The International Journal of Ancient Iranian Studies
July 2022, VOL. 2, NO. 3: 3-17. OPEN ACCESS
https://doi.org/10.22034/pa.2022.347319.1012

PERSICA ANTIQUA

RESEARCH ARTICLE

The Process of Constructing a Regular Hexagon in the Near East: From Neolithic Pottery to Euclid's Elements

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Abstract

A regular hexagon is one of the shapes introduced in Plane Geometry and refers to a hexagon with equal sides wherein the size of each angle is 120 degrees. This geometric shape, which can be quickly drawn today, was constructed over a long period in the millennia BC In the Late Neolithic period in Mesopotamia, the primary geometric shapes, including triangles, quadrilaterals, arcs, and circles, were additionally painted on the surface of pottery ware. Naturally, these shapes had been initially drawn by hand, and the sides of the polygons were not comprised of straight lines, or the circles had not been drawn perfectly. However, in the Chalcolithic age, geometric shapes moved away from handmade forms and approached standard ones. This standardization was not possible without drawing tools. In the meantime, the role of compasses or other objects with a similar use was of utmost importance because drawing a circle with such tools paved the way for drawing regular polygons. In fact, from the Late Neolithic, handmade triangles and arcs in the Near East, the first regular hexagon in the late second or the early first millennium emerged over several thousand years. Constructing this geometric shape with the help of standard circles and arcs has been well documented in the Near Eastern archaeological evidence. On the other hand, regular hexagons have been attributed to the second half of the first millennium in the history of mathematics. Therefore, this study reflected on the construction process of this geometric shape and dated its drawing hundreds of years back.

Keywords: Regular Hexagon; Near East; Neolithic; Pottery; Geometry.

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Article info: Received: 6 February 2022 | Accepted: 7 April 2022 | Published: 1 July 2022
Citation: Alirezazadeh, Mahdi; Heydarian, Mahmood. (2022). "The Process of Constructing a Regular Hexagon in the Near East: From Neolithic Pottery to Euclid's Elements". *Persica Antiqua*, Vol. 2 (3): 3-17.
https://doi.org/10.22034/pa.2022.347319.1012

Introduction

For millennia, geometric patterns have been utilized as decorative elements on painted pottery, rocks, building walls, clay, terracotta objects, and other household items in the Near East. One of these patterns is a polygon, particularly a hexagon, a fundamentally historic element throughout the Islamic art design and commonly used as an arithmetic object in metric algebra in Mesopotamia. Over a period from just before 6000 cal BC to a little after 5000 cal BC, the pottery of this region has been characterized by extensive and sometimes elaborate painted decorations (Campbell, 2010: 144). Since the Late Neolithic period, circular, spiral, and meander elements have been significant in art. They have even developed into seaweed and other soft living forms in this region. One might speak of a change from decoration into art, which, at the same time, is a step away from mathematical geometry processing (Hoyrup, 2000: 5). This type of decoration displays a strong interest in precisely informal patterns - enough to allow communication an authentically mathematical concentration on geometry.

Beyond noting that a particular motif has been further employed, this study addresses some questions: Was the hexagon associated with other motifs to derive a sense of context or meaning? Was this motif common in neighboring areas? In the hexagon case, does it correspond to the use of particular mathematical systems?

Indeed, some questions are probably too non-specific to allow meaningful answers, even if such responses could be formulated. Moreover, sources are hardly available that would enable ascertaining their validity.

In this study, a descriptive-comparative research design was implemented to investigate how the people of the ancient Near East understood and created a regular hexagon, as a six-sided polygon in which the total of the internal angles is 720° (Wenninger, 1974) and regularly constructed by any method (Ohochuku, 2016: 245; Fig. 1). Since stylistic evidence is so notable, this study analyzed these polygon motifs separately to determine whether they had been used, adopted, and developed in the study area from the Late Neolithic/Early Chalcolithic periods through the first millennium BC or not. Recognizing continuity in a motif in a region over time was also of interest as a first step. To realize the development and permanence of this motif, all the simultaneous with this motif in the given area needed to be included, and this process was continued for the other next periods. This method can be utilized for artifacts accordingly, and it can be correspondingly assumed as formal and semantic resemblance. Then, the differences in their style can also be discovered over time. This requires a historical sequence or a sequence of artifacts conducted in both time and place in the present study. Based on the chronology of the discovery layers, the oldest specimens were further identified in the areas, and historically, the following examples were arranged.

