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ʿAlī Muḥammad Iṣfahānī and the Logarithm of Numbers[1]

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Abstract

ʿAlī Muḥammad Iṣfahānī (1215/1800–1293/1876), was one of the famous scientific figures of Qajar period who passed the formative years of his scientific and professional life in Iṣfahān, i.e. before being acquainted with the European sciences through the modern institution of Dār al-Funūn. Several scientific works of Iṣfahānī were written in the style of the ancient mathematical treatises like *Miftāḥ al-Ḥisāb* by Kāshī or *ʿUyūn al-Ḥisāb* by Muḥammad Bāghir Yazdī, whereas his other works seem to have been written in the style of the pedagogic books of Dār al-Funūn. The historians have mentioned some titles among Iṣfahānī's works which unfortunately are not available today and therefore the attribution of some innovations to Iṣfahānī remains ambiguous. One of the innovations attributed to Iṣfahānī is the discovery of the extracting the logarithm of the numbers. This attribution is repeated in some historical Persian sources such as *The Diffused Articles* by Abu al-Ḥasan Furūghī, *The Algebra of Khayyām* by Gholamhossein Mosaheb, *The History of Islamic Sciences* by Jalal al-Din Homaee, etc. Nevertheless, the historian of mathematics, Abu al-Ghasim Ghorbani, believes that the attribution of discovery of the logarithm to Iṣfahānī could not be more than a legend, though after examining some historical facts, Abd al-Hossein Moshafi shows that this attribution is not far from reality. In this article, after introducing this mathematician and mentioning the previous debates about the subject, I will try to shed some light on these ambiguities with the help of the extant documents.

Keywords: ʿAlī Muḥammad Iṣfahānī, Ghīyāth al-Dīn Jamshīd Thānī, Najm al-Dawla, the discovery of logarithm in Iran, Muḥammad Bāghir Yazdī.

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Biography

ʿAlī Muḥammad ibn Muḥammad Ḥusayn Iṣfahānī, known as Ghīyāth al-Dīn Jamshīd Thānī, is a mathematician of the Qajar period who was born in 1215/1800 in Isfahan and died in 1293/1876 in Tehran. We can divide his life into two periods: the first period is when he lived in Isfahan and before his immigration to Tehran, the second period is after his immigration to Tehran in about 1268/1851. All we know about the first period of his life in Isfahan is that he had started to learn the ancient sciences, especially mathematics, in his native city, achieved a high level of mathematical knowledge and wrote precious treatises like *The Completion of the Spring-heads* [*of Arithmetic*] (*Takmilat al-ʿUyūn* [*al-Ḥisāb*]), and he had several students including his own children.

His eldest son, ʿAbd al-Wahhāb (1250/1834-1289/1872), was an astronomer and his second son, ʿAbd al-Ghaffār, known as Najm al-Dawla (1255/1839 or 1259/1843-1326/1908), was one of the representative scientific figures of the Qajar epoch. Not only ʿAbd al-Ghaffār was among the earliest graduate students of Dār al-Funūn, but also he became one of the principal mathematics teachers of Dār al-Funūn at an early age and produced several textbooks on several mathematical disciplines to be used as course books in Dār al-Funūn. It is said that Iṣfahānī had a daughter who was the mother of the famous physician Mīrzā ʿAlī Khān Nāṣir al-Ḥukamā – known as Aʿalam al-Mamālik – (Moallem Habib Abadi, II, 554), the first cataract surgeon in Iran.

Among Iṣfahānī's disciples, there is a certain Muḥammad ʿAlī Ḥusaynī Qā'inī Bīrjandī Iṣfahānī (1224/1809-1311/1893), another great mathematician of Qajar epoch, who in some of his treatises and commentaries mentions Iṣfahānī as his master.[1] Besides his own original works,[2] there are lots of important Arabic mathematical treatises – such as the Arabic translation of the *Conics* of Apollonius

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1. See mss. Malik, no. 0601, fol. 62ʳ and Majlis, no. 15537, fol. 116ʳ.
2. Like *A Survey of Sine and Tangent* (about how to derive and apply the sine and tangent of the numbers from the tables; ms. Central Library of University of Teheran, no. 462), *Mashāriq al-ʾAḍwā* (including a geometrical introduction and some chapters on sunrise and sunset, mss. Majlis, no. 6311; Majlis, no. 642/5; Majlis, no. 6310; Majlis, no. 2736/1), *Nahāya al-Īḍāḥ fī Sharḥ-i Bāb al-Masāḥa min* al-Miftāḥ (a long commentary on the chapter on areas and volumes of *Miftāḥ al-Ḥisāb* by Kāshī, mss. Majlis, no. 6383; Majlis, no. 15246; Majlis, no. 1533; Majlis, no. 223), etc.

