Journal of System Management
2019, Issue **2**, pp. **001-020**

Fuzzy Compromise Approach for Solving Interval-Valued Fractional Multi-Objective Multi-Product Solid Transportation Problems

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Abstract. In this paper, a fractional multi- objective multi-product solid transportation problem with interval costs, supply, demand, and conveyances is investigated based on fuzzy programming approach. To minimize the problem, the order relations that represent the decision maker's (DM) performance between interval costs, supply, demand and conveyances are defined by the right limit, and the left limit. Through the deterministic problem is obtained, a fuzzy programming approach is applied by defining membership functions. A linear membership function is being used for obtaining optimal compromise solution. Finally, a numerical example is given to the utility of the approach.

Keywords: Multi-Objective Multi-Product Solid Ttransportation Problem; Interval Numbers; Inexact Programming; linear Membership Function; Fuzzy Programming; Fractional Programming.

1. Introduction

The solid transportation problem (STP) is a generalization of the well-known transportation problem (TP) in which three-dimensional properties are taken into consideration in the objective and constraint set instead of source and destination. The STP was first stated by Shell (1955). Haley (1962) introduced a solution procedure for solving STP which is an extension of the modified distribution method. Pandian and Anuradha (2010) proposed a new method for solving STP based on the principle of zero point method introduced by Pandian and Natarajan (2010). Ammar and Khalifa (2014) introduced fuzzy multi- objective STP and determined the stability set of the first kind corresponding to the obtained solution. Ammar and Khalifa (2015) presented multi-objective multi- item STP with fuzzy numbers in the supplies, demands, capacity of conveyances, and costs. Khalifa (2015) studied multi-objective multi- item STP with possibility objective functions coefficients and determined objective multi- objective multi- item STP involving fuzzy numbers in the objective functions coefficients, and treatment the problem using the fuzzy programming technique and global criteria methods. Ida et al. (1995) studied multi- criteria STP involving fuzzy numbers. Using general fuzzy cost and time, Ojha et al. (2009) studied entropy based STP. Under stochastic environment, Yang and Yuan 2007 investigated a bicriteria STP. Kundu et al. (2014) investigated multi-objective STP under various uncertain environments. Rani and Gulati (2015) introduced fully fuzzy multi- objective multi- item STP and applied fuzzy programming approach to find the fuzzy optimal compromise solution of the problem. Kumar and Dutta (2015) studied multi- objective STP with fuzzy coefficients for the objectives and constraints and applied fuzzy goal programming for obtaining fuzzy optimal compromise solution. Under some restriction on transported amount, Baidya et al. (2016) introduced six new transportation models with breakability and vehicle cost. Jimenez and Verdegay (1999) solved fuzzy STP by applying an evolutionary algorithm based on parametric approach. Nagarajan et al. (2014) introduced a solution procedure for stochastic multi-objective interval STP. Cui and Sheng (2012) introduced an expected constrained programming for an uncertain STP problem. Uddin et al. (2018) developed a utility function using a fuzzy membership function that's by considering deviations of objective function from the goal value to obtain the target goal for a multiobjective TP. Fractional Programming (FP) is a decision problem arises to optimize the ratio subject to constraints. In real world decision situations, MOLFP programming arises very frequently. As, for instance, the ratio between inventory& sales, actual cost & standard cost, output& employee, measuring relative efficiency of decision making unit in the public/ or nonprofit sectors, Data Envelopment Analysis (DEA) & many other areas of economics, non- economics and indirect applications. Charnes and Cooper (1962) studied a linear fractional programming (LFP) problem and showed that it can be optimized by solving two linear programs. Ammar and Khalifa (2009) studied LFP problem with fuzzy parameters. Ammar and Khalifa (2004) introduced a parametric approach for solving multi-criteria LFP problem. Luhandjula (1984) applied fuzzy programming approach for solving MOLFP problem. Nykowski and Zolkiewski (1985) solved the MOLFP problem by converting it into the multi- objective linear programming (MOLP) problem. Gupta and Chakraborty (1999) have been introduced a methodology for a restricted class of MOLFP problem in the sense that there exists some values of decision variables for which the numerator is positive and the denominator is positive for all values of decision variables in the feasible region, and then applied fuzzy approach for solving the problem by defining a linear membership function. Radhakrishnan and Anukokila (2014) used fractional goal programming approach for solving STP with interval cost.