Research Background

The word geometry means measuring the Earth (Unwin, 2003: 131). According to Yaglom, geometry goes back more than 4,000 years, while the first scientific definition of geometry traces back to 1872 when Christian Felix Klein, the German mathematician, presented it (Yaglom, 1962). On the other hand, Rhodes believed that "geometry was discovered by the Egyptians and was measured by the Farmland," Later, the geometry went from Egypt to Greece, where it was fur-

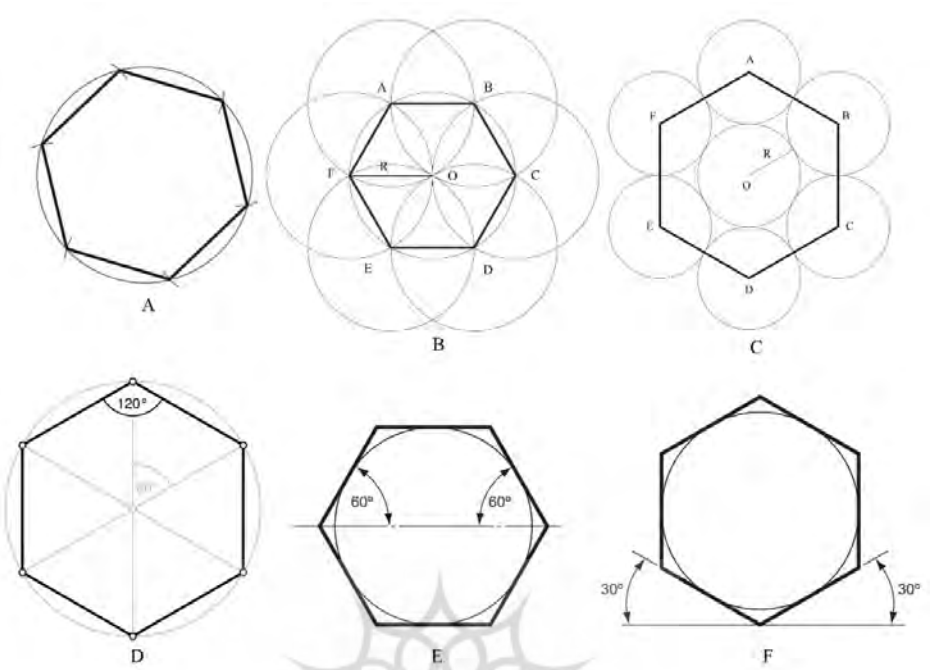

Fig. 1. Some Methods to Construct a Regular Hexagon.

ther developed. The role of the Greeks is also decisive. Still, the fact is that many cultures before and since the Greeks have mainly exploited mathematical operations from simple counting and measuring onwards and have even solved problems of differing degrees of difficulty. If the beginnings and the various stages of mathematics are shown via a hypothetical line, the start of this line in Greece is very vague and unclear; in contrast, the history of the much more ancient civilizations of Iraq (viz. Sumer, Akkad, and Babylon) in the years from 2500 to 1500 BC can provide the details (Hodgkin, 2005: 14).

Also, most mathematicians know at least a little about "Babylonian" mathematics, the Sexagesimal place-value notation (SPVN), written in a strange wedge-shaped script called cuneiforms, and even about the very accurate approximation to . It should be noted that it was in the late fourth millennium BC when writing probably began in the southern city of Uruk. According to Robson, accounting and the need to record mathematics led to the emergence of ancient Mesopotamian writing. In the third millennium, the SPVN emerged (from which the modern system of counting hours, minutes, and seconds was ultimately derived). In the second millennium, more advanced problems were also called the "Babylonian" mathematics (Robson, 2000: 149). However, the period of ancient Babylon (2000-1600 BC) is when the best mathematical documentation of Mesopotamia is available, including the problem of sequences and series on tablets (Melville, 2005a: 158-162). On the other hand, the question of area and sides seems very interesting. The ancient Babylonians were looking for the rate of change in the sides with a fixed area; they were dealing with quadratic problems,

indicating their high understanding of abstract algebra. The extent of thinking in the ancient Babylonians was even beyond their descendants after thousands of years (Melville, 2005b: 7 & 8). The ancient Babylonian clay tablets and the problem raised therein have been thoroughly studied and even taught as a history of mathematics at universities worldwide (Friberg, 2007).

On the other hand, the first three centuries of Greek mathematics commenced with the initial efforts at the demonstrative geometry by Thales in about 600 BC and culminated with the remarkable Euclid's Elements in about 300 BC (Eves, 1958: 77). Euclid's Elements were a very successful and systematic compilation of earlier works. However, this conclusion was not the onset of geometry. Although geometry has always played a prominent role in human life, it is an integral part of industry and engineering (Morling, 2010). Still, the origins of planar geometry have to be traced back to prehistoric pottery (Shulte, 1992: 106), where the invention of pottery provided a flat surface for drawing. Such a surface helped develop geometry over thousands of years.

