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Stock Portfolio-Optimization Model by Mean-Semi-Variance Approach Using of Firefly Algorithm and Imperialist Competitive Algorithm

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Abstract

Selecting approaches with appropriate accuracy and suitable speed for the purpose of making decision is one of the managers' challenges. Also investing decision is one of the main decisions of managers and it can be referred to securities transaction in financial markets which is one of the investments approaches. When some assets and barriers of real world have been considered, optimization of stock basket can't be solved easily, therefore, Meta-Heuristic approach is considered. In this regard, the main goal of this paper is to solve stock portfolio constrained optimization problem by using of Firefly Algorithm (FA) and Imperialist Competitive Algorithm (ICA). In order to do so, daily information of 25 accepted stocks in period of 2010-2016, in Tehran stock market have been used. Results show that Firefly Algorithm (FA) and Imperialist Competitive Algorithm (ICA) showed successful function in constrained optimization of stock portfolio and has acceptable accuracy in finding optimal answers in whole risk and returns levels. Also, the results of comparison of Cardinality Constrained Mean – Variance (CCMV) and Cardinality Constrained Mean – Semi -Variance (CCMSV) portfolios two using of Firefly Algorithm (FA) and Imperialist Competitive Algorithm (ICA) and considering to the findings of two criteria for assessing accuracy in stock basket optimization simultaneously show that Imperialist Competitive Algorithm (ICA) has high speed and accuracy for solving stock basket optimization and could have desirable interaction with real barriers of market. Moreover, there is high accuracy optimization of Cardinality Constrained Mean – Semi -Variance (CCMSV) compared to Cardinality Constrained Mean – Variance (CCMV).

JEL: G40, G11, C61

Keywords: Stock Constrained optimization; Firefly-Algorithm (FA); Imperialist-competitive Algorithm (ICA); Cardinality Constrained Mean – Semi -Variance (CCMSV); Cardinality Constrained Mean – Variance (CCMV).

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1. Introduction

Stock basket optimization concepts and diversification have become as a tools in order to financial markets and financial decisions improvement and perception. Harry Markowitz presented the main model of portfolio in 1952 which has based on modern portfolio theory. He claimed that portfolio goal is maximizing expected desirability instead of maximizing the expected returns. Markowitz assumed that all of investors are willing to have a combination of Floor & Ceiling returns; in fact, logical investors are following efficient portfolio because these portfolios cause maximizing of expected returns for certain level of risk or the least risk for expected returns [13]. To determine an efficient portfolio, expected returns and Standard deviation of returns should be determined for each portfolio.

Markowitz model has been used for this purpose as well as calculating of expected returns and its standard deviation. The basic assumptions of Markowitz form his model bases, which are as: investors make returns desirable, they know risk as undesirable, they make decision logically and making decisions which cause maximizing desirable returns. Therefore, desirability of investors is a function of expected returns and risk that both of them are main parameters of investments decisions. With the purpose of optimal portfolio selection, mean, as criterion of returns and variance, is as risk criterion. In other words, he assumed that the higher variance of investment, the higher possibility difference between real returns and expected returns. Markowitz suggested that investor is not only maximizing returns, but also willing to insuring of efficiency. He suggested, as justification of his claim "if investors have followed just maximizing of expected returns, they had invested in an asset with the most returns." While it can be perceived that investors have owned combination of portfolio securities transaction. Markowitz could inverted complicated problem and multi-dimension portfolio with several assets and various characteristics to two-dimensional mean-variance approach.

According to Markowitz formulation, selecting an efficient portfolio can begin with triple estimates analysis, as,

1. Expected returns for each share,
2. Expected variance –returns for each share,
3. Interactions among securities transaction returns which are measured by covariance criterion between returns of each share with other shares [13].

It can be noted that unlikely securities transaction risk with its returns, is not equal with its weighted mean of securities transaction risk.

Securities transaction risk is not only related to its completed securities transaction risk separately, but also it depends on rate of effects which comes from fundamental events, like macroeconomic events [33]. Another important point is that Markowitz has assumed that expected returns is a random variable with normal distribution. Assumption of returns normality mostly is true because many researches show that data distribution function has two thicker ends to normal function or returns distribution is skewed. The other problem is using of variance. In other words, when we use ~~of~~ this criterion, interests which have more distances from mean and desirable for investments are known as risk and is got more weight in optimization process to shares with stretched distribution function [24]. These problems caused more researches about optimal stock basket forming and many people tried to improve the Markowitz's pattern. Including he himself, Markowitz suggested then that analysis based on semi-variance compared to variance, is making better stock baskets [24].

