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Research Paper

Providing an Optimal Robust Portfolio Model with Mean- CVaR Approach

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ABSTRACT

The portfolio selection problem is one of the main investment management problems. In the portfolio selection problem, robustness is sought against uncertainty or variability in the value of the parameters of the problem. This paper has been conducted for Robust portfolio optimization based on the mean-cvar approach. And introduces the linear mean-cvar model as a criterion for calculating risk and provides an optimal Robust mean-cvar model. The Robust approach used in this research is the Bertsimas and Sim. In this approach, the Robust counterpart presented for a linear programming model remains linear, maintaining the advantages of the linear programming model in the optimal model. The model developed in this research is randomly selected by real data of 20 stocks of the S&P 500 index for three years, this development help the portfolio selection problem to consider uncertainty. Interval optimization is a modeling approach to consider parameter uncertainty in this paper. Considering uncertainty make the model more realistic. The results of the model show that this approach has computational efficiency and on the other hand proposed model produces a better solution from a risk and portfolio rate of return point of view.

1 Introduction

The practical relevance of portfolio selection models has constantly increased, since their introduction in the financial literature, due to the structural transfer of big private capitals toward investments generally not required by non-institutional operators. As a consequence, the interest of private and institutional investors in techniques and tools aimed at a more efficient forecast of the dynamics of securities prices and rational management of investment capital is hugely increased. The last aspect is the heart of this contribution which essentially consists of the application of robust optimization to the minimization of the conditional value at risk (CVaR) as a way to obtain efficient portfolios [7]. Classical portfolio selection models, still largely used for their conceptual simplicity and utility in applications, are based on a bi-criteria optimization scheme in which the goal is to form a portfolio in which expected

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return is maximized, while some index of risk is minimized [14]. Portfolio selection and portfolio management are the most important problems from the past that have attracted the attention of investors. To solve these problems, Markowitz. proposed his model which was named Markowitz or the mean-variance (MV) model. He believed that all investors want a maximum return and minimum risk in their investments. So, he presented his model that expresses investors want minimum risk for each level of expected return. Markowitz results in an area with an efficient frontier of return and risk. For which point along an efficient frontier, there is no point with higher return and less risk [13].
Most authors have tried to make optimal solutions to portfolio selection issues by balancing between return maximization and investment risk minimization. Given the assumption of the normal or abnormal return of assets, two different theories have been proposed. In modern theory, the distribution of return is assumed to be normal. Accordingly, the standard deviation is introduced as a risk measure. However, the research shows that the distribution of the asset's return is not normal. Given the assumption, risk measures vary from standard deviation to values at risk. stock portfolio optimization is one of the most important stages of the portfolio, stock selection is the most important stage of a proper investment [11].In the classical Markowitz, model risk is measured utilizing a dispersion measure, such as variance or standard deviation. More recently, starting from the observation that positive and negative deviations of the returns from their mean play a greatly asymmetric role in the investor's perception, financial practice and related theory showed increasing interest towards quantile-based measures, such as value at risk (VaR) [16]. Value at Risk, if studied in the framework of coherent risk measures, lacks subadditivity, and therefore convexity, in the case of general loss distributions (although it may be subadditive for special classes of them, e.g. for normal distributions). This drawback entails both inconsistencies with the well-accepted principle of diversification (diversification reduces risk) and greater problems from the point of view of numerical tractability [15].

To overcome these problems, recent literature on portfolio selection focused on coherent risk measures and in particular on conditional value at risk (CVaR) [16]. Another weak point of classical selection models has been recently illustrated: the optimization process leads to solutions that are likely to depend heavily on the parameter perturbations. As data are often, for several reasons, only known approximatively, this dependence makes the theoretical and numerical results highly unreliable for practical purposes [8]. This feature has been initially dealt with through the methods of stochastic programming and, in the last few years, with the help of a methodology that was recently introduced in the optimization literature [2,21]. In recent years, a body of literature is developing under the name of robust optimization to consider uncertainty in the value of parameters of the model. Soyster proposed a linear optimization model to construct a solution that is feasible for all data that belong in a convex set [18]. The solutions of the Soyster model are too conservative in the sense that it causes us to give up too much optimality for the nominal problem in order to ensure robustness. The second step forward in developing a theory for robust optimization was taken independently by Ben-Tal and Nemirovski [1]. They use an ellipsoidal uncertainty set. This model can adjust conservatism. However, this model is not linear which can be problematic in real-world problems. Another development on robust optimization has been done by Bertsimas and Sim [3]. This model is linear, applicable, and extendable to discrete optimization and can flexibly adjust the level of conservatism of the robust solutions in terms of probabilistic bounds of constraint violations. In this paper, we use Bertsimas and Sim methodology for the development of our model. There are some practical models of robust optimization in finance. El-Ghaoui et al proposed a robust portfolio model under the uncertainty of covariance matrix which is developed by semi-definite programming (SDP) and considers worst-case value-at-risk [5]. Tutuncu and Koenig developed a robust

