Alternative Conditions to Time Inconsistency Equilibrium of an International Monetary Policy

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Abstract

Monetary policy rule is an approach to avoid time inconsistency problem as regarded by new classical economist to choose a time plan for policy making in order to maximize households' wlll-being. The foundation of time inconsistency problem is not coincidence of expectations as an ex-ante variable, which is expected variable, with actual variable as an ex-post variable. Expectations in Finn Kydland and Edward Prescott as the 2004 laureates of the Nobel Memorial Prize in Economics, is rational and formed only by a representative agent because of the discretionary policy of benevolent planner. However the benevolent planner may be as an international planner. In this paper, we develop the model of Kydland and Prescott, by substituting the assumption of heterogeneous households (a domestic household and foreign household) instead of a representative agent and using heterogeneous beliefs. The recent assumption helps us to have an alternative time inconsistency equilibrium with at least two different sources of expectations, which is called *Dichotomy Sources of Expectations (DSE)* as the main contribution of this paper .We then use expectations-adjusted Phillips curve to see the conditions of time inconsistency of k percent monetary rule of Friedman in a framework of DSEss Model. The results show that expectations-adjusted Phillips curve in a society with DSE is not vertical and Friedman's k-percent rule may not be optimal. We find out that, not only an international benevolent planner but also a foreign household must set a rule to maximize the well-being of the world. Indeed, we need a multi-dimensional rule for any international monetary policy.
Keywords: Time Inconsistency, International Monetary Policy Rule, DSE

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1. Introduction

Finn Kydland and Ed Proooơ''s th 2004 uuraasss of the Nobel Memorial Prize in Economic Sciences have transformed the field of macroeconomics with two path-breaking contributions. One is the idea of time inconsistency of optimal policy rules. The second is their analysis of business fluctuations. Their analysis of business fluctuations is the result of a research effort that lasted more than two decades, but, the idea of time inconsistency of policy stems from a single paper, Kydland and Prescott (1977). The idea of time inconsistency is very important and powerful to open up a new line of research in macroeconomics (Tabelini, 2005).

Time inconsistency, describes a situation where decision-makrr's preferences change over time in such a way that what is preferred at one point in time (*t*) is inconsistent with what is preferred at another point in time(*t+1*). This problem is because of special specification of expectations formation. This term is the more commonly used in some cases. For example, in an individual choice, non-exponential discount function would lead to a time inconsistency problem (Fisher and Blanchard, 1993). Indeed if there are different time preferences during the time, there is a time inconsistency problem. Another use of time inconsistency is in economic policy making.

Mankiew (2003) has provided clear definition. He declares that "dssrr ppcccy bttween announcements (what policymakers say they are going to do) and actions (what they subsequently in fact do) is called the time inconsistency of poyyyÿ (Mankiew, 2003, P: 507). Also policymakers at the end of time t claim they are trying to decrease the natural rate of unemployment at the end of time *t+1*. But at the end of time *t+1* they did not succeeded. This is also time inconsistency problem.

Introducing time inconsistency problem would help new classical economics to accept the idea of short run instability by ignoring the rigidities of macroeconomics. This problem affects the cconomy's rransooơ through iim00a "myshows how nnn.. fccss poccymkkrrs' abttttE dd veerrrrrr inflation. Because time inconsistent policies can alter how the economy evolves over time and how the economy responds to shocks, it is important even in environments where inflation is low (Dennis, 2003).

The key point of time inconsistency is expectations formation. In order to show the effect of expectations formation, Kydland and Prescott (1977) specified a model which has two sections, a benevolent planner and a representative agent. They mentioned that expectations of a representative agent only depend on the announcement of the planner. So if the planer does discretionary policy, public as a representative agent could not make decisions at any point of time and this leads to short run instability in economics. One approach to avoid the problem of time inconsistency is monetary policy rules (Romer 2006).

Rules help public to make decisions more confident by converging expectations to the recommended monetary rate of stock money or interest rate derived by the rules. In this way sources of expectations are important. Kydland and Prescott (1977) assume that a planner is the only source of expectations, but it a restrictive assumption in a real world. Because when a group of households follow other households, expectations would not be formed only by the planner and behavioral facts such as belief and herding should be under consideration.

In this paper we substitute the assumption of a representative agent or homogenous household to heterogeneous household in Kydland and Prescott (1977). We use two households, named (*i*), a dominant household and household (*j*), a foreign household, with an international benevolent monetary planner. We suppose that there are societies which sources of expectations are different from each other. It supposes that expectations formation of household (*i*), depends on the behavioral of household (*j*) in the form of herding and contrarian behavior on one hand and benevolent planner on the other hand. Indeed there are two sources for formation of household (*i*'s expccooooos, pnnmrr and househodd (*j*). In contrast, expectation formation of household (*j*) only depends on the planner. So for household (*j*), planner is the only source of expectation which is familiar to the expectations formation of time inconsistency model of Kydland and Prescott. Indeed household (*i*) has two different sources of expectations which we call it DSE.