Generally, one can be said that investments approach in investment basket frame and with Markowitz and Sharp thoughts, has improved and it increased applying mathematic programming, accurate investment decision makings and have been presented different models with the purpose of investment guidance in the framework of investment basket using mathematic programming. Borrowing and lending capital options, as typical way~~s~~ of capital market, were ignored in the Markowitz model.

William Sharpe expanded Markowitz model in two dimensions, as this model involves assets without risk and possibility of borrowing and lending [13]. Sharpe approach is known as "Capital Market Line (CLM) that is kind of Capital Asset Pricing Model (CAPM). He showed that when a risky portfolio combines with a risk-free portfolio, achieved compound risk fits with relative capital which is invested in risky asset [32]. Arbitrage Theory which was presented in 1980 by Ross & Roll, is similar to capital asset pricing model. Arbitrage Theory says unsystematic risk with diversification can be omitted; therefore, this risk in the market with aero Arbitrage doesn't have any price [29, 30].

It must be expected when estimated requirements of Markowitz model are changed, a combination of efficient portfolios will change [13]. Changing in estimations is an optimization of model with the purpose of new weights. One can be noted that there are two problems for experts who estimate covariance between different industry stocks: High volume of information, their complexity [13].Markowitz accepted it and tried to solve it [24]. He suggested that instead of covariance, relation between rates of return in per share, is replaced with Stability index of the market. Sharpe (1964) followed Markowitz approach which was known as “Single indicator model.” Then multi-indicator models were introduced toward controlling of some effects of non-markets which caused change in the price of securities transaction. If other characteristics like the fixed costs and obscurity of the exchange transaction effect on selecting portfolio issues, the model of Markowitz losses its functions. Konno & Yamazaki have developed Mixed – Integer Linear Programming which absolute deviation is representing risk aversion. This pattern can’t response to investors’ requests without attention to Functional constraints [18].

Kellerer et al. (2000) have introduced some kind of Mixed – Integer Linear Programming which is able to encounter with fixed costs and obscurity of the exchange transaction [17]. Also one can be noted that combination of many stocks and High volume of exchange cause increasing of exchange cost. Some preconditions on Markowitz model cause increasing stock combination and the number of exchanges which causes increasing of cost. As the purpose of solving problem, Bertsimas et al. (1999) showed an approach that can form portfolio through Non-Linear Mixed – Integer Programming. That portfolio is close to a target portfolio which builds using of Quadratic Programming of Markowitz model. It has not only equal Liquidity with it and expected returns and its turnover is similar to target portfolio, but also it includes some advantages like Frictional Costs due to selecting less stock as possible[5]. Mansini & Speranza suggested that portfolio selection patterns assumed investment split as unlimited, while securities transaction are exchanged with certain number in real world. Therefore, they suggested that a Mixed – Integer Linear Programming with considering to corresponding constraints to least exchanges is used. It must be noticed that Marquez standard pattern doesn’t have constrains related to selected assets and also constraints related to floor and ceiling limit to investment in each asset [22]. Chang et al (2000) and Fernandez & Gomez (2007) have used Marquez corrected pattern Cardinality Constrained Mean – Variance (CCMV) [6, 11].

It can’t be expected that future stock returns by historical of returns are reflected well and accurate. Li & Xu (2009) supposed input variables of model with Cardinality Constrained Mean – Variance (CCMV) Random Variables because of promoting information accuracy and combining science and experience systematically and they did some essential changes on Markowitz model. This model has more flexibility and can adopt itself with rate of investor pessimist-optimist and can balance efficient frontier of Markowitz model due to this topic [19].