(*Kitāb al-Makhrūṭāt*),[1] some recensions[2] by Naşīr al-Dīn Ṭūsī,[3] *Tanqiḥ al-Manāzir* by Kamāl al-Dīn Fārsī,[4] *Miftāḥ al-Ḥisāb* by Kāshī,[5] etc. – in which one can find marginal notes and valuable supplementary explanations by Muḥammad ʿAlī Qāʾinī. These marginal notes show that he knew well the ancient mathematics. In fact, he knew European mathematics as well, since he mentions some European achievements in his own works.[6] Several famous mathematicians of the time were trained under supervision of Muḥammad ʿAlī Qāʾinī, such as Mīrzā ʿAbdullāh Riyāzī (d. 1311/1893), ʿAbd al-Ḥusayn ibn Muḥammad Mūsavī Dizfūlī and the sons of Farhād Mīrzā, the Qajar prince. The fact that a good part of the scientific life of these mathematicians was spent in Isfahan, show how this city was still alive in classical scientific learning at the time.

ʿAlī Qulī Mīrzā Iʿtiḍād al-Salṭanah, a prince of the Qajar dynasty and the minister of science of the time, considered himself as a disciple of Iṣfahānī. In his *Translation and Commentary of a Part of* The Chronology of Ancient Nations (*Tarjumi wa Sharḥ-i Bakhshī az* Āthār al-Bāqīya) by Bīrūnī, Iʿtiḍād al-Salṭanah claims that all his achievements in this book are due to the help of the great Master – the Bīrūnī of the time – ʿAlī Muḥammad Iṣfahānī[7]. Furthermore, in an extant autograph anthology by Iʿtiḍād al-Salṭanah, he praises the great scientific dignity and the expertise of Iṣfahānī and calls himself a student of Iṣfahānī.[8]

Another scientific figure whom we can consider as a student of Iṣfahānī is Mīrzā Jaʿfar Mushīr al-Dawla. Although none of the

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1. Library of Mashhad, Astān Quds, no. 5619.
2. The 13th century mathematician, Naṣīr al-Dīn al-Ṭūsī, made new Arabic editions of some classic mathematical sciences which were studied between Euclid's *Element* and Ptolemy's *Almagest*, known as the Middle Books, as well as a few original works by Islamic mathematicians.
3. See for example The Revision of Archimedes' *On the Sphere and the Cylinder* in ms. Majlis no. 6411; The Revision of Archimedes' *Lemmas* in ms. Mashhad, Astān Quds, no. 29374; The Revision of Menelaus' *Spherics* in ms. Majlis, no. 824.
4. Ms. Majlis, no. 168.
5. Ms. Majlis, no. 15537.
6. See the treatise *A Survey of Sine and Tangent*, ms. Central Library of University of Teheran, no. 462, fol. 1r.
7. Ms. Malik, no. 1471, fol. 6r.
8. Ms. Majlis, no. 1453, p. 26.

historical sources has considered him as one of the Iṣfahānī's students, based on an interesting treatise we can include him among Iṣfahānī's students. The treatise is entitled *The Questions Mīrzā Jaʿfar Khān Mushīr al-Dawla put to ʿAlī Muḥammad Iṣfahānī with the Answers*.[1] As we know, Mīrzā Jaʿfar Mushīr al-Dawla was one of the first Iranians who were sent to England in about 1230/1815 in order to study modern sciences and engineering. He was the first person in Iran to give a more or less technical account of the new (i.e. Newtonian) physics (Masoumi, p. 634). The treatise mentioned above is striking since we see that to have the answer to some scientific questions, Mīrzā Jaʿfar Mushīr al-Dawla – a person knowledgeable in modern sciences – addresses ʿAlī Muḥammad Iṣfahānī – a scholar who was rather known for his mastery of the ancient sciences. The questions discussed in the treatise are in the fields of geometry, arithmetic, physics (including gravity, etc.), and the answers by Iṣfahānī sometimes refer to the propositions from Euclid's *Elements* or the introduction of *The Elements of Astrology* (*al-Tafhīm*) by Bīrūnī.

We are not sure when exactly Iṣfahānī moved to Tehran. According to a well-known historical source, *al-Maʾāthir wal-Āthār* by Muḥammad Ḥasan Ṣanīʿ al-Dawla, it was Iʿtiḍād al-Salṭanah, the then minister of science, who invited Iṣfahānī to Tehran because of his knowledge and expertise (Ṣanīʿ al-Dawla, 1363, p. 261). In the extant anthology of Iʿtiḍād al-Salṭanah, the autograph note mentioned before, where the author considers himself as a disciple of Isfahani, bears the date 27th of Dhu al-Qaʿdah 1268/12th September 1852. Furthermore, at the end of a passage in an anthology written by Iṣfahānī himself, it is noted that he has written down this passage on 11th Shawwāl 1269/18th July 1853 in the village Farahzad [in the vicinity of Tehran] while the disease of Cholera was spread all over the Rey region.[2] Therefore, we can confirm that ʿAlī Muḥammad Iṣfahānī had immigrated to Tehran before 1269/1853, i.e. the time when the first modern pedagogical establishment in Iran, Dār al-Funūn, had just been inaugurated. As the

1. Mss. Majlis, no. 1453/42, Majlis, no. 81 (the folios are not numbered).

٢. هذا ما جرى عليه قلم العبد الجانى على محمد بن محمد حسين الاصفهانى في ليلة الأحد العاشر من شهر شوال المكرم ١٢٦٩ في قرية فرحزاد على سبيل الاستعجال مع تراكم الهموم والغموم حين استيلاء الوبا على بلاد الري وقي اللّه عباده منها. (Ms. Majlis, no. 81, the folios are not numbered) historian Muḥammad Ḥasan Ṣanīʿ al-Dawla reports in his *Mirʾāt al-Buldān*, in the beginning years of the foundation of Dār al-Funūn, Iṣfahānī accompanied Iʿtiḍād al-Salṭanah as the examiner of the students and the inspector of the administrative affaires in his visits of the school (Ṣanīʿ al-Dawla, 1367, p.1083).

