

## Portfolio optimization with robust possibilistic programming

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### Abstract

Portfolio selection is one of the most important financial and investment issues. Portfolio selection seeks to allocate a predetermined capital (wealth) over one or multiple time periods between assets and stocks in a such way that the wealth of investor (portfolio owner) is maximized the risks are minimized. In the paper, we first propose a mathematical programming model for Portfolio selection to maximize the minimum amount Sharpe ratios of portfolio in all periods (max-min problem). Then, due to the uncertain property of the input parameters of such a problem, a robust possibilistic programming model (based on necessity theory) has been developed, which is capable of adjusting the robust degree of output decisions to the uncertainty of the parameters. The proposed model has been tested on 27 companies active in the Tehran stock market. At the end, the results of the model demonstrate the good performance of the robust possibilistic programming model.

**Keywords:** Portfolio Optimization, Sharpe ratio, Robust Possibilistic Programming.

## Introduction

Portfolio formation and diversification of assets are among the most fundamental strategies for reducing and controlling investment risks. A good Portfolio selection with high returns and low risk is demanded by all investors. Hence, there are many models for Portfolio selection and many efforts have been made to improve these models. In fact, Portfolio selection of assets is one of the most important issues in the field of investment management.

Several optimization methods have been developed following Markowitz's innovation and his minimum risk model. These methods have tried to generate the highest quality portfolios in terms of risk and return, by adding more metrics in the target function and intelligent constraints. In recent years, in addition to the optimization of portfolios' risk and return, the discussion of the sustainability of results and the need for a gradual change in the weight of assets in the investment portfolio has been raised, given the existence of an uncertainty factor in the level of risk and return of financial assets. Also, using stochastic, fuzzy logic and robustification approaches, attempts have been made to optimize the level of uncertainty in addition to achieving the optimal combination of risk and return. Therefore, risk and return both play a vital role in the investment portfolio, so in designing each model for optimization, it is necessary to consider return and risk maximization in the target function simultaneously. This can be done in the form of a Multi-objective function or by targeting a measurement in a function that is a combination of risks and returns and to target the measurement of maximization/minimization. One approach is using sharpe; that is, a measurement having a risk and return at the same time. Therefore, the Sharpe statistics/ratio was selected.

$$\frac{R_p - R_f}{\sigma_p}$$

$R_p$  : Portfolio return

$R_f$  : Risk-free rate of return

$\sigma_p$  : Portfolio Standard deviation (risk measurement)

In addition to being considered as an appropriate ratio for assessing the relative performance of financial assets, the Sharpe ratio (Sharpe, 1963) maximization in the optimization portfolio model is equivalent to minimizing the risk of falling investment returns below a certain limit (Roy's Safety-First) in the most conservative mode; that is, when the form of the distribution

function is not known, and it is necessary to use Chebyshev inequality, where even the worst types of distributions (distributions with negative skewness, etc.) can be applied to the model, and inject robustification to the model in practice.

In another aspect, due to the competitive and risky space in the stock market, decision-making is often faced with a lack of information or uncertain information; therefore, the model programming should take this into account. Robust optimization is presented in recent years to deal with uncertainty, in which the optimization is addressed when the worst happens. The Robust approach was proposed to solve optimization problems with data uncertainty and has recently been widely explored and developed. The main advantages of this approach are as follows (Alem, Morabito, 2012):

1. Robust optimization is easier than the probabilistic approach in terms of solving the model.
2. There is no need for clear knowledge of the possibilistic distribution of data with uncertainty.

In the subject matter discussed in this paper, due to the incompleteness and unavailability of information, we face uncertainty in data that is of a kind of epistemic uncertainty (Asadujjaman, 2019), therefore, robust possibilistic programming has been used to model the problem. It is superior to possibilistic programming for the following reasons:

1. In robust optimization, the confidence level of constraint satisfaction is determined by the model itself and its value is optimal,
2. In robust optimization, the final answer has Optimality Robustness and Feasibility Robustness
3. Considering the objective function deviations due to the uncertainty of the parameters, it avoids heavy and irreparable costs for managers and investors. In the case of possibilistic programming, the following issues are not taken into consideration.

## Background and Literature review

So far, many research have been done in the optimization of stock portfolios due to the increasing development of global financial markets and the impact of this optimization on economic returns and profits.

In 2009, Huang used the combination of Fuzzy c-means (FCM), a variable-precision rough set (VPRS) model, Autoregressive with exogenous input (ARX) and a gray-system theory for Portfolio selection investing.

Soleimani et al. (2009) presented a genetic algorithm-based approach

with integer constraints and the market share of various industries to optimize Markowitz's MV model.

Chang et al. (2009) introduced a genetic algorithm to solve optimization problems with different risk measurements on the model.

Tiriaki et al. (2009) combined the fuzzy AHP with Portfolio selection. The main approach in AHP was to design and implement a model for combining corporate risk behavior with the investor risk class (low, medium and high), investor goals and internal and external factors.

Montazar et al. (2010) proposed a method for designing a fuzzy expert system for recommending and introducing investment portfolios in the Tehran Stock Exchange.

Zymler et al. (2011) combine a robust optimization portfolio with a classic insurance portfolio model to cover risks from rare events.