portfolio optimization problem formulated in a quadratic program (QP) [20]. Kawas and Theile developed a log robust portfolio model to consider the heavy-tailed property of stock prices [9]. Chen and Tan developed robust portfolio selection based on asymmetric measures of variability of stock returns. In this paper, they showed a robust model for the mean-cvar model [4]. Moon and Yao developed a robust mean absolute deviation model for portfolio optimization [12]. Miryekemami et al. using a genetic algorithm in an issue aimed at maximizing returns and stock liquidity show that the selected model provides good performance for selecting the optimal portfolio for investors with specific goals and constraints [11]. Rezaei and Elmi showed that the reaction of the stock price in the stock market was modeled by the behavioral finance approach.
The population of this study included the companies listed on the Tehran Stock Exchange. In order to forecast the stock price, the final price data of the end of December, March, June, and September 2006-2015 and the stock prices of 2014 and 2015 were analyzed as the sample. In this study, Bayes' rule was used to estimate the probability of the model change. Through this rule, the probability of an event can be calculated by conditioning the occurrence or lack of occurrence of another event. The results of model estimation showed that there is a probability of being placed in high-fluctuated regimes (over-reaction) and low-fluctuated (under-reaction of stock price) despite the shocks entering the stock market. In models with the -month’s final prices, it was proved that the real stock price had no difference from the market price [6]. ShahNazari et al developed Climate Policy Uncertainty and Power Generation Investments: A Real Options-CVaR Portfolio Optimization Approach. findings show that there is potential for investors to fully hedge their existing fossil fuel-based generation assets through the addition of on-shore wind capacity [17]. Liwei et al developed A CVaR-robust-based multi-objective optimization model and three-stage solution algorithm for a virtual power plant considering uncertainties and carbon emission allowances [10]. Tao et al developed the Optimal position of supply chain delivery window with risk-averse suppliers: A CVaR optimization approach. Obtained the results show that a numerical case is executed to compare the optimal position with minimum CVaR and the one minimizing the expected penalty and to illustrate the influence of several parameters on the optimal position [19].
Investors must always decide to get the best results in the financial markets. They seek the highest return and the lowest risk. Undoubtedly, providing models that are most compatible with the real world and financial markets will be of particular importance; because they allow financial managers and portfolio decision-makers to make reliable decisions. In fact, the importance of this issue becomes more apparent with the expansion of increasing competition in the financial markets. In Iran, little research has been done in the field of portfolio optimization using unfavorable risk criteria, and the criteria for evaluating the mean-Cvar ratio remain unknown. The reason for selecting these variables in this study is that the Mean-Cvar ratio itself is a new and fundamental approach in postmodern risk and mitigation risk approaches. Therefore, in a situation where the correct decision of managers and investors is considered and consequently their accurate understanding of the optimal portfolio is considered. In this regard, understanding the need for research on the issue of robust optimal portfolio model with the Mean-Cvar approach which is the postmodern perspective on investment management and financial theories has been examined.

1.1 Definitive Model of Mean-Cvar

Rockafellar and Ursayev established a new risk measure called Conditional value at risk (CVaR). Value at risk measures the minimum loss corresponding to a certain worst number of cases but it does not

quantify how bad these worst losses are. An investor may need to know the magnitude of these worst losses to discern whether there are possibilities of losing huge sums of money CVaR quantities this magnitude and is a measure of the expected loss corresponding to several worst cases, depending on the chosen Confidence level. Using CVaR makes the portfolio selection problem linear and when we solve it a minimum VaR is found since $CVAR \geq VAR$ [16] CVaR is derived as follows:
Let $f(X, \xi)$be the loss function of the portfolio. Usually, losses are in monetary terms, but we list losses in terms of returns (percentage). Given a confidence level α, CVaR is the expected value of all$(1-\alpha)\%$ losses and can be found using the following function:

$$CVaR_{\alpha}(x, \eta) = \eta + (1-\alpha)^{-1} \int_{\xi \varepsilon R^n} [f(x, \xi) - \eta]^{+} P(\xi) d\varepsilon$$