Materials and Methods

Using a descriptive-comparative research design, the primary purpose of this study was to collect data on hexagon motifs and classify them based on a time scale (from Neolithic to first millennium BC) and regionalism (viz. the Near East). This research design could provide dialectic answers to questions such as when or where a hexagon pattern had been extensively used and other questions mentioned above. A collection of photographs representing geometric decorations on various artifacts, mostly ceramics (i.e., metal, figurine, plaque, pendant, seal, etc.), essential for the arguments, were reproduced in the following. These materials spanned the time between the Late Neolithic/Early Chalcolithic periods until the first millennium BC. Such materials had been traditionally divided into different cultures or phases, namely, the Hassuna, Samarra, Halaf, Ubaid, Susiana, Hissar, Assyrian, Hittite, Urartian, etc. They were also presented as an elected collection of some well-known sites from the Eastern Mediterranean region to the east Iranian plateau, stretched across a vast area from the upper regions of Euphrates (namely, eastern Anatolia) to South Arabia and the Levant to Iran (Fig. 2). Therefore, this study could cover the essential prehistoric-historic artifacts of the ancient Near East. For this reason, it comprehensively referred to not only archaeological reports, books, and articles but also regional/local archaeological museums. This study was comparably based on designs on artifacts of related images, gathered as a collection in the era resources.

Many of these items were perfect or repaired, some had been broken, and the remaining items included items in fragments. Because of the poor quality of the images in some resources, simply the cases with recognizable motifs were recruited in this study. They were extracted by the CorelDRAW Graphics Suite 2017, with the capability of creating two-dimensional (2D) designs and drawing different shapes. Draw out the motifs; after opening each image of the objects in the CorelDRAW Graphics Suite 2017, by Curves tool. Therefore, they were separated from other motifs of the objects.

Early Exercises for Drawing Geometric Shapes

Fig. 3 shows the initial attempts to draw the geometric shapes in the Near East.

