Adv. Math. Fin. App., 2023, **8**(1), P. 305-317

Advances in Mathematical Finance & Applications

www.amfa.iau-arak.ac.ir

Print ISSN: 2538-5569

Online ISSN: 2645-4610

Doi: 10.22034/AMFA.2022.1930153.1592

Applied-Research Paper

Multiple Portfolio Optimization in Tehran Stock Exchange

Shadi Khalili Moghadam, Farimah Mokhatab Rafiei*, Mohamad Ali Rastegar, Hamed Aghayi Bejestan

Department of Financial Engineering, Faculty of Industrial Engineering & Systems, Tarbiat Modares University, Tehran, Iran.

ARTICLE INFO

Article history:
Received 2021-05-10
Accepted 2022-02-23

Keywords:
Mult portfolio optimization
Market impact
Tehran Stock Exchange

ABSTRACT

Managing a single portfolio is a basic assumption in most research. However, in reality, an advisor manages many accounts at the same time; therefore, there is a significant dependency among portfolios and a correlation between decisions on one portfolio with the performance of others, so the results of the multi-portfolio are different from classic models (single portfolio management, that portfolios are optimized independently) due to market impact and the trade dependency of one account to the other accounts. We propose a structural model to optimize accounts simultaneously, considering interdependences, decision correlation and mutual behavioral effects of managed portfolios. Moreover, to compare and analyze both single-portfolio and multi-portfolio approaches, real data from Tehran Stock Exchange in 1398 are used and the model is solved with GAMS. Results indicate that multi-portfolio optimization excels other approaches and consequence notable improvement in the perspective of the customer and advisor. Also, for the validation of the proposed model, the selected stocks are considered in pairs to solve the model and the results show the proper performance of the model with different stocks, thus indicating the validity of the model.

1 Introduction

Since the publication of Harry Markowitz's theory on selecting investment portfolios in 1952, portfolio optimization has played a significant role in making decisions in the financial industry [9]. In multi-portfolio management, the utility of the investment portfolio is increased through diversification. Every trader in financial markets makes efforts to maximize the possible investment returns considering risk levels in the short term or long term. Most research on multi-portfolio optimizations has assumed that portfolios have been managed personally by the owner or by a consultant regardless of the relationship between portfolios; however, these consultants were supposed to manage multiple portfolios [17]. The objective of multi-portfolio optimizations is different from single-portfolio optimization. In multi-

* Corresponding author.
E-mail address: *f.mokhatab@modares.ac.ir*

portfolio optimization, investment decisions for one client will affect the other clients. Therefore, instead of independent optimization of each portfolio, there must be a process for optimization in which making decisions will result in all clients' benefits [11]. Trading costs are an essential part of the objective function in modern rebalancing techniques of portfolios, leading to optimization. In multi-portfolio optimization, the trading costs of one portfolio are highly dependent on other portfolios' trading. It can be said that the trading costs of one portfolio are not only dependent on its portfolio trades but others. It is generally believed that when portfolio trades are made simultaneously, the trading costs will increase [13]. The fact that portfolio managers are responsible for their clients is emphasized in this research study. Due to the market impact and dependency of one portfolio on another, the performance of multiple portfolios is different from a single portfolio.

The aim of the paper is to propose a structural model based on real-world data; the model decreases deviations in prevalent portfolio optimization models which occur due to an independent optimization approach. Right now, in financial institutions, account managers that manage many accounts as a daily operation commonly use a single approach. This decreases the utility and proceeds of all accounts and incurs a significant loss to all stockholders. To remove these inefficiencies, we need a multi-step model in the context of simultaneous optimization. Obtaining transaction costs and allocating the cost to different accounts is the crucial step in this approach. The proposed model enables managers to deal with challenges and achieve more accurate and fair results. The main contribution of the current research is implementing a real model for multiple portfolio optimization in the Iranian stock market. This is different from previous studies which mostly have adopted an independent optimization approach and have ignored interdependencies that cause fundamental inefficiencies stemming from transaction costs and accounts' interdependencies. Having used simultaneous optimization with real data imposes considerable challenges such as calculating market impact costs and distributing the cost among different accounts. Moreover, researchers that have followed a multi-optimization approach are also in an experimental environment with just numerical examples while ignoring real-world complexities and data challenges.