By DSE, time inconsistency has been extended and the proposed rule based on this expectations formation approach would be different than before. With this alternative view, not only the planner, but also a foreign household must set a rule to avoid the problem of time inconsistency

and short run instability. By including two kinds of households, we could specify time inconsistency solution for three supposed societies.

The first society, we call domestic society, is a society where expectations formation is based on the mechanism of expectations formation of household (*i*). In this society we have DSE because of the existence of two sources of expectations. In second society, we call it foreign society, the expectations formation is based on the mechanism of expectations formation of household (*j*). There is thus no DSE because of only one source of expectations. In third society, we call it world society, its population comprises of combination of household (*i*) and household (*j*). Hence, in this society mechanism of expectations formation of households depends on both household (*i*) and household (j). So, we have DSE because of the existence of two sources of expectations in this society. Under these conditions, we hlen us "expccooo00s-ddjusddd Phillips curve to see the conditions of time inconsistency of Friedman's k-percent rule.

The paper proceeds as follow: In Section 2, we introduce related literature. In Section 3, we develop an augmented model of Kydland and Proooõ'' 1977 (DSE model). In Section 4, we survey the optimal conditions of Friedman's k-percent rule in time inconsistency equilibrium of DSE. In Section Five, we conclude.

2. Related Literature on Time Inconsistency Problem

Th ooneep of ''iim inoonssseency'' was usdd very early on to describe the changing plans of a consumer who regrets his early choices. Allais (1947) is the first contribution on these issues. He considered a consumer with exogenously changing tastes. Indeed iim inoonssseency'' ss hle reason of exogenous tastes. The problem of ensuring consistency in dynamic choice was first addressed in a seminal paper, by Strotz (1956).

He raised a question under what conditions an optimizing consumer would want to adhere to his earlier consumption plan, if he is allowed to reconsider such a plan at later dates. His answer was that, a time inconsistency arises if and only if the individual discounts the utility of future consumption with a non-exponential discount function. Indeed people would not obey their optimal plan of the present moment if they are allowed   their plans at future periods. Because

people are impatient, they give more weight to the earlier time as it gets closer, while causes time inconsistent behavior (also known as hyperbolic behavior).

Pollak (1968) showed that Strotz's result is incorrect. He uses a model with two types of consumers that expectations formation of both is different. The first kind of consumer is called "naive" as he is not aware of his time inconsistency. He knows that his future discounting today is different from discounting that believes tomorrow. On the other hand, the "sophisticated" hyperbolic consumer is aware of his time inconsistency and knows that his future discounting today is as same discounting that believes tomorrow.

The study of Pollak (1968) makes a room to study the idea of time inconsistency in behavioral economics. It means that psychological assumptions and interrelation among economic agents are important to analysis of time inconsistency equilibrium. These studies can be categorized in two groups. The first group refers to the studies that survey dynamic choice of decision making. Some of the issues related to time inconsistent preferences to present the problems of self-control such as Gul and Pesendorfer (2001, 2004) and difference of short and long run time preferences such as Laibson and Harris (2001). These studies show the relation of time preference in the form of hyperbolic discounting function. Second group focuses on the expectations formation of economic agents. In these studies have shown that the expectation of one group of people depend on the expectation of other people. For example McQueen and Roley (1993) and Boyd et al. (2005) investigate the impact of macroeconomic news on market prr ppppppss' earnings expectations. Carroll (2003) uses a model to show how people derive their expectations from news reports.

3. The Model
Here at first we introduce the time inconsistency model of Kydland and Prescott (1977) and then we introduce the model of DSE.

3.1. Time Inconsistency Model
We first introduce the definition of time inconsistency with T infinite horizon and then summarize it with two period of time (Kydlnad and Prescot, 1977, pp: 475-6).

3.1.1. Time Inconsistency Definition

Let $\pi = (\pi_1, \pi_2, \pi_T)$ be a sequence of policies for periods 1 to T and $x=(x_1, x_2, x_T)$ be the corresponding sequence for economic agents' decisions. An agreed-upon social objective function,

$$S(x_1, x_2, ... x_T, \pi_1, \pi_2, ... \pi_T) \qquad \text{(1-a)}$$

Equation (1-a) is assumed to hold. uurhlrr, 22005), decisions at period t depend upon all policy decisions and their past decisions as follow:

$$x_t = X_t(x_1, x_2, ...x_T, \pi_1, \pi_2, ...\pi_T), \\ t = 1, ... T \qquad \text{(1-b)}$$

In such a framework an optimal policy, if it holds, is that feasible π which maximizes (1-a) subject to constraints (1-b). The concept of consistency is defined as follows:

Definition: A policy π is consistent if, for each time period t, π_t maximizes (1-a), taking as given previous decisions, $(x_1, ..., x_{t-1})$ and that future policy decisions (π_s for s > t) are similarly selected.