The most significant mathematical work of Iṣfahānī is *The Completion of the Spring-heads* [*of Arithmetic*] (*Takmila al-ʿUyūn*) which is in fact a 'completion' of *Spring-heads* [*of Arithmetic*] (*ʿUyūn al-Ḥisāb*) of Muḥammad Bāghir Yazdī, the famous mathematician of 11th/17th century. We do not know when exactly Iṣfahānī wrote this treatise,[1] but we are almost sure that it was done before his immigration to Tehran. In this treatise, Iṣfahānī classifies the algebraic equations – whose degree is inferior or equal to three– into twenty-five groups consisting of six binomial equations, twelve trinomials and seven quadrinomials. This treatise which was written in Arabic – the traditional scientific language – is directly related to the algebraic tradition of Khayyām and Sharaf al-Dīn Ṭūsī (Rashed, p. 394).[2] According to Roshdi Rashed (Rashed, p. 400 and Masoumi, p.638):

> From an epistemological point of view, the most interesting fact to be learned from this work is that this mathematician, who evidently knew only a little about the development of mathematics in the eighteenth century, could arrive, only on the basis of his twelfth-century predecessors, at some results which were similar to those obtained by mathematicians of the seventeenth and eighteenth centuries; and to do this only through an arithmetical study of polynomial functions and not by analytical considerations.

Another treatise written by Iṣfahānī is entitled *The Division of Sphere by Means of Planes* (*Taqsīm-i Kura bi Suṭūḥ-i Mustawīya*). In this Persian treatise, of which an autograph manuscript – dated 1274/1858 – exists, the author intends to show how to obtain some

1. Although Roshdi Rashed claims that this treatise has been written in 1239/ 1824, he does not give any reason for this given date (Rashed, p. 294).
2. Ms. Nacéra Bensaou is working on the edition with a French translation and commentary of this treatise as a part of a PhD thesis in Paris-Diderot University.

elements of the segments (such as altitude, diameter, chord, etc.) in a sphere in order to calculate the segments' volumes. In the autograph extant copy, this treatise includes long tables copied by Iṣfahānī's eldest son, ʿAbd al-Wahhāb.[1]

There is also another work by Iṣfahānī which is entitled *The Extraction of the Table of Logarithm of Sines*. The problems discussed in this treatise show the extent to which Iṣfahānī was acquaintance with European mathematics. We will talk about this work later.

It is reported that Iṣfahānī wrote treatises on the music, on alchemy, on the science of the numbers and on the logarithms (Bamdad, pp. 482-483), most of which seem to be lost. Moreover, the French diplomat and philosopher, Comte de Gobineau (1816-1882), writes in his diaries that Mullā ʿAlī Muḥammad knew well the theory of music, although he played no musical instrument (Gobineau, p. 442).

Attribution of the Invention of Logarithm to Iṣfahānī

Historical Background

Since the attribution of invention of logarithm to ʿAlī Muḥammad Iṣfahānī has been always controversial among the Iranian historians, we will first examine the different opinions expressed in this regard.

One of the most ancient texts that attributes the invention of logarithm to Iṣfahānī is by a scholar of 19th and 20th century, ʿAbu al-Ḥassan Furūqī, in his *Disturbed Papers* (or *Diffused Articles*) (*Urāgh Mushawwash*/ *Maghālāt-i Mukhtalifa*) (independently published in 1330/1912; first published as a part of the *Persian Calendar*, extracted by ʿAbd al-Ghaffār Najm al-Dawla in 1328/1910 (Pakdaman, p. 346)). In this text, after having praised the high place of Iṣfahānī in ancient mathematics, Furūqī says that Iṣfahānī discovered, "thanks to his abundant knowledge and his keen intelligence, on the basis of the elements of mathematics inherited from Iranian and Muslim scientists 80 years ago", lots of algebraic rules and the "foundations of the science of logarithm", but unfortunately his discoveries did not propagate, since they coincided with the introduction of the modern western sciences (Furūqī, pp. 38-39). The text of Foroughi implies that the discoveries of

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1. See ms. Majlis, no. 2138.

Isfahani, including his “discovery” of logarithm, go back to about 1250/1830, which is when he still lived in Isfahan.