2 Literature Review

In order to select the optimal portfolio, Markowitz has deployed a single portfolio optimization model, which suggests the investor select a portfolio with the lowest risk based on the variance. This method was a milestone in considering diversification in financial decisions, resulting in a tradeoff between stock return and risk [9]. Multi-portfolio optimization was first introduced by O'Cinneide in [11]. They noticed suspicious transactions between portfolios concerning justice and potential rebalancing profit. They considered multi-portfolio optimization as a simple combination of each client's portfolio in which the trading cost equals the sum of their trading requirements. The objective function of their proposed model was to maximize social welfare. It reflects the entire trades and scenarios on one optimization model. They stated that their multi-portfolio optimization models solved multiple transactions and would bring justice since it caused a competitive balance for portfolios' liquidity. O'Cinneide et al. [11] believed that multi-portfolio optimization would lead to the same decisions that the clients will make with liquidity. This idea is not necessarily correct since a personal investor and portfolio manager (or investment consultant) cannot access the competitor's decisions. Cash allocation in multi-portfolio optimization follows the Pareto principle so that by increasing for a client, another client will suffer, so both clients' portfolios are optimal. Stabs et al. [9] reviewed the multi-portfolio optimization techniques. They investigated the advantages and disadvantages of Cournot-Nash and Collusive Solutions and presented an integrated framework that includes both approaches. The objective functions of all portfolios will be maximized to maximize the welfare in a collusive solution; in Cournot-Nash, besides maximizing welfare, each portfolio will be optimized by assuming that the other portfolios' transactions are constant. As mentioned earlier, fair trade cannot benefit one client and losses for another, and this research concentrates on fairness among portfolios. The definition of this fairness is dependent on the portfolio managers and the offers they make for their clients. Salvesberge et al. [13] emphasized that multi-portfolio optimization requires precision because there might be unfairness in clients' portfolios. Some portfolio managers may be more interested in some portfolios, which may result in the loss of others. They deployed the Cournot-Nash approach to optimize the objective function of each portfolio while the others are constant. Their model showed a Cournot-Nash balance among portfolios. The cost of market impact was calculated using a nonlinear convex function of transaction volume. This function is in the form of a power function (x^{γ}) with a power of 2.

Yancu et al. [17] proved that Cournot-Nash is not only appropriate for trade fairness, but it also would not result in an optimal answer since portfolios are considered a fake transactions and do not comply with the Securities and Exchange rules; therefore, the results cannot be trusted. First, if the mutual effects among multiple portfolio transactions are ignored, the rebalancing advantages will be decreased. Second, there would be a considerable profit in rebalancing and optimizing multiple portfolios. Third, it is important how to share information to bring fairness in profit distribution among portfolios. Market impact in their presented approach considers the above-mentioned. The cost of market impact is not external and cannot be divided; however, it is considered a random variable in the model. It can be said that the cost of market impact in each portfolio is considered internal rather than external. They also discussed the generalizability of cross-market impact cost and multi-step implementation.