Sajjadi and Seyyed Hosseini (2011) proposed a multi-period dynamic fuzzy model for Portfolio Selection of stocks, in which borrowing and lending are possible in real terms (different rates of cash borrowing and lending).

Jun and Lu (2012) used a Mini-Max -based robust ranking model in integer programming.

The constraints in this model are generated using a network streaming model, and ultimately used for portfolio optimization.

Looking deeper into robust investment portfolios, Fabozzi et.al. (2014) analyzed the behavior of these portfolios formed with Robust Optimization. Their research suggested that by increasing robust optimization of investment, optimal weights would be directed towards that portfolio, whose variance is described to the highest by specific factors.

Wu Chang Kim et al. (2014) introduced a new approach to the robustation portfolio of the minimum variance to control the kurtosis and skewness (third and fourth torque) without the aid of higher torques. The main idea in this article is that the robust investment portfolio, in the worst-case scenario, is prone to skewness and opposed to kurtosis.

Pishvae, Razmi, and Torabi (2012) used robust possibilistic programming to design a supply chain.

Pishvae and Kalantari (2012) also used robust possibilistic programming for the primary programming of the drug supply chain.

Millt and Takkapi (2015) presented robust models to meet the needs of investors looking for a global minimum variance portfolio and a rule against robust uncertainty.

In this paper, the Monte Carlo simulation showed the robust portfolio superiority to unrobust portfolios in different dimensions of the risk and variance based adjusted return. They found that the robust investment portfolio had a minimum variance, lower turnover, and a Sharpe ratio compared to traditional portfolios.

Zulfagar and Ayoub (2015), based on a study conducted on the Karachi stock exchange in the area of using the robust downside index, showed that the use of this statistics, especially in relation to assets whose curtailment curves are greater than those of kurtosis, are much better performing compared to Markowitz's Mean-Variance Model.

Balabas and Balabas (2016) put forward the concept of ambiguity with risk in their paper to create a robust portfolio optimization model, and in particular to solve the shortcomings of capital asset pricing models.

Han, Zia and Lee (2016) developed the robust asymmetric model of the absolute mean standard deviation that covers asymmetry in the returns distribution.

They tested various strategies for robustification in emerging markets and falling markets and showed that the model was able to identify lucrative stocks.

In the recent period, and since 2009, Some important points of foreign research include:

A) Among the researches, the use of the Sharpe index as a performance measurement was very high and had a significant difference with other methods. The most significant measurements are CVAR, Torque, kurtosis, Alpha and Treynor ratio.

B) Mathematical modeling, Fuzzy logic, and genetic algorithm are the most widely used algorithms to optimize portfolios of assets. Other algorithms include particle swarm optimization approach, quadrilateral programming model, expert system methodology and goal programming.

### Research gap

The literature review identifies important research gaps. Despite decades of research on investment optimization and risk management, there is still no acceptable tool for risk measurement (The fundamental nature of risk in the

financial field has made it impossible to achieve such a model, since any model of a universal nature will change the behavior of investors in financial markets and ultimately reduce the performance of the model.)

The development of new Investment Portfolio Selection approaches and models with more comprehensiveness that deals with differences, conflicts and is more flexible in dealing with the risk phenomenon (goals (with no definitive means to measure it)) is a perpetual gap.

The research also aims to develop a new model with Robust Optimization and Fuzzy Logic as the goal of Investment Portfolio Selection.

The question now is, what method for portfolio design (in terms of type, volume, and number of financial assets used in the portfolio) should be used in this model, which can maintain its credibility and efficiency over an acceptable time frame, despite the ongoing Volatilities of financial markets.

None of the research on stock portfolio programming in uncertainty has used the robust possibilistic programming approach to deal with this issue. Using this approach will make the model responses determined so that the feasibility robustness and optimality robustness are guaranteed and, consequently, the cost of implementing real-world decisions are reduced. Therefore, sources of uncertainty in the stock market should be effectively managed. And, in order to manage the uncertainty surrounding this environment and to have sufficient confidence in the results, robust programming must be done so that managers can be sure of their results and reduce the risk of their decision making. Programming robust is one of the new and reliable approaches.

### **Research methodology**

Dantzig et al. (1993) proposed a standard framework for multi-period asset allocation problems. They assume risky assets in the capital market; trading periods, linear transaction costs for trading stock and one riskless asset e.g. risk-free deposit, and we have used this framework for portfolio making and objective function based on sharpe ratio.

