Visual and Structural Analysis of Fractal Geometry in the Sheikh Lotfollah Mosque Ornaments (Isfahan- Iran)

[1]*Hengame Rezazade

1*M.A., Department of Arts and Architecture, Yadegar-e- Imam Khomeini (RAH) Shahr-e-Rey Branch, Islamic Azad University, Tehran, Iran.

ABSTRACT: The Islamic era is the period of new styles and beyond its own time. Shamsa, Muqarnas, Star, geometric motifs, and decorative elements have fractal nature, which was used as a tool to express the architect's idea by repeating the same components in a spatial dimension. We will reach the effect of Islamic impacts on fractal architecture and semantic cognition, and by finding the roots of theories about fractal with an analytical-descriptive approach, we will understand the relationship between fractals and how they are used by architects. Our focus is on comprehensive study and calculation, not only in ornaments but also in structure, to find patterns of fractal form, to represent coordination between components. First, we proved and extracted the fractal properties and patterns between the constituent elements, including self-similarity, repeatability, small-scale, and non-integer dimension, using visual analysis, and then we set up the Box-counting analysis technique with two purposes of calculating fractal dimension and finding their relationship. Aware of the mathematical proportions and relationships between the components of nature, the architect of the Sheikh Lotfollah Mosque has displayed the geometric sequence limit with an ascending equation in the interior ornaments of the dome. His goal is both to create the world of spirituality on a micro-scale and make it understandable for an observer. Thus, in this way, he has sometimes expressed his concept through mathematics, proportions, and sometimes by showing beliefs in concepts such as unity in diversity, all of which in their essence have concepts consistent with fractal forms.

Keywords: Sheikh Lotfollah Mosque; Fractal, Islamic ornaments; Islamic architecture; Box-counting technique.

INTRODUCTION

Over the past few decades, fractal variations and patterns with their natural origin have attracted much attention. Architects have done a lot of research in this area to find methods to utilize them in their designs in addition to accessing fractal patterns in nature to generate sustainable architectures like the patterns of the fractal components of nature. As an example, sunflower seeds, following the internal order, are repeated and grown together in such a way that they accommodate the largest number of seeds with the least space and absorb the most sunlight, and this is one of the stable patterns of nature. In Islamic architecture, especially in the architectural style of religious buildings, this concept can be seen in the ornaments of Muqarnas[1], Knots[2] (Gereh), and Decagram[3] (Shamsa). Hundreds of years ago, the Muslim artist combined Islamic geometry, religious belief, and art to create a new form of art that included repetition of similar patterns and shapes. The reason for choosing mosque as our case of study in the Islamic period is their high importance and fundamental roles of these buildings in the eyes of society and governments, which gained intense attention to it particularly in architecture to represent the glory of their religious buildings to the world beside maintaining their findings and styles as a legacy for the future. A building with religious use, the architect of which has a secret code behind it, deserves a profound and comprehensive study. Therefore, by obtaining the characteristics of fractal shapes and studying previous researches, we will examine the Sheikh Lotfollah Mosque during the Islamic architecture accurately. The step-by-step process from the entrance to the central space under the main dome is based on the fact that it is fractal and

*Corresponding Author Email: rezazadehengame@gmail.com

the interaction between fractal and Islamic architecture. In writing this paper, we try to answer the following questions by analyzing the architectural geometry of Sheikh Lotfollah Mosque: Does the geometry, form, and structure of the Sheikh Lotfollah Mosque follow the rules of fractal shapes, and is it only specific to the patterns of ornaments under its dome? Can traces of the concept of fractals be found in the way they are created and arranged? And how is it organized? For what reason and for what purpose did the architect use fractal geometry to design this mosque? (In other words, how did he propose to the viewer the relationship between fractal features and the intellectual foundations of plurality in diversity, infinity, paradise, and dynamism? Is the harmony and order of the endogenous elements of the fractal patterns of Islamic motifs also seen among its structural components?

The purpose of the research in the period of Islamic architecture and extract the fractal pattern from it will be done to understand the relationship and impact of Islamic beliefs on architecture in terms of fractal geometry and fractal semantic cognition in the architecture of this era. The aims of this research are as follows: Calculation and study of how to apply fractal geometry in elements and building structure related to the Islamic period of Iran. Clarifying the importance of fractal forms and the role of nature and its laws as a source of inspiration for expressing the idea of the architect in the Islamic period. Achieving how fractals of structural components behave with others.

