.79272 (30)
12	0.91024 (54)	0.99707 (07)	0.81958 (12)	45	0.93768 (18)	0.54237 (63)	0.79045 (34)
13	0.92387 (50)	1.00000 (01)	0.83541 (07)	46	0.56211 (61)	0.76738 (25)	0.54671 (60)
14	0.95107 (03)	0.86567 (14)	0.84284 (04)	47	0.93631 (20)	0.60991 (54)	0.78250 (41)
15	0.93207 (36)	0.62502 (51)	0.77167 (51)	48	0.93188 (37)	0.66970 (43)	0.77798 (47)
16	0.88389 (56)	0.85447 (17)	0.76842 (56)	49	0.93335 (29)	0.89288 (11)	0.79737 (26)
17	0.94834 (05)	0.63035 (49)	0.82622 (09)	50	0.93611 (21)	0.62876 (50)	0.79625 (28)
18	0.93907 (17)	0.76685 (26)	0.79738 (25)	51	0.93208 (35)	0.86486 (15)	0.79182 (33)
19	0.93289 (31)	0.85039 (18)	0.78719 (36)	52	0.92698 (45)	0.72180 (34)	0.77089 (52)
20	0.99686 (01)	1.00000 (01)	0.95932 (01)	53	0.94000 (15)	0.60771 (55)	0.80322 (19)
21	0.94043 (11)	0.72459 (33)	0.79825 (23)	54	0.48670 (62)	0.52096 (65)	0.46832 (62)
22	0.93514 (24)	0.56742 (60)	0.78086 (43)	55	0.88484 (55)	0.99011 (08)	0.76936 (55)
23	0.95928 (02)	1.00000 (01)	0.84948 (03)	56	0.92492 (49)	1.00000 (01)	0.87298 (02)
24	0.92791 (42)	0.85722 (16)	0.79739 (24)	57	0.93759 (19)	0.64735 (46)	0.79007 (35)
25	0.91562 (53)	1.00000 (01)	0.78661 (37)	58	0.93593 (22)	0.80257 (23)	0.80471 (18)
26	0.92964 (39)	0.52903 (64)	0.82469 (11)	59	0.94041 (12)	0.57755 (59)	0.80802 (17)
27	0.92559 (48)	0.71879 (35)	0.77364 (50)	60	0.94233 (09)	0.60133 (57)	0.80320 (20)
28	0.94165 (10)	0.82761 (21)	0.81780 (13)	61	0.92841 (41)	0.68029 (41)	0.78358 (39)
29	0.92636 (47)	0.87474 (13)	0.76151 (57)	62	0.63938 (59)	0.73806 (30)	0.63419 (58)
30	0.94865 (04)	0.70716 (37)	0.83456 (08)	63	0.93418 (25)	0.73995 (29)	0.77980 (44)
31	0.92765 (43)	0.62423 (52)	0.77836 (46)	64	0.93070 (38)	0.80689 (22)	0.78154 (42)
32	0.94021 (14)	0.64584 (47)	0.81004 (15)	65	0.93344 (28)	0.74730 (28)	0.79260 (31)
33	0.92847 (40)	0.88777 (12)	0.78345 (40)	ρ	- 0.045	1.000	0.238

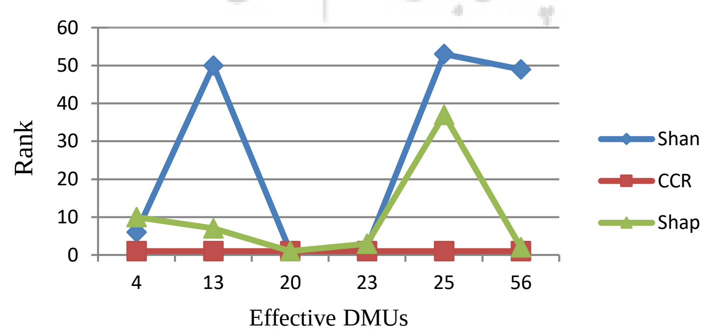

Fig 2: Comparative Ranks of Effective DMUs

5 Conclusions

In multiple criteria decision-making, the weight of criteria is a dominant factor in the ranking of alternatives. The use of subjective methods based on the opinion of experts and decision-makers, especially in conditions of a high diversity of criteria, makes it possible to determine the unrealistic values of criteria and thus deviate from the actual ranking results. Also, using objective methods based on mathematical models may lead to the production of disproportionate and irrational weights. Therefore, developing a new model that does not depends not only on decision makers' opinions but also overcomes the weaknesses of existing objective methods is fully justified. In this research, an objective method based on Shapley value in collaborative games has been used. This model, in addition, to have a simple and understandable mathematical procedure, by determining the value of criteria in a coalition game and with the participation and agreement of all criteria (players) not based on the opinion of each of them, offers more logical and fair weights. In this paper, a large-scale numerical case study of 65 banks has been used to explain the efficiency of the proposed method. First, in the decision matrix, the weight of criteria is calculated once based on the Shapley value and again based on the Shannon entropy method, which is the most widely used objective method for determining the weight of criteria.

Then the alternatives are ranked using the TOPSIS technique. To compare the rankings, the input-oriented CCR multiplier model has been used. Using this method as an impartial judge is that in addition to not needing the opinion of DMs in determining weights, inputs, and outputs (criteria), they can choose the best weights by avoiding undesirable weights. Finally, the rankings obtained from the TOPSIS technique are compared to the rankings obtained from CCR model using the Spearman rank correlation coefficient. The results show a significant correlation between Shapley value-based rankings by the CCR method than rankings based on Shannon entropy. The results of the present study, compared to several studies that have used the Shannon entropy method for weighting, have the potential to create more equitable weights. Therefore, using the proposed method will have a critical capability to be used in real issues of multi-criteria decision-making, especially in the banking sector. A second research subject may concern designing weighting methods according to solutions of other coalitional games, such as nucleolus, core, etc.

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