Another example of this attribution is found in *The Algebra of Khayyam* by the famous mathematician and historian, Gholamhossein Mosaheb (1328/1910-1399/1979). Mosaheb cites ʿAbd al-Ghaffār Najm al-Dawla according to whom his father, Iṣfahānī, could discover the benefits of logarithm by contemplating on a specific phrase which is found in the treatise *Spring-heads of Arithmetic* (*ʿUyūn al-Ḥisāb*) by Muḥammad Bāghir Yazdī “who had been a contemporary of Shah Ismāʿīl or Shah Sulaymān, the Safavid kings” (Mosaheb, pp. 160-161; Najm al-Dawla, p. 8).[1] The phrase to which Najm al-Dawla refers is the following:

“If we want to know the [amount of an] axis of a given arc, we multiply the sine of the half of that arc by itself, and divide the result by the sine of thirty degrees, i.e. the half of the half of the diameter. The result of the division is the axis of that arc. For example: we want to know the axis of fifty degrees. We take the sine of twenty-five degrees, it is 96259482. We multiply it by 2, it becomes 192518964. In fact, it is like the square of the sine of twenty-five degrees. We subtract from it the sine of thirty degrees, which is 9698700; there remains 95529264. Which is like the result of the division of the square of the sine of twenty-five degrees by the sine of thirty degrees. Therefore, it is the axis of fifty degrees.”[2] (Najm al-Dawla, p. 8)

1. As a matter of fact, this citation of Najm al-Dawlat – quoted by Mosaheb – is extracted from his treatise *The Tables of the Logarithm of the Integers from 1 to 1000* (*Jadāwil-i Logārītm-i Aʿdād-i Ṣaḥīḥ az 1 tā 1000*), preserved in Malik National Library, no. 11615, pp. 8-9. I would like to thank Ms. Fatemeh Keighobadi, from Institute of History of Science in University of Tehran, for providing me with a copy of this treatise.

٢. «وإذا أردنا أن نعلم سهم قوس معلومه، نضرب جيب نصف تلك القوس في القوس في نفسه ونقسم الحاصل على جيب ثلاثين درجة، أعني نصف نصف القطر. فالخارج من القسمة سهم تلك القوس. مثاله: أردنا أن نعرف سهم خمسين درجة. أخذنا جيب كه درجة، فكان ٩٦٢٥٩٤٨٢. ضعفناه، فصار ١٩٢٥١٨٩٦٤. فهو بمنزلة مربع جيب كه درجة. نقصنا منه جيب ل درجة وهو ٩٦٩٨٩٧٠٠، بقي ٩٥٥٢٩٢٦٤. وهو بمنزلة الخارج من قسمة مربع جيب كه على جيب ل درجة. فهو سهم ن درجة.»

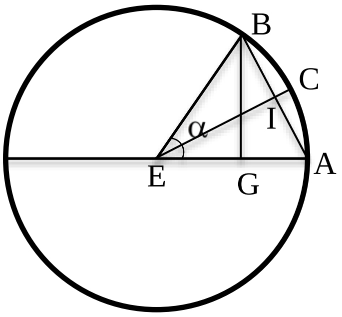

Figure 1

To rewrite this phrase into modern notation, we consider *AB*, the given arc, *AG* is then the desired axis (Figure 1).

$$\overset{\triangle}{ABG} \sim \overset{\triangle}{AEI} \Rightarrow \frac{AG}{AB} = \frac{AI}{AE}$$

$$\frac{AG}{\frac{1}{2}AB} = \frac{AI}{\frac{1}{2}AE} \Rightarrow AG = \frac{AI^2}{\frac{1}{2}r} = \frac{r^2 \sin^2 \frac{\alpha}{2}}{\frac{r}{2}}$$

for $\alpha = 50°$ and $r = 1$:

$$\log(\sin 25°) = -10 + 9.6259482$$

$$2\log(\sin 25°) - \log(\sin 30°) = \log \frac{\sin^2 25°}{\sin 30°}$$

$$\log AG = [(2 \times 9.6259482) - 9.6989700] - 10$$

$$\log AG = 9.5529264 - 10$$ [1]

After having cited the phrase in question, Najm al-Dawla remarks that when Mīrzā Naṣīr Ṭabīb – "a great scholar in many sciences who added marginal notes to this treatise of Yazdī" – confronted this phrase, he wrote in the margin that he cannot understand it. Najm al-Dawla continues that this remained unknown until his father discovered its concept and composed a treatise on how to extract simply the logarithm of numbers, and that this discovery was made in Isfahan in 1240/1824, that is, in a period when there were not still any communications with the Europeans (Mosaheb, pp. 160-162; Najm al-Dawla, p. 9). It should be noted that at the end of his book, Mosaheb adds some supplementary notes. One of the notes concerns the text of Najm al-Dawla on the specific phrase of Muḥammad Bāghir Yazdī. Mosaheb confesses that although he has tried to obtain a copy of the treatise *Spring-heads of*

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1. It should be noted that the numbers which are employed in the specific phrase are in agreement with the numbers in the logarithm tables of Jean-Baptiste Morin (see *Trigonometriae Canonicae Libri Tres*, Paris, 1633, p. 168) or Henry Briggs' tables (see Roegel, D., *A Reconstruction of the Tables of Briggs and Gellibrand's Trigonometria Britannica (1633)*, [Research Report] 2010, p. 172) which are in the excess of the amount of 10.