Literature Review

There are various studies and researches regarding fractals and their geometric characteristics of Sheikh Lotfollah Mosque, most of which emphasize on ornaments of the interior dome. Regarding Islamic elements, mosques, geometric patterns, and their characteristics there are distinctive interior sources: "Analysis of the origins and concepts of geometric patterns of the Islamic period in the Persian art" (Abeddoost & Kazempour, 2016), "the mosque, manifestation of holy architecture" (Haghtalab & Karvan, 2012), "Fractal nature and its effect on architecture" (Karami Mofrad, 2017). Moreover in "Analysis the application of the natural geometry and the fractal in parametric architecture: the interior ornament of the dome of the Sheikh Lotfollah Mosque", they properly analyzed and studied algorithmic patterns of interior dome's ornaments (Mostaghni & Alimoradi, 2016). In another study, Pudine seeks to examine the meanings and dimensions of fractal geometry in Iranian architecture and motifs (Pudine, 2016). Recent studies in connection with fractal definitions, theories of experts, and techniques used globally can be mentioned in these studies: "Fractal models in architecture: A case of study" in which the writer investigated relationships between architecture a fractal theory in addition to calculating the Robin house's fractal dimension (Sala, 2001), in "fractal geometry and architecture design: case study review", they explained the theory of fractal geometry, presents the current position of this architecture

and designed examples (Lu et al., 2012). In the computational part, we have used the formulas and calculations performed by the Box-counting method in articles like "A method of compositional fractal analysis and its application in Islamic architectural Ensembles" which has attempted to examine the ensemble of Islamic architecture by visual and dimension fractal analysis to recognize the consistency of fractal features in case studies like Bibi-Khanym and Taj mahal (Shishin & Aldeen Ismail, 2016). Previously, researches related to fractal and Sheikh Lotfollah Mosque has been limited to the ornaments under its dome and has not been comprehensively studied. Therefore, the need for complete research seems integral. For the first time in this research, an Iranian-Islamic building will be applied to calculate the fractal dimensions, which will be novel in this respect. Besides, fractal has been reflected everywhere in this building to reach the architect's ultimate goal.

MATERIALS AND METHODS

The method of this research is analytical-descriptive, which includes ways to describe conditions and phenomena under study. Data collection is done by evaluating and analyzing the existing works and matching its features with the information that has been done by observing and overviewing the related library documents, researches, data collection, computer networks, and measuring the studies of buildings and books available in the Islamic architecture period in Iran. We argue that Islamic patterns in the building correspond to the concepts obtained, and with the help of drawing software, we reach the angles and numbers that patterns follow to repeat their fractal. Four features of fractal geometry structure, including self-similarity, repetition, small scaling, and non-integer dimension are considered as the principle and we analyze these features and their adaption to the concept of fractal geometry in separate tables. We will see a process referral to mathematical formulas and equations, which is the main nature of fractals, and we encountered an interesting issue that with the ornaments and patterns under the inner dome, the formula for the total geometric sequence limit can be calculated. To sum up, the research process is one in two stages which is illustrated in Fig.1.

Theatrical Framework
Fractal Analysis Technique in Architecture

Fractal analysis for Sheikh Lotfollah Mosque will be carried out on a small scale and related to a single building to investigate the building's self-similarity features and fractal dimensions. According to the studies carried out in the field of techniques, we first analyze the Lotfollah Mosque's structure on a small scale because our goal is to extract fractal features from a single building. Then we identify and prove the self-similar elements and by box-counting, we calculate the fractal dimension of the section, façade, plan, and ornaments. Finally, to figure out how
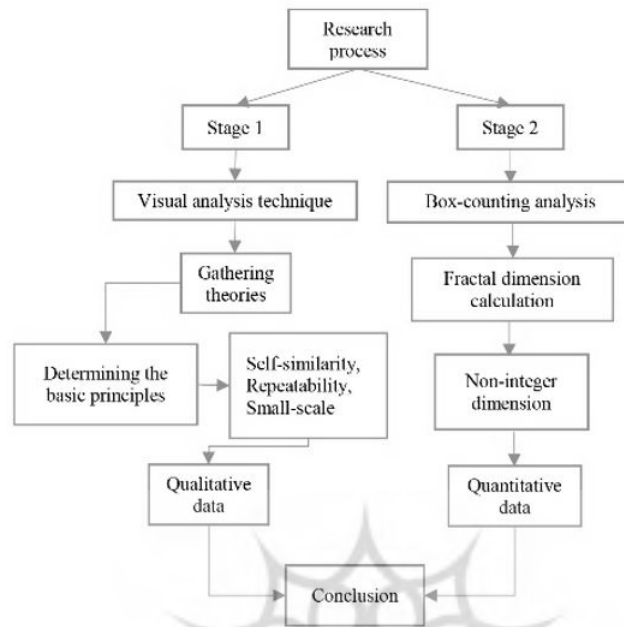
Fig. 1. Total research process in analyzing the fractal aspects of the Sheikh Lotfollah Mosque

the fractal components relate to each other, we display the line equations obtained from each graphically. Similarly, we used this part of analysis techniques from research with the article" A method of compositional fractal analysis and its application in Islamic architectural ensembles" to seek the relationship of structural components with each other (Shishin & Aldeen Ismail, 2016).

Fractal Geometry

Fractals are formed by a repetition of patterns, shapes, or a mathematical equation. The formation is dependent on the initial format. Not only in nature, but fractals are also seen in the study of various disciplines such as physics, mathematics, economics, medicine, and architecture. For a variety of reasons, in different cultures and geography, many times the fractal pattern had reflected on creating the architecture (Alik & Ayyildiz, 2016). Fractal forms in terms of geometry can be classified as four main features of fractal geometry through the following theories:

Self-similarity: Visual self-similarity on any scale "Since all levels of magnification, fractals are similar", the self-similarity feature is a prominent characteristic of the fractal forms, whereas it indicates the similarity between the parts that form the shape with the whole shape, i.e. the part of the whole is so similar to the whole (Elgohari, 2019). That can be explained as small parts of the object, which sequentially are similar to the whole object (Osama et al., 2014).