Finally, it can be said that Modern Portfolio Theory (MPT) is based on efficient markets' hypothesis and logical investors. However, there can be found any markets which are against to this rules and there is not a Modern Portfolio Theory. These markets are called "abnormal." Behavioral finance explains these abnormal in several forms. This branch presents theories due to explanting inefficient of markets and enological behaviors. Frijns et al. (2008) showed that according to behavioral concepts, mean-variance model can be expanded that leads to involved Socio – Demographic Variables. His results of those expansion showed that evaluation of individual ability, his profession, age, and sex can be specified in a stock portfolio-optimization to risk and returns, risk-free returns rate level, individual risk, and market beliefs [12]. Ponsich et al, (2012) in their research as" investigation of multi-goals evaluation algorithms in the purpose of solution for optimization portfolio problem and other applies in economics and financial issues" stated that coincidence of development and optimizing multi-purpose algorithms and existed complex formulas in economics and financial lead to two-way courtesy for both society. Selected classification for this research shows the difference between optimizing portfolio problem and uses [27]. Hanen & Faou (2015) investigated o mean-value in risk with random fluctuations, making decision for selecting optimal combination selection from a risky property and especial using of maximizing utility function with VaR limitation with losses proportional to limited current output. The results of this research showed that VaR limitation reduced investments amounts in risky property gradually, and fluctuation has an important effect on optimization response [15]. Yin et al. (2015) showed a research of heterogeneous Multiple Population Particle Swarm Optimization Algorithm (HMPPSO) with the purpose of solving selection sample model of Markowitz mean-variance. This algorithm is according to multiple population strategy which whole population divided to several sub-population and whole sub-population were completed several kind of optimization of particle mass. The relation between sub-population is performed in regular distances which cause information exchange [38].Liu et al (2016) in "multi-year validity of optimization portfolio model with bankrupt control and reuse" investigated about selecting sock box with bankrupt control in fuzzy investment environment and they used stock box optimization of combined particle mass algorithm. Its results show that particle mass algorithm is effective in selected issues [20]. Mishra et al. (2016) in “prediction of mean-variance model for selecting limited property portfolio” have investigated on selecting limited portfolio with multi-objective evolutionary algorithm. In this research, optimization of the mass of self-regulating multi-objective particles is an effective way of portfolio optimization. For this purpose, multi-objective predictive prediction, mean-variance model was supposed that is a solution for Markowitz mean-variance model that is used for limited portfolio optimization problem [26]. Tilahun & Ngnotchouye (2017) in “Firefly algorithm for discrete optimization issues” has investigated changes done firefly algorithm in the purpose of solving optimization issues with especial discrete variables. So, developing in the using of firefly algorithm for optimization issues of binary numbers, integers and also mixed variables are studied. In this research, firefly algorithm was corrected and used for optimization issues with discrete variables [35].

According to increased importance of investments in financial baskets, two Meta-Heuristic Algorithms in order to solving stock basket optimization problem with the number of stock constraint and also limitation of floor and ceiling rate of each stock with efficient frontier have been shown in this article. Also, in this article Cardinality Constrained Mean – Semi -Variance (CCMSV) and Cardinality Constrained Mean – Variance (CCMV) have been investigated. The only difference between two last patterns is about minimizing of risk aversion.

For the purpose of solving this problem using of Firefly-Algorithm (FA) and Imperialist-competitive Algorithm (ICA) with daily data of 25 companies in Tehran securities transaction stock in period of 2010-2016 have been used.

2. Investments basket optimization patterns

The main weakness of Markowitz model was that it doesn’t let the participation transaction aspects of real world like maximum size of the portfolio, minimum stock, and (etc.) Fernandez & Gomez corrected Markowitz model with a stock portfolio-optimization floor and ceiling constraint for variables and they developed “Cardinality Constrained Mean – Semi -Variance (CCMSV)”; we will talk about this pattern.

Measuring of maximizing expected returns (R), decreasing expected risk (Q) as standard deviation are the goals of Markowitz model [21].
With portfolio returns (R_p) and portfolio varianceσ_p^2, maximizing equation is as:

$$Max\ (\lambda . E(R_p) - (1-\lambda).\sigma_p^2) \quad (1)$$

Portfolio returns achieves from combination of mean of with each share weight [25, 28, 36], which is relation 2,

$$E(R_p) = \sum_{i=0}^{n} W_i\, E(R_i) \quad (2)$$

Where, $E(R_p)$ is expected returns rate of portfolio; W_i is weight of each share in portfolio which is specified by algorithm; $E(R_i)$ is expected returns rate of i asset and n is the number of selected assets for forming portfolio.
Shares which are existed in portfolio can have direct or indirect relation with together. This relationship is announced by equity correlation coefficient and used by portfolio risk calculation [28]. Portfolio risk can be achieved by:

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} W_i W_j \sigma_i \sigma_j p_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{n} W_i W_j \sigma_{ij} \quad (3)$$

for i=j p_{ij} =1

Where, σ_p^2 is portfolio variance; σ_i is standard division of i asset; σ_j is standard dilation of j asset; p_{ij} is correlation coefficient between i and j assets, W_i is weight of i asset in portfolio; W_j is weight of j asset in portfolio and n is number of selected assets for portfolio forming.

$$\sum_{i=1}^{n} w_i = 1 \quad (4)$$

$$0 \leq W_i \leq 1 \quad (5)$$

As if expected returns of each asset is E (R_i) , standard deviation of asset is equal of σ_i and W_i is weight of each asset.
In the mathematic model of elation(1), which is the main model of this article, is weighting parameter (λ) that its amount can change in [0-1] i.e $\lambda \in [0,1]$) , which indicates an implicit interaction with risk and returns. (4) & (5) equations are constraints for weights which shouldn't violated.
Also, stock returns vary in different periods and it doesn't have constant process. Therefore, fluctuating and variability are undeniable parts of stock returns during time. According to fluctuation and variability, the returns of future periods are not trusted. Unreliability to future stock returns makes risky investment [21].

Generally, volatility of investment returns is called investment risk. In other words, the more change of returns of investment, the more risk of investment. Criterion for measuring of whole risk of stock is relation (3). For the purpose of more accurate, consider to mean-variance model as:

$$Min: \sum_{i=1}^{n} \sum_{j=1}^{n} W_i W_j \sigma_{ij}$$

$$\text{s.t: : } \sum_{i=0}^{n} W_i E(R_i) = E(R_p), \quad (6)$$

$$\sum_{i=1}^{n} w_i = 1,$$

$$W_i \geq 0 \ ,(\mathrm{i} = 1, \dots, \mathrm{n})$$

We will represent variety assets portfolio. Practically, making decision about asset weight is done with decision on asset and can perceive risk variety better if we select combination of small assets. According to definition of portfolio, investment basket generally and stock basket in particular, investment basket means combination of invested assets by micro investor(individual) or Macro investor (institution). As previously stated, we can make stock baskets with optimization models and using of portfolio modern which has the least risk in compare of expected returns or has the most returns in compare of expected risk. As above, Markowitz had invented a method in which risk of a stock basket is function of each stock variance, its covariance with other stock and stock percentage in basket.

As above, expected returns and portfolio variance can be calculated from (2), (3). Therefore, Markowitz model can be written as problem with dual-purpose function as the following,

$$(7)$$

$$\begin{cases} Max: \sum_{i=0}^{n} W_i E(R_i) \\ Min: \sum_{i=1}^{n} \sum_{j=1}^{n} W_i W_j \sigma_i \sigma_j p_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{n} W_i W_j \sigma_{ij} \end{cases}$$

s.t:

$$\sum_{i=1}^{n} w_i = 1$$

$$0 \leq w_i \leq upper\ (i) \ , i = 1, \dots, n$$

Where, $E(R_p)$, expected portfolio returns rate; $E(R_i)$,expected returns rate of i asset; σ_p^2 ,portfolio variance; σ_i,standard deviation of i asset; σ_j standard deviation of j asset; p_{ij} correlation coefficient between i and j assets; W_i weight of i asset in portfolio; W_j weight of j asset in portfolio and σ_{ij} covariance between returns of i and j assets as $i, j \in \{1, \dots, n\}$. so, $\sigma = (\sigma_{ij})\ , i = 1, \dots, n\ \ , j = 1, \dots, n$ represents variance-covariance matrix n.n assets returns (n: number of selected assets for portfolio forming).

One can be noted that in model (1), the first objective function of a linear function is on W, but the second objective function is as a quadratic function on W.

According to misgivings, Fernandez and Gomez corrected Markowitz model with a stock portfolio-optimization constraints of number of selected stock and floor and ceiling constraint for variables and the developed Cardinality Constrained Mean – Semi -Variance (CCMSV). We will discuss applied constraints:

- **Cardinality Constraint**

Existed stocks number in portfolio are often constrained or specified by given amount. This constraint can be shown through binary variable z_i (i.e if i asset exists in portfolio, it is equal of 1, if it is not existed, it is equal of 0).

$$\sum_{i=1}^{n} Z_i \leq \mathrm{K} \qquad \text{Where} \qquad Z=\begin{Bmatrix} 1 & if\ w_i > 0 \\ 0 & if\ w_i = 0 \end{Bmatrix} \tag{8}$$

