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Forming Efficient Frontier in Stock Portfolios by Utility Function, Risk Aversion, and Target Return

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Abstract

Asset allocation has always been a challenging issue / for individuals and businesses to survive in our competitive world. One of the famous businesses, which has an enormous impact on people's lives worldwide, is the pension industry. Pension funds- as Defined Benefit, Defined Contribution, or others- accept reserves from contributors and try to invest them in a way to keep up with their obligations in the future or even pay more than that. The equity market has been one of the good choices for investment as pension funds try to reach a particular rate of return to maximize their wealth while considering not crossing red lines in taking risks. This paper will detail the new mathematical model for finding optimal stock portfolios using Generalized Co-Lower Partial Moment as a risk measure to minimize portfolio optimization. On the other hand, it introduces new tailored Expected Utility as a performance metric to maximize in this model. The proposed model's issue against previous studies is considering risk aversion and target rate of investment return as two significant investor characteristics. This is based on price returns' simulation of candidate stocks in TSE while using accurate and nonparametric Probability Density Function in historical data analysis.

Keywords: Risk Aversion, Generalized Co-Lower Partial Moment, Target Rate of Return, Portfolio Optimization, Reference Dependent Utility Function

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Introduction

Any individual or institution is always concerned about preserving or increasing its wealth by a set of actions like investing in a profitable business. Pension funds are in a company to get contributions from firms or the government to help contributors in retirement or disability. This business is under the influence of many political, economic, and social factors, making predefined goals for these funds harder to achieve and necessary actions more complicated. The stock market in any country can be one of the alternatives pension funds may invest in to preserve or even increase the value of assets to keep up their obligation in time of need. Investment managers in these funds face so many challenges in preparing an optimal portfolio among candidate stocks. Price fluctuation of stocks is a big issue that may cause risk of portfolio devaluation, so they always try to measure this risk. Defined Benefit (DB) pension funds are the type of fund obliged to pay the minimum level of benefits regardless of the investment portfolio's performance. Their liabilities also restrict them in making strategic portfolio decision same as other funds (Hoevenaars, R.P.M.M., Molenaar. R.D.J., Schotman, P.C., Steenkamp, T.B.M., 2008). Pension funds need to analyze the impact of various political, economic, and social factors on their assets and liabilities. One of the important results of this actuarial analysis is concluding the rate of return they must earn from asset allocation.

Another issue any investor, before any investment, needs to know is the level of risk aversion. The market crisis in 2008 amplified financial risks for all businesses in the US, such as pension funds, insurance companies, financial intermediaries, etc. Bruno (2008) reported that significant corporate funds lost over $100 billion during the first week of October 2008. That dramatic loss for pension funds prompts whether they have undertaken excessive investment risk and how they can determine the risk (An, H., Huang, Z., Zhang, T., 2013). Investment managers in pension funds, like other institutions, must keep an eye on investment risk and return at the same time in portfolio selection and management. Since the equity market is one of the critical financial markets, they can assign a proportion of assets to earn considerable capital gain from investment in better stocks. For this purpose, they can calculate the utility of any investment outcome for a pension fund by defining an appropriate utility function. The advantages of using utility functions to form portfolios primarily relate to them providing a mechanism for directly evaluating all potential outcomes. They do this by attaching a score to each point in the distribution, as mentioned before. The expected utility can then be estimated as the probability-weighted average of these scores. The overall desired utility scores they deliver can evaluate portfolios (Warren, G, J., 2019).

Literature Review

Practitioners and academics in the field of investment are always looking for good models to better understand the stock's risk and return. As mentioned before, investment managers in pension funds are the same as other practitioners. Various studies related to academics tried to propose an optimal model for investment management in different markets for pension funds. Bodie (1991) studied asset management and shortfall risk for pension funds. He discussed the reasons for investing and not the investment equity market. He also mentioned that investing in equities will increase the volatility of plan assets, and on the other hand, it may also increase their return. Hoevenaars et al. (2007) studied the strategic asset allocation for pension funds subject to inflation and real interest rate risk in different markets. They compared the optimal mean-variance portfolio for the asset-liability investor with an asset-only portfolio in different investment horizons. They also mentioned that the form of pension plans' utility function deserves examination, which they didn't address in their paper. Rauh (2009) studied the asset allocation of DB pension plans in the presence of risk-shifting and risk-management incentives. They empirically concluded that funds with low funding ratios allocate a more significant share of their assets to safer securities, such as fixed incomes. Overfunded ones invest more in equities; thus, sponsors with high default risk and low funding levels generally undertake low investment risk, consistent with the risk management hypothesis. He concluded his findings by studying among firms in The United States with low inflation and interest rate, so these findings may not be fully variable in weaker economies.

The other issue, which has always been studied alongside asset allocation outcomes in pension funds, is investment risk measurement modeling. Jin et al. (2006) worked on this issue based on US firms' data. They empirically examined whether systematic equity risk of US firms as measured by beta is good enough for reflecting the risk of their pension plan. In addition, they paid attention to the impact of off-balance sheet and non-operating risks on the risk of a DB pension funds equity. Their findings showed that equity betas of firms accurately reflect the betas of their pension assets and liability. An et al. (2013) tried to uncover determinants of corporate pension funds' risk-taking strategy. They examined the dynamic nature of corporate pension risk by some questions about pension risk, bankruptcy risk, tax status to lighten the relationship between pension risk and sponsor's financial performance. They concluded that Pension risk is also significantly affected by the intensity of labor unionization and various sponsor incentives, including maximizing tax benefit, justifying pension accounting choice, and restoring financial liquidity.