η: VaR
ξ: random variable
$Z^{+} = \max\{z, 0\}$

The Mean-CVaR portfolio selection can be formulated as a linear programming problem when scenarios of future returns are available. Since r is the return matrix, rX is the portfolio returns. Therefore the losses will be $-rX$. The problem tries to find the expected Value of the worst $(1-\alpha)\%$ losses. The following linear program would solve the problem:

$\tilde{r}_{ij}$	Uncertain return on j share in scenario i
E_0	specic expected return for the portfolio
μ_j	the mean return of the securities for j scenario $\mu_j = \frac{\sum_{i=1}^{s} \tilde{r}_{ij}}{s}$
α	ondence level
X_j	Percentage of investment per share j
η	Var
y_i	Auxiliary variable that measures the value between the missing scenario i and Va

$$\min \eta + \frac{1}{(1-\alpha)^s} \sum_{i=1}^{s} (y_i) \tag{1}$$

s.t

$$y_i \geq \sum_{j=1}^{n} \left[\left(-r_{ij} X_j\right) - \eta \right] \qquad i = 1,2, \dots, s \tag{2}$$

$$y_i \geq 0 \qquad i = 1,2, \dots, s \tag{3}$$

$$\acute{x}\mu = E_0 \tag{4}$$

$$\sum_{j=1}^{n} x_j = 1 \tag{5}$$

$$x \geq 0 \tag{6}$$

1.2 Robust Counterpart of the Mean-Cvar Model

The proposed model in the second case is related to the uncertainty of the model. This model is based on the model of robust optimization proposed by Ben-Tal and Nemirovski [2]. Consider the following deterministic linear optimization model.

$$\begin{aligned} &\text{Min } cx + d \\ &\text{s.t.:} \\ &Ax \leq b \end{aligned} \tag{7}$$

Based on the research of Ben-Tal and Nemirovski, the uncertainty linear optimization model, which includes a set of linear optimization problems, is defined as follows.

$$\begin{aligned} &\text{Min } cx + d \\ &\text{s.t.:} \\ &Ax \leq b \\ &c, d, A, b \in U \end{aligned} \tag{8}$$

In this model, the parameters C and A in the uncertainty set U are assumed to be variable. The vector x is a robust answer to a problem (8) if it can satisfy all the constraints with an uncertainty parameter belonging to the set U. According to Ben-Tal and Nemirovski, the robust model of the problem is defined as follows.

$$\text{Min } \left\{ \hat{C}(x) = \sup_{(c, A \in u)} [cx + d] : Ax \leq b, \quad \forall c, A \in U \right\} \tag{9}$$

The optimal solution to problem (9) will be the optimal solution to problem (7). This optimal solution satisfies the constraints of the problem for all possible data in the set U and guarantees the optimality of the objective function in such a way that even in the worst case it is not more than $\hat{C}(x^*)$ Problem (9) is a semi-infinite linear optimization problem and is considered computationally impossible. However, for convex sets, it seems that Model (9) will become a convex polynomial problem that can be computed. For a better understanding, the following model for the studied problem can be defined as follows:

$$\begin{aligned} &\text{Min } fy + cx \\ &\text{s.t.:} \\ &Ax \leq dy \\ &y, x \in \{0,1\} \end{aligned} \tag{10}$$

The vectors d, r, C, and f correspond to the parameters of the problem, respectively. Similarly, all variables zero and one are defined in the form of y and x vectors. To describe the robust model corresponding to the above model, parameters d and c are assumed to be indefinite. It is assumed that each of these indefinite parameters can be changed within a defined framework. The general form of this framework can be defined as follows:

$$u_{box} = \left\{ \xi \in R^n : \left| \xi_t - \bar{\xi}_t \right| \leq \rho G_t \quad t = 1, \dots, n \right\} \tag{11}$$