3.1.2. Time Inconsistency Equilibrium with two-period

According to Kydland and Prescott (1977), the inconsistency of the optimal plan is demonstrated by a two-period example. The model is as below:

$$S(x_1, x_2, \pi_1, \pi_2) \\ \textit{Subject} \qquad\qquad\qquad\qquad\qquad\qquad to \\ x_1 = X_1(\pi_1, \pi_2) and \; x_2 = X_2(x_1, \pi_1, \pi_2)$$

S is an agreed-upon social objective function. π_1, π_2 are polices at time (1) and time (2). x_1, x_2 are households' *economic* decisions at time (1) and (2). Time inconsistency would be explained as the Equation (1-c), which is the first order condition of social objective function optimality:

$$\frac{\partial s}{\partial x_2}.\frac{\partial x_2}{\partial \pi_2} + \frac{\partial s}{\partial \pi_2} + \frac{\partial X_1}{\partial \pi_2}\left[\frac{\partial s}{\partial x_1} + \frac{\partial s}{\partial x_2}.\frac{\partial X_2}{\partial x_1}\right] = 0 \qquad \text{(1-c)}$$

The policy is consistent only if either the effect of π_2, upon x_1 is zero (i.e., $\frac{\partial X_1}{\partial \pi_2} = 0$) or the effect of changes in x_1, upon S both directly and indirectly through x_2 is zero (i.e., $[[\frac{\partial s}{\partial x_1} + \frac{\partial s}{\partial x_2}.\frac{\partial X_2}{\partial x_1}]$ = 0). Hence, according to conditions of time consistency, $\frac{\partial X_1}{\partial v_2} = 0$ or $[\frac{\partial s}{\partial x_1} + \frac{\partial s}{\partial x_2}.\frac{\partial X_2}{\partial x_1}] = 0$, we know that time consistency means a situation in which preferences of a household is constant during two periods. Indeed one of the important results of the Equation (1-c) is that preferences of household or discounting factor would not change during the time and planner is the only factor of preference changing and would create the problem of time inconsistency. So if pnnmrr's announcements at time t (expected plan) overlap its actions at time t+1 (actual plan) there is a time consistency equilibrium.

3.2. DSE's Model

According to third basic Friedman (1981)ss principles of optimal quantity of money, a monetary theory must distinguish between ex-ante and   ex-post. Thus, all variables used here would be categorized in two classes of ex-ante and ex-post ones. The first we define exact definition of DSE.

Definition: For same household, when sources of expectations formation are not completely the same, there is a DSE. Indeed heterogeneous expectations in the framework of DSE are because of different sources of expectations.

3.2.1. DSE's Assumptions

-There are only two periods: t and $t+1$ in this economy.
-There are two households, household (*i*), a follower and household (*j*), a leader household with a benevolent planner or government.
-Economy is in equilibrium at time t and there is no excess demand at this time.
- Households have the ability to evaluate their expectations in the form of price.
-The planner has no commitment for operation of policy rules.
-Households (*i*) and (*j*) consider the effect of future monetary policy with the Fisher equations with constant velocity of money and output.
-Expectations of household (*i*) and household (*j*) are endogenous rational expectations.
-Households' expectations should incorporate knowledge that households have and believe to be economically relevant,
-Household will not persist with methods of expectations formations that generate "Systematically" incorrect forecasts and which agents know to be systematically incorrect.

These two latest principals of expectations formation recognize that expectations are bound up with knowledge, learning, and error recognition. These two latest axioms help us to use endogenous rational expectations.

3.2.2. Concept of DSE model

If we relax the assumption of homogenous agents, we could make a society with at least two agents with more than one source of expectations. Therefore, expectation formation does not only hold by the planner but also by herding behavior of some people. We could explain time inconsistency problem with two sources of expectations which nature of one source depends on the behavioral actions. These models separate sources of expectations and develop the dimension of expectations formation of time inconsistency by focusing on the behavioral characteristics.

3.2.3. Specification of DSE Model

As explained previously, we assume two households are replaced with a representative agent. In order to formulate expectations, we should consider two different models. A model shows the process of households' expectations formation while another model shows the process of pnnmr's action. The first model is "sub-mod and hh sccond ss called encompassing model.