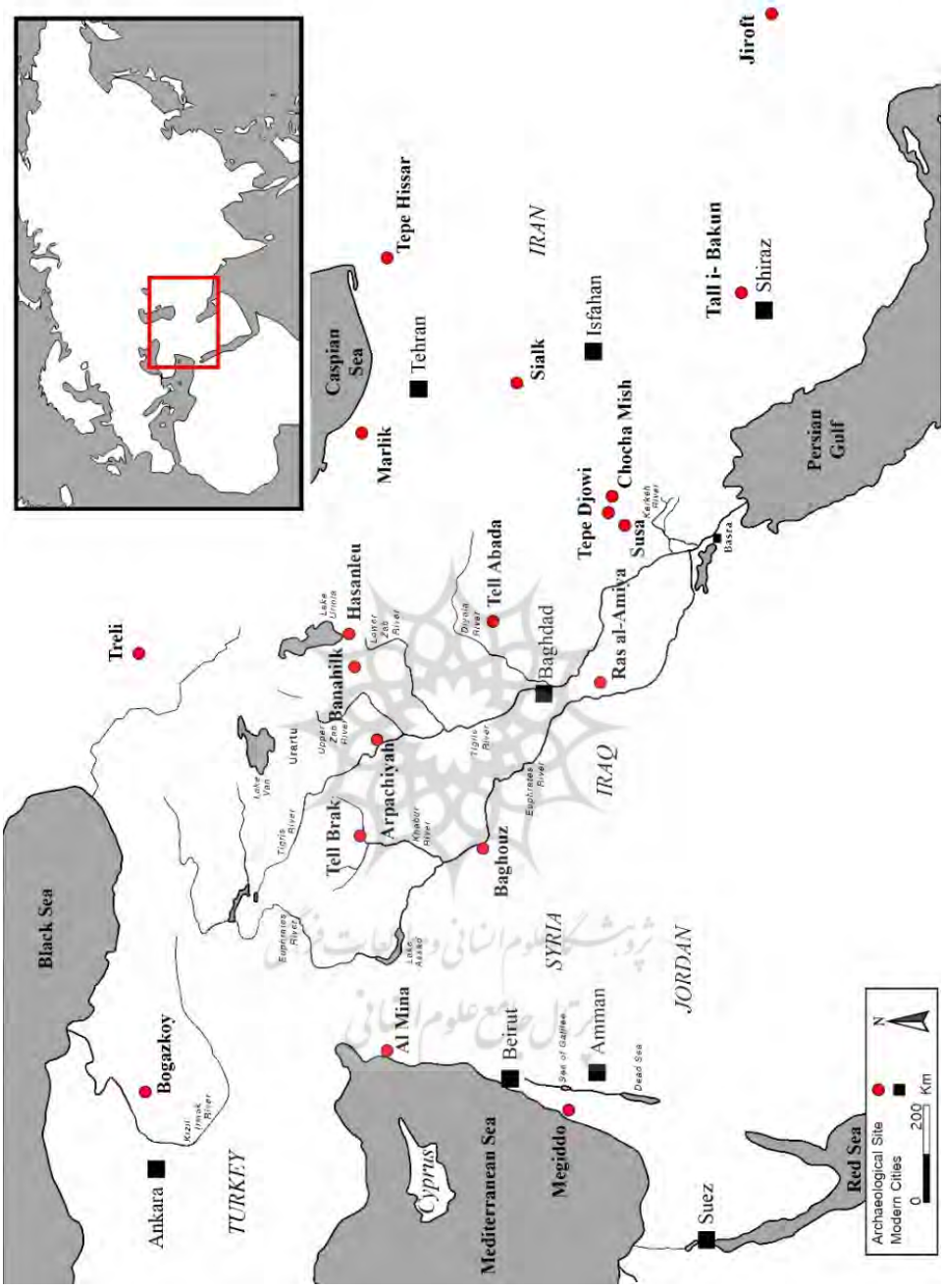

Fig. 2. Study Area and Archaeological Sites Mentioned in the Text.

As shown in Fig. 3A, there is a practice of triangle drawing. The triangles on the top row are solid, and the bottom row ones are hollow. However, in the second row in Fig. 3B, several geometric shapes drawn together in different sizes can be observed. The pottery-maker seemed to be well acquainted with the concepts

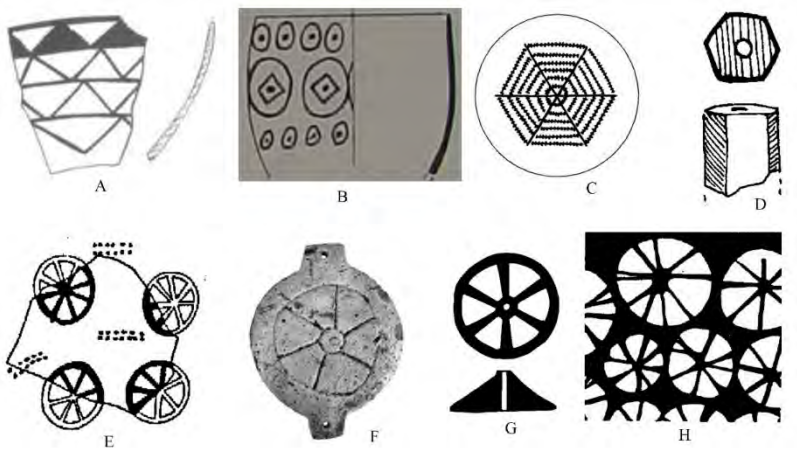

Fig. 3. Initial Drawings of Geometric Shapes and Unequal Sectors of a Circle.

Table 1. Description of Samples Mentioned in Fig. 3

Fig. 3	Site	Chronology	Geometric Shapes	Reference
A	Sialk	Sialk I	Triangle	Ghirshman, 1938a: P.L. XLIII, A-1
B	Tepe Djowi	Periode II	Circle and Diamond	Dollfus, 1983: Fig. 33, 12
C	Baghouz	Samarran	Triangle, Circle, and Hexagon	Braidwood, et al. 1944
D	Tall-e Bakun A	Level I (?)	Hexagon	Alizadeh, 2006: Fig. 62, A
E	Tall Bakun A	Level III	Circle Division	Langsdorff and McCoown, 1942: Plate 22, 3
F	Tell el-Mutesell-im (Megiddo)	Surface	Circle Division	Lamon and Shipton, 1939: Plate 77, 7
G	Chogha Mish	Late Middle Susiana	Circle Division	Alizadeh, 2008: Fig. 83, A
H	Arpachiyah	Halaf	Circle Division	Mallowan and Rose, 1935: FIG. 78, 26

of circles and diamonds. In Fig. 3C, the concentric hexagons are located within a larger circle. The geometric shapes were not only in the pottery decoration but also, as Fig. 3D shows, they (herein, hexagons) were considered on other types of archaeological evidence such as beads, seals, etc. The examples in Figs. 3E-H illustrates the division of a circle into even parts, which probably reveals the early stages of dividing circles into different parts. One reason to emphasize the division of circles is that it is one of the basic steps in drawing hexagons, especially regular ones. However, these forms revealed that their creators had acquired a relative understanding of shapes such as triangles, circles, hexagons, and circle divisions. Table 1 presents the archaeological information about Fig. 3.

Types of Regular Hexagon Patterns

As depicted in Fig. 1, different methods can be exploited to draw a regular hexa-

Fig. 4. The First Method of Drawing a Regular Hexagon, Circles, and Arcs in Archaeological Evidence.

gon. By examining the archaeological evidence of the Near East, some methods can be accordingly identified. Concerning studies on these ancient documents, two patterns were distinguished for constructing a regular hexagon (Figs. 1B & C). These patterns have evolved over a long period (i.e., several thousand years). These two patterns are first introduced in the following, and then the relevant archaeological evidence is presented.