Where$\bar{\xi}_t$ is the nominal value ξ_t of the t parameter of the vector c (it is a 1-dimensional vector), respectively. Also, the two positive values of G_t and ρ indicate the degree of uncertainty and the level of uncertainty, respectively. According to the above description, the robust model corresponding to the model will be as follows:

$$\begin{aligned}
&\mathrm{Min}\ \mathrm{Z}\\
&\mathrm{s.t.}:\\
&\mathrm{fy}+\mathrm{cx}\le \mathrm{z},\quad \forall \mathrm{c}\in \mathrm{u}_{\mathrm{Box}}^{\mathrm{c}}\\
&\mathrm{Ax}\le \mathrm{dy},\quad \forall \mathrm{A}\in \mathrm{u}_{\mathrm{Box}}^{\mathrm{A}}\\
&\mathrm{y},\mathrm{x}\in\{0,1\}
\end{aligned}\tag{12}$$

Ben-Tal et al. showed that in a limited framework the robust model could be transformed from a semi-infinite problem to a balanced problem in which the set U_{box} is replaced by a finite set U_{box}. In this problem, set U_{box} contains the maximum values in the set U_{box}. To show the formability of the problem(12), the constraints of the problem must be made formable. So for the first limitation, we have the:

$$\mathrm{cx}\le \mathrm{z}-\mathrm{fy},\ \ \forall \mathrm{c}\in \mathrm{u}_{\mathrm{Box}}^{\mathrm{c}}|\mathrm{u}_{\mathrm{Box}}^{\mathrm{c}}=\{\mathrm{C}\in \mathrm{R}^{\mathrm{n_c}}:|\mathrm{C_t}-\bar{\mathrm{C}}_{\mathrm{t}}|\le \rho_{\mathrm{c}}\mathrm{G}_{\mathrm{t}}^{\mathrm{c}}\quad \mathrm{t}=1,\dots,\mathrm{n_c}\}\tag{13}$$

The left side of the above inequality has an indeterminate parameter while all the parameters on the right are definite. Therefore, the controlled form of the semi-infinite inequality will be as follows:

$$\begin{aligned}
&\sum_{\mathrm{t}}(\bar{\mathrm{C}}_{\mathrm{t}}\mathrm{x_t}+\eta_{\mathrm{t}})\le \mathrm{z}-\mathrm{fy}\\
&\rho_{\mathrm{c}}\mathrm{G}_{\mathrm{t}}^{\mathrm{c}}\mathrm{x_t}\le \eta_{\mathrm{t}},\quad \forall \mathrm{t}\in\{1,\dots,\mathrm{n_c}\}\\
&\rho_{\mathrm{c}}\mathrm{G}_{\mathrm{t}}^{\mathrm{c}}\mathrm{x_t}\ge -\eta_{\mathrm{t}},\quad \forall \mathrm{t}\in\{1,\dots,\mathrm{n_c}\}
\end{aligned}\tag{14}$$

For the second constraint of the model (12), the semi-infinite controlled equation is as follows:

$$\begin{aligned}
&\mathrm{a_i x}\le \mathrm{d_i y},\quad \forall \mathrm{i}\in\{1,\dots,\mathrm{n_a}\}\quad \forall \mathrm{a}\in \mathrm{u}_{\mathrm{Box}}^{\mathrm{a}}|\mathrm{u}_{\mathrm{Box}}^{\mathrm{a}}\\
&\qquad\qquad =\{\mathrm{a}\in \mathrm{R}^{\mathrm{n_a}}:|\mathrm{a_i}-\bar{\mathrm{a}}_{\mathrm{i}}|\le \rho_{\mathrm{a}}\mathrm{G}_{\mathrm{i}}^{\mathrm{a}}\quad \mathrm{i}=1,\dots,\mathrm{n_a}\}\\
&\bar{\mathrm{a}}_{\mathrm{i}}+\rho_{\mathrm{a}}\mathrm{G}_{\mathrm{i}}^{\mathrm{a}}\le \mathrm{d_i y},\quad \forall \mathrm{i}\in\{1,\dots,\mathrm{n_a}\}
\end{aligned}\tag{15}$$

The relations below show the general form of the linear robust model for the hypothetical problem.

$$\begin{aligned}
&Min\ Z\\
&s.t.:\\
&\sum_{t}(\bar{C}_t x_t+\eta_t)\le z-fy\\
&\rho_c G_t^c x_t\le \eta_t,\quad \forall t\in\{1,\dots,n_c\}\\
&\rho_c G_t^c x_t\ge -\eta_t,\quad \forall t\in\{1,\dots,n_c\}\\
&\bar{a}_i+\rho_a G_i^a\le d_i y,\quad \forall t\in\{1,\dots,n_a\}\\
&y,x\in\{0,1\}
\end{aligned}\tag{16}$$