The rest of the paper is organized as : In section 2; some preliminaries need are presented. In section 3, a multi- objective multi- product solid transportation problem with interval costs, supply, demand is formulated. In section 4, a fuzzy programming approach for solving the problem is given. In section 5, an interactive procedure for obtaining the optimal compromise solution is suggested. In section 6, A numerical example is given for illustration. Finally, some concluding remarks are reported in section 7.

2. Preliminaries

In order discuss our problem conveniently, we shall state some necessary results on interval arithmetic (see, Moore ,1979 ; Kauffmann and Gupta, 1988).

Let $I(R) = \{[a^L, a^U]: a^L, a^U \in R = (-\infty, \infty), a^L \leq a^U\}$ denote the set of all closed interval numbers on$R$, where a^L is the left limit and $a^U$is the upper the right limit.

Definition1. Assume that: $A = [a^L, a^U], B = [b^L, b^U] \in I(R)$, we define:

(i) $[a^L, a^U](+)[b^L, b^U] = [a^L + b^L, a^U + b^U]$ (1)

(ii) $[a^L, a^U](-)[b^L, b^U] = [a^L - b^U, a^U - b^L$ (2)

(iii)
$[a^L, a^U](\cdot)[b^L, b^U]$
$= [\min(a.^L\, b^L, a^L.b^U, a^U.b^L, a^U.b^U), \max(a.^L\, b^L, a^L.b^U, a^U.b^L, a^U.b^U)]$ (3)

(iv)
$$[a^L, a^U](/)[b^L, b^U] = \left[\min\left(\frac{a^L}{b^L}, \frac{a^L}{b^U}, \frac{a^U}{b^L}, \frac{a^U}{b^U}\right), \max\left(\frac{a.^L}{b^L}, \frac{a^L}{b^U}, \frac{a^U}{b^L}, \frac{a^U}{b^U}\right)\right] \quad (4)$$

(v) $k[a^L, a^U] = \begin{cases}[ka^L, ka^U], k \geq 0 \\ [ka^U, ka^L], k < 0\end{cases}$ (5)

Where, $k \in R$.

(vi) The order relation $\leq^{LU}$ in $I(R)$ is defined by:
$[a^L, a^U](\leq^{LU})[b^L, b^U]$ if and only if $a^L \leq b^L, a^U \leq b^U$, (6.1)

$[a^L, a^U](<^{LU})[b^L, b^U]$ if and only if
$[a^L, a^U](\leq^{LU})[b^L, b^U]$, and $[a^L, a^U](\neq)[b^L, b^U]$. (6.2)

Proposition1. (Ishibuchi andTanaka (1990)).
If $A = [a^L, a^U](\leq^{LU})B = [b^L, b^U]$, then $A = B$.

Proposition2. (Ishibuchi andTanaka (1990)).
$A = B$ if and only if , (6.1), and (6.2) hold.

Proposition3. (Gupta and Chakraborty, 1999).
If $d^T > 0$, and $d_0 > 0$, then

$N=\frac{c^Tx+c_0}{d^Tx+d_0}, x\geq 0$ has the maximum value $\bar{N}=max\{c_i/d_i, c_0/d_0, i=1,2,\ldots,n\}$and the minimum value $\underline{N}=\min\{c_i/d_i, c_0/d_0, i=1,2,\ldots,n\}$.

3. Problem formulation and solution concepts

A fractional multi- objective multi- product solid transportation problem is formulated as follows

$$\text{Min}Z_r=\frac{N_r(x)}{M_r(x)}=\frac{\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\left[\left(c_{ijk}^{rp}\right)^L,\left(c_{ijk}^{rp}\right)^U\right]x_{ijk}^p}{\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\left[\left(d_{ijk}^{rp}\right)^L,\left(d_{ijk}^{rp}\right)^U\right]x_{ijk}^p},$$
$$r=1,2,3,\ldots,S$$

Subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{K}x_{ijk}^p\leq\left[\left(a_i^p\right)^L,\left(a_i^p\right)^U\right]=A_i;\forall i,p,$$
$$\sum_{i=1}^{m}\sum_{k=1}^{K}x_{ijk}^p\geq\left[\left(b_j^p\right)^L,\left(b_j^p\right)^U\right]=B_j;\forall j,p,$$
$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ijk}^p\leq[(e_k)^L,(e_k)^U]=C_k;\forall k,$$
$$x_{ijk}^{rp}\geq 0;\forall i,j,k,p. \tag{7}$$

Where $p(=1,2,\ldots,l)$ products can be transported from m origins $A_i(i=1,2,\ldots,m)$ to $n$destination $B_j(j=1,2,\ldots,n)$ by means of $C_k(k=1,2,\ldots,K)$ conveyances, and $r(=1,2,\ldots,S)$ objectives are to be minimized.