Arithmetic (*ʿUyūn al-Ḥisāb*) by Muḥammad Bāghir Yazdī in order to verify the specific phrase, he did not succeed. Therefore he believes that the specific phrase attributed to Yazdī is an addition by someone else, since this type of calculation has no other examples in the works of Muḥammad Bāghir Yazdī (Mosaheb, pp. 273-274).[1]

About fifty years after the publication of the first edition of Mosaheb's book, and after the diffusion of the idea of the discovery of logarithm by ʿAlī Muḥammad Iṣfahānī in the historical books such as *The History of Islamic sciences* by Jalal al-Din Homaee (Homaee, p. 144), another Iranian historian, Abu al-Ghasim Ghorbani, attacked this idea and published a note to show that this attribution is not more than a legend. Ghorbani points out that he has checked the treatise of Yazdī carefully, but he has not find the phrase cited by Najm al-Dawla. So he guesses that this citation is probably an addition by one of Yazdī's readers who lived several years after the discovery of the logarithms in Europe and who had access to the European logarithm tables (Ghorbani, 1365, p. 139).

The last attempt to elucidate this problem was made by the Iranian mathematics diligent teacher, Abd al-Hossein Mos-hafi.[2] He took into consideration the beginning part of Najm al-Dawla's note, according to which Yazdī had been a contemporary of "Shah Ismāīl" or "Shah Suliymān", the Safavid kings. As Mos-hafi points out, these two Safavid kings were not contemporaries. On the other hand, given the fact that Yazdī had a grandson with the same first and last name, i.e. Muḥammad Baghir Yazdī, who wrote a commentary on *Spring-heads of Arithmetic* (*ʿUyūn al-Ḥisāb*), the very treatise of Yazdī the grandfather to which Najm al-Dawla refers, Mos-hafi presumes that the phrase quoted by Najm al-Dawla belongs to the commentary of the *Spring-heads of Arithmetic* (*ʿUyūn al-Ḥisāb*), namely *Explanation of the Difficulties of Spring-heads of Arithmetic* (*Sharḥ-i Mushkilāt-i* ʿUyūn al-Ḥisāb) by Yazdī the grandson (Mos-hafi, p. 536). This hypothesis becomes more plausible when Mos-hafi reminds that not

1. It is remarkable that in the second edition of his book in 1339 (SH)/1960, Mosaheb omitted the parts concerning the quotation from Najm al-Dawla and the discovery of logarithm by his father.
2. I would like to thank Dr. Mohammad Bagheri for introducing me Mos-hafi's remarks in the magazine *Danesh va Mardom*, no. 8-9.

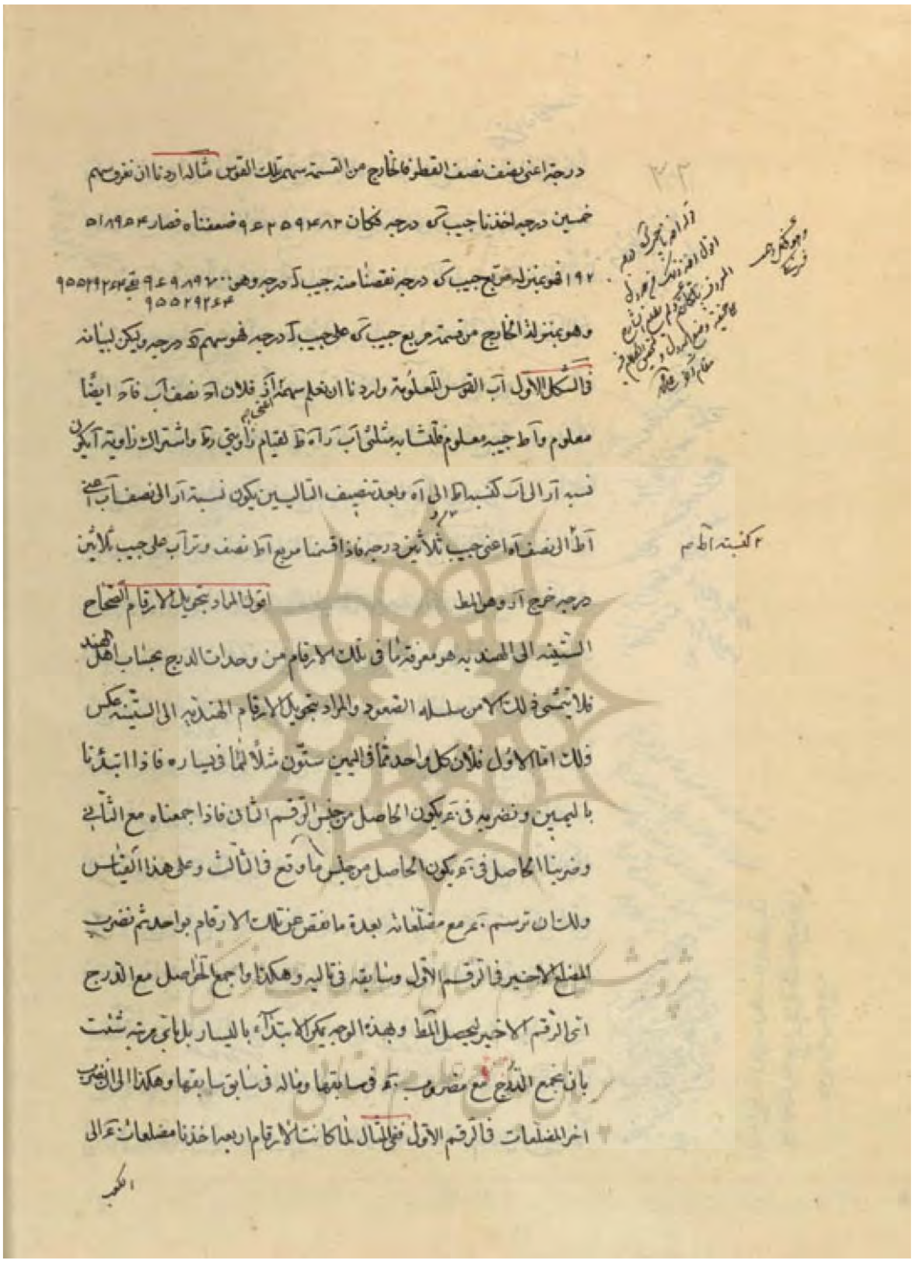
only Yazdī the grandson was a contemporary of Shah Sulaymān[1], but also he introduced some European mathematical results in his commentary on the *Spring-heads of Arithmetic*.[2] As a result, as Mos-hafi presumes, the phrase quoted by Najm al-Dawla was probably due to Yazdī the grandson – in his commentary – and it is written on the basis of European logarithm tables which were accessible at the time. Unfortunately, since Mos-hafi did not have access to any manuscripts of the treatises of Yazdī the grandfather or Yazdī the grandson, he could not improve upon this conjecture. As for the attribution of invention of logarithm to Iṣfahānī, Mos-hafi assumes that since Iṣfahānī had been always eager to solve difficult mathematical problems, one can assume that he had probably discovered the function of logarithm by contemplating on this specific phrase which he eventually had found in the treatise of Yazdī the grandson. Moreover, Mos-hafi suggests that Isfahani had calculated a logarithm table himself which is said to be still kept by his descendants (ibid, p.537).