Small- scaling: Similar to self-similarity, scaling is the process of reduction and/or progression of elements (Osama et al., 2014). In a set with fractal shapes, the smaller subsets are the same as the larger sets that make up the entire fractal shape.

Repetition (balance): Fractal components and elements are repeated on different scales. This repetitive rhythm in fractal shapes leads it to balance (Mobini & Fathollahi, 2014).

Decimal dimension: To explain the concept of fractal dimension, it is necessary to understand what we mean by dimension in the first place. A line has dimension one, a plane has dimension two, and a cube has dimension three (Nurujjman et al., 2017). Euclidean geometric forms are regular and have integer dimensions (one, two, and three, for line, surface, and volume respectively). A fractal line has a dimension between one and two (Sala, 2013). Hence, here, in this paper, we can consider that the fractal form's dimension is non-integer.

Fractal in Islamic ornaments

Ornaments in Iranian architecture are a fundamental part of the spatial qualities, and they could be primarily divided into the form of two-dimensional ornaments such as Knots, arabesque motifs, tile working, and three-dimensional ornaments including Muqarnas (Kiani & Amiriparyan, 2016). Inspired by nature, Muslim artists reflected its architecture and geometry.

The sophisticated geometry involved in dome interiors shows how artists try to express their feelings and emotions, as well as their beliefs and philosophy, through complex geometrical

73

designs involving repetition, rhythm, pacing, scale, and color combination. The construction of stucco[4] domes shows that they also were aware of the geometry of three-dimensional Euclidean space. The designs reveal, through self-similarity, that the artists had a sense of fractal geometry (Sarhangi, 1999). From point of view of Iranian architect, abstract art manifest unity in plurality. Iranian architects used the fractal geometry for several reasons including use of geometry for creation of diversity, collection, analysis of structural stability, use of this geometry to create details at various scales, visualization of heaven and human living environment, and geometric motifs as unity in plurality (Pudine, 2016).

RESULTS AND DISCUSSION

Sheikh Lotfollah Mosque

Sheikh Lotfollah Mosque was built with the architecture of Mohammad Reza Isfahani, in the Islamic architecture period in the Safavid kingdom. By examining the studies and adapting them to the fractal characteristics, self-similarity, repetition and small scaling of this mosque can be deduced from the following cases.

Muqarnas: The definitions of Muqarnas, which explicitly state that this structural and decorative element is fractal, are as follows:

The synthesis of geometry and rhythm is likewise found, in a form that is not simply linear but fully spatial, in a very typical element of Islamic architecture, namely, the Muqarnas which are described by the very approximate term of "Stalactites" (Burckhardt, 2009).

To using fractals as construction details under the domes and in the upper parts of the entrance arches named Muqarnas (Abdelsalam & Ibrahim, 2019). Muqarnas are the result of the repetition of an element that has become rhythmically smaller from bottom to top and is generally a subset of the general shape, which is a symbol of the sky and stars. The altar[5] (Mihrab) of this mosque, located in the main courtyard, has Muqarnas with Arabesque and Islamic motifs on the inside. The lines are in harmony with the original shape and the arrangement of ornaments and tiles are regular (Fig. 2). In particular, the basis for the development of the base model on a Muqarnas pattern is composed of three main elements including similarity, change in scale, and iteration (Kiani & Amiriparyan, 2016).

Decagram (Shamsa): they are the result of symmetry and repetition of self-similar forms in many decorative elements in Islamic monuments and signify the means of sun and stars in an abstract language. Examples of it can be seen at the top of the entrance Muqarnas, corridors, Mihrab, and dome of Sheikh Lotfollah Mosque.

Hierarchy of spaces and mathematical patterns: In Sheikh Lotfollah Mosque, the sequence of spaces and the way they are connected induces something more than a normal place, which is the role of separation from the material world and connection to the spiritual world (Mehrali, 2013). The width of the two continuous shells of this dome is 12 and its height is 32 meters. At the top is a perforated circle with 16 lattice windows and ornaments. After the inscriptions on the stem of the dome, 32 rhombus-shaped motifs (ornaments) can be seen, which become smaller as they get closer to the top. This is the relationship that the architect has used between the numbers in designing. 4walls to 8 sides, 8 to 16 windows and 16 have been turned into 32 elements (Fig. 3).

In summarize, the numerical pattern used in the dome's interior ornaments in Sheikh Lotfollah Mosque is regular as described in table1, which led to the derivation of the equation of geometric sequence from it.

Carefully following the sequence above, we find that it has a factor of 2 between the two consecutive numbers. If we multiply each sequence by 2, the next number is obtained. A set of numbers whose division of each sentence into its previous sentence is equal to a constant value is called a geometric sequence, q is the value of the ratio. Each geometric progression is represented by its Nth sentence, which is called the general sentence (Equation 1).