Yang et al. [16] combined multi-portfolio optimization with a non-cooperative game theory approach. They deployed the Cournot-Nash balance model to maximize social welfare. The constraints in their model include the time limit of holding a stock, a short sale is not allowed, budget constraints, turnover limit, and transaction volume on different portfolios. It was concluded in their model that when all portfolios are limited, a unique Cournot-Nash balance shows up. That is the reason for using Nash balance in their model. Yancu and Trichakis [17] presented a comprehensive review of the existing methods in the finance industry. They reviewed previous related studies and presented three things that financial service providers may encounter. Jing Fu [4] proposed an information pooling game for multi-portfolio optimization which differs from the classical ones in several aspects, with a key distinction of allowing the clients to decide whether and to what extent their private trading information is shared with others, which directly affects the market impact cost split ratio. The empirical results suggest that within this framework, information pooling has a non-negative impact on all participants' perceived fairness, although it may hurt some account's realized benefit compared to a null information pool. Ji et al. [5] proposed a class of stochastic risk budgeting multi-portfolio optimization models that impose portfolio as well as marginal risk constraints. The models permit the simultaneous and integrated optimization of multiple sub-portfolios in which the marginal risk contribution of each individual security is accounted for. A risk budget defined with a downside risk measure is allocated to each security. Zhang et al. [19] considered market impact cost in the multi-portfolio optimization model. The main contribution of their study was using Conditional Value-at-Risk (CVaR) for risk measurement and modeling market impact cost in a joint optimization framework. The study proposed a model while market impact costs accounted as the unique feature of the model. Results show joint optimization model incurs less market impact cost than the independent decision. Yu et al. [18] developed a target-oriented framework that optimizes the rebalancing trades and the market impact costs incurred by trading jointly with consideration of target and distributional uncertainty. To evaluate multiple portfolios' uncertain payoffs in achieving their targets, they first proposed a type of performance measure, called fairness-aware multi-participant satisficing (FMS). In MPO, they focused on the FMS criterion with the underlying risk measure being conditional value-at-risk. Lampariello et al. [6] analyzed a Nash equilibrium problem arising when trades from different accounts are pooled for execution. They introduced a multi-portfolio model and state conditions for the monotonicity of the underlying Nash equilibrium problem. Monotonicity makes it possible to treat the problem numerically and, in the case of nonunique equilibria, to solve hierarchical problems of equilibrium selection. They also gave sufficient conditions for the Nash equilibrium problem formulation to be a potential game.

3 Methodology

3.1 Problem Statement

In the presented optimization framework in this research, a financial consultant manages two different portfolios simultaneously. As mentioned earlier, when a financial consultant manages multiple portfolios, all transactions are performed by that consultant in the optimization process. The costs of market impact are considered as a sum of portfolio transactions and are not divided separately. The allocation of total cost among different portfolios is another difference between multiple and single portfolio optimization. In order to investigate multiple portfolio optimization comprehensively, a model with variance risk has been designed in five steps which helps the portfolio manager select the optimal portfolio combination.

3.2 Variables

Indices, parameters, and variables which have been used in this research are introduced in this section.

- k: portfolio index (client’s account)
- n: number of portfolios
- j: stock index
- m: number of stocks
- C_i: total capital
- $\bar{r}_{p_k}$: expected return
- Σ: return covariance matrix among stocks
- p: stock price

- k_i: risk-taking percentage for each person
- σ_i: the lowest risk level for each portfolio (the result of the first step in the model)
- x_i: vector of the purchase amount
- t(x): market impact cost (the result of the transaction)
- U_i: utility (net return of portfolio)
- $F_{(U1, U2, \dots, Un)}$: welfare function

3.3 Assumptions

The following assumptions are considered in the proposed model.

- Balancing of portfolios is considered in a determined period.
- In portfolio optimization, rebalancing is implemented for two portfolios by the consultant.
- There are three risky stocks that can be traded in portfolios.
- All transactions are assumed in a single period for all portfolios. For example, transaction costs in all portfolios are other than zero.
- All sale and purchase orders in 'n' portfolios will become one sale and purchase order by the consultant in a rebalancing period.
- Cross-trading is possible where the transactions in all portfolios must be done via the market, and no internal transaction is allowed in this model.
- After optimization, the market impact cost is divided into the portfolios with the related ratios.
- Single portfolio optimization is performed to maximize the net profit of U_i, which is calculated as the return of the portfolio minus market impact cost. In multi-portfolio optimization, the net return of U_i is calculated by solving a multi-objective problem.
- Transaction costs related to the purchased stock depend on other portfolios, even if the consultant separately makes the sale and purchase orders.
- A short sale is not possible in this research.

3.4 Modelling

In this section, the five-step model is provided to optimize multiple portfolios. As mentioned, variance is used as a risk factor in the optimization model and market impact is considered a function of the trading volume.