An examination of the extant manuscripts of the *Explanation of the Difficulties of the Spring-heads of Arithmetic* (*Sharḥ-i Mushkilāt-i ʿUyūn al-Ḥisāb*) by Yazdī the grandson confirms the accuracy of Mos-hafi's conjecture: the phrase cited by Najm al-Dawla is found there. Fortunately, among the extant manuscripts of this treatise, there is a one kept in the library of Majlis (no. 6174/1) with marginal notes by a certain Mīrzā Naṣīr and a few glosses with the signature of Iṣfahānī at the end.[3]

1. As stated by Ghorbani, who possessed the autograph copy of *Explanation of the Difficulties of Spring-heads of Arithmetic* (*Sharḥ-i Mushkilāt-i* ʿUyūn al-Ḥisāb), the author had started to compose his treatise when "Shah Sulaymān" was still alive, because he had written the name of "Shah Sulaymān" in the introduction. But after completing the work in 1106/1694, he revised it by crossing the name "Sulaymān" and writing the name "Husayn" above it (Ghorbani, 1375, p. 440).

2. In his treatise, Yazdī, the grandson, mentions some rules in European mathematical books, according to which the amount for the number "pi" is 3,14, 159 265 358 979 323 847 (Ms. Majlis, no. 6174, fol. 53r).

3. This signature which comports the first name of Iṣfahānī, i.e. ʿAlī Muḥammad, is the same signature which is found in the autograph copy of the anthology of Iṣfahānī (see ms. Majlis, no. 81, the folios are not numbered).



Ms. Majlis, no. 6174/1, fol. 32r: The specific phrase by Yazdī the grandson and the marginal note by Iṣfahānī with his signature (ʿAlī Muḥammad) at the end.

Besides, one of the glosses of Iṣfahānī is found in the margin of the very phrase quoted by Najm al-Dawla. This gloss is as follows:

> He says: "We take the sine of twenty-five degrees."
>
> I say: "we take it from the table known as logarithm table, which is actually sine and tangent. The commentator did not know the truth of the table's arrangement and we should investigate the statement elsewhere."[1]

It is conceivable that this copy of the *Explanation of the difficulties...* – preserved in the library of Majlis – would be the same copy which was in the hands of Mīrzā Naṣīr, Isfahānī and Najm al-Dawla. However, if we assume that this "Mīrzā Naṣīr" is the same "Mīrzā Naṣīr Ṭabīb" about whom Najm al-Dawla talks, in this particular manuscript then there is no marginal note of the latter ending by the phrase: "I cannot understand it". Furthermore, the gloss of Isfahānī indicates that he was def nelyt famil arw thEuropean logarithm tables. Therefore, the claim of Najm al-Dawla concerning the invention of logarithm by his father is not confirmed by this manuscript.