Abourashchi et al. (2014) tried to investigate about solvency risk of DB pension plans to propose a new approach in measuring their solvency positions in the presence of extreme market movements. In their study, they considered the fat-tail nature of asset returns, discount rate change over time, and the dynamic nature of the correlation between discount rates and asset returns. The paper concluded that to estimate future solvency scenarios better, it is critical to model both asset returns and discount rates jointly. This conclusion seems a little challenging in economies with low information efficiency to determine fundamental discount rate changes for pension funds over time but forming a model to take the fat-tailed nature of asset returns into account in these economies seems essential and applicable. It seems necessary to use a suitable Probability Density Function (PDF) to analyze asset returns' nature. There have been so many studies in this area, but our focus is papers that can help study the fat-tailed nature of asset returns and estimate their PDF more accurately. For this purpose, nonparametric estimation methods such as kernel are preferred. The kernel density (Parzen-Rozenblatt window) estimation method is a well-known nonparametric method for estimating probability density for a random variable. Rozenblatt (1956) defined the naïve kernel density estimator by using simply a bin centered at variable. Since then, different academics have tried to increase the accuracy of this estimation method with consideration of new variables. Botev et al. (2010) studied mathematically kernel density estimators based on diffusion processes with the plug-in method for optimal bandwidth selection. When considering this method, it would be adversely affected by the normal reference rule (Devrˆoye, L., 1997 and Jones et al., 1996) to propose the meth free from the arbitrary normal reference rules. Figueroa-Lopez and Li (2020) tried to justify the optimal convergence rate of the kernel estimator, and they proceeded to show optimal bandwidth by proposing plug-in type bandwidth. By considering previous studies about the pros and cons of using plug-in type bandwidth in kernel estimators, Darestani et al. (2021) introduced a new risk metric using kernel density estimation via linear diffusion in Generalized Co-Lower Partial Moment (GCLPM) for forming an optimal stock portfolio. One of the advantages of the proposed model in this study was considering investors' risk aversion and target rate of return while trying to use a more accurate estimator for the probability density function.

The other topic academics and practitioners may be concerned about in portfolio optimization for Institutional investors such as DB pension funds is modeling utility function. Romaniuk (2007) studied the determination of optimal asset allocation for pension plans considering expected utility maximization of final wealth for fund managers. He concluded that DB pension funds, which must generate a minimum guarantee, must choose an investment policy to maximize the expected utility of surplus of assets over fund's liability so that they increase asset value above the predetermined value. Chen and Hao (2012) focused on modeling the asset allocation problem by Markov regime-switching method for a pension plan manager looking forward to maximizing the expected utility of the difference ratio between the benefit and contribution rates of the total salary until ruin. Finally, Warren (2019) studied utility functions in forming a portfolio. He discussed three utility functions as power utility and two variations of reference-dependent utility and their applications in investment management for DB pension funds. In his paper's conclusion, he stated that his key contention is that utility functions should be selected and parameterized with tailoring to the investor. Therefore it seems chosen utility function should consider target returns of the fund's investment plan, especially underfunded ones, to improve the funding ratio in the future when other factors might not be in favor.

The main contribution of this paper in comparison with other research is taking essential variables such as investor's target rate of return and risk aversion into account in implying the investment efficient frontier. The additional contribution in this research refers to proposing a new mathematical equation for calculating portfolio risk in asset allocation. On the other hand, Warren (2019) showed us that using the utility function in portfolio optimization for DB pension funds is applicable. Therefore, this paper used a mixture of the utility function and proposed risk measure to form a new portfolio optimization model in DB pension funds.

Research Methodology

This part sets out the methodology adopted in this study, and it presents proposed models. A quantitative approach is employed in this paper to study introducing a new model of forming an efficient frontier for stock portfolios for institutional investors such as DB pension funds. Literature reviews showed us that there is a possibility of focusing on essential variables discussed separately but has not been discussed mathematically altogether informing stock portfolios. This study will focus on altering the reference criterion for measuring deviations, resizing the downside deviations, estimation models to reach a more realistic model for measuring risk and reward. Some other papers study this alternation in derivatives (Lien and Tse, 2000; Doganoglu et al., 2007) but do not inform efficient frontier in the stock market.

As mentioned in previous sections, the focus in building the proposed model has been on introducing a more accurate probability density function, a new risk measure based on two important variables, and introducing consistent performance measures with risk measures as an alternative to expected return. This study presents a new method for finding the optimal bandwidth in the estimation of the probability distribution. Then, it would use General semi-variance to calculate Generalized Co-Lower Partial Moment as the risk measure. It also used reference-dependent utility in ratio form as reward representative to maximize and Generalized Co-Lower Partial Moment as a risk measure to minimize; so, in the proposed model, different variables necessary to build the model will be introduced.

1. Model Variables

Table1. Review on Important Variables In Proposed Model

Item	Name	Short introduction
1	Generalized Co-Lower Partial Moment	This variable has been calculated by the proposed model for risk measurement in this research and has been used as a risk metric in the portfolio selection model.
2	Curvature parameter on losses	This variable is defined as representative of the pension fund's loss aversion, and it has been used to calculate GCLPM and expected utility.
3	Curvature parameter on gains	This variable is defined as representative of the pension fund's loss aversion, and it has been used to calculate expected utility.
4	Reference Dependent Utility	This variable can be implied in a different form or ratio form and can represent the investment outcome for pension funds.
5	Weighting Parameter on Gains	This variable will justify the scale of upside deviation of stock's price return in time t from the target rate of return.
6	Weighting Parameter on Losses	This variable will justify the scale of downside deviation of stock's price return in time t from target rate of return.
7	Target Rate of Return	This variable is the rate of return for the investor as a target and has been considered the upper bound in model integral for calculation risk.
8	Gaussian Kernel	As one of the kernels for density estimation, this kernel considers stochastic processes whose finite-dimensional distributions are multivariate Gaussians.
9	Asymptotic Mean Integrated Squared Error	The asymptotic mean integrated squared error (AMISE) is an optimality criterion function used to assess the performance of a kernel density estimator.
10	Plug-in Bandwidth	This method, which usually compares with the Cross-Validation method, has been used to optimize asymptotic mean integrated squared error.
11	Stock daily price volatility	This variable shows the volatility of stock price and is used in the model for measuring the risk.