To solve the problem (7), the following conditions must be satisfied:

$$\sum_{i=1}^{m}\left[\left(a_i^p\right)^L,\left(a_i^p\right)^U\right]\geq\sum_{j=1}^{n}\left[\left(b_j^p\right)^L,\left(b_j^p\right)^U\right], p=1,2,\ldots,l$$

and

$$\sum_{k=1}^{K}[(e_k)^L,(e_k)^U]\geq\sum_{p=1}^{l}\sum_{j=1}^{n}\left[\left(b_j^p\right)^L,\left(b_j^p\right)^U\right].$$

Definition2. (Interval- valued efficient solution). Apoint$x^*\in X(A_i,B_j,E_k), i=1,2,\ldots,m;\ j=1,2,\ldots,n; k=1,2,\ldots,K,$ is said to be interval- valued efficient solution to the problem (7) if and only if there does not exist another$x \in X(A_i, B_j, E_k)$, such that:$Z_r(x) \le Z_r(x^*)$, and $Z_r(x) < Z_r(x^*)$ for at least one r.

It follows that the problem (7) can be rewritten as follows

$$\text{Min}(Z_r)^U = \frac{\left(N_r(x)\right)^U}{\left(M_r(x)\right)^L} = \frac{\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\left(c_{ijk}^{rp}\right)^U x_{ijk}^p}{\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\left(d_{ijk}^{rp}\right)^L x_{ijk}^p}, r = 1,2,3,\dots,S$$

Subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^p \le \left(a_i^p\right)^U; \forall i, p,$$
$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^p \ge \left(a_i^p\right)^L; \forall i, p,$$
$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^p \le \left(b_j^p\right)^U; \forall j, p,$$
$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^p \ge \left(b_j^p\right)^L; j, p,$$
$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^p \le (e_k)^U, \forall k,$$
$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^p \ge (e_k)^L; \forall k,$$
$$x_{ijk}^{rp} \ge 0; \forall i, j, k, p. \tag{8}$$

4. Fuzzy programming approach for solving the problem

Bellman and Zadeh (1970) introduced three basic concepts: fuzzy goal (G), fuzzy constraints (T), and fuzzy decision (D) and explored the applications of these concepts to the decision making under fuzziness. Their fuzzy decision is defined as follows:

$$D = G \cap T \tag{9}$$

This problem is characterized by the membership function

$$\mu_D(x) = min(\mu_G(x), \mu_T(x)), \tag{10}$$

To define the membership function of the problem (8), let us follow:

Calculate the individual minimum as:

$$((Z_r)^U)(Z_r)^{U^\bullet}{}_{min} = \min\{(Z_r)^U(x): x \in X\}, \tag{11}$$

The individual maximum as:

$$((Z_r)^U)(Z_r)^{U^\bullet}{}_{max} = \max\{(Z_r)^U(x): x \in X\}, \tag{12}$$

Where,$X$is the feasible region of the problem (8).

On the basis of definition of $((Z_r)^U)_{min}$, and,$((Z_r)^U)_{max}$, the membership functions for the problem(8) as follows (Biswal (1992)):

$$\mu_r\left(\left(Z_r(x)\right)^U\right) = \begin{cases} 1, & \left(Z_r(x)\right)^U \le \left(\left(Z_r\right)^U\right)_{\min}, \\ \dfrac{\left(\left(Z_r\right)^U\right)_{\max} - \left(Z_r(x)\right)^U}{\left(\left(Z_r\right)^U\right)_{\max} - \left(\left(Z_r\right)^U\right)_{\min}}, & \left(\left(Z_r\right)^U\right)_{\min} \le \left(Z_r(x)\right)^U < \left(\left(Z_r\right)^U\right)_{\max}, \\ 0, & \left(Z_r(x)\right)^U \ge \left(\left(Z_r\right)^U\right)_{\max}, \end{cases} \tag{13}$$