a) Sub-Model

Discounting of future is the important part of intertemporal choice between consuming now or later. Indeed, decision making with care of discounting of future expectations is formed by the theory of time preference which were found in the work of Böhm-Bawerk (1889) and Fisher (1930). They both were trying to find the question of why people look for rewards for saving money by lending it to others and why they are willing to pay compensation in order to borrow money. The core of their theory is a difference between investment and spending. Indeed, the nature of intertemporal choice is the distinction between immediate enjoyment and possible greater deferred enjoyment (Nyhus and Webley, 2006). In principle, discounting factor is an important factor for expectations formation:

$$\pi_{t+1}^{hi} = \alpha_t . \frac{\pi_{t+1}^{g}}{1 + \rho_t} + (1 - \alpha_t)\eta_t \pi_{t+1}^{hj} \Rightarrow \pi_{t+1}^{hi}$$
$$= \alpha_t \lambda_t \pi_t^{g}$$
$$+ (1 - \alpha_t)\eta_t \pi_{t+1}^{hj} \Rightarrow \pi_{t+1}^{hi} \quad (2)$$
$$= \alpha_t \lambda_t \pi_t^{g}$$
$$+ (1 - \alpha_t)\eta_t \pi_{t+1}^{hj}$$

where π_{t+1}^{hi} is expected inflation of household (i) which is formed at time t and is an ex-ante variable. π_t^{g} is actual inflation at time t or the present value of future monetary policy. π_{t+1}^{g} is actual inflation at time $t+1$. ρ_t is discounting rate and $(1+\rho_t)$ is discounting factor and is shown by λ_t. π_{t+1}^{hj} is expected inflation of household (i) which is formed at time t. So, π_{t+1}^{hi} is influenced by the discounted future value of monetary policy of planner at time t and expectations of household (j).

Equation (2) stands for expectations of one group depending on the expectations of another. For instance, the information is disseminated through weblog or other media. This equation helps us to make a model of expectations formation in which source of expectations is not unique. In this equation there are two sources of expectations (planner and household (j)). η_t is the factor of herding or contrarian behavior. Herding and contrarian occurs in a situation, whnn agen''s private information in swamped by the information derived from directly observing ohlrr's cctoons (Park and Sabourian, 2010).

There are many studies such as Bannerjee (1992, 1993); Contand and Bouchaud (1999), Stauffer and Sornette (1999), Iori (2002), Markose et al. (2004) and LeBaron et al. (2009) that show herding can lead to large price fluctuation and expectations. Here we suppose that If$\eta_t > 0$, thus we have herding behavior and if $g_t < 0$, we have contrarian behavior. We suppose that household (i) has herding behavior being influenced by household (j). If $\eta_t = 0$, it means that household (j) has no effect on the formation of household (i)'s expectations. α_t is the weighed source of expectation that household (i) considers it as formation of the expectations from two sources. $\alpha_t \in [0,1]$. If $\alpha_t = 1$ or 0 we have only one source of expectation but at $\alpha_t = 1$ the only source of

expectation is planner and if $\alpha_t = 0$, the only source of expectations formation of household (i) is another sources. If $0 < \alpha_t < 1$, we have two sources of expectations and it means that household (i) has the ability to form their weight of expectations formation from two different sources of expectations.

$$\pi_{t+1}^{hj} = \frac{\pi_{t+1}^{g}}{1 + \rho_t} \Rightarrow \pi_{t+1}^{hj} = \lambda_t \pi_t^{g} \qquad (3)$$

where π_{t+1}^{hj} is the expected inflation of household (j) which forms at time t and is an ex-ante variable. π_{t+1}^{hj} is only influenced by the present value of future monetary policy. Indeed this formation of expectation is consistent with the conventional time inconsistency of Kydland and Prescott (1977).

b) Encompassing Model
$$\pi_{t+1}^{g} = \gamma_t \pi_t^{g} \qquad (4)$$
and
$$\pi_t^{g} = \gamma_t^{-1} . \pi_{t+1}^{g} \qquad (5)$$

where π_{t+1}^{g} is an actual inflation at time t+1 and is an ex-post variable. This variable shows the actual performance of the planner. γ_t is the multiplier coefficient of monetary policy. According to (4) and (5), planner would affect the actual inflation. We suppose that planner affects the inflation by Fisher equation with the assumption of constant velocity of money.

3.2.4. The Relation between Ex-ante and Ex-post Variables
According to Equation (3), λ_t is a discounting factor or interest rate. This factor depends on a rule that people generally prefer present to future. According to Böhm-Bawerk, interest rate has subjective and objective cases. Diminishing marginal utility of income over time (DMU of income) and time preference are subjective cases and round about production is objective causes of interest rate (Cohen, 2011).We have illustrated reasons of discounting factor introduced by Böhm-Bawerk (Figure 1).

According to Figure 1, we would have $\lambda_t= R+D+T$, where R is roundabout of production, D

is diminishing marginal utility of income over time and T is time preferences.