Fig. 2: Entrance Muqarnas and Altar (Mihrab) of the Sheikh Lotfollah Mosque

Fig. 3: Internal space of the dome of the Sheikh Lotfollah Mosque

Table 1: Numerical patterns of interior elements of Sheikh Lotfollah Mosque's dome

Interior elements of the dome	Number of elements	Nth Sentence
4 walls	$2^2 = 4$	2^n
8 sides	$2^3 = 8$	2^{n+1}
16 windows	$2^4 = 16$	2^{n+2}
32 rhombus-shaped ornaments	$2^5 = 32$	2^{n+3}

$$a_1, a_1 (\times q), a_2 (\times q), a_3 (\times q), a_4 (\times q) \dots a_n \quad , \quad a_1, a_1 q, a_1 q^2, a_1 q^3 \dots a_1 q^{n-1} \Rightarrow a_n = a_1 a^{n-1}$$

Equation 1: the formula of geometric progression sentences

The sequence extracted from the geometry of interior ornament of the dome of Sheikh Lotfollah Mosque ascending. Like some fractal patterns that have reached their final shape with exponential order and ascending growth. The use of mathematical laws in the construction and design of these important urban buildings in the Islamic era was done with special purposes, both abstractly to express the designer's aim with concepts such as unity in plurality and structurally based on precise calculations.

Proportions and numerical regular pattern: In the geometry of the sunflower, the seeds are wrapped around a hypothetical axis, with two types of spirals, one clockwise and the other counterclockwise (Fig. 4), which has proportion and numerical pattern that are very similar to the elements under the dome of the Sheikh Lotfollah Mosque.

Inspiration from nature: The natural world, the highest manifestation of divine wisdom, created by the Absolute Unity as a reasonable and therefore mathematical reality, is reflected by the sacred architectural structures in a magical and symbolic language to express divine order, harmony, and beauty (Hosseinzade, 2017). Islamic architecture represents a successful example in extracting the mathematical proportions and the fractal geometry of the natural organisms (Abdelsalam & Ibrahim, 2019). The interior ornament of the dome of the mosque s of the Islamic period is often solar and stellar, and thus indicates the late acquaintance of the arch, the dome, and the sky (Hillenbrand, 19, 2001). Decorative and structural elements such as muqarnas, Islamic motifs, Shamseh, elements under the dome, and its hemispherical space, all have a natural origin.

Dynamic architecture: In the geometry of this dome, we will see that by changing the observer's position relative to the dome, in most cases we will see a change in the architectural position, while the object is fixed, and we will turn our eyes to the center (Mostaghni & Alimoradi, 2016). So far, by analyzing all the components used in this building, we have summarized in Table 2.

Plan of the Sheikh Lotfollah Mosque

The building of this mosque is based on a quadrangle and in the upper parts they are connected to the octagon and in the end, they are circularly connected to the stem of the dome. In this mosque, with a 45-degree turn, the original shape is rotated and the spaces divided around it are repeated at a constant angle. By analyzing these results, it can be concluded that the repetition of self-similar spaces and micro-scales with spatial features concerning the basic shape of the whole plan of the
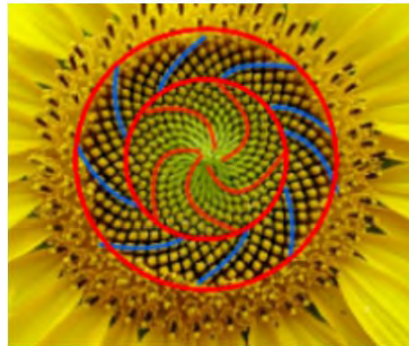
Fig 4: Geometrical pattern of sunflower in nature

Table 2: summary of extracted elements with fractal geometry

Extracted elements	Features of related fractal patterns
- Muqarnas - Decagram (Shamsa) - Islamic star - Geometry motifs of ornaments and tiles	Inspiration from nature. Proportions and numerical regular patterns. The repetitive rhythm, harmonic, and balance. Visual similarity at any scale. Non-linear characteristic of shapes.

mosque (square), creates a variety of space that visitors find themselves in a dynamic space by passing through these spaces branching from the dome of the Mosque (Fig. 5). Here, the architect's goal in designing the space plan, as in other cases such as ornaments, is to create the concept of unity while a plurality and the dependence of smaller components in terms of use and appearance with the main space indicates the artist's harmony with his religious beliefs to build a mosque. By passing the visitor through similar spaces on a smaller scale, which is decorated with fractal roles and tiles, he forms a space similar to the less detailed ones to gradually prepare people to enter the main space (the world of spirituality).

Spaces were designed purposefully in Islamic architecture, and the architect sought to convey his message to the viewer with tools such as fractals that evoked emotions and created a connection with them.