### **Non-deterministic model of stock portfolio optimization**

In the real world, especially in capital markets, many of the parameters of the problem are subject to change over time and the definitive assumption of these parameters during programming causes errors and problems. In the underlying question, it is assumed that the stock return parameter and, hence, Sharpe ratios

are not definite numbers and are predictable fuzzy numbers. Given the dynamic nature and Volatility of some of the important parameters (Sharpe ratio and stock return), the following equations provide a method for modeling imprecise parameters, and can be defined by their four prominent points (Pishvaei et al. 2012):

$$\tilde{S}_{it} = (S_{it(1)}, S_{it(2)}, S_{it(3)}, S_{it(4)})$$

$$\tilde{R}_{it} = (R_{it(1)}, R_{it(2)}, R_{it(3)}, R_{it(4)})$$

$\tilde{S}_{it}$  represents the fuzzy sharpe ratio in time period t, and  $\tilde{R}_{it}$  represents fuzzy rate of return in time period t

### The robust possibilistic programming model

The evaluation of definitive parameters for long-term decision-making is difficult and sometimes impossible. Even if one could estimate a possibilistic distribution function for these two parameters, these parameters may not have the same behavior as the past data. Different approaches, including possibilistic programming, have been used to address the uncertainty. It should be noted that the uncertainty parameters are suited for the possibilistic functions, such as triangular or trapezoidal possibilistic functions, based on inadequate data or knowledge and experience of modeling decision makers. Therefore, in this paper, uncertain parameters are considered as fuzzy data at any time when it changes in a long-term programming horizon. If the possibilistic programming method is used, in order to control the level of confidence in creating these uncertain limits, the concept of decision can achieve the minimum level of assurance as a safe margin for any of these constraints. To do this, two fuzzy standard methods and practices are commonly used. It is worth noting that the optimistic fuzzy (NEC) indicates the optimistic probability level of an uncertain event involving uncertain parameters, while the pessimistic fuzzy (POS) indicates a pessimistic decision about an uncertain event. However, it is more conservative to use a pessimistic fuzzy, that is, we assume that the decision has a pessimistic (conservative) constraint to create uncertainties; Currently, based on the ambiguous parameters mentioned and the use of the expected value for the objective function and the pessimistic action for uncertain constraints, the obvious equivalent of the uncertain model can be formulated. To do this, one must first consider the abbreviation for the proposed model (tanaka,2000):

$$\text{Max } w = x \tag{1}$$

s. t.:

$$dx \geq x' \quad (2)$$

$$Bx = bx' \quad (3)$$

$$Ex \leq Sy \quad (4)$$

$$y \in \{0,1\}, x, x' \geq 0 \quad (5)$$

It is assumed that vectors  $d$  and  $b$  are presented in the non-deterministic parameters in the above model. Regarding the generic non-deterministic finite program, the expected value of the pseudo-objective and fuzzy function is obtained, respectively, for dealing with the objective function and the uncertain limit. Now with the abbreviation, the basic possibilistic programming model is as follows:

$$\text{Max } E[w] = x \quad (6)$$

s. t.:

$$\text{NEC}\{\tilde{d}x \geq x'\} \geq \alpha \quad (7)$$

$$\text{NEC}\{Bx = \tilde{b}x'\} \geq \beta \quad (8)$$

$$Ex \leq Sy \quad (9)$$

$$y \in \{0,1\}, x, x' \geq 0 \quad (10)$$

In which  $\beta$  and  $\alpha$  control the minimum degree of certainty for establishing a non-deterministic constraint with a pessimistic decision-making approach. Regarding the distribution of the trapezium probability for ambiguous parameters, the general form of relations 6-10 can be defined as follows (tanaka,2000):

$$\text{Max } E[w] = x \quad (11)$$

s. t.:

$$((1 - \alpha)d^1 + \alpha d^2)x \geq x' \quad (12)$$

$$Bx \geq \left( \left( \frac{\beta}{2} \right) (b^4) + \left( 1 - \frac{\beta}{2} \right) (b^3) \right) x' \quad (13)$$

$$Bx \leq \left( \left( 1 - \frac{\beta}{2} \right) (b^4) + \left( \frac{\beta}{2} \right) (b^3) \right) x' \quad (14)$$



$$Ex \leq Sy \quad (15)$$

$$y \in \{0,1\}, x, x' \geq 0 \quad (16)$$

In the possibilistic programming models, the minimum level of confidence to establish a non-deterministic constraint should be determined in terms of decision preferences. As seen, in the proposed model, the objective function is not sensitive to the deviation from its expected value, which means that gaining robust solutions in the possibilistic programming model is not guaranteed. In such cases, there may be a high risk in many real cases of decision-making, especially in strategic decisions that the robustness of the solution is vital. In fact, possibilistic programming has important shortcomings. In probabilistic programming, the constraint satisfaction level is a parameter determined by the decision maker, which does not optimize the confidence level. In the possibilistic programming model, there is little interest in the feasibility of robustness and optimality robustness. On the other hand, the lack of attention to the deviations of the objective function due to the uncertainty of the parameters can lead to irreversible costs for managers and organizations, and this must not be considered in possibilistic programming. Therefore, Pishvae et al. (2012) proposed a robust optimizing program called robust possibilistic programming using the concept of robust optimization. This approach takes advantage of both robust optimization and possibilistic programming, which clearly distinguishes it from other programming uncertainty approaches. The robust possibilistic programming form in the previous model is as follows:

$$\begin{aligned} \text{Max } E[w] = & x - \eta_1(d^2 - (1 - \alpha)d^1 - \alpha d^2)x - \\ & \eta_2\left(b^4 - \left(1 - \frac{\beta}{2}\right)b^3 - \frac{\beta}{2}b^4\right)x' - \eta_2\left(b^4 - \left(1 - \frac{\beta}{2}\right)b^4 - \frac{\beta}{2}b^3\right)x' \end{aligned} \quad (17)$$