This constraint is composed as the purpose of portfolio management is facilitated and costs of portfolio management are decreased. The above inequality is quite common [16, 21, and 31]. Also, it can be introduced a floor constraint [7,8]:

$$K_{min} \leq \sum_{i=1}^{n} Z_i \leq K_{max} \tag{9}$$

But it can be represented as equal form [2, 34]. In this article, it is used of equal or of his constraint:

$$\sum_{i=1}^{n} Z_i = \mathrm{k} \tag{10}$$

One can be noted that portfolio sock number constraint can be considered as objective function [1].

- **Floor & Ceiling Constraint**

With these constraints, there will be a minimum and maximum (γ_i &ε_i, respectively) to each weight of asset that maintain in portfolio as $W_i \geq 0, \varepsilon_i \leq W_i \leq \gamma_i$. In other words, portfolio share for a certain asset can change in given period [7].

$$\varepsilon_i \leq W_i \leq \gamma_i \ \ , (\mathrm{i} = 1, \dots, \mathrm{n}) \tag{11}$$

With a stock portfolio-optimization selected assets to have problem,

$$Min: \lambda\left[\sum_{i=1}^{n}\sum_{j=1}^{n} Z_i W_i Z_j W_j \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{n}\sum_{j=1}^{n} Z_i W_i R_p\right] \quad (12)$$

s.t:

$\sum_{i=1}^{k} w_i = 1,$

$\sum_{i=1}^{n} Z_i = K,$

$\varepsilon_i \leq W_i \leq \gamma_i \quad ,(i = 1, \dots, k)$

$0 \leq \varepsilon_i < \gamma_i \leq 1$

$Z_i \in [0,1] \quad ,(i = 1, \dots, n)$

$W_i \geq 0 \quad ,(i = 1, \dots, k)$

Z_i is decision variable about investment in each stock. If z_i=1, it meant i stock will put in basket. Combination of stocks which will be in basket, due to this constraint, is k_s and are floor and ceiling constraint for i variable, respectively.

As above, λ is parameter which is in [0, 1]. As if λ=0, optimal portfolio equation without risk, will be maximum expected returns and if λ =1, optimal portfolio equation without risk, will be minimum expected returns and investor is as conservative. In fact, we can specify efficient frontier with changing λ from zero to 1. We achieved a new point for each λ on efficient frontier which can be made efficient frontier if we conjoin those points together. In fact, different combination of expected returns and desirable stock basket variance form a line in chart which is called "efficient frontier." Because each points represents the best possible scenario for expected returns and certain variance. As a matter of fact, investor, using efficient frontier, can select optimal portfolio according to his expected returns/risk portfolio requests [11].

Between all critics to modern portfolio theory, we can announce variance is not suitable criterion for risk and distribution of assets' returns does not match with normal distribution. Because both suitable criteria with the purpose of risk measuring and normal distribution play a very important role in selecting optimal portfolio. For that reason, it is tried to use criterion that match with investors' perception and corresponding hypothesis fits with reality. According to experts, semi-variance is one of the criteria for risk.

If in (12), s-variance replaced with variance,

$$Min: \lambda\left[\sum_{i=1}^{n}\sum_{j=1}^{n} Z_i W_i Z_j W_j \Sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{n}\sum_{j=1}^{n} Z_i W_i R_p\right] \quad (13)$$

s.t: $\sum_{i=1}^{k} w_i = 1,$

$\sum_{i=1}^{n} Z_i = K,$

$$\varepsilon_i \leq W_i \leq \gamma_i \quad ,(\mathrm{i} = 1, \dots, \mathrm{k})$$
$$0 \leq \varepsilon_i < \gamma_i \leq 1$$
$$Z_i \in [0,1] \quad ,(\mathrm{i} = 1, \dots, \mathrm{n})$$
$$W_i \geq 0 \quad ,(\mathrm{i} = 1, \dots, \mathrm{k})$$

In the above problem, Estrada equation is used for semi-variance calculation [9], which is:

$$\mathrm{MSV} = \Sigma_{ij} = \frac{1}{\mathrm{k}} \sum_{\mathrm{t}=0}^{\mathrm{k}} \left(\mathrm{Min}\left[(\mathrm{r_{it}} - \mathrm{E(r)}, 0).(\mathrm{r_{jt}} - \mathrm{E(r)}, 0) \right] \right) \tag{14}$$

Generally, collection of Cardinality Constrained Mean – Semi -Variance (CCMSV) and Cardinality Constrained Mean – Variance (CCMV) are combination of Integer Linear Programming and Dual Programming. For accurate solving of this kind of problems, there is not efficient algorithms in mathematical programing. In this article, we considered the investigation of specification possibility and portfolio forming by Meta-Heuristic techniques which is aimed on forming optimal portfolio and recognizing efficient frontier investment.