We suppose that government would affect the ex-post (actual) inflation with monetary policy. Diminishing marginal utility of income over time depends on the relative under provision of present versus future income.
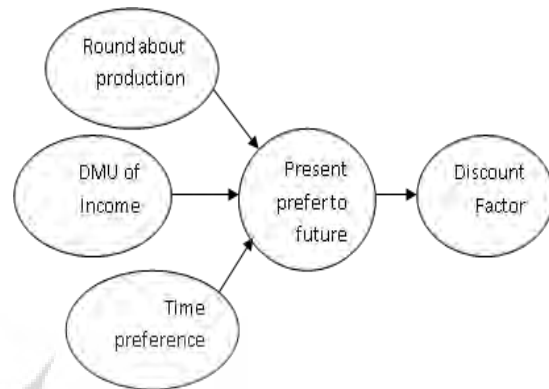

Figure 1: Formation of discounting factor inspired by Böhm-Bawerk
Source: Authors

Households compare present income to future expected income so inflation would affect the future income and would affect the agent preferences. If government affects the inflation by monetary policy we suppose that this effect could be shown by a multiplier coefficient of γ_t.So we could illustrate this effect byFigure2.
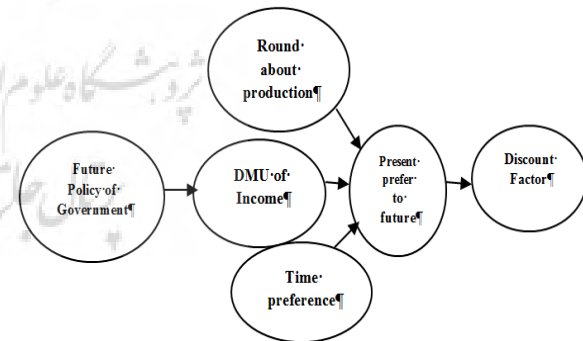

Figure 2: Future Policy of Government effect on discounting factor
Source: Authors

According to Figure 2, we could define the term $\lambda_t = \gamma_t$. But if government makes the policy without commitment, households will be influenced by pessimistic or optimistic belief to run the policy. Indeed, policy with commitment

has definite effect but policy without commitment has no definite effect and this uncertainty depends on the probability which we call it belief to future policy of government. Belief[1] is an idea, concept, or value that an individual holds, with some probability to be true. This term may refer to our understanding of how world functions (MacFadyen, 2006).

In Rational Beliefs Equilibrium (RBE), money is non-neutral. This rationality is, in contrast to the rational expectations results of money neutrality and policy ineffectiveness, leads to a scenario in which monetary policy has an impact on the real economy and price volatility. Rational Beliefs agents have different beliefs and different predictions about the effect of any particular monetary policy (Motolese, 2000). The standard rational expectations, changing in information has exogenous fundamental but an alternative perspective is that changing in belief would change decisions in an endogenously mechanism. So, diverse beliefs would be called Endogenous Uncertainty (Kurz and Motolese, 2008). Households have optimistic and pessimistic beliefs about future (De Grauwe, 2010).

Hence, a future policy of government would affect expected future income and this would cause changing DMU of income and finally changing the discounting factor and present decision making. Now, we would define as $\lambda_t = \gamma_t . \theta_t$ where θ_t is the degree of belief to future policy of government. However, if we suppose that we have diverse beliefs, belief to future policy of government and belief to another household's behavior. Thus, we would write the following:

$$\lambda_t = \gamma_t . \theta_t + \varepsilon_t . \eta_t \qquad (6)$$

ε_t is agent's belief to others. According to equation (6), disconting factor of the agent is a function of beliefs to government, beliefs to another agent (θ_t and $\%$), monetary policy of government (γ_t) and herding and contrarian behavioral (η_t). If households have perfect or full belief to monetary policy of government, $\theta_t = 1$ and if there is no any belief to some agents behavior, $\varepsilon_t = 0$. Hence, if $\theta_t = 1$ and $\varepsilon_t = 0$ or $\eta_t = 0$, then $\lambda_t = \gamma_t$. With diverse beliefs such

as θ_t and ε_t, households could predict endogenously and with η_t and γ_t households could predict exogenously. Indeed this specification would consider both exogenous and endogenous mechanism of a rational expectation.

3.2.5. Models of Expectations Formation
Here we specify three models of expectations formation of households. The first model is assumed that it is based on the expectations formation of household (*i*), while it means two sources of expectations exist. The second model is based on the expectations formation of household (*j*). This model is close to Kydland and Prescott (1977). In third model, it is assumed that there exists a combination of first and second models by which a half of expectations is close to household (*i*) while another is close to household (*j*).

a) First Model: Domestic Society
If we substitute Equation (6) to Equation (2), we have:

$$\pi_{t+1}^{hi} = \alpha_t(\gamma_t . \theta_t + \varepsilon_t . \eta_t)\pi_t^g \\ +(1 - \alpha_t)\eta_t\pi_{t+1}^{hj} \qquad (7)$$

Substituting π_t^g from (5) to (7), we have:

$$\pi_{t+1}^{hi} = \alpha_t(\gamma_t . \theta_t + \varepsilon_t . \eta_t)\gamma_t^{-1} . \pi_{t+1}^g \\ +(1 - \alpha_t)\eta_t\pi_{t+1}^{hj} \Rightarrow \pi_{t+1}^{hi} = \alpha_t(\theta_t + \qquad (8) \\ \varepsilon_t\gamma_t^{-1}\eta_t)\pi_{t+1}^g + (1 - \alpha_t)\eta_t\pi_{t+1}^{hj}$$

where if $\theta_t = 1$, $\eta = 0$, $\varepsilon_t = 0$ and $\alpha_t = 1$, π_{t+1}^{hi} would be equal to π_{t+1}^g.