s. t.:

$$\left((1 - \alpha)d^1 + \alpha d^2\right)x \geq x' \quad (18)$$

$$Bx \geq \left(\left(\frac{\beta}{2}\right)(b^4) + \left(1 - \frac{\beta}{2}\right)(b^3)\right)x' \quad (19)$$

$$Bx \leq \left(\left(1 - \frac{\beta}{2}\right)(b^4) + \left(\frac{\beta}{2}\right)(b^3)\right)x' \quad (20)$$

$$Ex \leq Sy \quad (21)$$

$$y \in \{0,1\}, x, x' \geq 0 \quad (22)$$

$$0.5 \leq \alpha, \beta \leq 1 \quad (23)$$

In the first objective function, equation 17 of the first expression refers to the expected value of the first objective function, using the mean values of the non-deterministic parameters of the model. The second, third, and fourth lines indicate the total cost of the deviation from the non-deterministic parameter. Hence, the parameter  $\xi$  is the weight function of the objective function,  $\eta_1$ , and  $\eta_2$ , the penalty for not estimating the uncertainty parameter.

The parameters  $\beta$  and  $\alpha$  represent the correction factor at the fuzzy numbers, based on pishvae (2012) which should be between 0.5 and 1.

### research findings

In this section, a dynamic model is designed to invest in a limited number of financial assets (Tehran Stock Exchange and risk-free deposits) over a period of time and with a specified cash budget and at the end of each period on the basis for risk and return data, investments reviewed, some sold, and some new assets purchased. One of the best benchmarks and measurements for the selection of a mass of stocks and financial assets is the Sharpe ratio of each share/asset. In order to achieve real diversification, the weight assigned to each share in the portfolio of investment placed in a certain range (Floor and ceiling) (fabozzi,2007) has been determined, and we have attempted on designing and presenting a conservative portfolio consistent with the facts of real capital markets.

### Definitive modeling

$S_{it}$	Sharpe ratio of $i^{\text{th}}$ share in the time period t
$R_{it}$	Return of $i^{\text{th}}$ share in the time period t
$R_{(n+1)t}$	Cash profit (risk free deposit) in the time period t
$U_{it}$	The maximum weight of the share $i^{\text{th}}$ in the portfolio in the time period t
$L_{it}$	The minimum weight of the share $i^{\text{th}}$ in the portfolio in the time period t
K	Number of authorized shares in the portfolio
cb	Purchase fee (about 0.5% of transaction value)
cs	Sales fee (about 0.6% of transaction value)

**Decision variable**

$Y$	Continuous variable
$X_{it}$	Weight of $i^{\text{th}}$ share in the time period $t$
$X_{(n+1)t}$	Cash amount in the time period $t$
$\sigma_t$	Standard deviation of portfolio in time period $t$
$W_{it}$	Weight of $i^{\text{th}}$ share in the time period $t$ (The result of the $\frac{X_{it}}{\sum_{j=1}^{n+1} X_{jt}}$ )
$Y_{it}$	Sale amount of $i^{\text{th}}$ share in the time period $t$
$Z_{it}$	Purchase amount of $i^{\text{th}}$ share in the time period $t$
$F_{it}$	Binary variable, (if $F_{it} = 1$ then $Z_{it} > 0$ , and $F_{it} = 0$ then $Z_{it} = 0$ )

$$\max(\min_t \frac{\sum_{i=1}^n w_i R_{it} - R_{(n+1)t}}{\sqrt{\sum_{i=1}^n \sum_{j=1}^m (\sigma_{ijt} * w_{it} * w_{jt})}}) \quad (24)$$

s. t.:

$$L_{it} F_{it} \leq \frac{X_{it}}{\sum_{j=1}^{n+1} X_{jt}} \leq U_{it} F_{it}, \quad t = 1, 2, \dots, T, i = 1, 2, \dots, n \quad (25)$$

$$X_{it} = (1 + R_{i(t-1)})X_{i(t-1)} - Y_{it} + Z_{it}, \quad t = 1, 2, \dots, T, i = 1, 2, \dots, n \quad (26)$$

$$X_{(n+1)t} = (1 + R_{(n+1)(t-1)})X_{(n+1)(t-1)} + \sum_{i=1}^n (1 - cs)Y_{it} - \sum_{i=1}^n (1 + cb)Z_{it}, \quad (27)$$