Following the fuzzy decision of Bellman and Zadeh (1970) with the linear membership function (13) a fuzzy programming model to the problem (8) can be written as follows:

$$Max \min_{r=1,2,\ldots,S} \{\mu_r((Z_r)^U)\},$$

Subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^p \le \left(a_i^p\right)^U, i = 1,2,\ldots,m; p = 1,2,\ldots,l,$$

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^p \ge \left(a_i^p\right)^L, i = 1,2,\ldots,m; p = 1,2,\ldots,l,$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^p \le \left(b_j^p\right)^U, j = 1,2,\ldots,n; p = 1,2,\ldots,l,$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^p \ge \left(b_j^p\right)^L, j = 1,2,\ldots,n; p = 1,2,\ldots,l,$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^p \le (e_k)^U, k == 1,2,\ldots,K,$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^p \ge (e_k)^L, k == 1,2,\ldots,K,$$

$$x_{ijk}^{rp} \geq 0; \forall i, j, k, p, \; 0 < \lambda \leq 1. \tag{14}$$

Problem (14) can be transformed into the following problem

$$\underset{x}{Max}\lambda$$

Subject to

$$\mu_r((Z_r(x))^U) \geq \lambda,$$

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^{p} \leq \left(a_i^p\right)^U, i = 1,2,\ldots,m; p = 1,2,\ldots,l,$$

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^{p} \geq \left(a_i^p\right)^L, i = 1,2,\ldots,m; p = 1,2,\ldots,l,$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^{p} \leq \left(b_j^p\right)^U, j = 1,2,\ldots,n; p = 1,2,\ldots,l,$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^{p} \geq \left(b_j^p\right)^L, j = 1,2,\ldots,n; p = 1,2,\ldots,l,$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^{p} \leq (e_k)^U, k == 1,2,\ldots,K,$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^{p} \geq (e_k)^L, k == 1,2,\ldots,K,$$

$$x_{ijk}^{rp} \geq 0; \forall i, j, k, p, 0 \leq \lambda \leq 1. \tag{15}$$

Where, λ is an auxiliary variable.
By the transformation$y = xt$, the problem (15) becomes

$$\underset{x}{Max}\lambda$$

Subject to

$$\mu_r(t(N_r(y/t))^U) \geq \lambda,$$

$$t(M_r(y/t))^L \leq 1,$$

$$\sum_{j=1}^{n}\sum_{k=1}^{K} (y/t)_{ijk}^{p} \leq \left(a_i^p\right)^U, i = 1,2,\ldots,m; p = 1,2,\ldots,l,$$

$$\sum_{j=1}^{n}\sum_{k=1}^{K} (y/t)_{ijk}^{p} \geq \left(a_i^p\right)^L, i = 1,2,\ldots,m; p = 1,2,\ldots,l,$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K}(y/t)_{ijk}^{p} \leq \left(b_j^p\right)^U, j = 1,2,\dots,n; p = 1,2,\dots,l,$$
$$\sum_{i=1}^{m}\sum_{k=1}^{K}(y/t)_{ijk}^{p} \geq \left(b_j^p\right)^L, j = 1,2,\dots,n; p = 1,2,\dots,l,$$
$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}(y/t)_{ijk}^{p} \leq (e_k)^U, k == 1,2,\dots,K,$$
$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}(y/t)_{ijk}^{p} \geq (e_k)^L, k == 1,2,\dots,K,$$
$$y_{ijk}^{rp} \geq 0, t > 0; \forall i,j,k,p; 0 < \lambda \leq 1. \tag{16}$$

5. A solution procedure

Step 1: Calculate the individual minimum and maximum of each objective function under the given constraints.

Step 2: Define the membership function, $\mu_r((Z_r(x))^U)$, and, $r = 1,2,\dots,S$, as mentioned in equation (13).

Step 3: Construct the fuzzy programming problem (14), and its equivalent linear programming problem (15).

Step 4: Solve problem (16) by using integer-programming approach to obtain an integer optimal compromise solution and hence evaluate the S objective functions at the resulted optimal compromise solution.