It means that expected inflation for household (*i*) at time *t+1*, which is formed at time *t*, would be the same as actual inflation of *t+1*, if and only if household (*i*) is not influenced by the behavior of household (*j*), $\eta_t = 0$, $\theta_t = 1$, household (*i*) has no belief to the household (*j*)ss behavior, $\varepsilon_t = 0$, and a planner is the only source of expectations ($\alpha_t = 1$). More specifically, herding behavior would be significant to make the expected inflation (as an ex-ant variable), which is equals to actual inflation (as an ex-post variable). In this society, we would have DSE if $\alpha_t \epsilon(0,1), \eta_t \neq 0$ and $\theta_t \epsilon(0,1)$ and $\varepsilon_t \neq 0$.

b) Second Model: Foreign Society
If we substitute Equation (6) to Equation (3), we have:

$$\pi_{t+1}^{hj} = (\gamma_t . \theta_t + \varepsilon_t\eta_t)\pi_t^g \qquad (9)$$

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[1] To see more about belief, impact of belief, modeling the impact of belief diversity, see Kurz and Arrow (2010).

Replacing π_t^g from (5) to (9), we have:

$$\pi_{t+1}^{hj} = (\theta_t + \varepsilon_t \gamma_t^{-1}\eta_t)\pi_{t+1}^g \qquad (10)$$

where if $\theta_t = 1$, $\varepsilon_t = 0$ and $\eta_t = 0$, π_{t+1}^{hj} equals π_{t+1}^g. It means that expected inflation for household (j) which is formed at time t to predicted inflation at time $t+1$,would be the same as the actual inflation at time $t+1$, if and only if household (j) has belief to the planner, $\theta_t = 1$, and has no belief to another household behavior , $\varepsilon_t = 0$.

c) Third Model: World Society
We suppose that in a world society expectations formation is the combination of expectations formation of household (i) and household (j). Indeed, fifty percent of people hold expectations like household (i) while the others hold like household (j). Hence, the expected inflation for this society would be aggregated to these expectations formation as $\left(\frac{N}{2}\pi_{t+1}^{hi} + \frac{N}{2}\pi_{t+1}^{hj}\right)$, where N is population of the society.

4. Optimal Conditions of Friedman's k-percent Rule in Time Inconsistency Equilibrium with DSE
In this section we first rewrite all expectations formations of three supposed societies in a framework of time inconsistency. We call it time inconsistency of DSE. Then we use Expectations-Adjusted Phillips Curve, to show conditions of time consistency of eeeedmnn's k-percent.

4.1. Time Inconsistency of DSE
Here we are going to show how an ex-ante variable (expected inflation) equals to an ex-post variable (actual inflation) in a framework of DSE. According to Fischer and Blanchard (1993), time inconsistent solutions are obtained when expectations and actions do not happen together. Thus discrepancy between actual inflation and expected inflation would be time inconsistency.

4.1.1. Time Inconsistency in Domestic Society
Now we show the time inconsistency of household (i) through the following equation:
$$\pi_{t+1}^{hi} - \pi_{t+1}^g = \alpha_t(\theta_t + \varepsilon_t \gamma_t^{-1}\eta_t)\pi_{t+1}^g + (1-\alpha_t).\eta_t\pi_{t+1}^{hj} \qquad (11)$$

$$\pi_{t+1}^g \Rightarrow \pi_{t+1}^{hi} - \pi_{t+1}^g = (\alpha_t(\theta_t + \varepsilon_t \gamma_t^{-1}\eta_t) - 1)\pi_{t+1}^g + (1-\alpha_t).\eta_t\pi_{t+1}^{hj}$$

If equation (11) equals *zero*, it means there is time consistency equilibrium in domestic society. Thus policy with the expectations formation like household (i) is time consistency if ($\eta_t = 0$), ($\theta_t = 1$), ($\alpha_t = 1$) and $\varepsilon_t = 0$.

In equation (11), if $\alpha_t\epsilon(0,1)$, we have time inconsistent problem because of DSE, however, if $\alpha_t = 1, \theta_t \neq 0$ or $\varepsilon_t \neq 0$,   we have time inconsistent problem because of the degree of belief to planner and household.

4.1.2. Time inconsistency in Foreign Society
Now we could show time inconsistency equilibrium of household (j) or foreign society through the following the equation:
$$\pi_{t+1}^{hj} - \pi_{t+1}^g = (\theta_t + \varepsilon_t \gamma_t^{-1}\eta_t)\pi_{t+1}^g - \pi_{t+1}^g$$
$$\Rightarrow \pi_{t+1}^{hj} - \pi_{t+1}^g = (\theta_t + \varepsilon_t \gamma_t^{-1}\eta_t - \pi_{t+1}^g \qquad (12)$$

If Equation (12) is equal to zero, we have time consistency equilibrium. Thus, policy with the expectations formation of household (j) is time consistency. If $\theta_t = 1$, $\varepsilon_t = 0$ and/or $\eta_t = 0$, DSE has no effect on time inconsistency equilibrium in foreign society, but belief, herding and contrarian behaviors are important. As time inconsistency in this society is close to Kydland and Prescott (1977), it is now extended by behavioral assumptions such as belief, herding or contrarian.