According to the index, decision parameters and variables expressed, the mathematical planning model of the Mean-Cvar robust counterpart is as follows:

$$min\ Cvar = Z \tag{17}$$

$s.t.$:

$$\eta + \frac{1}{(1-\alpha)S}\sum_{i=1}^{S}(y_i + \vartheta_i) \le Z \tag{18}$$

$$-\vartheta_i \le \sum_{j=1}^{N}\left(-r_{ij}x_j\right) - \eta \le \vartheta_i, \quad \forall i \tag{19}$$

$$y_i \ge \sum_{j=1}^{N}\left(-r_{ij}x_j(1+\rho)\right) - \eta, \quad \forall i \tag{20}$$

$$\sum_{j=1}^{N}\left(\mu_j x_j\right) = E_0 \tag{21}$$

$$\sum_{j=1}^{N} x_j = 1 \tag{22}$$

$$x_j, y_i, \vartheta_i \ge 0, \quad \forall i, j \tag{23}$$

Table 1: Changes in The Objective Function and The Return on The Portfolio versus The Robust Cost of the Robust Counterpart Model

Γ		**Mean-Cvar**	**Return**
0		0.01	0.03
1		0.0095	0.03
2		0.0085	0.03
3		0.007	0.03
4		0.0063	0.04
5		0.0061	0.04
6		0.006	0.05
7		0.0058	0.05
8		0.0056	0.05
9		0.0055	0.06
10		0.0053	0.06
11		0.0052	0.06
12		0.0051	0.06
13		0.005	0.07
14		0.0048	0.07
15		0.005	0.07
16		0.0048	0.08
17		0.0047	0.08
18		0.0046	0.07
19		0.0047	0.08
20		0.0047	0.08

2 Mean-Cvar Computational Results

Now a numerical example for Mean-Cvar optimization with the approach of Bertsimas and Sim will be presented and compared with the definite case. In the example, 20 stocks under the 500 S&P index are randomly selected for three years from 2018 to 2020, and the monthly returns of these stocks are used for analysis. The obtained data are historical data and these data are used to form the optimal portfolio of stocks. However, in estimating the expected return of each stock, the relevant uncertainty should be taken into account. Therefore, in order to consider the uncertainty, a robust approach to Bertsimas and Sim is used for modeling, and a 20% volatility (average monthly volatility) is considered. The role of a parameter Γ in the constraints is to adjust the degree of robustness against the conservative level of the answer that controls the level of robustness for the objective function.

The value of Γ is the value of the uncertainty parameter of the research. In this research, we have 20 uncertainty portfolios that are specified up to level 20. In the solved model and constraints with uncertainty, there are 20 coefficients with uncertainty, so the protection level up to 20 is examined. It has been proven that if the protection level changes up to the number of uncertainty parameters, the feasibility of a robust answer is guaranteed. The relevant robust model is developed using the relations (23) to (17) which due to the linearity of the resulting model, with the help of one of the common software for solving research problems in operations, the solution model and the results of Table 1 have been obtained which is shown in Fig. 1.

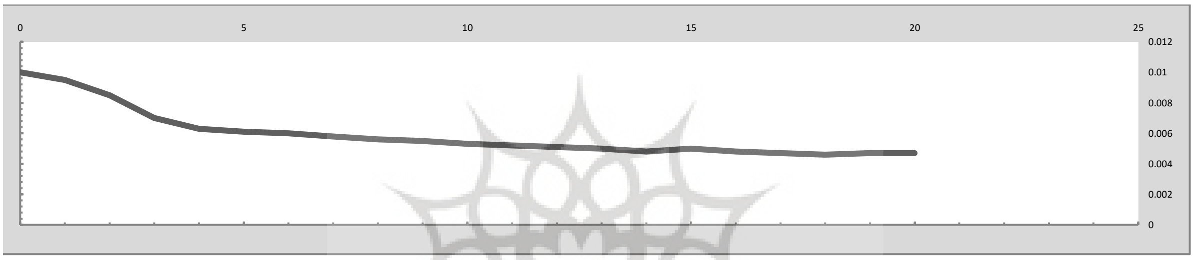

Fig. 1: Changes in The Objective Function and the Return on The Portfolio versus The Robust Cost of the Robust Counterpart Model