2. Models

We developed our stock portfolios by calculating their risk based on the generalized co-lower partial moment, which will be explained as follows. For building the risk variable in this research, Generalized Semi Variance has been used as the main body of the model while using a Gaussian kernel density estimator via linear diffusion with implementing plug-in type bandwidth selection for achieving optimal estimation. For this purpose, Asymptotic Mean

Integrated Squared Error has been considered as validation criteria. Furthermore, in this research, we used a Gaussian kernel to calculate lower partial moment for each stock to imply the estimation function as (1).

$$f(y) = \frac{1}{N\varphi}\sum_{i=1}^{N} g(\frac{y - R_\varphi}{\varphi}) \tag{1}$$

For GCLPM calculation, we proposed a Gaussian kernel density estimator via linear diffusion with plug-in bandwidth selection as a density function (Darestani et al., 2021). As a result, the lower partial moment can be calculated as follows (2).

$$\vartheta\left(R_T, n, R_\varphi\right) = \int_{-\infty}^{R_t} (R_T - y)^\alpha \frac{1}{N\varphi}\sum_{i=1}^{N} g(\frac{y - R_\varphi}{\varphi}) dy \tag{2}$$

Where R_t is the target rate of return, N is the number of historical observations, and φ is kernel bandwidth. Thus, one of the decisive variables that influence model calculation critically and should be paid attention to is bandwidth. As mentioned before, Mean Integrated Squared Error is one of the suitable variables that could help assess kernel density estimation performance (Lien and Tse, 2000), which can be estimated through (3).

$$MISE(\varphi) = \frac{1}{N^2\varphi}\sum_{i=1}^{N}\sum_{i=1}^{N} g^* \left(\frac{y - R_\varphi}{\varphi}\right) + \frac{2}{N\varphi} g(0) \tag{3}$$

Two widely used methods for finding optimal bandwidth are the plug-in method and cross-validation, which both have pros and cons. Figueroa-Lopez and Li (2020) shoed in their research that the plug-in method runs significantly faster than cross-validation. As to the accuracy of the kernel estimator, simulation results show that, in almost all sampling frequencies, the plug-in method outperforms the cross-validation method. Therefore, the proposed bandwidth to estimate the distribution optimally is plug-in bandwidth (4) (Botev et al., 2010).

$$\varphi^* = \left(2\pi N\left(\psi_{0,2} + \psi_{2,0} + 2\psi_{1,1}\right)\right)^{-1/3} \tag{4}$$

In which $\psi_{i,j}$ is:

$$\psi_{i,j} = (-1)^{i+j} \int_{\mathbb{R}^2} f(x) \frac{\partial^{2(i+j)}}{\partial x_1^i \partial x_2^j} f(x)\, dx \tag{5}$$

So bivariate PDF for each selected stock can be Proposed by (6) and GCLPM by (7).

$$\hat{f}(x,y) = \frac{1}{N\varphi_x \varphi_y} \sum_{i=1}^{N} K\left[\frac{x_i - x}{\varphi_x}, \frac{y_i - y}{\varphi_y}\right] \tag{6}$$

$$\tau_\alpha(R_T, R_i, R_j) = \int_{-\infty}^{R_t} \int_{-\infty}^{+\infty} (R_T - R_i)^{\alpha-1} (R_T - R_j) dF(R_i, R_j) \tag{7}$$

As one of the essential characteristics of DB Pension funds, the future performance of an investment is so important to meet the obligations. Therefore, we chose the Monte Carlo method to simulate stock's daily prices in the next three years (Warren, G, J., 2019). In our applied method, asset returns are simulated as proportional increments of constant drift and constant volatility stochastic processes, thereby approximating continuous-time geometric Brownian motion.

$$\frac{ds}{s} = \mu dt + \sigma dz = \mu dt + \sigma\varepsilon\sqrt{dt} \tag{8}$$

Where ε represents a random drawing, σ is the standard deviation of the asset price, μ is the expected rate of return, and S is the asset price.

On the other hand, we propose using a tailored reference-dependent utility function to measure stock portfolio performance, which defines the difference between projected and target outcomes. For this purpose, the ratio form of the reference-dependent utility function is chosen to be more consistent with defined benefit pension funds (Warren, G, J., 2019 and Blake et al., 2013). As a result, the function can be measured as follows:

$$
\begin{cases}
\gamma\left[\left(\dfrac{W}{W_t}\right)^{\alpha}-1\right] & R > R_T \\
\lambda\left[\left(\dfrac{W}{W_t}\right)^{\beta}-1\right] & R < R_T \\
0 & R = R_T
\end{cases}
\tag{9}
$$

Where β is the curvature parameter on losses, α is the curvature parameter on gains, ɣ is the weighting parameter on gains, λ is the weighting parameter on losses, R_T is the target rate of return, and W_t is the wealth of investor in time t.

3. Statistical Population for Investigation

The statistical population is composed of all firms listed on the Tehran Stock Exchange during the years 2010-2017. This sample needs to meet the following conditions and corrections:

1- They were listed on Tehran Stock Exchange between October 13, 2013, and October 13, 2018.

2- They are not included in financial intermediate and investment companies.

3- To increase sample reliability, stocks with trading days lower than 250 days in this period were deleted from the sample.

4- All of the incidental effects of equity capital raising on stock price volatility in this period were adjusted.

5- The effect of dividend payments to shareholders on stock price volatility in this period was adjusted.

After conducting adjustments, 215 firms remained as the statistical population for empirical study in this research.

Research Findings

As we developed our models in computational tools, we tried to reach a new efficient frontier considering mentioned factors and variables and examining their stability, altering the proxies, especially the investor's target rate of return. In this research, there has been no intention to examine the outperformance of the proposed model comparing other portfolio optimization. Still, the primary purpose was originating the new mathematical model to use important variables, which has been often neglected in different models and seems necessary to consider. As to the stability of the estimations and implied efficient frontiers, calculation results have been categorized in three scenarios with three iterations in each scenario to compare the changes in statistical characteristics of numerical results.

In the first step, we tried to derive the important metrics of the understudy DB fund (Iran Social Security Organization) for use in risk and performance determination based on the previous actuarial study conducted by the International Labor Organization (ILO), which led to 3 scenarios for an annual target rate of return as 0.2, 0.3 and 0.4. Then, we needed to use figures for other key variables such as curvature parameter on losses and gains and weighting parameter on losses and gains as assumptions, which can be investigated in further empirical studies later (Warren, G, J., 2019 and Blake et al., 2013).