Just as Salehi et al. in a related study describe how to communicate with the viewer, here observer goes through this process of perception. "The manner of creation and perception of the work and the manner of presentation and the advent of reality in architecture is examined by determining nature and the manner of emotions, perceptions, and cognition" (Salehi et al., 2016).
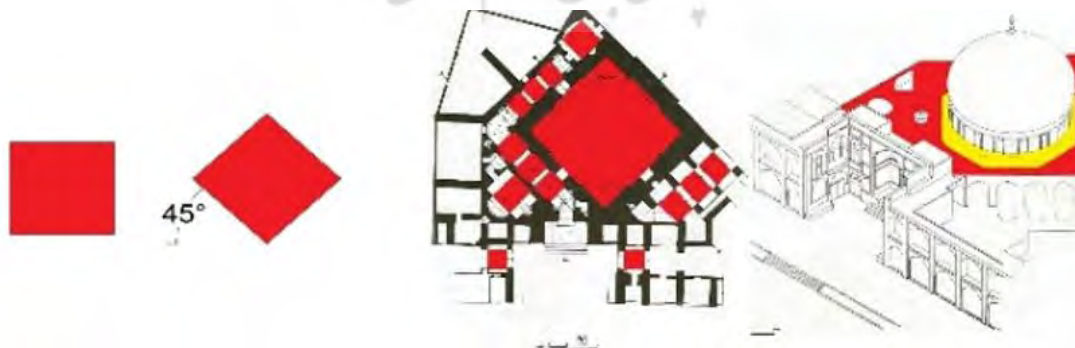

Fig. 5: Rotate the initial plan at a 45-degree angle and self-similar spaces around the central space (main plan's source: Pirnia, 2008).

Method of Visual Analysis in Small Scale

A visual analysis of the architectural monument is an observation of the buildings to identify their self-similar elements which are the fractal patterns or identification of mathematical fractal forms. This is an important stage of the study because the visual analysis helps to achieve some important tasks. First, it will determine whether the fractal models are present (like Sierpinski's "napkin" or the Menger sponge[6], etc.) in the plans, elevations, and other significant parts of the building. These fractal properties aids to study the building itself (Shishin & Aldeen Ismail, 2016). Adapting self-similar elements of the Sheikh Lotfollah Mosque with four main features of fractals as determined before, we have collected the extracted results in table 3 by drawings and details.

Box-counting Analysis Method

In all self-similar constructions, there is a relationship between the scaling factor and the number of smaller pieces that the original construction is divided into. This is true for fractal and non-fractal structures, the relationship is the power-law (Equation 2). (Bovill, 1996, 24).

$$a = \frac{1}{(S)^D}$$

Equation 2: Formula for fractal dimension

Another is the box-counting dimension where the iterative procedure is used (Parashar & Bandyopadhyay, 2014) (Equation 3).

$$D_b = \frac{[\log(N(s_2) - [\log(N(s_1)]]}{[\log\left(N\left(\frac{1}{s_2}\right)\right) - \log\left(N\left(\frac{1}{s_1}\right)\right)]}$$
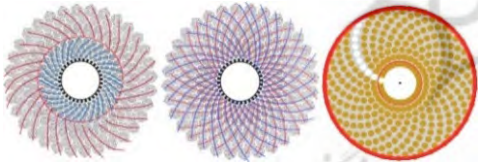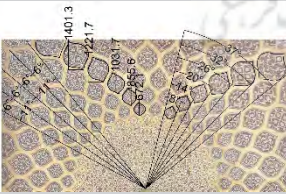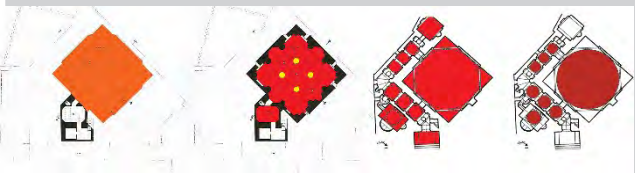
Equation 3: Formula for Box-counting dimension.

D: Mandelbrot's fractal dimension (for non-fractal structure the exponent D is an integer)
S: display the grid size of square boxes over the image
N: count the number of boxes that contain some of the images.

First, we count the number of boxes of section (Fig. 6), elevation (Fig. 7), plan (Fig. 8), and interior ornaments of the dome (Fig. 9) which contain a piece of image with different

Table 3: Analyzing and extracting the fractal geometry of the Sheikh Lotfollah Mosque

Fractal characteristic	Extracted fractal geometry
	Repeat the interior ornament of the dome in a fixed spiral pattern (Self-similarity, proportion, and balance).
	"Similar" means that the relative proportions of the internal angles and shape's sides remain the same (Sala, 2013). Repetition and growth of the dome's patterns under regular and proportional angles.
	Nonlinear and spatial geometry, scale change, and rhythmic repetition to achieve the overall shape are visible in the main stages of Muqarnas formation.
	The self-similar branched shapes, rhythm, and hierarchy of scaled spaces are all fractal attributes (Abdelsalam & Ibrahim, 2019).

grid sizes, and according to the formula, we have calculated the required mathematical exponents in tables 4 to 7.