- Step 1

Each portfolio will be optimized separately by minimizing variance as its objective function.

$$minZ = \sum_{i=1}^{m}\sum_{j=1}^{m}\frac{x_{ki}p_i}{C_k}\frac{x_{kj}p_j}{C_k}cov(\bar{r}_i,\bar{r}_j) \quad (1)$$

$s.t$:

$$\sum_{j=1}^{m} x_{kj} p_j \bar{r}_j \geq \bar{r}_{p_k} C_k \qquad \forall k \qquad (2)$$

$$\sum p_j x_{kj} \leq C_k \qquad \forall k \qquad (3)$$

$$x_{kj} \geq 0 \qquad \forall k, j \qquad (4)$$

This step is literally the Markowitz model. The objective function (1) minimizes the variance. The first constraint (2) guarantees to meet the minimum expected return on investment for the client. The second constraint (3) indicates that the purchase amount of the client cannot be more than his capital and the last constraint (4) Indicates positive buying and no short sale. In this step, the model seeks to calculate the variance for each portfolio separately. Therefore, market impact is not considered as each portfolio was analyzed independently. In fact, this step is the same as optimizing the portfolio in an independent mode.

- Step 2

Single portfolio optimization considers the net utility as an objective function. The constraints are deployed to calculate the upper limit of the variance. Market impact is now considered, and the net expected utility is maximized for independent portfolios. In this step, the model considers portfolios' transactions but assumes that are not effective on each other.

$$\max\{\sum_{j=1}^{m} \bar{r}_j \frac{x_{kj} p_j}{C_k} - \sum_{j=1}^{m} (x_{kj})^{y_j}\} \qquad \forall k \qquad (5)$$

s.t:

$$\sum_{i=1}^{m} \sum_{j=1}^{m} \frac{x_{ki} p_i}{C_k} \frac{x_{kj} p_j}{C_k} cov(\bar{r}_i, \bar{r}_j) \leq k_k \sigma_k \qquad \forall k \qquad (6)$$

$$x_{kj} \geq 0 \qquad \forall k, j \qquad (7)$$

The objective function (5) shows that the portfolios are being optimized separately. It calculates the utility via the difference between the return on investment and market impact. Constraint (6) indicates each client's risk regarding the lower limit concluded from the previous step.

- Step 3

In this step, the optimized orders for each stock resulting from step 2 are aggregated for all n portfolios. In step 2, the net utility was maximized for each portfolio; however, it must be noted that not considering the effect of other portfolios will considerably decrease market impact costs.

$$U_k^{IND} = \sum_{j=1}^{m} \bar{r}_j \frac{X_{kj}^{IND} p_j}{C_k} - \sum_{j=1}^{m} \frac{X_{kj}^{IND}}{\sum_{a=1}^{n} X_{aj}^{IND}} (\sum_{a=1}^{n} X_{aj}^{IND})^{y_j} \qquad \forall k \qquad (8)$$

In this equation (8), purchase orders of each stock (j) were aggregated to calculate the total cost of market impact.

- Step 4

In this step, the effect of different portfolios' transactions on each other has been considered. Thus, the total market impact is allocated to all portfolios, and by subtracting it from the transaction return (calculated in step 2), the utility can be concluded. The results of this step are closer to reality since the effect of transactions on each other is considered.

The real net utility for each portfolio (i) is calculated as follows:

$$U_k^{IND} = \sum_{j=1}^{m} \bar{r}_j \frac{X_{kj}^{IND} p_j}{C_k} - \sum_{j=1}^{m} \frac{X_{kj}^{IND}}{\sum_{a=1}^{n} X_{aj}^{IND}} (\sum_{a=1}^{n} X_{aj}^{IND})^{y_j} \quad \forall k \qquad (9)$$