Table 2. Numerical Assumptions In Portfolio Risk And Performance Determination

Item	Variable	Amount
1	Risk Averseness Degree	2
2	Curvature parameter on gains (in comparison with R_T)	0.44
3	Curvature parameter on losses (in comparison with R_T)	0.88
4	Weighting Parameter on Gains (in comparison with R_T)	1
5	Reference Dependent Utility (in comparison with R_T)	4.5

In the second step, the price time series for all 215 firms changed to daily price return based on (10).

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{10}$$

$$W_t = W_0 \prod_{i=1}^{t} (1 + R_i) \tag{11}$$

For conducting mathematical calculations based on (6), bivariate PDF correspondent to each two selected stocks estimated by coding the model in MATLAB software. For instance, the PDF figure for stock named "Electric Khodro Shargh" is plotted vertically and horizontally as Fig. 1.

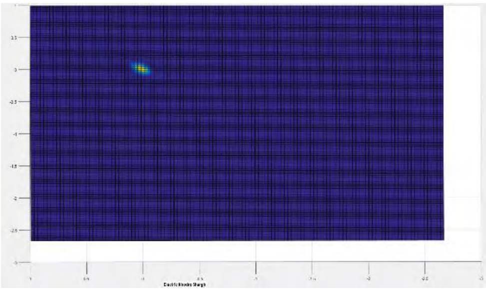

Figure 1.a. Vertical Plot For Presentation The PDF Of "Electric Khodro Shargh"

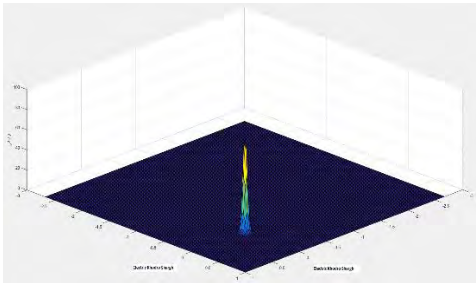

Figure 1.b. Horizontal Plot For Presentation The PDF Of "Electric Khodro Shargh"

The figure of joint PDF for stocks named "Electric Khodro Shargh" and "Alborz Darou" is plotted vertically and horizontally as Fig. 2.

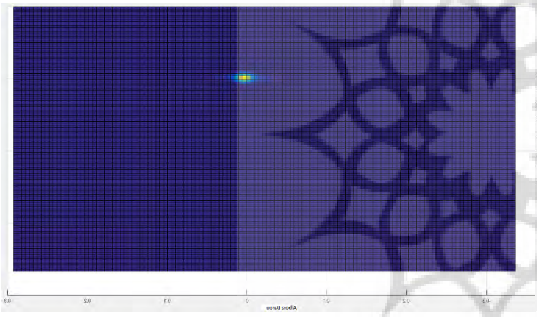

Figure 2.a. Vertical Plot for Presentation the Joint PDF of "Electric Khodro Shargh" and "Alborz Darou"

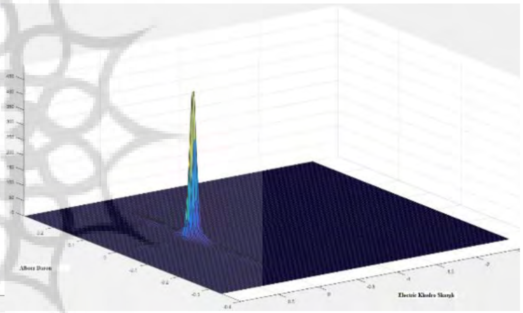

Figure 2.b. Horizontal Plot for Presentation the Joint PDF of "Electric Khodro Shargh" and "Alborz Darou"

In the second step, the lower partial moment for each stock and generalized co-lower partial moment for each selected stock were calculated, giving us a 215×215 matrix as risk matrix.

In the next step, the reference-dependent utility in ratio form was calculated for each stock, and a matrix 1×215 consists of results considered for conducting the efficient frontier. For this purpose, we determined three scenarios in the study of behavior in our mathematical model. We prepared 5,000 iterations for changing stocks' weights in each portfolio to achieve the efficient frontier in each scenario. In scenario 1, we assumed 20 percent as the target annual return rate and other variables as Table 2.

Scenario 1, Iteration 1.

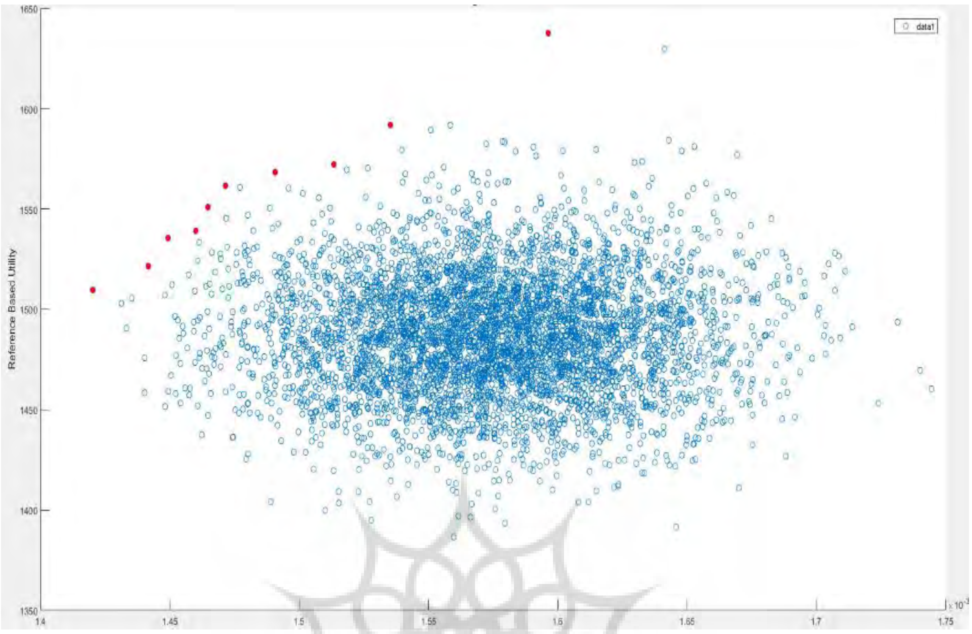

Figure 3. The Result Of Investigation Of 5,000 Different Portfolios In First Iteration Of Scenario 1

The result specification of scenario 1 in the first iteration was as Table 3.