$$t = 1, 2, \dots, T$$

$$k_l \leq \sum_{i=1}^n F_{it} \leq k_u, \quad t = 1, 2, \dots, T \quad (28)$$

$$X_{(n+1)0} = 1000 \quad (29)$$

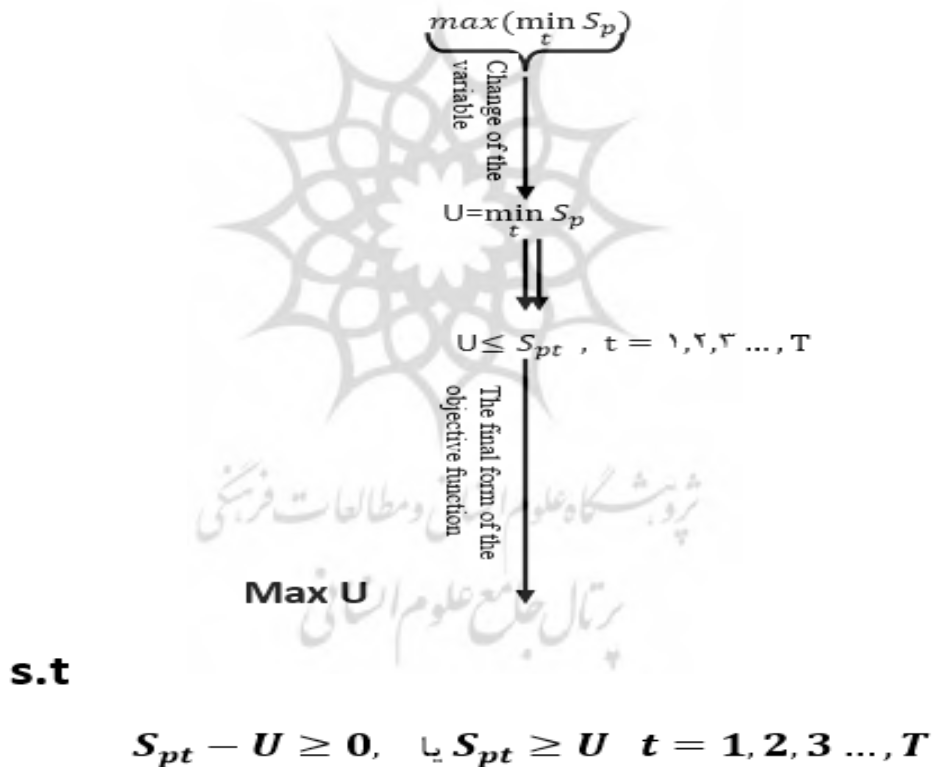
$$X_{it}, Y_{it}, Z_{it} \geq 0, \quad t = 1, 2, \dots, T, i = 1, 2, \dots, n + 1 \quad (30)$$

$$F_{it} = \{0, 1\} \quad t = 1, 2, \dots, T, i = 1, 2, \dots, n + 1 \quad (31)$$

Relation 24 represents the objective function of the model. Constraint 2<sup>o</sup> specifies that the weight of each share must not exceed the set minimum and maximum values. Constraint 2<sup>٦</sup> defines the weight of each share in each period, based on the share weight in the previous period and the amount of the transaction.

Constraint 2<sup>٧</sup> specifies the amount of cash. Constraint 2<sup>٨</sup> specifies upper and lower bound of the number of shares to be selected per period, Constraints 30 and 31 also specify the range of decision variables.

This model is nonlinear due to its objective function type and is not easily solvable by software packages. Therefore, with the help of changing the variable and some mathematical actions, this optimization model is applied to the change of target function and final shaping.



$$\max y \quad (32)$$

s. t.:

$$\frac{\sum_{i=1}^n w_i R_{it} - R_{(n+1)t}}{\sqrt{\sum_{i=1}^n \sum_{j=1}^m (\sigma_{ijt} * w_{it} * w_{jt})}} \geq y \quad , \quad t = 1, 2, \dots, T \quad (33)$$

$$L_{it} F_{it} \leq \frac{X_{it}}{\sum_{j=1}^{n+1} X_{jt}} \leq U_{it} F_{it}, \quad t = 1, 2, \dots, T, i = 1, 2, \dots, n \quad (34)$$

$$X_{it} = (1 + R_{i(t-1)})X_{i(t-1)} - Y_{it} + Z_{it}, \quad t = 1, 2, \dots, T, i = 1, 2, \dots, n \quad (35)$$

$$X_{(n+1)t} = (1 + R_{(n+1)(t-1)})X_{(n+1)(t-1)} + \sum_{i=1}^n (1 - cs)Y_{it} - \sum_{i=1}^n (1 + cb)Z_{it}, \quad (36)$$

$$t = 1, 2, \dots, T$$

$$k_l \leq \sum_{i=1}^n F_{it} \leq k_u, \quad t = 1, 2, \dots, T \quad (37)$$

$$X_{(n+1)0} = 1000 \quad (38)$$

$$X_{it}, Y_{it}, Z_{it} \geq 0, \quad t = 1, 2, \dots, T, i = 1, 2, \dots, n + 1 \quad (39)$$

$$F_{it} = \{0, 1\} \quad t = 1, 2, \dots, T, i = 1, 2, \dots, n + 1 \quad (40)$$

### The proposed robust possibilistic programming model for stock portfolio optimization

The purpose of this section is to allow constraint  $\alpha$  to exceed a certain level. Given that a pessimistic fuzzy is used to ensure greater reliability, then relation  $\alpha$  is converted as follows:

$$NEC\left(\sum_{t=1}^T \sum_{i=1}^n \tilde{S}_{pt} X_{it} \geq y\right) \geq \alpha \quad (41)$$