The First Method: If seven circles of the equal radius (R) are drawn following pattern B in Fig. 1, then the hexagons obtained by connecting the centers of the outer circles become regular (Ohochuku, 2016: 245). However, today's way it is has been an evolutionary process over thousands of years. The ancient evidence in the Near East also well documents this process. Fig. 4 shows a collection of this evidence. In the first step, the circle's circumference is divided into four parts using four arcs (Figs. 4A-G). As observed, these parts are not equal. Therefore, the artist is not expected to take advantage of an object or a device such as a compass. The artist seems to have drawn these motifs by hand.

It should be noted that there is a flower with four petals unequal in this case. The four-leaf flower from Mesopotamia to the Indus Valley has also been drawn and practiced. In the second step (Figs. 4K-O), the arcs have expanded and become more varied. At this point, such arcs are still hand-drawn, but utilizing a compass cannot be imagined once again. However, the samples P-T in Fig. 4 seem to be drawn with an object whose perfor-

Table 2. Description of Samples Mentioned in Fig. 4

Fig. 4	Object Type	Site	Chronology	Reference
A	Pottery	Kot Diji	Harapa	Khan, 1965: Plate XVI
B	Pottery	Tell Brak	Samera-Halaf	Mallowan, 1947: Plate LXXX, 11
C	Pottery	Kot Diji	Mehrgarh VII (Middle of the third millennium BC	Khan, 1965:fig. 11,1
D	Pottery	Ghogha Mish	Middle Susiana	Alizadeh, 2008: Fig. 39, C
E	Pottery	Tell Brak	Samera Level	Mallowan, 1947: plate LXXX, 13
F	Pottery	Kot Diji	Mehrgarh VII (Middle of the third millennium BC	Khan, 1965:fig.14, 5
G	Pottery	Arpachiyah	Halaf	Mallowan and Rose, 1935: Plate XVII, b
H	Pottery	Ghogha Mish	?	Alizadeh, 2008: Fig. 3a, C
I	Pottery	Tell Brak	Fourth millennium BC	Mallowan, 1947: plate LXIV
J	Pottery	Tell Brak	Fourth millennium BC	Mallowan, 1947: plate LXIV
K	Pottery	Ras al 'Amiya	Halaf	Stronach, 1961: Plate XLV, 6
L	Pottery	Arpachiyah	Halaf	Mallowan and Rose, 1935: Fig. 60, 3
M	Pottery	Ras al 'Amiya	Ubaid	Stronach, 1961: Plate XLI, 7
N	Pottery	Ghogha Mish	Early Middle Susiana	Alizadeh, 2008: Fig. 39, A
O	Pottery	Ghogha Mish	Early Middle Susiana	Alizadeh, 2008: Fig. 38, L
P	Stone	Santa Trega	late 2nd century BC	Ruibal, 2006: Fig. 11
Q	Pottery	Kot Diji	Mehrgarh VII (Middle of the third millennium BC).	Khan, 1965: fig. 14, 10
R	Ivory plaque	Megiddo	Stratum IV. 1000-800 BC	Lamon and Shipton, 1939: Plate 115, 2
S	Pottery	Tell Abada	Hajji Muhammad or Ubaid 2	Jasim, 1983: fig. 10, 1
T	Golden Cup	Marlik	Late second millennium - early first millennium BC	Negahban, 1996: plate 20, 10

mance resembles the modern-day compasses. The six-leaf flower of this shape also has petals of equal sizes because the arcs sweep part of a circular circumference. One sample from Tepe Marlik (Fig. 4T) was accordingly compared with an ideal design drawn in the CorelDRAW Graphics Suite 2017 (Fig. 5), demonstrating that the circles drawn on this object were in a geometrically perfect form. The hexagons created on them were regular (that is, they had six equal sides and six

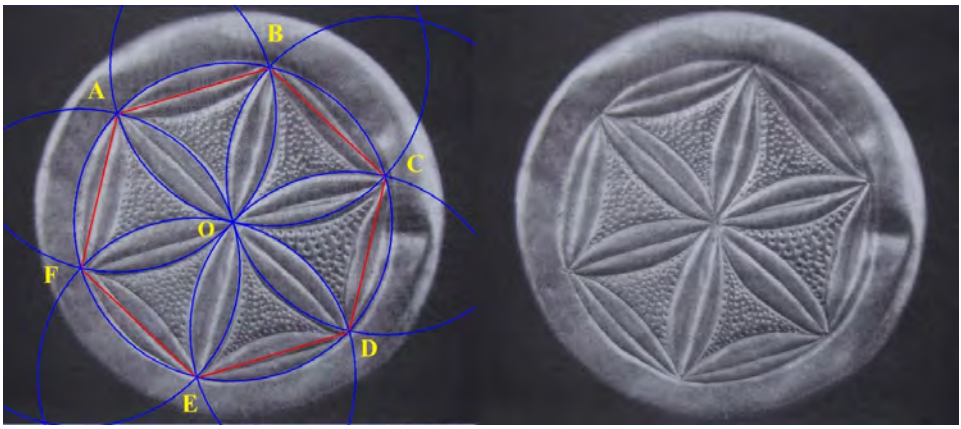

Fig. 5. Compare the Floor Design of the Marlik Cup with the Ideal Arcs Draw in CorelDraw Software and Regular Hexagon (ABCDEFA).