Table 3. Result Specification of Scenario 1 in First Iteration

Item	Variable	GCLPM	Reference Dependent Utility
1	**Min**	0.00142	1387
2	**Max**	0.001744	1638
3	**Mean**	0.001575	1487
4	**Median**	0.001574	1487
5	**Mode**	0.00142	1387
6	**Standard Deviation**	0.00004648	28.93

Scenario 1, Iteration 2:

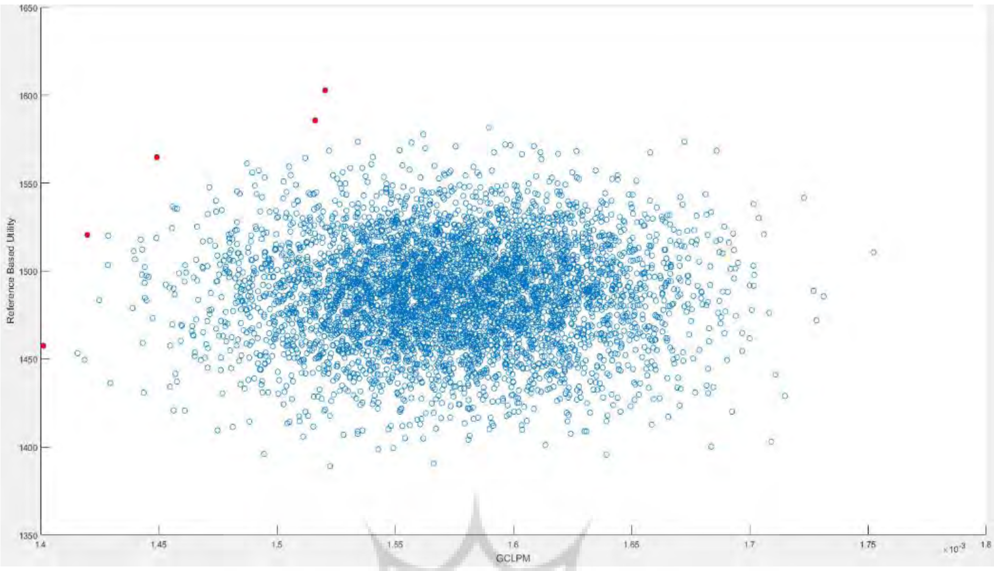

Figure 4. The Result Of Investigation Of 5,000 Different Portfolios In Second Iteration Of Scenario 1

The result specification of scenario 1 in the second iteration was as Table 4.

Table 4. Result Specification of Scenario 1 in Second Iteration

Item	Variable	GCLPM	Reference Dependent Utility
1	**Min**	0.001401	1389
2	**Max**	0.001752	1603
3	**Mean**	0.001574	1487
4	**Median**	0.01573	1488
5	**Mode**	0.001401	1389
6	**Standard Deviation**	0.00004717	29.38

Scenario 1, Iteration 3:

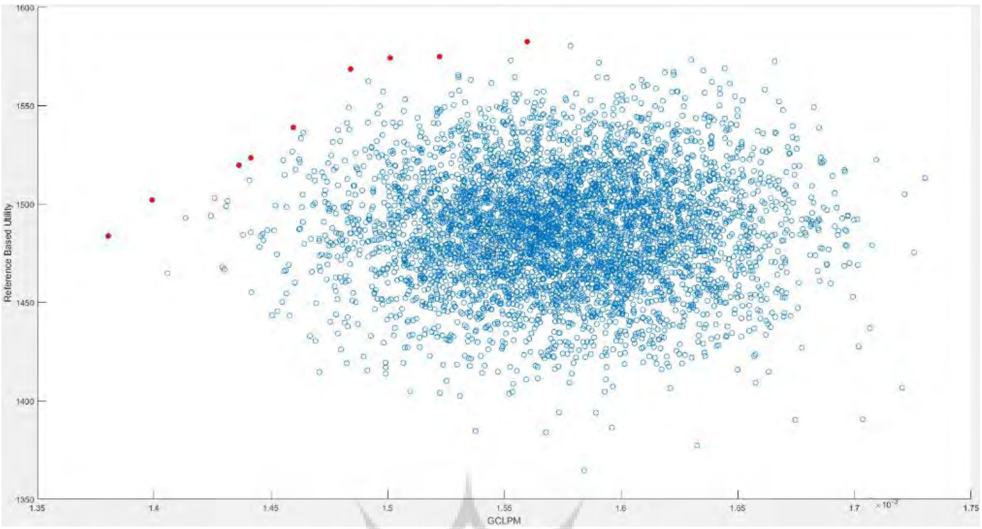

Figure 5. The Result Of Investigation Of 5,000 Different Portfolios In Third Iteration Of Scenario 1

The result specification of scenario 1 in the third iteration was as Table 5.

Table 5. Result Specification of Scenario 1 in Third Iteration

Item	Variable	GCLPM	Reference Dependent Utility
1	**Min**	0.00138	1364
2	**Max**	0.00173	1582
3	**Mean**	0.001575	1487
4	**Median**	0.001574	1487
5	**Mode**	0.00138	1364
6	**Standard Deviation**	0.00004669	29.18

As it can be implied, there is stability in results across different iterations for one scenario. Still, investigation on changing the target rate of return helps us trace the magnitude of variability in the mathematical model results. For this purpose, we assumed 30 percent and 40 percent as other target rates of returns in scenarios 2 and 3.