By calculating the dimension of fractals in checkered images, non-integer numbers were obtained that demonstrate the

decimal dimension of this mosque's components. Finally, to find the harmony between the fractal components in the structure, i.e. the section, elevation, plan, and interior ornaments of the dome, we find the figures of log (S) in the
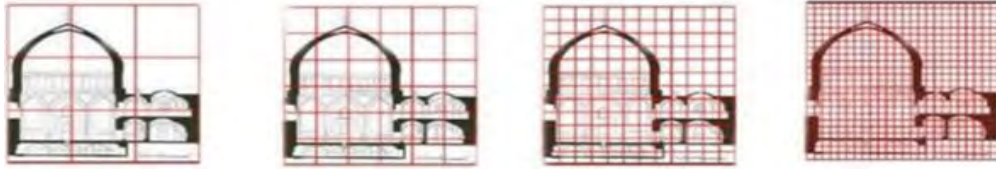
Fig. 6: Grids places over the section of the Sheikh Lotfollah Mosque

Table 4: Box-counting analysis of section

Step	Grid size (S)	Marked boxes (N)	Log (S)	Log (N)
1	24	8	1.380	0.903
2	12	28	1.079	1.447
3	6	101	0.778	2.004
4	3	351	0.477	2.545

$$D(24-12) = \frac{\log(28) - \log(8)}{\log(24) - \log(12)} = \frac{0.544}{0.301} = 1.807$$

$$D(12-6) = \frac{\log(101) - \log(28)}{\log(12) - \log(6)} = \frac{0.557}{0.301} = 1.850$$

$$D(6-3) = \frac{\log(351) - \log(101)}{\log(6) - \log(3)} = \frac{0.541}{0.301} = 1.797$$

Fig. 7: Grids places over the elevation of the Sheikh Lotfollah Mosque

Table 5: Box-counting analysis of elevation

Step	Grid size (S)	Marked boxes (N)	Log (S)	Log (N)
1	24	11	1.380	1.041
2	12	34	1.079	1.531
3	6	104	0.778	2.017
4	3	400	0.477	2.602

$$D(24-12) = \frac{\log(34) - \log(11)}{\log(24) - \log(12)} = \frac{0.49}{0.301} = 1.627$$

$$D(12-6) = \frac{\log(104) - \log(34)}{\log(12) - \log(6)} = \frac{0.486}{0.301} = 1.614$$

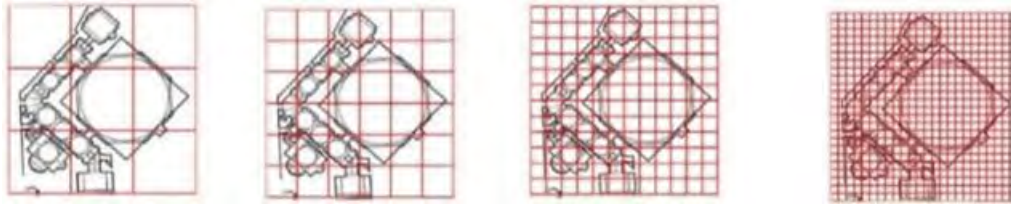$$D(6-3) = \frac{\log(400) - \log(104)}{\log(6) - \log(3)} = \frac{0.585}{0.301} = 1.943$$

Fig. 8: Grids places over the plan of the Sheikh Lotfollah Mosque

Table 6: Box-counting analysis of the plan

Step	Grid size (S)	Marked boxes (N)	Log (S)	Log (N)
1	24	9	1.380	0.954
2	12	24	1.079	1.380
3	6	84	0.778	1.924
4	3	295	0.477	2.469

$$D(24-12) = \frac{\log(24) - \log(9)}{\log(24) - \log(12)} = \frac{0.426}{0.301} = 1.415$$

$$D(12-6) = \frac{\log(84) - \log(24)}{\log(12) - \log(6)} = \frac{0.544}{0.301} = 1.807$$

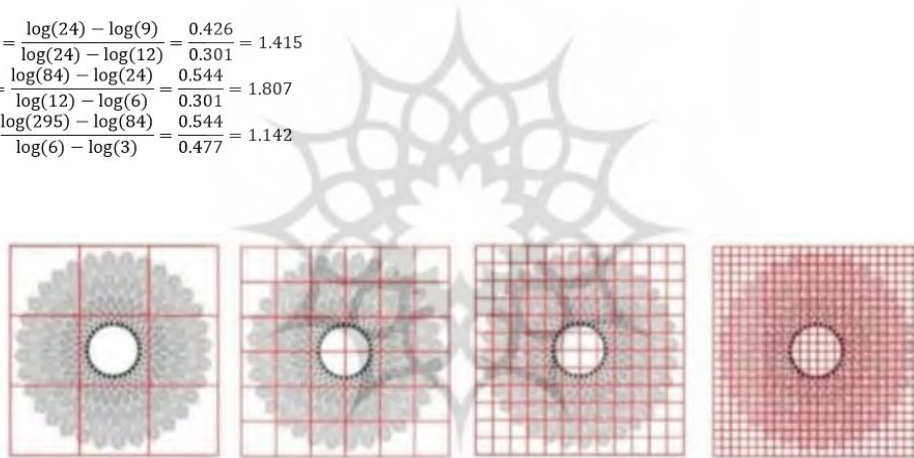$$D(6-3) = \frac{\log(295) - \log(84)}{\log(6) - \log(3)} = \frac{0.544}{0.477} = 1.142$$

Fig. 9: Grids places over the interior ornament of the dome of the Sheikh Lotfollah Mosque