According to the above, the robust possibilistic programming model is as follows:

$$\text{MAX} \left( y - \eta \left( \frac{\sum_{i=1}^n w_i R^2_{it} - R_{(n+1)t}}{2 \sqrt{\sum_{i=1}^n \sum_{j=1}^m (\sigma_{ijt} * w_{it} * w_{jt})}} - (1 - \alpha) \frac{\sum_{i=1}^n w_i R^1_{it} - R_{(n+1)t}}{2 \sqrt{\sum_{i=1}^n \sum_{j=1}^m (\sigma_{ijt} * w_{it} * w_{jt})}} - \alpha \frac{\sum_{i=1}^n w_i R^2_{it} - R_{(n+1)t}}{2 \sqrt{\sum_{i=1}^n \sum_{j=1}^m (\sigma_{ijt} * w_{it} * w_{jt})}} \right) \right) \quad (42)$$

s.t

$$\alpha * \frac{\sum_{i=1}^n w_i R^2_{it} - R_{(n+1)t}}{2 \sqrt{\sum_{i=1}^n \sum_{j=1}^m (\sigma_{ijt} * w_{it} * w_{jt})}} + (1 - \alpha) * \frac{\sum_{i=1}^n w_i R^1_{it} - R_{(n+1)t}}{2 \sqrt{\sum_{i=1}^n \sum_{j=1}^m (\sigma_{ijt} * w_{it} * w_{jt})}} \geq y, \quad (43)$$

$$t = 1, 2, \dots, T, i = 1, 2, \dots, n$$

$$L_{it} F_{it} \leq \frac{X_{it}}{\sum_{j=1}^{n+1} X_{jt}} \leq U_{it} F_{it}, \quad t = 1, 2, \dots, T, i = 1, 2, \dots, n \quad (44)$$

$$X_{it} \geq \left( \left( \frac{\beta}{2} \right) (1 + R^4_{i(t-1)}) + \left( 1 - \frac{\beta}{2} \right) (1 + R^3_{i(t-1)}) \right) X_{i(t-1)} - Y_{it} + Z_{it}, \quad t = 1, 2, \dots, T, i = 1, 2, \dots, n \quad (45)$$

$$X_{it} \leq \left( \left( \frac{\beta}{2} \right) (1 + R^3_{i(t-1)}) + \left( 1 - \frac{\beta}{2} \right) (1 + R^4_{i(t-1)}) \right) X_{i(t-1)} - Y_{it} + Z_{it}, \quad t = 1, 2, \dots, T, i = 1, 2, \dots, n \quad (46)$$

$$X_{(n+1)t} = (1 + R_{(n+1)(t-1)}) X_{(n+1)(t-1)} + \sum_{i=1}^n (1 - cs) Y_{it} - \sum_{i=1}^n (1 + cb) Z_{it}, \quad t = 1, 2, \dots, T \quad (47)$$

$$k_l \leq \sum_{i=1}^n F_{it} \leq k_u, \quad t = 1, 2, \dots, T \quad (48)$$

$$X_{(n+1)0}=1000 \quad (49)$$

$$X_{it}, Y_{it}, Z_{it} \geq 0, \quad t = 1, 2, \dots, T, i = 1, 2, \dots, n + 1 \quad (50)$$

$$F_{it} = \{0, 1\} \quad t = 1, 2, \dots, T, i = 1, 2, \dots, n + 1 \quad (51)$$

$$0.5 \leq \alpha, \beta \leq 1 \quad (52)$$

Relation 42 shows the objective function of robust possibilistic based on the proposed model. Constraints 43, 45 and 46 are also rewritten according to robust possibilistic programming rules. Other constraints are the same as the proposed definitive model.

### Computational results

In this section, at first, 27 companies active on the iran stock market were selected in 6 time periods(weekly) for problem-solving. The model was solved using GAMS software and BONMIN Solver and with the help of the 3 GB RAM, the Core 2 Duo CPU system and the outputs of the problem are also shown. The sensitivity analysis was performed on some of the parameters of the model and the objective function and decision variables were compared. This sensitivity analysis has two main objectives: a) Creating managerial insights and new scientific achievements; b) Ensuring the validity of the model (Sensitivity analysis on some parameters does not really mean much and does not lead to new knowledge. But this analysis and observing the change in the value of the objective function help us make sure that his model is valid and not technically problematic.)

### Problem-Solving in a Real Sample of Companies in Stock market

Considering the uncertainty assumption of some of the model parameters such as Sharpe ratio of stock and stock returns at different time periods, the mentioned parameters are considered as a trapezoidal fuzzy. Other parameters of the model are based on the information available from the stock market. Table 1 shows the definite parameters used to solve the model (27 companies active in the stock exchange).

Table 1. The definitive parameters used in problem-solving

Parameter	Value	Parameter	Value
Number of authorized shares per Portfolio	5	Purchase fee	0.005
The maximum weight of the share	0.5	Sales fee	0.006
The minimum weight of the share	0.1	Initial investment	1000

According to table 1, the maximum objective function in the above problem is 1520.056.

### Sensitivity analysis

- **Number of authorized shares per portfolio**

Initially, the sensitivity of the problem is analyzed on the difference between minimum and maximum number of authorized stocks in each portfolio. Thus, the objective function and the computational time are shown in Table 3.