Scenario 2, Iteration 1:

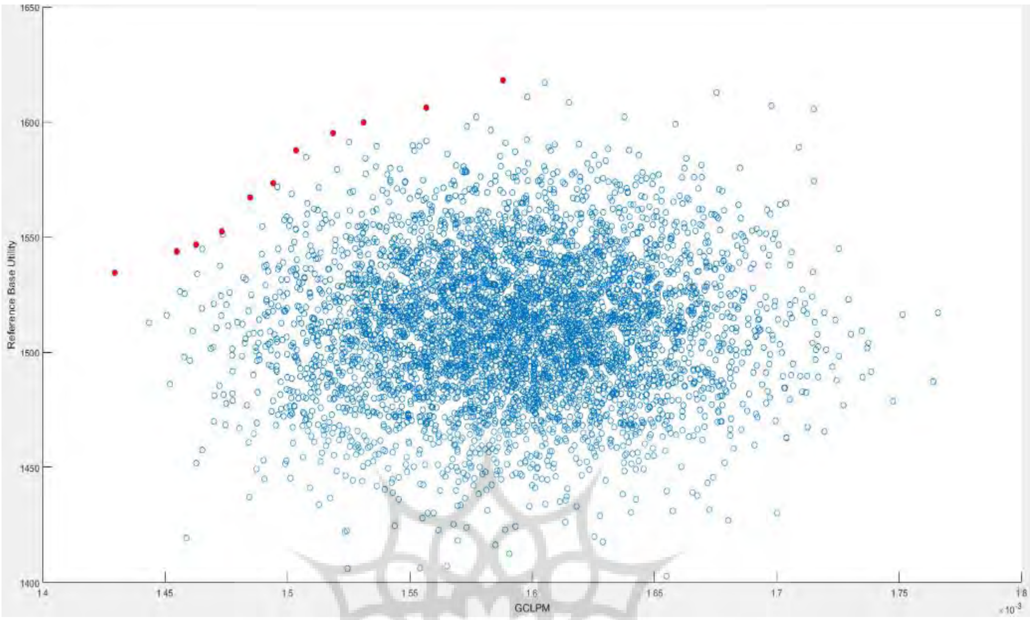

Figure 6. The Result Of Investigation Of 5,000 Different Portfolios In First Iteration Of Scenario 2

The result specification of scenario 2 in the first iteration was as Table 6.

Table 6. Result Specification of Scenario 2 in First Iteration

Item	Variable	GCLPM	Reference Dependent Utility
1	**Min**	0.001429	1403
2	**Max**	0.001766	1618
3	**Mean**	0.001592	1511
4	**Median**	0.001592	1511
5	**Mode**	0.001429	1403
6	**Standard Deviation**	0.00004824	30.18

Scenario 2, Iteration 2:

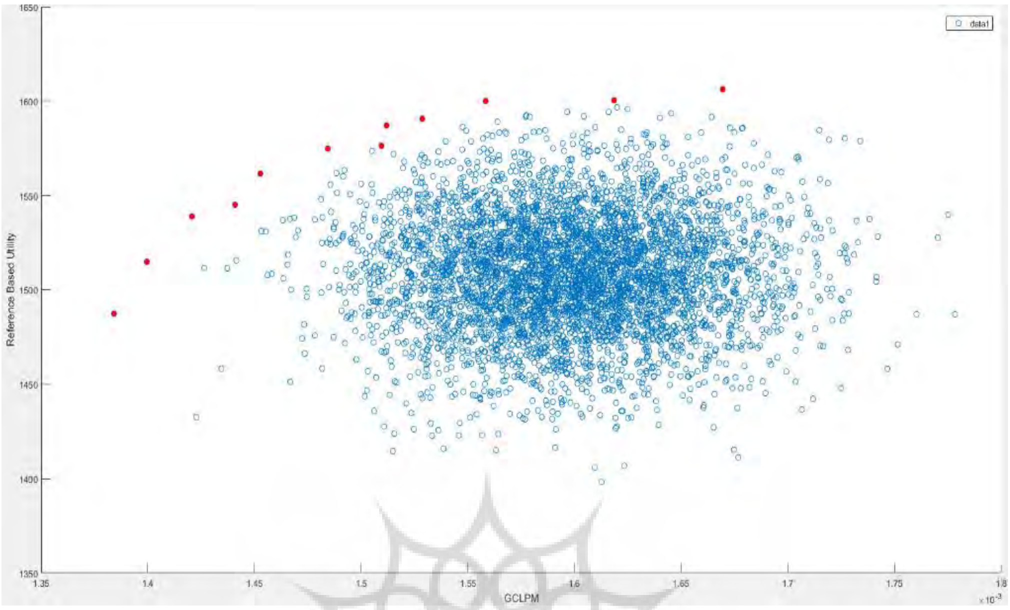

Figure 7. The Result Of Investigation Of 5,000 Different Portfolios In Second Iteration Of Scenario 2

The result specification of scenario 2 in the second iteration was as Table 7.

Table 7. Result Specification of Scenario 2 in Second Iteration

Item	Variable	GCLPM	Reference Dependent Utility
1	**Min**	0.001384	1398
2	**Max**	0.001778	1606
3	**Mean**	0.001598	1509
4	**Median**	0.001598	1509
5	**Mode**	0.001384	1398
6	**Standard Deviation**	0.0000487	29.74

Scenario 2, Iteration 3:

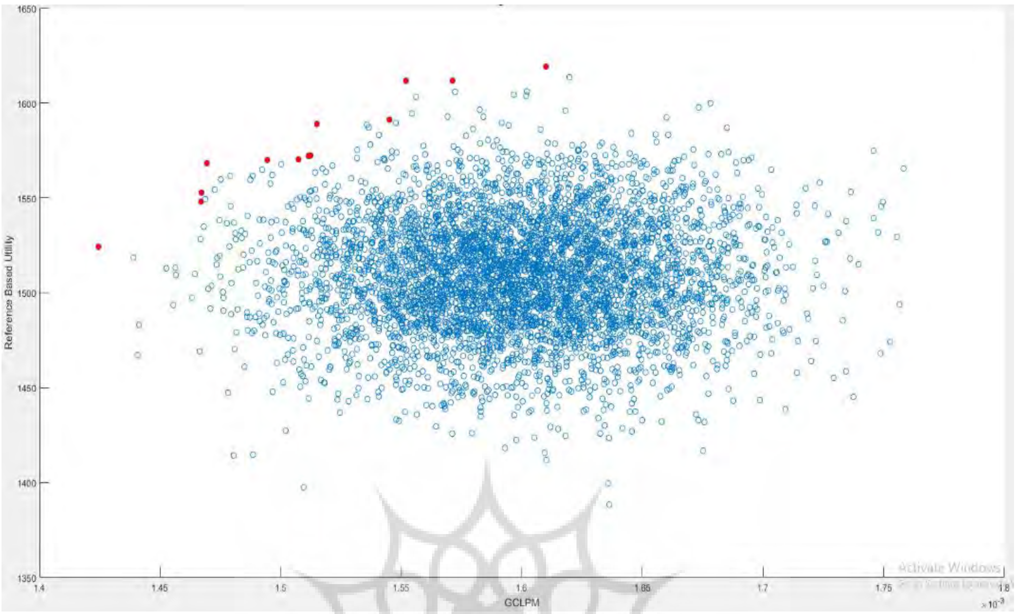

Figure 8. The Result Of Investigation Of 5,000 Different Portfolios In Third Iteration Of Scenario 2

The result specification of scenario 2 in the third iteration was as Table 8.

Table 8. Result Specification of Scenario 2 in Third Iteration

Item	Variable	GCLPM	Reference Dependent Utility
1	**Min**	0.001424	1388
2	**Max**	0.001758	1619
3	**Mean**	0.001598	1510
4	**Median**	0.001598	1510
5	**Mode**	0.001424	1388
6	**Standard Deviation**	0.00004799	29.62

In the final step, we conducted 5,000 iterations three times again in the third scenario to test the reliability and sustainability of results for reassurance. The figure result can be seen in Fig 8, 9, and 10.

Scenario 3, Iteration 1:

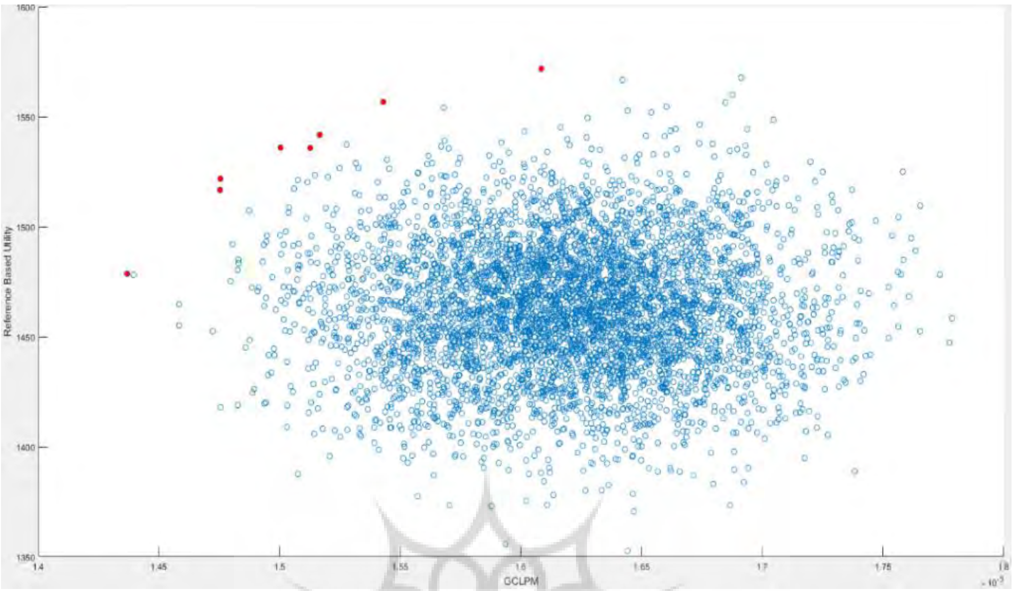

Figure 9. The Result Of Investigation Of 5,000 Different Portfolios In First Iteration Of Scenario 3

The result specification of scenario 1 in the first iteration was as Table 9.

Table 9. Result Specification of Scenario 3 in First Iteration

Item	Variable	GCLPM	Reference Dependent Utility
1	**Min**	0.001437	1353
2	**Max**	0.001779	1572
3	**Mean**	0.001619	1463
4	**Median**	0.00162	1463
5	**Mode**	0.001437	1353
6	**Standard Deviation**	0.0004979	29.21

Scenario 3, Iteration 2:

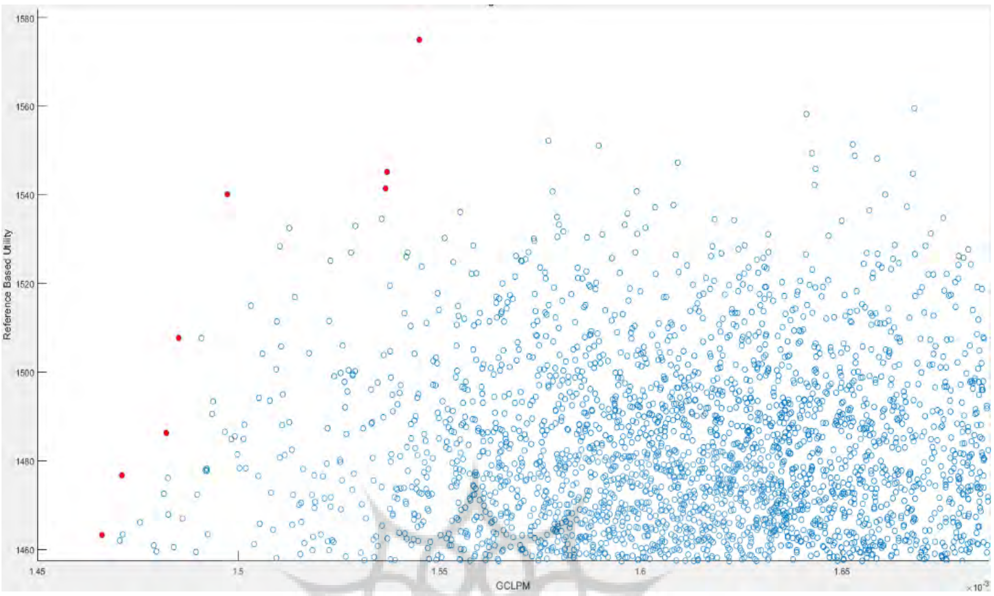

Figure 10. The Result Of Investigation Of 5,000 Different Portfolios In Second Iteration Of Scenario 3

The result specification of scenario 3 in the second iteration was as Table 10.