angles equivalent to 120 degrees). Finally, Table 2 presents the archaeological information related to Fig. 4.

The Second Method: Another method for drawing a regular hexagon is the same as that displayed in Fig. 1C. In this method, six circles of the equal radius (R) are tangent to a circle, whose radius is R too, located between these circles. A hexagon can be obtained if straight lines in a series connect the centers of the six outer circles. This hexagon is regular under the rules governing planar geometry. This approach seems to have been practiced in the Near East. Fig. 6 shows the archaeological evidence collection, wherein there were attempts to fit the seven circles following the pattern in Fig. 1C. Fig. 6A also presents a pottery ware from the Arpachiyah. On the outer surface of the pottery ware, circular motifs are further drawn in three rows, with the circles of each row tangent to the bottom rows.

Nevertheless, this dish does not show the pattern expected (Fig. 1C), and it has taken steps along that path. On the other hand, Fig. 6B shows a stone vessel belonging to the Jiroft region. The goat carved in this dish is fed by plants whose flowers resemble the pattern concerned. However, the circles carved on it are far apart. The same is true for objects E and G. Despite this, Fig. 6E, which represents the painted pottery from Tepe Hissar, fits well with the pattern in question. The point to note is that the radius of the circles drawn in these objects is unequal. This weakness has been remedied in the copper seal obtained from Tepe Hissar IIB (Fig. 6F). The objects G and, H also represent a more recent design of this pattern. Thus, as cited earlier, method C mentioned in Fig. 1 has been practiced in the Near East and can be followed at various stages in the ancient evidence. Finally, Table 3 outlines the archaeological characteristics of these objects, such as site, chronology, and object type.

According to the two methods described, it becomes clear that modern methods had been practiced for drawing regular hexagons in the prehistoric period of the Near East (from the half period to the first millennium BC). Advances in geometry for constructing standard shapes have further evolved over several thousand years. As shown in Fig. 4, the circles may have been shaped purely ornamentally, and these are not perfect geometric circles. Nonetheless, as observed in sample T (Fig. 4), the circles and the arcs on this object were standard put,

Fig. 6. The Second Pattern of Drawing a Regular Hexagon, and The Circles Tangent to The Central Circle.

Table. 3 Description of Samples Mentioned in Fig. 6

Fig. 6	Object Type	Site	Chronology	Reference
A	Pottery	Arpachiyah	Halaf	Mallowan and Rose, 1935: FIG. 66, 1
B	Stone	Jiroft	?	Majidzadeh, 2003: 34
C	Pottery	Brak	Nuzu Ware	Mallowan, 1947: Plate XXVI, 24
D	Stone	?	?	Miroschedji, 1973: Planche VIII, g
E	Pottery	Tepe Hissar	Hissar IC	Schmidt, 1937: Plate XII, DH 46
F	Copper Seal	Tepe Hissar	Hissar IIB	Schmidt, 1937: Plate XXVIII, H2183
G	Golden Jewelry	Ashur and Bogazkoy	Middle Assyria 1244-1208 BC	Maxwell-Hyslop, 1980: Fig. 1, 3
H	Bronze	Treli	$8^{th} - 7^{th}$ BC	Tsetskhladze, 2005: Fig. 2, 4-5

the prehistoric (namely, the late second millennium and the early first millennium BC) artists had exploited compasses to draw these circles and arcs. They even-