Step 5: Stop.

6. Numerical example

Consider the following problem with $p = 1,2 = i = k, j = 1,2,3$

$$MinZ_1 = \frac{\sum_{p=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2}\left[\left(c_{ijk}^{1p}\right)^L, \left(c_{ijk}^{1p}\right)^U\right]x_{ijk}^{p}}{\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\left[\left(d_{ijk}^{1p}\right)^L, \left(d_{ijk}^{1p}\right)^U\right]x_{ijk}^{p}},$$

$$MinZ_2 = \frac{\sum_{p=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2}\left[\left(c_{ijk}^{2p}\right)^L,\left(c_{ijk}^{2p}\right)^U\right]x_{ijk}^p}{\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\left[\left(d_{ijk}^{2p}\right)^L,\left(d_{ijk}^{2p}\right)^U\right]x_{ijk}^p},$$

Subject to

$$\sum_{j=1}^{3}\sum_{k=1}^{2}x_{1jk}^1 \le [(a_1^1)^L,(a_1^1)^U] = [24,26]\sum_{j=1}^{3}\sum_{k=1}^{2}x_{2jk}^1 \le [(a_2^1)^L,(a_2^1)^U] = [32,35]$$

$$\sum_{j=1}^{3}\sum_{k=1}^{2}x_{1jk}^2 \le [(a_1^2)^L,(a_1^2)^U] = [34,37]$$

$$\sum_{j=1}^{3}\sum_{k=1}^{2}x_{2jk}^2 \le [(a_2^2)^L,(a_2^2)^U] = [28,30]$$

$$\sum_{i=1}^{2}\sum_{k=1}^{2}x_{i1k}^1 \ge [(b_1^1)^L,(b_1^1)^U] = [16,19]\sum_{i=1}^{2}\sum_{k=1}^{2}x_{i1k}^2 \ge [(b_1^2)^L,(b_1^2)^U] = [23,25]$$

$$\sum_{i=1}^{2}\sum_{k=1}^{2}x_{i2k}^1 \ge [(b_2^1)^L,(b_2^1)^U] = [20,22]\sum_{i=1}^{2}\sum_{k=1}^{2}x_{i2k}^2 \ge [(b_2^2)^L,(b_2^2)^U] = [18,19]$$

$$\sum_{i=1}^{2}\sum_{k=1}^{2}x_{i3k}^1 \ge [(b_3^1)^L,(b_3^1)^U] = [15,18]\sum_{i=1}^{2}\sum_{k=1}^{2}x_{i3k}^2 \ge [(b_3^2)^L,(b_3^2)^U] = [17,19]$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ij1}^p \le [(e_1)^L,(e_2)^U] = [48,51]\sum_{p=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3}x_{ij2}^p \le [(e_2)^L,(e_2)^U] = [53,56]$$

$x_{ijk}^{rp} \ge 0; \forall i,j,k,p.$

Where, the unit transportation penalties are given in Tables 1-8 as follow:

Table 1. Penalties/costs c_{ijk}^{11}

i	j			j		
	1	2	3	1	2	3
1	[7, 9]	[6, 10]	[12, 14]	[11, 13]	[8, 10]	[8, 12]
2	[10, 12]	[7, 9]	[13, 15]	[11, 13]	[8, 10]	[16, 18]
k	1			2		

Table 2. Penalties/costs d_{ijk}^{11}

i	j			j		
	1	2	3	1	2	3
1	[2, 4]	[1, 3]	[4, 6]	[5, 7]	[4, 8]	[7, 9]
2	[3, 5]	[7, 9]	[11, 13]	[8, 12]	[6, 10]	[16, 18]
k	1			2		

Table 3. Penalties/costs c_{ijk}^{12}

i	j			j		
	1	2	3	1	2	3
1	[10, 12]	[8, 10]	[10, 12]	[12, 14]	[6, 10]	[9, 11]
2	[12, 14]	[8, 12]	[14, 16]	[16, 18]	[10, 12]	[13, 15]
k	1			2		

Table 4. Penalties/ costs d_{ijk}^{12}

i	j			j		
	1	2	3	1	2	3
1	[7, 9]	[7, 9]	[12, 14]	[11, 13]	[8, 10]	[8, 12]
2	[10, 14]	[7, 9]	[13, 15]	[11, 13]	[8, 10]	[16, 18]
k	1			2		