Table 10. Result Specification of Scenario 3 in Second Iteration

Item	Variable	GCLPM	Reference Dependent Utility
1	**Min**	0.00146	1354
2	**Max**	0.001789	1575
3	**Mean**	0.00162	1463
4	**Median**	0.00162	1463
5	**Mode**	0.00146	1354
6	**Standard Deviation**	0.00004913	28.9

Scenario 3, Iteration 3:

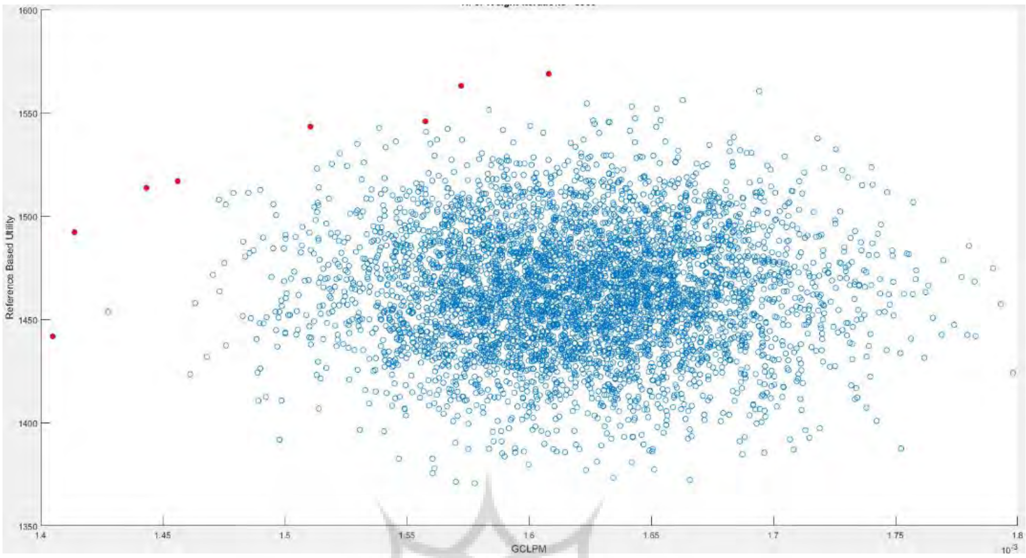

Figure 11. The Result Of Investigation Of 5,000 Different Portfolios In Third Iteration Of Scenario 3

The result specification of scenario 3 in the third iteration was as Table 11.

Table 11. Result Specification of Scenario 3 in Third Iteration

Item	Variable	GCLPM	Reference Dependent Utility
1	**Min**	0.001405	1371
2	**Max**	0.001798	1569
3	**Mean**	0.00162	1464
4	**Median**	0.001619	1463
5	**Mode**	0.001405	1371
6	**Standard Deviation**	0.00005	29.16

As upper information presents, sustainability and consistency can be seen in model outcomes, which assures the proposed model can give us reliable, efficient frontiers with considering new variables in risk and utility measurement for portfolio optimization.

Conclusion

Investment portfolio optimization has always been an issue for people since they want to protect or raise their wealth. Since stock markets started to work and companies registered in stock exchanges, people and other companies showed interest in investing their money by purchasing the shares of registered companies in local or global stock exchanges. With passing time and a growing presence of more companies in this market, big firms with huge wealth, such as banks, insurance companies, and pension funds, began to consider stock markets as eligible investment channels and entered this market to capture more significant gains. These institutional investors must be more cautious and accurate in informing investment portfolios because they must pay predetermined amounts to their contributors. Therefore, they need to consider various factors in their investment, which target rate of return as one of the outcomes in actuarial calculations is one of the known examples.

As some investors might be underfunded to pay their dues, they would have more constraints in their investment, and losses would hurt their situation to meet obligations more than overfunded ones. Despite willingness in these situations, the ability of these investors to take risks would decrease, and their risk aversion would surge, so this factor can alter outcomes and should be considered in the model. Our proposed model in measuring the risk and performance of a fund's stock portfolio consists of two mentioned factors, which would improve the calculation's accuracy. It also offers a new method in probability density estimation to decrease the errors based on the nonparametric estimation methodology. In this study, we found out:

- According to the proposed mathematical model and findings, there is a suitable risk measure in forming a stock portfolio considering their target return and risk aversion. To examine reliability to use this model in the Tehran Stock Exchange, we chose a case of empirical study, and our findings showed that there is sustainability in measuring the risk by using GCLPM in different target rates of returns as a sensitivity analysis.
- The critical issue in using a mathematical model for portfolio optimization is consistency between variables that measure the risk and performance of a stock portfolio. Our proposed performance measure considers both target return and risk aversion highlighting the positive difference between price return and target return on the adjusted scale and vice versa.

Suggestion

There is an excellent opportunity to discuss related outcomes for investment optimization in the context of specific institutional investors such as DB pension funds, which was not in our study scope. It would help enhance the quality and efficiency of investments.

1- One of the essential areas researchers can work on is parameterizing the risk aversion degree of DB funds to their risk tolerance by stress tests and using that in improving the accuracy of our proposed optimization model.

2- The other item that may help to increase the efficiency of the model is including some other actuarial variables affecting the portfolio selection concepts to achieve better outcomes.

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