4.1.3. Time Inconsistency in World Society
If we have N population who are divided into household (i) and household (j), we have:
$$\left(\frac{N}{2}\pi_{t+1}^{hi} + \frac{N}{2}\pi_{t+1}^{hj}\right) - \pi_{t+1}^g$$
$$= \frac{N}{2}[\alpha_t(\theta_t + \varepsilon_t \gamma_t^{-1}\eta_t)\pi_{t+1}^g + (1-\alpha_t)\eta_t\pi_{t+1}^{hj}) + (\theta_t + \varepsilon_t \gamma_t^{-1})\pi_{t+1}^g \qquad (13\text{-}a)$$
$$] - \pi_{t+1}^g \Rightarrow \frac{N}{2}[(\alpha_t + 1)(\theta_t + \varepsilon_t \gamma_t^{-1}\eta_t)\pi_{t+1}^g + (1-\alpha_t)\eta_t\pi_{t+1}^{hj}) - \pi_{t+1}^g$$

Now through normalized population (N=1), we have:

$$\frac{1}{2}\pi_{t+1}^{hi} + \frac{1}{2}\pi_{t+1}^{hj} - \pi_{t+1}^{g}$$
$$= \frac{1}{2}\big[(\alpha_t + 1)(\theta_t \quad \text{(13-b)}$$
$$+ \varepsilon_t\gamma_t^{-1}\eta_t)\pi_{t+1}^{g} + (1 - \alpha_t)\eta_t\pi_{t+1}^{hj})\big] - \pi_{t+1}^{g}$$

According to Equation (13-b), if $\varepsilon = 0, \theta = 1$ and $\eta = 0$ and/or $\alpha_t = 1$, we have time consistency equilibrium. As Equation (13-b) shows, we need augmented considerations to avoid inconsistency problem when sources of expectations are not unique.

4.2. Expectations-adjusted Phillips Curve, Time Consistency and Friedman's k-Percent Rule

Here we survey the relation of the k percent of monetary rule in a framework of DSE.

4.2.1. Expectations-adjusted Phillips Curve in a Framework of Time Consistency

In a framework of "expccooooœs-ddjusddd Phillips, we have:

$$\pi_{t+1}^{e} - \pi_{t+1}^{g} = -\varphi(u - u_n) \qquad \text{(14)}$$

where π_{t+1}^{e} is expected inflation which is formed at time t. π_{t+1}^{g} is actual inflation at time $t+1$. u is actual unemployment and u_n is natural unemployment. φ is an exogenous parameter. The nnuuurll -r""""", "cceerrroooons or "xxpccooooœs-ddjusddd Phiiiips ss vrr π_{t+1}^{g} is equal to π_{t+1}^{e}.

Proposition (1): "Expectations-ddjusddd Phlllpps is vertical; or k percent of monetary rule is optimal if and only if lump-sum taxes or government transfer payments equals the expected inflation of a society.

Proof: According to the Friedman's k-percent rule, we have $M_t = (1 + \tau).M_{t-1}$. τ is lump-sum taxes or government transfer payment at a properly chosen rate. M_t and M_{t-1} are stock of money at time t and time t-1. Normalizing this equation by P_t and manipulate the right side, we then have:

$$\frac{M_t}{P_t} = (1 + \tau).\frac{M_{t-1}}{P_{t-1}}.\frac{P_{t-1}}{P_t} \qquad \text{(15)}$$

If we consider $\frac{M_t}{P_t} = k$ in Equation (15), we then have $P_t = (1 + \tau).P_{t-1}$ or $P_{t+1} = (1 + \tau).P_t$. As we know that actual inflation at time $t+1$ is:

$\pi_{t+1}^{g} = \frac{p_{t+1} - P_t}{P_t} = \tau$. Hence, we rewrite Equation (14) as bellow:

$$\pi_{t+1}^{e} - \tau = -\varphi(u - u_n) \Rightarrow \tau$$
$$= \pi_{t+1}^{e} - \varphi(u - u_n) \qquad \text{(16)}$$

This equation is the condition of optimality of k percent of Friedman rule in a consistency solution.

Therefore, the Friedman's k-percent rule is an optimal policy or "xxpccooooœs-ddjusddd Phiiiips curve is vertical when $\tau = \pi_{t+1}^{g}$. Indeed lump-sum taxes or government transfer payment should be equal to the expected inflation for a society.

4.2.2. The k percent of Friedman's k-percent rule

Proposition (2): Time consistency conditions of Friedman's k-percent rule in a framework of DSEss model are different.