A general view of Isfahānī's extant works on logarithm

In looking for the traces of logarithm among the extant works of Iṣfahānī, two titles in the catalogues of the manuscripts in the libraries of Iran deserve attention. The first one is entitled *Treatise on Extracting the Table of Logarithm of the Sine from this Logarithm* (*Dar Istikhrāj-i Jadwal-i Lukārītm-i Jayb az īn Lukārītm*) and, as far as we know, has reached us through two manuscripts. The second one, which is an anonymous treatise, is entitled *The Tables of Logarithm in Arithmetic* (*the Science of the Numbers*) (*Jadāwil-i Logārītm dar Ḥisāb* (*ʿIlm al-Aʿdād*)), and it has reached us through a unique manuscript. I will show the reasons for attribution of this second treatise to Iṣfahānī later.

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١. قوله: "أخذنا جيب كه درجة." أقول: "أخذ ذلك من جدول المعروف بلكاريتم وهو ظل وجيب. ولم يطلع الشارح على حقيقة وضع الجدول ولتحقيق الكلام فيه مقام آخر."

The *Treatise on Extracting the Table of Logarithm of the Sine from this Logarithm* (*Dar Istikhrāj-i Jadwal-i Lukārītm-i Jayb az īn Lukārītm*)

As mentioned above, two manuscripts of this short treatise exist. One of them, (Malik National Library, no. 601/5), is very likely copied by Isfahānī's disciple, Muḥammad ʿAlī Qāʾinī Bīrjandī. The reason that I ascribe its handwriting to Bīrjandī is that first of all, both for the characters and the numerals, it is the same as Bīrjandī's handwriting in his autograph extant works – such as *A Survey of Sine and Tangent* [1]–; secondly, at the end of the second copy of this treatise (Majlis, no. 2736, fol. 166), which seems to have been copied from the first copy, the copyist, ʿAbd al-Ḥusayn ibn Muḥammad Mūsavī Dizfūlī, asserts that he has copied this treatise from the copy of his master, Muḥammad ʿAlī Qāʾinī Bīrjandī. Unlike the first copy, which is not dated, this second copy is dated 24 Shaʿbān 1286/ 29 November 1869. It is remarkable that at the beginning of the treatise, it is written in the marge: "from the benefits of our master, Ghīyāth al-Dīn Jamshīd Thānī". For this reason, this treatise is sometimes cited among the works of Muḥammad ʿAlī Qāʾinī Bīrjandī, more precisely, an outcome of one of Isfahānī's treatises by Qāʾinī Bīrjandī himself (see Haeri, p. 614; Arshi, p. 133). But it is not true, since the last phrase of this treatise implies that it is totally from ʿAlī Muḥammad Iṣfahānī.[2]

Now, let us look at the content of this treatise.

The three principle parts of this treatise are as follows: 1) calculating the sine of the degrees, 2) calculating the logarithm of the sine of one degree as it has been presented in the book,[3] 3) presentation of Isfahānī's rule in order to calculate the logarithm of the sine of one degree.

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1. See ms. Central Library of University of Teheran, no. 462.

٢. فالرسالة من جملة افادات سيدنا المعظم واستادنا الأفخم الأكرم، مولانا علي محمد دامت أفضاله.

3. Iṣfahānī does not cite the book's name or author. It is probable that one of the European logarithm tables circulated among the students and professors in Dār al-Funūn, and Iṣfahānī refers to these tables.

1) In the first stage, Isfahānī explains how to obtain the sine of the degrees. As an example and in order to clarify the method, he illustrates how to find the sine of one degree, which results in 0.0174524061.[1]

2) In the second stage, he talks about "the method of the author of the book",[2] according to which one needs to use the logarithms of 174 and 175 from the available table, i.e. 2.24054925 and 2.24303805[3] respectively, in order to obtain the difference of the fractional parts which is 248880. Now we have to multiply 248880 by the rest of the fractional part, i.e. 524061; the result is 130427.

To obtain the fractional part, all we need is to add 130427 to the fractional part of the logarithm of 174; the result is 24185352.

Since the decimal part is 8,[4] the final result is 8.24185352.

Isfahānī points out that this result, which is obtained by the rule explained by the author, is not accurate enough, since the number registered in the table has eighteen units more than this number.[5]

He continues to tell that he has invented a rule according to which one could obtain the result with the same accuracy as that of the number registered in the extant table.

3) In the third stage, he explains his own method. Instead of using logarithm of 174, Isfahānī suggests to find logarithm of 174.5 as he reveals in his treatise later. He writes the results of his calculations for finding the logarithms of the numbers of the interval {174.0, …, 174.6} in a table. Then he registers the differences between the results of each two rows in the table, as follows:

1. Except for a few cases that the integer part of number is separated by a "comma", usually there is no decimal mark in the treatise. But in order to keep the calculations justifiable, I put the decimal mark wherever it is needed.

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3. As I mentioned before, usually there is neither decimal mark nor integer part in writing the decimal numbers.

4. It is written in the margin that since the fractional part constitute of a number of nine digits, so the integer part is 8.