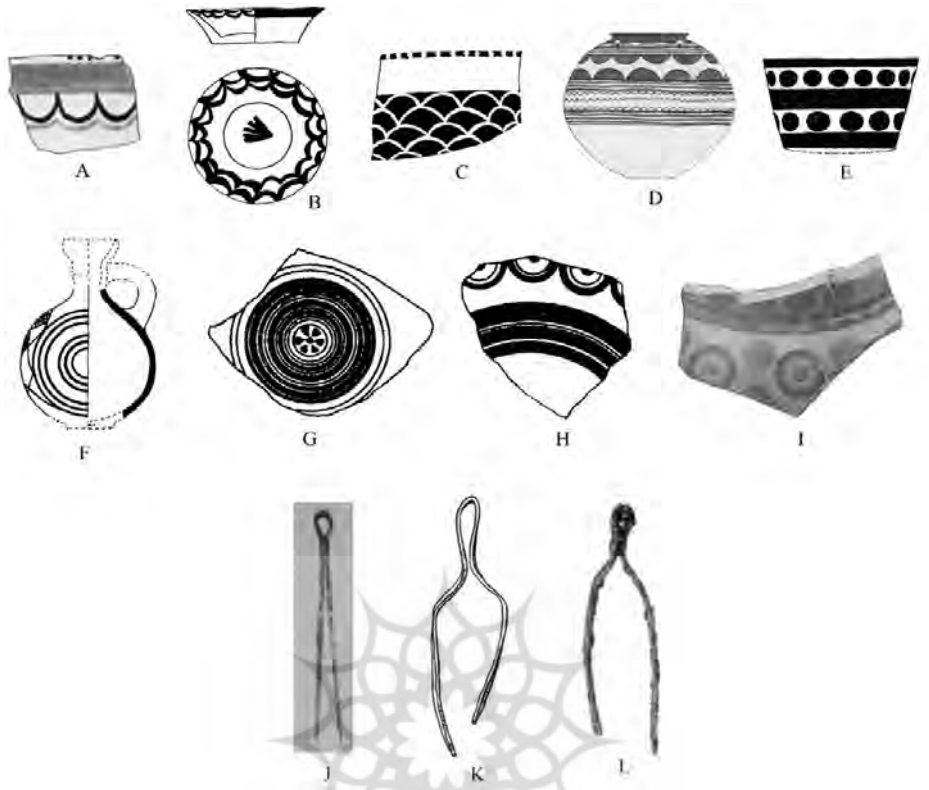

Fig. 7. Handmade Arcs and Curves (A, B, C); Non-Ideal Semicircles and Circles (D, E), Ideal Circle (F-I), and maybe Drawing Tools (J-L).

tually succeeded in creating hexagons (Fig. 5). In fact, from the first attempts to divide circles into equal parts by hand and without measuring instruments (Figs. 3E-H), the circle circumference has been divided into six equal parts utilizing a compass (Fig. 4T).

Fig. 7 also shows the same evolutionary process in geometric drawings. Besides, Fig. 7A displays a pottery shard from the Chagar Bazar related to the Halaf-Ubaid Transitional period. Of course, the arcs on this pottery shard are drawn by hand. However, the entire outer edge in Fig. 7B consists of two rows of interconnected arcs. In Fig. 7C, several rows of arcs are intertwined. Fig. 7D also shows a pottery shard from the Tell Brak. There are even rows of semicircles in two separate lines, and the pottery shard from the Susa in Fig. 7E reveals complete circles. However, such circles are different in size, and they are not ideal.

In some cases, they are closer to an oval than a circle. Thus, the artist's pottery work (A-E) has hand-painted motifs. Conversely, it appears to draw circles in Fig. 7F wherein an object such as a compass (or one with a similar function) has been utilized. Dozens of concentric circles plotted in Fig. 7G analogously suggest this. The circles in Figs. 7H & I are ideal too. All points on the perimeter of these circles are located at the same distance from their centers. Therefore, it was assumed that there was a compass.

Table 4. Description of samples mentioned in Fig. 7

Fig. 7	Object Type	Site	Chronology	Reference
A	Pottery	Chagar Bazar	Late-stage of T-Halaf	Mallowan, 1936: Plate II, 1
B	Pottery	Chagar Bazar	Level II & III < 2000 B.C	Mallowan, 1936: Fig. 20, 16
C	Pottery	Brak	Level 1-Nuzu Ware	Mallowan, 1947: Plate LXXVI, 25
D	Pottery	Brak	Level 3	Mallowan, 1947: Plate LXIX, 7
E	Pottery	Susa	Susiana b,c	Le Breton, 1957: Fig. 6, 30
F	Pottery	Megiddo	Stratum V	Lamon and Shipton, 1939: Plate 36, 14
G	Pottery	Megiddo	Stratum IV	Lamon and Shipton, 1939: Plate 29, 108
H	Pottery	Megiddo	Stratum II	Lamon and Shipton, 1939: Plate 29, 109
I	Pottery	Al Mina	Iron age?	Taylor, 1959: Plate XXI, 3a-1
J	Bronze	Sialk	Sialk VI	Ghirshman, 1938b: PL XXIV, 4
K	Bronze	Chagar Bazar	Ca. 3000 B.C	Mallowan, 1936: Fig. 8, 19
L	Bronze	Megiddo	Stratum V	Lamon and Shipton, 1939: Plate. 84, 21

The tools shown in Fig. 7J-L may have been further used as compasses and typical applications such as tongs. It should be noted that the signs of the tool use could be traced to the ancient Near Eastern evidence. Fig. 8 depicts evidence collection, where non-ideal arcs end up in the perfect circle in a process.