- Step 5

In this step, the model seeks to optimize the portfolios simultaneously (multi portfolios optimization). The objective function indicates the max-min between social welfare (total utilities) and fairness (fair allocation of utilities). Market impact is allocated to different portfolios. $f(U_1, U_2, \dots, U_n)$ is a welfare function that is formed as below:

$$f(U_1, U_2, \dots, U_n) = \min \left\{ \frac{U_i - U_i^{IND}}{U_k^{IND}} \right\} \qquad (10)$$

U_{IND} is the real net utility for each portfolio (i) separately, while the net utility for all portfolios can be calculated by:

$$U_i = U_i^x - t_i \qquad (11)$$

The result of step 5 includes the optimal allocation of the capital among the stocks for each portfolio (i), the divided market impact cost to each portfolio (i), and the real utility for each portfolio (i).

$$\max \{\min \left\{ \frac{\sum_{j=1}^{m} \bar{r}_j \frac{x_{kj} p_j}{C_k} - \sum_{j=1}^{m} t_{kj} - U_k^{IND}}{U_k^{IND}} \right\} \} \qquad (12)$$

s.t:

$$\sum_{i=1}^{m} \sum_{j=1}^{m} \frac{x_{ki} p_i}{C_k} \frac{x_{kj} p_j}{C_k} cov(\bar{r}_i, \bar{r}_j) \le k_k \sigma_k \quad \forall k \qquad (13)$$

$$\sum_{i=1}^{m} \bar{r}_i \frac{x_{ki} p_i}{C_k} - \sum_{i=1}^{m} \frac{x_{ki}}{\sum_{a=1}^{n} x_{ai}} (\sum_{a=1}^{n} x_{ai})^{y_i} \ge U_k^{IND} \quad \forall k \qquad (14)$$

$$x_{kj} \ge 0 \qquad (15)$$

The objective function (11) maximizes the minimum increase in net utility compared to the resulting utility in step 4. The first constraint (12) limits the client risk concerning the lower limit, which was concluded from previous steps. The last constraint (10) guarantees an increase in the utility compared to the utility in its independent mode.

4 Data Analysis and Empirical Result

Based on the proposed model, real data was analyzed and the results were evaluated using sensitivity analysis.

4.1 Data Preparation

Due to the financial market functions, financial data are abundant and easily available. Data was provided from Tehran Stock Exchange (TSE) and transactions within. According to the literature, the most appropriate data for this research is stocks with higher liquidity and transaction volume. Therefore, three stocks with high liquidity and transaction volume in 2019 have been selected, including VEBMELLAT, FOOLAD, and VEKHARAZM. Market impact is also considered a quadratic nonlinear function of the trading volume.

4.2 Results

The financial consultant deals with two accounts (portfolios), and three stocks. It is assumed that these stocks have not been bought before for these portfolios. The risk aversion coefficient (k_i) for the clients is considered between 1 to 4. The model is executed by the BARON solver, and the results are provided. As mentioned in the previous chapter, the variance was calculated while the portfolios are considered independent, and market impact is not considered (step 1). The values in tables 1, …, 4 are the result of such an assumption.

Table 1: Risk Comparison with 1st,2nd, and 3rd Stocks

Portfolio	Risk		
	Step 1	Step 2	Step 5
1	0.00008889	0.0001789	0.0001789
2	0.00008876	0.0001678	0.0001678

Table 2: Risk Comparison with 1st and 2nd Stocks

Portfolio	Risk		
	Step 1	Step 2	Step 5
1	0.000409	0.000704	0.000704
2	0.000409	0.000704	0.000704

Table 1: Risk Comparison with 1st and 3rd Stocks

Portfolio	Risk		
	Step 1	Step 2	Step 5
1	0.000336	0.000615	0.000516
2	0.000098	0.000142	0.000142

Table 2: Risk Comparison with 2nd and 3rd Stocks

Portfolio	Risk		
	Step 1	Step 2	Step 5
1	0.000319	0.000637	0.000637
2	0.000419	0.000517	0.000517