2.1 Expected Return of Mean-Cvar Model

In this section, the risk based on the robust model of the Mean-Cvar model is presented. According to the expected return determined at the level of 0.001, 0.002, and 0.003, the Mean-Cvar linear model has been examined. To consider the uncertainty, the robust approach of Bertsimas and Sim is used for modeling and a 20% volatility (average monthly volatility) is considered. In the solved model and constraints with uncertainty, there are 20 coefficients with uncertainty. The level of protection up to 20 has been examined, which is shown in Table 2 according to the level of uncertainty shown in Figs. (2), (3), and (4). Figs. 2, 3, and 4 are presented to examine the rate of change in the return on the portfolio by considering different values of Γ In the solved model and the constraints with uncertainty, there are 20 uncertain coefficients; therefore, the level of protection up to 20 is examined. It has been proven that if the protection level changes up to the number of uncertain parameters, the feasibility of a robust answer is guaranteed. The information in Tables 1 and 2 indicates the sensitivity of the model to data volatilities and uncertainty. The first row of Table 1 is data with zero protection level, meaning that the uncertain parameters do not fluctuate. The results of Tables 1 and 2 show the ability of the proposed model concerning the data uncertainty in the problem. As can be seen, with increasing Γ_0 , the values of the objective function have not improved and in fact show that as Γ_0 increases, the answers become more conservative. When Γ_0 is assumed zero, no volatility is allowed, and in fact forΓ_0 , the answer to the problem is no volatility.

Table 2: Return on Financial Portfolio by Considering Different Amounts μ_0 and Γ based on The Mean-Cvar Robust Counterpart Model

Γ	μ		
	0.003	**0.002**	**0.001**
0	0.01	0.0085	0.009
1	0.0074	0.0072	0.01
2	0.0075	0.0078	0.0095
3	0.0074	0.0076	0.0085
4	0.0074	0.0072	0.007
5	0.007	0.0056	0.0063
6	0.0064	0.0055	0.0061
7	0.0062	0.0056	0.006
8	0.0061	0.0052	0.0058
9	0.0063	0.0051	0.0056
10	0.0062	0.005	0.0055
11	0.0061	0.0051	0.0053
12	0.0065	0.0053	0.0052
13	0.0061	0.0054	0.0051
14	0.0064	0.0051	0.005
15	0.0063	0.0052	0.0048
16	0.0065	0.0054	0.0047
17	0.0064	0.0054	0.0047
18	0.0063	0.0054	0.0046
19	0.0064	0.0053	0.0047
20	0.0064	0.0054	0.0047

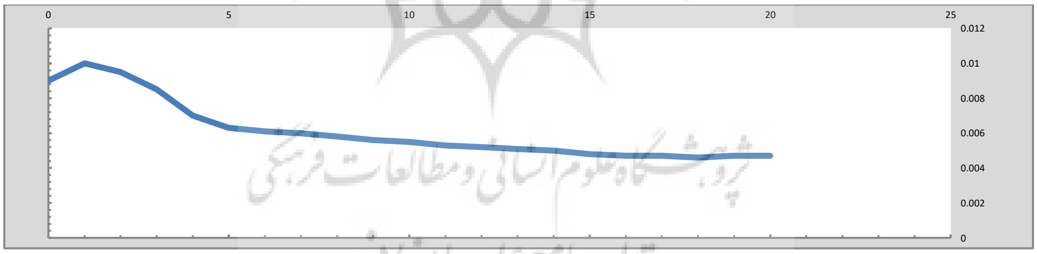

Fig. 2: Changes in The Objective Function and The Return on the Portfolio versus **Γ** Including Expected Return 0.001

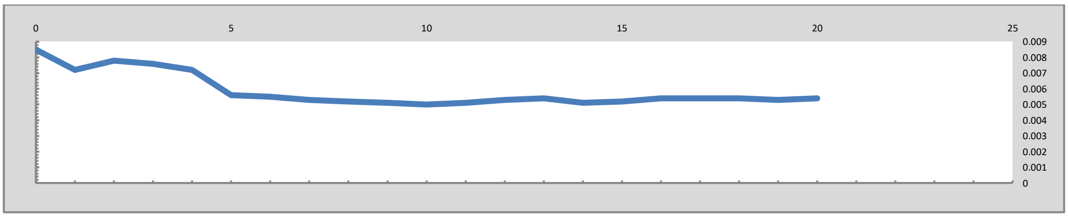

Fig. 3: Changes in The Objective Function and The Return on The Portfolio versus **Γ** Including Expected Return 0.002