As can be seen, the trend of handmade geometric shapes and curves to standard arcs in the Near East can be traced (Fig. 8). The arcs and the circles are thus drawn with tools such as compasses. It is unclear whether these measuring instruments were invented independently to satisfy geometric purposes or were merely derived from other devices such as tongs. However, according to the evidence presented in this study, it seems that compasses were employed in the late second millennium and the early first millennium BC in the Near East (Figs. 8K & L). Accordingly, compasses and standard arcs and circles paved the way for drawing regular shapes, such as hexagons. As already mentioned, if seven circles of equal radius are put together consistent with the patterns B and C in Fig. 1, we can draw from this pattern. The same thing had been done in the Marlik site where the hexagons were regular (Fig. 8L) because the circles and the arcs were ideal, owing to measuring and drawing tools such as compasses.

Conclusion

The invention of pottery has given humans a smooth surface to put more of their imaginations on it. The motif on the pottery is thus very diverse. A bird's-eye view of the development may be adequate to learn any lesson from this story. The Old Near East Middle Neolithic period also introduces simple patterns, i.e., line, zigzag, etc. No efforts have also been made to achieve geometric coherence between various parts of decorations. From the start of pure decoration, the

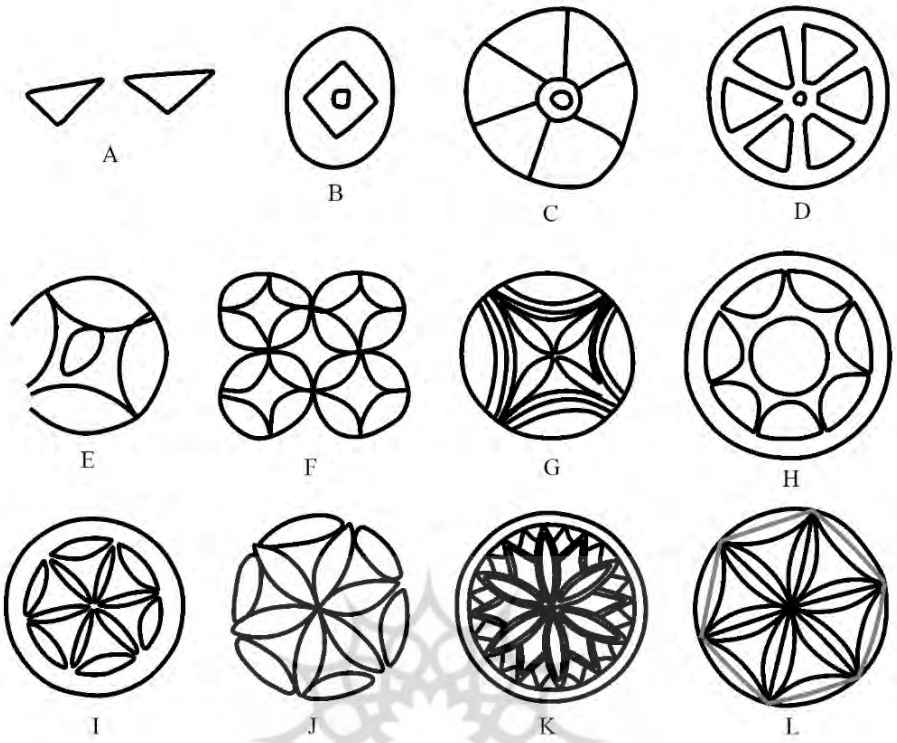

Fig. 8. The Process of Handmade Geometric Shapes and Curves to Standard Arcs, Circles, and Regular Hexagon in The Near East.

geometric pattern develops into structural experiments. During the Chalcolithic period, an artistic concern undergoes a qualitative leap, and the motifs are divided into animals, humans, plants, geometric shapes, and other types. However, the Early Chalcolithic bloom is a high-level manifestation of a general cultural substrate where straight lines, circles, as well as quadratic, hexagonal, and octagonal (and even abstract) symmetries, are essential in the Late Chalcolithic period, a high level of regularity develops into genuine mathematical structuring. As mentioned earlier, drawing non-standard and handmade geometric shapes such as triangles, squares, and circles is present in the Near East's prehistoric pottery. As Fig. 8 shows, the first non-standard arcs turned into perfect circles, but it happened for thousands of years. However, the role of drawing tools such as compasses is decisive. Standard circles also provided the basis for constructing regular shapes such as hexagons. Thus, throughout thousands of years from the handmade arcs of the Chagar Bazar in the Chalcolithic period, the standard circle and, therefore, the regular hexagons of the Marlik in the Iron Age were achieved. This regular hexagon is more than 500 years older than what is supposed in the history of mathematics regarding regular hexagons.

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