Fig. 1: Risk Comparison with 1st and 3rd Stocks

Fig. 2: Risk Comparison with 2nd and 3rd Stocks

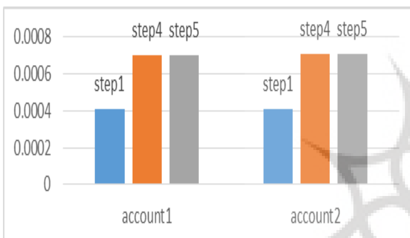

Fig. 3: Risk Comparison with 1st and 2nd Stocks

Fig. 4: Risk Comparison with all Stocks

The market impact has been considered in the model's second step, and the net utility of the portfolio was maximized separately. In this step, it was assumed that the transactions of each portfolio were not effective on the others, and the portfolios have been optimized independently, so it can be said that this model is far from reality. In step 2, risk has been increased compared to step 1 because the market impact is considered. The market impact cost is low in this step because the inter-effect of transactions in multiple portfolios are not considered, and as a result, the utility is increased.

In step 4, although the portfolios are optimized separately, the inter effect of transactions is considered. The market impact was allocated to the portfolios in related ratios and then subtracted from the return of the transactions (calculated in step 2), which finally equals utility. As shown in tables 5, 6, and 7, when the market impact cost increases and the return on investment is constant, the utility will decrease compared to step 2. It can be said that the result of this step is closer to reality. In step 5, multiple portfolios are optimized by calculating the max-min function of social welfare and fairness (fair allocation of utilities). Market impact is also allocated to the portfolios with related ratios. The results indicated that the final utility of all portfolios is increased in comparison with step 4. This confirms the better performance of the multi-portfolio optimization models due to the decrease in market impact in the transactions.

Table 3: The Comparison with 1st, 2nd, 3rd Stocks

Portfolio	Market impact cost			Utility			Improvement
	Step 2	Step 4	Step 5	Step 2	Step 4	Step 5	
1	0.0000455	0.000107	0.000105	0.03295	0.03289	0.03290	0.00095
2	0.0000821	0.000143	0.000133	0.03297	0.03285	0.03287	0.003

Table 4: The Comparison with 1st and 2rd Stocks

Portfolio	Market impact cost			Utility			Improvement
	Step 2	Step 4	Step 5	Step 2	Step 4	Step 5	
1	0.000728	0.011	0.01	0.3227	0.3199	0.39	0.219
3	0.000888	0.012	0.01	0.3211	0.3177	0.39	0.227

Table 5: The Comparison with 2nd and 3rd Stocks

Portfolio	Market impact cost			Utility			Improvement
	Step 2	Step 4	Step 5	Step 2	Step 4	Step 5	
2	0.000119	0.000267	0.000267	0.3988	0.2973	0.3773	0.2690
3	0.000486	0.000638	0.000410	0.3151	0.3136	0.3759	0.1985

The comparison of market impact along the steps are shown in figure 5, 6, 7, and 8. As it was stated, the presented model in step 5 has decreased market impact compares to step 4. The low market impact on step 2 is because of unreal conditions.

Fig. 5: Market Impact with 2nd& 3rd Stocks

Fig. 6: Market Impact with 1st& 3rd Stocks

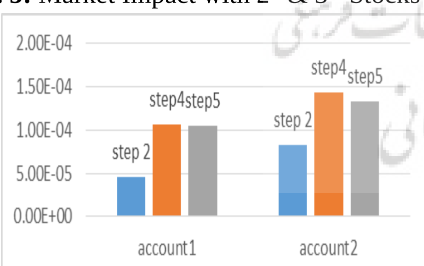

Fig. 7: Market Impact with 1st& 2nd Stocks

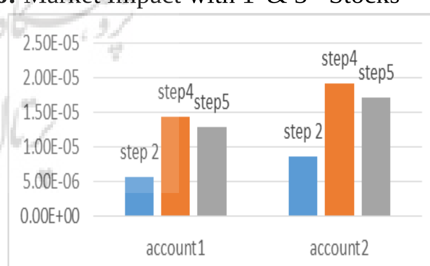

Fig. 8: Market Impact with All Three Stocks

The net utility of the model in different steps is indicated in figures 9, 10, 11, and 12.