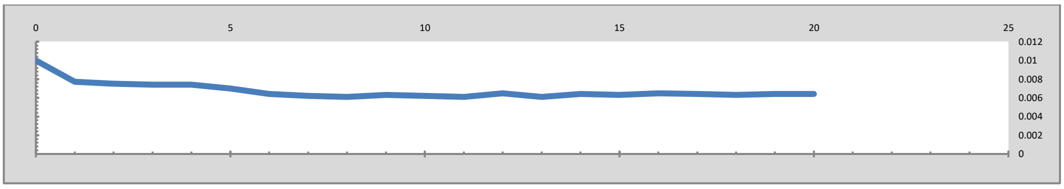

Fig. 4: Changes in The Objective Function and The Return on The Portfolio versus **Γ** Including Expected Return 0.003

3 Conclusions

The issue of choosing the optimal portfolio has always been one of the most important issues in modern economics. Extensive efforts are being made every day to improve the methods of analysis and stock in the world's financial markets. Efforts to improve stock analysis methods have led to the emergence of new methods that, along with previous methods, seek to find an answer to the desire to maximize individual profits in financial markets. Therefore, in the present study, a robust optimal model with a Mean-Cvar approach has been presented. Although the issue of portfolio selection goes back to Markowitz's initiative, the use of uncertain approaches to approach the real world in modeling portfolio selection is still new. On the other hand, most of the literature in this field has dealt with risk from a classical and modern perspective and has not focused on the postmodern perspective; they have only compared the previous models with the postmodern perspective, and no research has compared the postmodern models to each other. According to the studies conducted in this study, few models have been presented considering all the conditions and assumptions mentioned in the present study. In addition, in previous studies on portfolio selection models, methods such as regression, specific assumptions for the type of distribution, or time series models have been used to predict future stock returns and neural networks. However, in this research, an optimal model has been used using robust optimization, taking into account the Mean-Cvar, which is a postmodern perspective. In this research, it is tried to achieve a model that has a Mean-Cvar approach on the one hand and has an advantage over the optimization of the final model on the other hand, and on the other hand, the final model is linear. One of the most important features of financial markets is their uncertainty.
One of the most important components of financial market uncertainty is a risk. Therefore, the motivation for choosing the subject of this research is to use a methodology that makes it possible to model financial issues in accordance with reality. One of the most important concerns of modeling is the adaptation of mathematical models to reality, and in the real world uncertainty is one of the definite cases. Hence, we can understand the importance of including data with uncertainty in mathematical models. In real-world applications of linear optimization, no one can ignore the possibility that a small uncertainty in the data could make a typical optimal solution completely meaningless from a practical point of view. In robust modeling, the risk of misuse or misuse of uncertain data is much lower. Robust means that the output of the model should not be too sensitive to the exact values of the parameters and the input of the model. Another advantage of these models is that they can be easily converted to integer programming models. The optimal portfolio linear model can be used extensively in portfolio management. The purpose of this study was to provide an overview of portfolio optimization using the Mean-Cvar approach; so that if this relationship is determined, an optimal portfolio can be formed based on which the portfolio risk can be estimated. The model presented in this research, on the one hand, has Mean-Cvar as a negative risk approach and has an advantage over the optimization of the final model, and on the other hand, the final model is linear.

In this study, the issue of portfolio selection was examined by considering the uncertainty of input data. The risk measure used in this study is Mean-Cvar, which is one of the newest and most up-to-date risk measures in investment issues. In order to consider the data uncertainty, the Bertsimas and Sim robust optimization approach was used. One of the most important features of the proposed model is the existence of random dominance in this model. Experimental analysis of the results shows that the robust model presented in practice is robust against data volatility and, more importantly, offers more flexibility in financial analysis for investment. The model presented in this research is a linear programming model with computational advantages. In this research, by considering the uncertainty parameter of return in the initial certainty linear model, using Bertsimas and Sim method robust optimization has become a non-certain model and a robust counterpart. Given that the problem is convex, it has a universal optimization answer and the optimal answer is definite. The model presented in this study was an unrealistic return without considering uncertainty and had high dispersion and volatility, which by considering the uncertainty in the model return variable and providing a robust counterpart, the omega model had less volatility. Volatilities in the optimal omega model are influenced by uncertainty and data that can be affected by the political factors of underdeveloped financial crises and other factors.

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