Table 5. Penalties/ costs c_{ijk}^{21}

i	j			j		
	1	2	3	1	2	3
1	[5, 7]	[4, 6]	[8, 10]	[4, 8]	[5, 7]	[7, 9]
2	[7, 9]	[5, 7]	[7, 9]	[6, 8]	[9, 11]	[9, 11]
k	1			2		

Table6. Penalties/ costs d_{ijk}^{21}

i	j			j		
	1	2	3	1	2	3
1	[7, 9]	[5, 9]	[12, 14]	[10, 13]	[8, 10]	[6, 12]
2	[10, 14]	[7, 9]	[13, 15]	[11, 13]	[8, 10]	[16, 18]
k	1			2		

Table 7. Penalties/ costs c_{ijk}^{22}

i	j			j		
	1	2	3	1	2	3
1	[7, 9]	[5, 9]	[12, 14]	[11, 13]	[8, 10]	[8, 12]
2	[9, 13]	[6, 8]	[13, 15]	[12, 14]	[8, 10]	[16, 18]
k	1			2		

Table 8. Penalties/ costs d_{ijk}^{22}

i	j			j		
	1	2	3	1	2	3
1	[1, 3]	[2, 6]	[7, 11]	[9, 11]	[8, 12]	[8, 10]
2	[10, 12]	[13, 15]	[7, 11]	[9, 13]	[7, 11]	[13, 17]
k	1			2		

Referring to the problem(8), the equivalent multi- objective ordinary problem is

$$Min(Z_1)^U = \frac{\sum_{p=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2}\left(c_{ijk}^{1p}\right)^U x_{ijk}^p}{\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\left(d_{ijk}^{1p}\right)^L x_{ijk}^p},$$

$$Min(Z_2)^U = \frac{\sum_{p=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2}\left(c_{ijk}^{2p}\right)^U x_{ijk}^p}{\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\left(d_{ijk}^{2p}\right)^L x_{ijk}^p},$$

Subject to

$$x_{111}^1 + x_{121}^1 + x_{131}^1 + x_{112}^1 + x_{122}^1 + x_{132}^1 \le 26,$$
$$x_{211}^1 + x_{221}^1 + x_{231}^1 + x_{212}^1 + x_{222}^1 + x_{232}^1 \le 35,$$
$$x_{111}^2 + x_{121}^2 + x_{131}^2 + x_{112}^2 + x_{122}^2 + x_{132}^2 \le 37,$$
$$x_{211}^2 + x_{221}^2 + x_{231}^2 + x_{212}^2 + x_{222}^2 + x_{232}^2 \le 30,$$
$$x_{111}^1 + x_{211}^1 + x_{112}^1 + x_{212}^1 \ge 16,$$
$$x_{111}^2 + x_{211}^2 + x_{112}^2 + x_{212}^2 \ge 23,$$
$$x_{121}^1 + x_{221}^1 + x_{122}^1 + x_{222}^1 \ge 20,$$
$$x_{121}^2 + x_{221}^2 + x_{122}^2 + x_{222}^2 \ge 18,$$
$$x_{131}^1 + x_{231}^1 + x_{132}^1 + x_{232}^1 \ge 15,$$
$$x_{131}^2 + x_{231}^2 + x_{132}^2 + x_{232}^2 \ge 17,$$
$$x_{111}^1 + x_{121}^1 + x_{131}^1 + x_{211}^1 + x_{221}^1 + x_{231}^1 + x_{111}^2 + x_{121}^2 + x_{131}^2 + x_{211}^2 + x_{221}^2 + x_{231}^2 \le 51,$$
$$x_{112}^1 + x_{212}^1 + x_{122}^1 + x_{222}^1 + x_{132}^1 + x_{232}^1 + x_{112}^2 + x_{212}^2 + x_{122}^2 + x_{222}^2 + x_{132}^2 + x_{232}^2 \le 56,$$
$$x_{ijk}^p \ge 0; \forall i, j, k, p.$$