Proof: According to Equations (11), (12) and (13), time consistency of a eeeœdmnn's rule in a framework of DSE depends on $\alpha_t, \theta_t, \varepsilon_t, \gamma_t, \eta_t$ and N. Here we extract three conditions of τ for the three supposed societies.

Condition (1): Domestic Society
According to Equation (11), we have

$$\pi_{t+1}^{hi} - \pi_{t+1}^{g} = (\alpha_t(\theta_t + \varepsilon_t\gamma_t^{-1}\eta_t) - 1)\pi_{t+1}^{g}$$
$$+ (1 - \alpha_t).\eta_t\pi_{t+1}^{hj} \qquad \text{(17)}$$

The consistency for this society means that $(\pi_{t+1}^{g} = \tau)$ is equal to π_{t+1}^{hi}. Substituting π_{t+1}^{g} by τ, we have $(\alpha_t(\theta_t + \varepsilon_t\gamma_t^{-1}\eta_t) - 1)\tau + (1 - \alpha_t).\eta_t\pi_{t+1}^{hj} = 0$. Thus, we would conclude τ as below:

$$\tau = \frac{-(1 - \alpha_t).\eta_t\pi_{t+1}^{hj}}{(\alpha_t(\theta_t + \varepsilon_t\gamma_t^{-1}\eta_t) - 1)} \qquad \text{(18)}$$

Condition (2): Foreign Society
According to Equation (12), we have $\pi_{t+1}^{hj} - \pi_{t+1}^{g} = (\theta_t + \eta_t\varepsilon_t\gamma_t^{-1} - 1)\pi_{t+1}^{g}$.
The consistency for this society means that $\tau = 0$ or $\theta_t + \varepsilon_t\gamma_t^{-1}\eta_t - 1 = 0$. Thus, we have

$$\theta_t + \varepsilon_t\gamma_t^{-1}\eta_t - 1 = 0 \Rightarrow \gamma_t = \frac{\varepsilon_t\eta_t}{1 - \theta_t} \qquad \text{(19)}$$

Condition3: World Society
According to equation (13) we have:

$$\frac{1}{2}\pi_{t+1}^{hi} + \frac{1}{2}\pi_{t+1}^{hj} - \pi_{t+1}^{g}$$

$$= \frac{1}{2}\big[(\alpha_t + 1)(\theta_t + \varepsilon_t\gamma_t^{-1}\eta_t)\pi_{t+1}^{g} + (1 - \alpha_t)\eta_t\pi_{t+1}^{hj})\big] - \pi_{t+1}^{g} \qquad (13\text{-}b)$$

The consistency for this society means that $(\pi_{t+1}^{g} = \tau)$ would be equal to $\frac{1}{2}\pi_{t+1}^{ii} + \frac{1}{2}\pi_{t+1}^{ij}$. Thus, we have τ as below:

$$\tau = \frac{\frac{1}{2}(\alpha_t - 1)\eta_t\pi_{t+1}^{hj}}{\frac{1}{2}(\alpha_t + 1)(\theta_t + \varepsilon_t\gamma_t^{-1}\eta_t) - 1} \qquad (20)$$

According to Equations (18), (19) and (20), time consistency conditions of Friedman's ru   in a framework of DSE model are different.

5. Conclusion

A benevolent international planner would use monetary policy rule to avoid time inconsistency problem to maximize the well-being of households by choosing a time plan for a policy. A significant upshot is that the benevolent planner is unable to make binding commitments. This lack of commitments obligates us to follow the term of belief to accept the effect of future policy. Another important thing is the rationality. If we accept that herding behavior is also rational, we could add another source of expectations that can explain how expectations of one group depend on the others.

This paper developed the expectation aspects of Kydland and Prescott (1977) in three supposed societies with different mechanism of expectations formation because of different sources of expectations. A model with different sources of expectations, as a main contribution of this paper, is called DSE which has different monetary policy rule considerations. Kydland nnd Proooo''s contribution to the normative policy making has been rule rather than discretion. It means that a source of expectation (planner) must set a rule to avoid of time inconsistency.

The contribution to the DSE in an international monetary economy is that both planner and a foreign household should set a rule to avoid the problem of time inconsistency. It means that we need a multi-dimensional monetary policy rule to avoid the problem of time inconsistency in real world. Therefore, to design a multidimensional rule, we need hegemony for monetary policy which is on the principles of widespread humanity values, including all international varieties.

Another contribution of the DSE refers to the conditions of optimality Friedman's k-percent rule in   fram   work of "expccoooooos-ddjusddd Phillips curve. Indeed, with an only international monetary planner, "expccoooooos-ddjusddd Phillips curve in a framework of DSE models may not be vertical because it depends on many behavioral considerations. Hence, different behavioral actions because of different sources of expectations need different considerations to design monetary policy rule and the planner must consider behavioral actions in all societies.

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