Fig. 9: Net Utility with 1st and 3rd Stocks

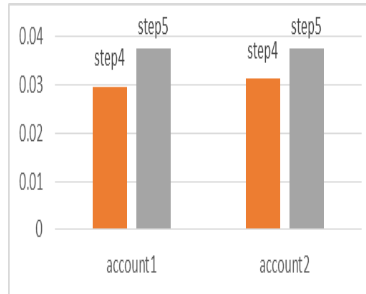

Fig. 10: Net Utility with 2nd and 3rd Stocks

Fig.11: Net Utility with 1st and 2nd Stocks

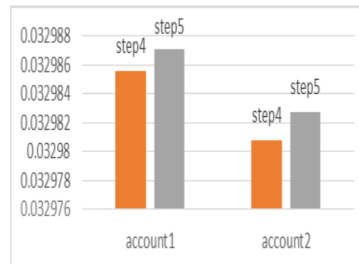

Fig. 12: Net Utility with All Stocks

In the first step, the utility of each account has obtained. However, market impact costs are ignored and this causes bias in utility calculations. In the second step, market impact costs are considered, and the optimizations are performed independently for each account. Since mutual effects have been removed, and market impact costs have been underestimated, once more we face unreliable outputs. As outputs indicate, utilities are higher than the next calculations. Accounts' interdependencies stem from selling and liquidating process which has an impact on the yield of other accounts. To adjust this, the estimated market impact cost should be allocated fairly among different accounts. Therefore, in steps three and four, the overall market impact cost which is already obtained by optimizing accounts independently is allocated to accounts with the proposed (pro rata) method. Then the utility of each account has been calculated. Having interdependent impacts on modeling, results in a more accurate estimation for utilities of accounts, in fact, it indicates the (real) utility that is achieved in the case of independent optimization.

The proposed model of the study is introduced in the final step. At this pace, a simultaneous model aligned with a novel allocation framework has been performed. The main advantage of the proposed structure is that, instead of having independent optimization for each account, we have a simultaneous model which can take into account all transaction costs and interdependencies among individuals. In addition, the market impact and corresponding imposed costs are estimated and allocated fairly to the accounts. As described in the results, of the model, through sharing accounts information, the market impact costs are reduced, and the utility of each account is increased which indicates an improvement in the proposed model. The model is established on the market realities and practical operations rather than theoretical estimated and non-practical concepts; therefore, it could be a practicable tool in the hands of portfolio managers and financial institutions to estimate real and applicable results. In order to validate the proposed model, the stocks are considered in pairs and the problem is solved several times, the results show the proper performance of the model in each problem.

5 Conclusion

The objective of this research is to provide a model for simultaneously optimizing multiple portfolios especially risk modeling and allocation of transaction costs in multi-portfolio optimization. The optimization of multiple portfolios investigates the optimization from different aspects, making it more appropriate and different from Markowitz's classic method. Multi-portfolio optimization was emphasized in the related literature; however, there is a wide gap considering market impact, variance risk, and utility. Allocation of market impact costs and risk calculation were addressed through a five-step optimization method. Variance is used in this research to calculate risk. Data is collected from Tehran Stock Exchange in 1398 and used to evaluate the model. Real stock market data with real-life constraints were provided for the first time in multi-portfolio optimization. The data was analyzed using GAMS software. The proposed five-step method for multi-portfolio optimization with the real stock market data was considered effective where the market impact costs and fairness are addressed. The model results showed that multi-portfolio optimization decreases the costs compared to single-portfolio optimization while it increases the total utility considerably.

Despite studies in this area, many gaps are still observed. For example, future studies can focus on a higher number of stocks and therefore increase the validity of the model. Extension of the single-period multi-portfolio optimization to a multi-period one is a potential future study. Many risk factors can be replaced in this model (including C-VAR) in future studies. Also, the use of metaheuristic methods can measure the performance of the model in a huge amount of data, which indicates its suitability for managers to make decisions in brokerages.