Where,

$$\left(c_{ijk}^{1p}\right)^U = \begin{pmatrix} 9x_{111}^1 + 10x_{121}^1 + 14x_{131}^1 + 12x_{211}^1 + 9x_{221}^1 + 15x_{231}^1 + 13x_{112}^1 + 10x_{122}^1 + 12x_{132}^1 \\ +13x_{212}^1 + 10x_{222}^1 + 18x_{232}^1 + 12x_{111}^2 + 10x_{121}^2 + 12x_{131}^2 + 14x_{211}^2 + 12x_{221}^2 \\ +16x_{231}^2 + 14x_{112}^2 + 10x_{122}^2 + 11x_{132}^2 + 18x_{212}^2 + 12x_{222}^2 + 15x_{232}^2 \end{pmatrix}$$

$$\left(d_{ijk}^{1p}\right)^L = \begin{pmatrix} 2x_{111}^1 + 1x_{121}^1 + 4x_{131}^1 + 3x_{211}^1 + 7x_{221}^1 + 11x_{231}^1 + 5x_{112}^1 + 4x_{122}^1 + 7x_{132}^1 \\ +8x_{212}^1 + 6x_{222}^1 + 16x_{232}^1 + 7x_{111}^2 + 7x_{121}^2 + 12x_{131}^2 + 10x_{211}^2 + 7x_{221}^2 \\ +13x_{231}^2 + 11x_{112}^2 + 8x_{122}^2 + 8x_{132}^2 + 11x_{212}^2 + 8x_{222}^2 + 16x_{232}^2 \end{pmatrix}$$

$$\left(c_{ijk}^{2p}\right)^U = \begin{pmatrix} 7x_{111}^1 + 6x_{121}^1 + 10x_{131}^1 + 9x_{211}^1 + 7x_{221}^1 + 9x_{231}^1 + 8x_{112}^1 + 7x_{122}^1 + 9x_{132}^1 \\ +8x_{212}^1 + 11x_{222}^1 + 11x_{232}^1 + 9x_{111}^2 + 9x_{121}^2 + 14x_{131}^2 + 13x_{211}^2 + 8x_{221}^2 \\ +15x_{231}^2 + 13x_{112}^2 + 10x_{122}^2 + 12x_{132}^2 + 14x_{212}^2 + 10x_{222}^2 + 18x_{232}^2 \end{pmatrix}$$

$$\left(d_{ijk}^{2p}\right)^L = \begin{pmatrix} 7x_{111}^1 + 5x_{121}^1 + 12x_{131}^1 + 10x_{211}^1 + 7x_{221}^1 + 13x_{231}^1 + 11x_{112}^1 + 8x_{122}^1 + 6x_{132}^1 \\ +11x_{212}^1 + 8x_{222}^1 + 16x_{232}^1 + 1x_{111}^2 + 2x_{121}^2 + 7x_{131}^2 + 10x_{211}^2 + 13x_{221}^2 \\ +7x_{231}^2 + 9x_{112}^2 + 8x_{122}^2 + 8x_{132}^2 + 9x_{212}^2 + 7x_{222}^2 + 13x_{232}^2 \end{pmatrix}$$

The steps of the solution procedure as follow:

1. $(Min(Z_1)^U)_{max}$ $(Min(Z_1)^U)_{min}$ $\left(Min\left(Z_2\right)^U\right)_{\max} = 4.5\, (Min(Z_2)^U)_{min}$

2. The membership functions are:

$$\mu_1((Z_1(x))^U) = \begin{cases} 1, (Z_1(x))^U \leq 0.9375, \\ \dfrac{4.5 - (Z_1(x))^U}{3.5625}, 0.9375 \leq (Z_1(x))^U < 4.5, \\ 0, (Z_1(x))^U \geq 4.5, \end{cases}$$

$$\mu_2((Z_2(x))^U) = \begin{cases} 1, (Z_2(x))^U \leq 0.615, \\ \dfrac{4.5 - (Z_2(x))^U}{3.885}, 0.615 \leq (Z_2(x))^U < 4.5, \\ 0, (Z_2(x))^U \geq 4.5, \end{cases}$$