Table 2. Changes in the amount of objective function and computational time by changing the difference between upper and lower number of authorized stocks

The number of companies that have been purchased at least once during six periods	Computational time	The value of the objective function	Number of difference between upper and lower
15	5.003	2.23	3
19	5.191	2.54	4
22	7.215	2.833	5
23	7.961	3.024	6
23	7.053	3.198	7

For simplicity, Figures 1 and 2 show the change in the objective function and the number of companies involved in stock purchases, with changes in the number of authorized shares per portfolio



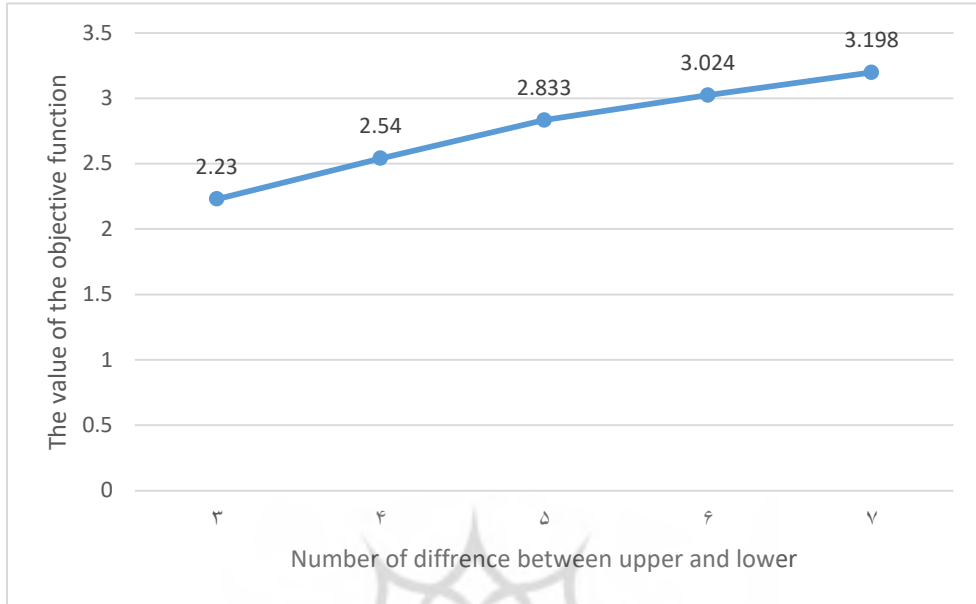


Figure 1. Change the objective function value by changing the difference between upper and lower number of authorized stocks

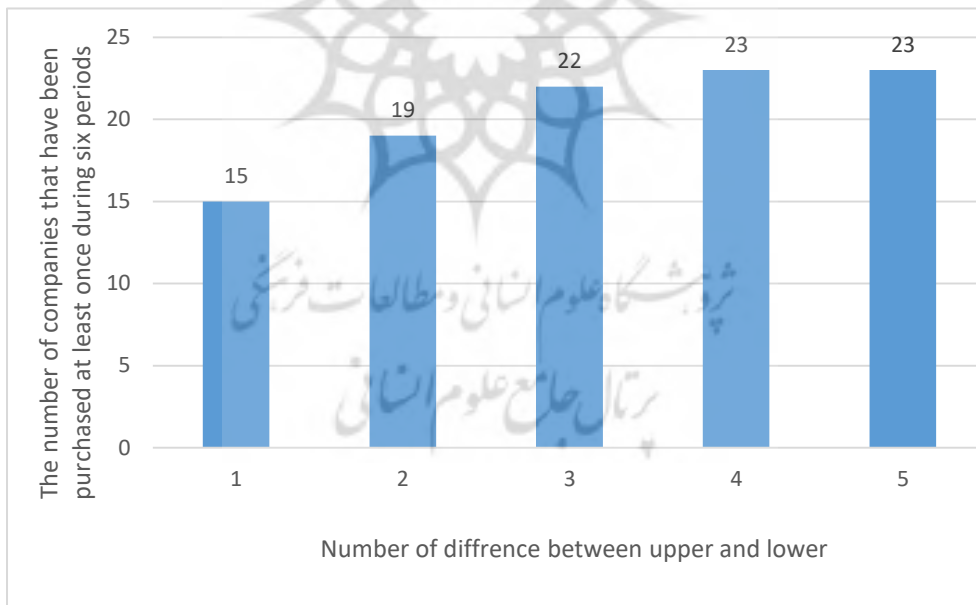


Figure 2. Chart of total companies by changing the difference between upper and lower number of authorized stocks

- **Risk-free rate of return**

In the remainder of this section, with the assumption of constant consideration of 5 shares in each portfolio of purchases in each time period, the objective function and the computational time of the problem solving are calculated by changing the risk-free rate of return. Table 3 shows the changes in these indicators in different amounts of the risk-free rate of return.

Table 3. Change the value of the objective function and computational time by changing the risk-free rate of return

Amount of change in the value of the objective function (%)	Computational time	The value of the objective function	Return of risk-free deposit
-0.00237	6.915	2.527	0.11
-0.00119	7.677	2.530	0.12
0.00000	7.215	2.533	0.13
0.00158	7.024	2.537	0.14
0.00276	7.168	2.540	0.15
0.00393	10.292	2.543	0.16

According to the results of the above table, with the increase of risk-free rate of return, the objective function is increased and with the increase of 1% of the risk-free rate of return, the total objective function is linearly increased. For this purpose, Figure 3 illustrates this change in risk-free returns.