Table 7: Box-counting analysis of the interior ornament of the dome

Steps	Grid size (S)	Marked boxes (N)	Log (S)	Log (N)
1	24	9	1.380	0.954
2	12	32	1.079	1.505
3	6	118	0.778	2.071
4	3	426	0.477	2.629

$$D(24-12) = \frac{\log(32) - \log(9)}{\log(24) - \log(12)} = \frac{0.551}{0.301} = 1.830$$

$$D(12-6) = \frac{\log(118) - \log(32)}{\log(12) - \log(6)} = \frac{0.566}{0.301} = 1.880$$

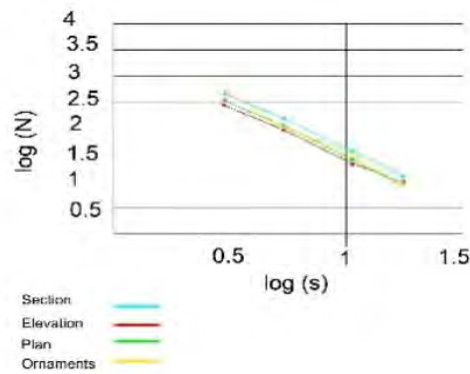$$D(6-3) = \frac{\log(426) - \log(118)}{\log(6) - \log(3)} = \frac{0.558}{0.301} = 1.853$$

Fig. 10: Compatibility between the main structural elements of Sheikh Lotfollah Mosque

table on the x-axis and for log (N) on the y-axis. The resulting line equations illustrate the slope of the almost identical lines and the compatibility and coordination of the constituent components (Fig. 10).

CONCLUSION

In this paper, the Sheikh Lotfollah Mosque was studied and according to the obtained tables, we extracted fractal geometry alongside fractal dimensions, comprehensively. During a step-by-step process, we were able to answer the research questions in the form of the following results:

- First, with the technique of visual analysis and the study of theories and definitions in the field of widely used elements

in mosque ornaments, we collected them by analysis and objective presentation in tables, and examined fractal patterns in the entrance Muqarnas to the interior ornaments in terms of self-similarity, reproducibility (repetition) and small-scaling properties, and by drawing angles and sizes, we concluded that smaller components are a piece of a larger one. Under internal angles commensurate with the same repetition in the same direction, they are regularly scaled to reach a single point. Hence, fractal can be extracted and calculated from its structure. As described at the beginning, the main focus of this research was to find fractal patterns in all aspects of this building to have a new word compared to previous researches.

- In general, rhythmic attributes, self-similar forms, and fractal divisions have all been used to emphasize the function of the

Table 8: Summary of fractal geometry extracted from the Sheikh Lotfollah Mosque

The manifestation of fractal in the language of Islamic architecture	Fractal features	Extracted elements
Unity in diversity	Repeat geometric patterns using repetition, transfer, and symmetry on one level to achieve a single shape.	- Regular numerical patterns of interior ornaments of the dome. - Geometric sequence in elements under the dome. - Shamsa
Dynamic architecture	Use self-similarity and repetition of components to achieve the overall shape. Rotation and distribution of small-scale spaces relative to the main space.	- Geometry motifs (rhombus-shaped elements under the dome) - Hierarchy of particles and spaces
Infinity (a symbol of the infinite world)	Using the similarity of scales change, repetition, and non-linearity due to its spatial dimension. Inspiration from nature.	-Muqarnas of entrance and altar - Shamsa - Geometry motifs of ornaments
The embodiment of the spiritual world	Inspiration from nature (the sky, stars, and the sun).	-Ornamental motifs - Muqarnas - Shamsa

mosque and to reflect the purpose of the architect. From the beginning, we see simpler fractals with fewer components than the interior space of the main dome, which has more intricate fractal motifs than other parts of the mosque, which shows the architect's language to express the concept of unity and forces the observer to ponder in a spiritual space. How to expand the spaces around the main space and arrange them with specific goals to complete the process of perception and structure of the observer by gradually passing through small scale spaces than the main space, in a fractal sequence. The geometrical sequence extracted from patterns of interior ornaments of the dome is like some fractal patterns that have reached their final shape with exponential order and ascending growth. Purposed can be done both abstractly to express the designer's aim with concepts such as unity in plurality and structurally based on precise calculations.

- For the designer to believe in the unity of the Creator, in the construction of this dome, interior ornaments with smaller-scale components have been used to finally joined the whole collection, which is the same as unity. In a way, the semicircular form of the mosque's dome makes visitor's eyes always pass through the hierarchy that smaller components have created and drawn to the peak and unity of existing fractal unity.

- The figures obtained from the Box-counting technique prove both the decimal properties of fractals and, with their linear equations, and the manner of component's behavior towards each other. The endogenous order in fractal forms that makes them sustainable, here too, has led to the creation of sustainable architecture with harmonious components.

Table 8 summarizes these thoughts in the form of fractals.

ENDNOTES

1. An ornamental element in Islamic architecture with three-dimensional behavior.

2. Regular geometric shapes known in Persian as Gereh.

3. ten-pointed Islamic star with asymmetrical geometry which is known as a Shamsa which means "Sun" in Persian.

4. Stucco is a typically Persian art form for the decoration of dome interiors (Sarhangi, 1999).

5. The prayer-niche, or Mihrab, is indisputably a creation of sacred art and has become in practice a regular element in the liturgy, though not an indispensable one (Burckhardt, 90, 2009).