3. Solve the problem corresponding to problem (16) as

Max λ

Subject to

$$\begin{pmatrix} 9y_1 + 10y_2 + 14y_3 + 12y_4 + 9y_5 + 15y_6 + 13y_7 + 10y_8 \\ +12y_9 + 13y_{10} + 10y_{11} + 18y_{12} + 12y_{13} + 10y_{14} + 12y_{15} \\ +14y_{16} + 12y_{17} + 16y_{18} + 14y_{19} + 10y_{20} + 11y_{21} + 18y_{22} \\ +12y_{23} + 15y_{24} - 3.5625\lambda + 4.5t \end{pmatrix} \geq 0,$$

$$\begin{pmatrix} 7y_1 + 6y_2 + 10y_3 + 9y_4 + 7y_5 + 9y_6 + 8y_7 + 70y_8 \\ +92y_9 + 8y_{10} + 11y_{11} + 11y_{12} + 9y_{13} + 9y_{14} + 14y_{15} \\ +13y_{16} + 8y_{17} + 15y_{18} + 13y_{19} + 10y_{20} + 12y_{21} + 14y_{22} \\ +10y_{23} + 18y_{24} - 3.885\lambda + 4.5t \end{pmatrix} \geq 0,$$

$$\begin{pmatrix} 2y_1 + y_2 + 4y_3 + 3y_4 + 7y_5 + 11y_6 + 5y_7 + 4y_8 \\ +7y_9 + 8y_{10} + 6y_{11} + 16y_{12} + 7y_{13} + 7y_{14} + 12y_{15} \\ +10y_{16} + 7y_{17} + 13y_{18} + 11y_{19} + 8y_{20} + 8y_{21} + 11y_{22} \\ +8y_{23} + 17y_{24} \end{pmatrix} \geq 1,$$

$$\begin{pmatrix} 7y_1 + 5y_2 + 12y_3 + 10y_4 + 7y_5 + 13y_6 + 11y_7 + 8y_8 + 6y_9 \\ +11y_{10} + 8y_{11} + 16y_{12} + y_{13} + 2y_{14} + 7y_{15} + 10y_{16} + 13y_{17} \\ +7y_{18} + 9y_{19} + 8y_{20} + 8y_{21} + 9y_{22} + 7y_{23} + 13y_{24} \end{pmatrix} \geq 1,$$

$$y_1 + y_2 + y_3 + y_7 + y_8 + y_9 - 26t \geq 0,$$
$$y_4 + y_5 + y_6 + y_{10} + y_{11} + y_{12} - 35t \geq 0,$$
$$y_{13} + y_{14} + y_{15} + y_{19} + y_{20} + y_{21} - 37t \geq 0,$$
$$y_{13} + y_{17} + y_{18} + y_{22} + y_{23} + y_{24} - 30t \geq 0,$$
$$y_1 + y_4 + y_7 + y_{10} - 16t \geq 0,$$
$$y_{13} + y_{16} + y_{19} + y_{22} - 23t \geq 0,$$
$$y_2 + y_5 + y_8 + y_{12} - 20t \geq 0,$$
$$y_{14} + y_{17} + y_{20} + y_{23} - 18t \geq 0,$$
$$y_3 + y_6 + y_9 + y_{12} - 15t \geq 0,$$
$$y_{15} + y_{18} + y_{21} + y_{24} - 17t \geq 0,$$
$$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_{13} + y_{14} + y_{15} + y_{16} + y_{17} + y_{18} - 51 \geq 0,$$
$$y_7 + y_8 + y_9 + y_{10} + y_{11} + y_{12} + y_{19} + y_{20} + y_{21} + y_{22} + y_{23} + y_{24} - 56t \geq 0,$$

$$y_q \geq 0; \forall q, t > 0; 0 \leq \lambda \leq 1.$$

The solution is

$$x_{121}^1 = 1.38, x_{132}^1 = 0.14, x_{111}^2 = 0.90,$$
$$Z_1 = [1.3225, 2.734], Z_2 = [0.74, 1.785].$$

7. Concluding remarks

In this paper, fractional multi- objective multi- product solid transportation problem with interval costs, supply, demand, and conveyances has been investigated based on fuzzy programming approach. The advantages are that the problem with interval- valued allows the DM to deal with a situation realistically. To deal with the minimization problem, the order relations who represent the decision maker's (DM) performance between interval costs, supply, demand and conveyances has been defined by the right limit, the left limit, the center and the width of intervals. Through the deterministic problem is obtained, a fuzzy compromise approach has been applied by defining membership functions. A linear membership function has been used for obtaining optimal compromise solution.

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