Acknowledgment

The authors acknowledge with grateful appreciation the kind assistance and financial support provided by Tehran Stock Exchange Company

References

[1] Aghamohammadi, R., Tehrani, R., Raad, A., *Portfolio Optimization Based on Semi Variance and Another Perspective of Value at Risk Using NSGA II, MOACO, and MOABC Algorithms*, Advances in Mathematical Finance & Applications, 2022, **7**(1), P.115-131. Doi: 10.22034/AMFA.2020.1907264.1479

[2] Esmaeili, B., Souri, A., Mirlohi, S., M., *Higher moments portfolio optimization with unequal weights based on Generalized Capital Asset Pricing Model with Independent and Identically Asymmetric Power Distribution*, Advances in Mathematical Finance & Applications, 2021, **6**(2), P.263-283. Doi: 10.22034/AMFA.2020.1909590.1484

[3] Fabozzi, F. J., Focardi, S. M., and Kolm, P. N., Q*uantitative equity investing: Techniques and strategies*, John Wiley & Sons. 2010

[4] Fu, J., *Information pooling game in multi-portfolio optimization*, Contributions to Game Theory and Management, 2017, **10**, P. 27-41.

[5] Ji, R., Lejeune, M. A., *Risk-budgeting multi-portfolio optimization with portfolio and marginal risk constraints*, Ann Oper Res, 2018, **262**, P.547–578.

[6] Lampariello, L., Neumann, CH., Ricci, J.M., Sagratella, S., *Equilibrium selection for multi-portfolio optimization*, European Journal of Operational Research, 2021, **295**, P.363-373.

[7] Mousavi, S. H., Mehregan, N., Yousefi, M. R., *Presentation of a Mathematical Model for Optimal Portfolio in the Form of a Dynamic Stochastic General Equilibrium Model for Economy of Iran*, Advances in Mathematical Finance & Applications, 2022, **7**(1), P.81-96. Doi: 10.22034/AMFA.2020.1884853.1338

[8] LIU, Y. J., ZHANG, W. G., ZHAO, X. J., *Fuzzy multi-period portfolio selection model with discounted transaction cost*, Soft Computing, 2016, P.1-17.

[9] MARKOWITZ, H., *Portfolio selection*, The journal of finance, 1952, **7**, P.77-91.

[10] MEHLAWAT, M. K., *Credibilistic mean-entropy models for multi-period portfolio selection with multi-choice aspiration levels*, Information Sciences, 2016, P.345, 9-26.

[11] O'CINNEIDE, C., SCHERER, B., XU, X., *Pooling trades in a quantitative investment process*, The Journal of Portfolio Management, 2006, **32**, P.33-43.

[12] O'CONNELL, J. F., *Welfare economic theory*, Auburn House, 1982.

[13] SAVELSBERGH, M. W., STUBBS, R. A., VANDENBUSSCHE, D., *Multiportfolio optimization: A natural next step*, Handbook of portfolio construction, Springer, 2010.

[14] STUBBS, R. A., VANDENBUSSCHE, D., *Multi-portfolio optimization and fairness in allocation of trades*, White paper, Axioma Inc. Research Paper, 2009, 13.

[15] RASTEGAR, M. A., EGHBALREIHANI, N., *Order Splitting Strategy to Reduce Market Impact in Tehran Stock Exchange*. Financial Research Journal, 2019, **21**, P. 321-346.

[16] YANG, S. C., LIN, T. L., CHANG, T. J., CHANG, K. J., *A semi-variance portfolio selection model for military investment assets*, Expert Systems with Applications, 2011, **38**, P.2292-2301.

[17] YANCU, D. A., TRICHAKIS, N., *Fairness and efficiency in multiportfolio optimization*, Operations Research, 2014, **62**, P.1285-1301.

[18] Yu, G., Cai, X., Zhuoyu Long, D., Zhang, L., *Multi-portfolio Optimization: A Fairness-aware Target-oriented Model*, Risk Management eJournal, 2020.

[19] Zhang, G., Zhang, Q., *Multiportfolio optimization with CVaR risk measure*, J of Data, Inf. And Manag, 2019, **4**, P.91-106.