5. The table of which Isfahani talks does not exist in the text. But the final result he obtains is 2418553 and he says that this is the number found in the table.

number	logarithm	difference[1]
174.0	2.2405492500	0.0002495181
174.1	2.2407987682	0.0002493763
174.2	2.2410481444	0.0002492345
174.3	2.2412973873	0.0002490927
174.4	2.2415464716	0.0002489509
174.5	2.2417954225	**0.0002488091**
174.6	2.2420442316	

Isfahānī pursues the operation used in the previous stage in order to obtain a more accurate approximation for the amount of the sine of the logarithm of one degree: he multiplies the difference between the logarithms of 174.5 and 174.6 by the rest of the fractional part, to get

$$0.0002488091 \times 0.2406 = 0.0000598634454 .$$

Adding this result to the fractional part of the logarithm 174.5, he gets

$$0.0000598634454 + 0.2417954225 = 0.2418552859$$ [2]

which is more accurate regarding the number registered in the table (0.2418553).

Now the question is how Isfahānī has obtained the numbers in each row of the logarithm column. The way he does this is not very clear, so we only repeat the general lines of his reasoning. Isfahānī considers another table which contains the logarithms of the three numbers 174, 175 and 176. Then he calculates the two differences between these three logarithms and, at last, he calculates the difference of these two differences. He continues the calculation by multiplying the difference in the fractional part and adding the result to the logarithm of the previous number, so that he obtains the resulting number in each raw.

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1. In the extant manuscripts, the numbers registered in this column are without the decimal mark and the zeros in the fractional part.
2. Note that I replace the decimal part of logarithm of 174.5 by zero, in order to keep Iṣfahānī's calculations uniformly.

From Isfahānī's explanation, we can conclude that he had at his disposal a European book on logarithm, containing tables of logarithms and a method for calculating the logarithm of a given number. The problem which faced Isfahānī was that when he calculated a logarithm by the method proposed by the author of the book, the result he obtained was different from what already existed in the table. So he invents a method of interpolation to calculate the logarithm with the same accuracy as that of the tables. His method is the following: instead of taking the logarithms of 174 and 175, he divides the interval in 10 parts to obtain the series {174.1, 174.2 …}. Then he calculates the logarithms of the numbers in this series till 174.6. His method of calculation is little bit obscure, but it seems that he uses the method employed in his European source. That is probably why he does not explain it in much detail.

***The Tables of Logarithm in Arithmetic* (*the Science of the Numbers*)**
Unlike the previous short treatise on logarithm, this treatise is in about 280 pages, most of which are different types of long logarithm tables. There are also a few pages which include notes and the calculations by the author, as well as some explanations on how the values of some of the tables are calculated.

As I mentioned before, this treatise which survives in a unique manuscript in the library of Majlis (no. 1531) is anonymous; nevertheless, we can demonstrate its attribution to Isfahānī with the help of three pieces of evidence:

First of all, the handwriting of the notes diffused in the whole manuscript is the same as Isfahānī's handwriting in his autograph extant treatises such as *The Division of Sphere by the Planes* (*Taqsīm-i Kura bi Suṭūḥ-i Mustawīya*) (ms. Majlis, no. 2138). Secondly, on the margin of page 137, the author employs a numerical example to illustrate how to calculate the sine of an arc (in a sphere) given its altitude. This numerical example and the related explanation are exactly repeated in *The Division of Sphere by the Planes* (ms. Majlis, no. 2138, fol. 4). Thirdly, on the one hand, when in the introduction of his treatise *The Tables of the Logarithm of the Integers from 1 to 1000*, Najm al-Dawla cites some of the works of his father – Isfahānī –, he mentions a long treatise entitled *The Science of the Numbers* (Najm al-Dawla, p.9). On the other hand, on the title page of this anonymous treatise, its title is given in a different hand as "*The Book of the Logarithm Table in Arithmetic*" (*Kitāb Jadwal Logārītm dar Ḥisāb*), and below it, it is written in quotation marks "The Science of the Numbers" (ʿIlm al-Aʿdād). Therefore, based on all these pieces of evidence, it is very likely that the author of this anonymous treatise is Isfahānī and probably this treatise is the very treatise which is mentioned by Najm al-Dawla as *The Science of the Numbers* among the works of Isfahānī. Now the question here is what are all these long tables about? As an illustration of one of the simplest types of these tables, let the given numbers a and b be the logarithms of x and y respectively. Isfahānī explains how to apply the tables in order to find the log (x-y) or log (x+y) in terms of a and b. This treatise deserves an independent study.

Conclusion

In the extant treatises and notes of Isfahānī, there is no explicit mention of the discovery of logarithm properties independent of the European achievements. Nevertheless, as we do not have access to all of his scientific works, especially those which have been written in Isfahan, we cannot draw a definite conclusion. What we can presume as the probable innovation of Isfahānī in logarithm, is his more refined method of interpolation in the first treatise. If we consider Isfahānī as the author of the second treatise on logarithm, we can claim that he has apparently developed the application of logarithm through various tables, which were seemingly designed by himself.

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