6. It is one of the volumetric types of fractal series which has the features of fractal series in surface and line.

REFERENCES

Abdelsalam, M., & Ibrahim, M. (2019). Fractal dimension of Islamic Architecture: The case of the Mameluke Madrasas: Al-sultanHassan Madrasa, *Journal of Science Gazi University.* 32 (1). 27-37

Abeddoost, H., & Kazempour, Z. (2016). Analysis of the origins and concepts of geometric patterns of the Islamic period in the Persian art. *Journal of Negarineh Islamic Art.* 3(10). 41- 58

Alik, B., & Ayyildiz, S. (2016). Fractal and fractal design in architecture. *13th international conference "standardization, prototypes and quality: A means of Balkan countries' collaboration".* November 3-4, (PP. 282-291). Romania: Transilvania University of Brasov.

Bovill, C. (1996). *Fractal Geometry in Architecture and design.* Boston: Birkhauser.

Burckhardt, T. (2009). *Art of Islam, Language and meaning.* Foreword by Seyyed Hossein Nasr, World Wisdom, Inc.

Elgohari, A. (2019). The Transformation of Using Fractal Forms between Islamic and Digital Architecture from a Sustainable Approach (Case study: King Abdullah Petroleum Studies & research center, Riyadh, K.S.A), *The Academic Research Community publication.* 3 (3), 87-98. DOI:10.21625/Archive.v3i3.526.

Haghtalab, T., & Karvan, F. (2012). The mosque, manifestation of holy architecture. *Haft Hesar Journal of Environmental Studies.* 1(1). 21- 28.

Hillenbrand. R. (2001). *Islamic Architecture: Form, Function and Meaning.* (Translated by: Bagher Ayatollahzadeh Shirazi). Tehran: Rowzanehnashr publications.

Hosseinzade, H. (2017). Sacred geometry in nature and Safavid architecture, *Research conference architecture and urbanism Islamic history of Iran.* DOI: MEMRICONF01-252.

Lu, x., Clement-Croome, D.& Viljanen, M. (2012). Fractal geometry and architecture design: Case study review. *Chaotic Modeling and Simulation (CMSIM),* 2, 311- 322

Karami Mofrad, R. (2017). The nature of fractal and its influence on architecture. *Conference of Iranian Islamic and Historical Architecture and Urbanism Research,* Shiraz.

Kiani, Z., & Amiriparyan, P. (2016). The structural and spatial analyzing of fractal geometry in organizing of Iranian traditional architecture. *Procedia-social and behavioral sciences,* 216, 766-777.

Mehrali, M. (2013). Sheikh Lotfollah Mosque, Mohammadreza Isfahani's architecture, Isfahan, *Journal of Honar-E Memari Magazine,* 31, 94-99.

Mobini, M., & Fathollahi, N. (2014). Investigating the position of fractal geometry in art and how it appears in the visual arts, *Journal of Paykareh Research in visual art,* 3(6), 7-23. DOI: 10.22055/ pyk.2015.13204.

Mostaghni, A., Alimoradi, M. (2016). Analysis the application of the nature geometry and the fractal in parametric architecture: The interior ornament of the dome of the Sheikh Lotfollah Mosque, *Architecture and Urban planning Journal*, 8(16), 103-122. DOI: 10.30480/ AUP.2016.284.

Nurujjaman, M. D., Hossain, A. & Ahmed, P. (2017). A review of fractal properties: mathematical approach. *Science Journal of applied mathematics and statistics.* 5(3), 98-105. DOI: 10.11648/J. SJAMS.20170503.11.

Osama, A., Sherif, L. & Ezzeldin, Sh. (2014). Fractal geometry in architecture: from formative idea to superficial skin design. *Archdesign Journal.* 14, 39-49.

Parashar, R., & Bandyopadhyay, A. (2014). Fractals, Architecture and Sustainability. *Journal of Recent research in science and technology*, 6(1), 93-96.

Pirnia, M. (2008). *Stylistics of Iranian architecture.* (Edited by Gholamreza Memarian), Tehran: Souroush Danesh publisher.

Pudine, H. (2016). A study on dimensions of fractal geometry in Iranian architecture. Journal of Urban Management, 15(42). 258-266.

Sala, N. (2001). Fractal Models in Architecture: A Case of Study. *Proceeding International Conference on "Mathematics for Living"*, November 18-23, (pp. 266-272). Jordan.

Sala, N. (2013). Fractal geometry and superformula to model natural shapes*, Journal of research and reviews in applied science.* 16(4). 78-92.

Salehi, S.; Naghizadeh, M.; & Habibi, F. (2016). The relationship between artists and audience in the architectural work as a creator of cognition factor. *International Journal of Architecture and Urban Development*, 6(3), 35-46.

Sarhangi, R. (1999). The sky within mathematical aesthetics of Persian dome interiors. *Nexus Network Journal,* 1 (1&2), 87-98 . DOI: 10.1007/s00004-998-0007-z.

Shishin. M. Y., & Aldeen Ismail, Kh. J. (2016). A method of compositional fractal analysis and its application in Islamic architectural Ensembles. *International Electronic Journal of Mathematics Educatio*n, 11(5), 1087-1100.