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Figure 3. Change in the percentage of total profit by changing the risk-free rate of return

• **Uncertainty rates**

Due to the nature of possibilistic Robust optimization model, the uncertainty rate is implemented as a decision variable in modeling, which is included in the objective function and computational time calculated in  $\beta$  and  $\alpha$  between 0.5 and 1. This section is shown in Table 4 by changing the uncertainty rate.

Table 4. Change the value of the objective function and computational time by changing the uncertainty rate

Computational time	The value of the objective function	$\beta$	$\alpha$
6.173	0.850	0.5	<b>0.9</b>
5.374	0.852	0.6	
5.844	0.854	0.7	
5.125	0.856	0.8	
5.721	0.858	0.9	
6.337	0.860	1	
3.985	1.553	0.5	<b>0.95</b>
4.146	1.552	0.6	
4.879	1.550	0.7	
4.457	1.549	0.8	
4.15	1.548	0.9	
4.436	1.547	1	
7.215	2.533	0.5	<b>1</b>
8.552	2.528	0.6	
6.113	2.523	0.7	
6.555	2.518	0.8	
6.148	2.513	0.9	
7.421	2.508	1	

According to the results of Table 4, the maximum profit earned in the uncertainty rate  $\alpha$  and  $\beta$  is 1 and 0.5, respectively. Figure 4 shows the trend of variations in the objective function value at different rates of uncertainty.

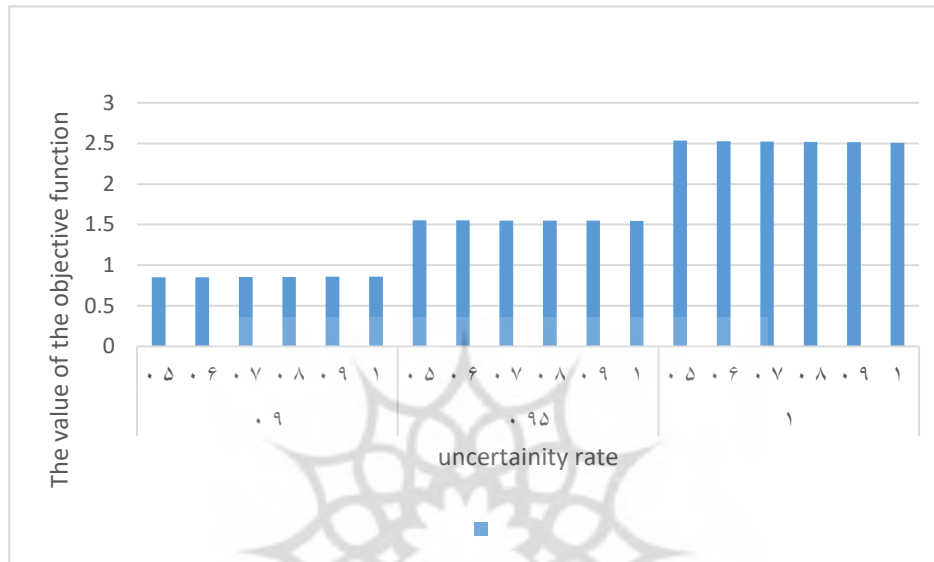


Figure 4. Change the value of the objective function with changes in uncertainty rate

## Conclusion and suggestions

In today's competitive atmosphere, it's important and essential to design a robust model for stock portfolio selection. Over the past decades, sudden Volatilities and the issue of coping with their adverse effects on the stock market have become a major challenge for financial managers of organizations or investors. Due to the uncertain nature of input parameters in the stock market, in this paper, we have developed a new robust possibilistic programming model based on the Sharpe ratio to deal with the uncertainty of the parameters and the low quality of the decisions made by this factor. In the following, a real problem based on the fuzzy data of 27 companies active in the stock market have been presented to show the performance of the proposed model, as well as the high-performance and functionality of the robust possibilistic programming model. Finally, it's worth noting that according to the outputs of the model, determining the exact amount of fines (penalty) in the possibilistic programming model is very important because fines are the main factor in the performance of the model and the determination of the confidence levels of non-deterministic parameters.

Other robust optimization approaches can also be used in future research. One can also use the opinions of various experts to increase the credibility of describing the sensitive parameters.

In the end, it is suggested to use risk-based minimization models, in particular, using more precise and comprehensive risk assessment measures such as the Estimator of Garman- Klass (garman,1980) and Parkinson's (Parkinson,1980), in order to take full advantage of Robust possibilistic programming capabilities. Additionally, adding an integer constraint to the weight of each asset/share in the investment portfolio helps the proposed portfolio of the model in the real world to be easily implemented and prevent the purchase of a very small shareholding (micro trade).



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#### Bibliographic information of this paper for citing:

Amiri, Maghsoud & Heidary, Mohammad Saeed (2018). Portfolio optimization with robust possibilistic programming. *Iranian Journal of Finance*, 2(4), 41-63.

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