3. Modern methods or Meta-Heuristic

Classic and Numerical techniques have two main weaknesses: positional and process inability on discrete issues. It can be said that classic techniques can find locality optimization and can't find global optimization based on complicated and multi-dimensions issues and/or issues which have distortion, distinctness, noise, and disturbance of information characteristics or issues that dmension is discontinuous and complicated nonlinear equations [4]. According to this technique, in the purpose of solving local answers, there is not any thought and if they get to fairly optimal answer, they stop it and suggest it as comprehensive and global answer. For the purpose of these troubleshooting, there are variety of techniques like Meta-heuristic. Meta-heuristic techniques are represented to solve those problems. Although there is not any guarantee, test of these techniques in engineering, economics, financial, and … show that if they perform correctly and their internal parameters are chosen suitable and according to problem it can be got more suitable answers like their classic counterparts

If we have the best design, we can have the best results; in other words, they had built for compensating optimization techniques shortcomings. They have been planned like they "get out" of local optimal and get to global optimal. In other words, although Meta-heuristic techniques do accidently research, possibility of trapping in local optimal is decrease.

Heuristic algorithms are algorithms which generally were inspired from nature in which can solve nonlinear with constraint. There are some properties of heuristic techniques:

- It doesn’t need to hypothesis on issue, like distinctness, convexity, and etc. therefore it can be done on extensive issues.
- Generally, heuristic techniques are global and without convexity.
- This technique is for continuous and discrete issues. However, this is very suitable for discrete problems.
- There is not any support based on mathematical on performance and convergence which get to optimal answer but it shows itself in convergence act.

Firefly Algorithm (FA) and Imperialist Competitive Algorithm (ICA) are the kinds of heuristic algorithms.

3.1 Firefly Algorithm (FA)

Firefly Algorithm (FA) was designed by Xin-She Yang (2008) benchmarking of the luminance of the firefly. They used rhythmic brightness of firefly for attract hunt and mating. Brightness pattern in each firefly is different. This brightness can be a protective mechanism for firefly. Rate of brightness and distance of brightness cause attraction of the couple together. Each bit is a firefly and is updated a multi-dimension research with attract dynamically and according to science about firefly and its neighbors. The searching process is really surprising in which a firefly is compared with all other fireflies, if it has low brightness in compare of other firefly, its mate will choose the other one. This causes that bits attract to other bit with more brightness(light) and if there is more brightness in other algorithm frequency, bits move to a bit that has more brightness. Search steps depend on maximum of frequencies. There are main rules in the FA algorithm:

- All of the fireflies are unisex, as a firefly attracts other ones.
- Attractiveness of firefly is adopted to its brightness. So every low brightness fireflies will move to high brightness firefly. The more distance, the less brightness. If none of the fireflies have high brightness, firefly will move randomly.
- The brightness of firefly is specified amount of target function.

According to FA algorithm, Brightness intensity changes (I), attractiveness (β) are important parameters for each firefly. Due to constant brightness attraction coefficient, their relation is function of r (distance between two fireflies):

$$I(r) = I_0 e^{-\gamma r^2} \quad (15)$$

I_0 is brightness intensity in r=0.

Due to adopting attraction of firefly with brightness, firefly attraction is:

$$\beta(r) = \beta_0 e^{-\gamma r^2} \quad (16)$$

Where β attraction in r=0.

In relations (15) and (16), ɣ is brightness intensity absorb in each firefly. This parameter effects on specifying of convergence rate and algorithm behavior.

Distance between i firefly and j firefly is,

$$r_{ij} = |X_i - X_j| = \sqrt{\sum_{k=1}^{d} (X_{i,k} - X_{j,k})^2} \quad (17)$$

Where, Xi, Xj are location of i and j firefly. Also, d is number of independence variable of optimization problem (problem dimension).

Moving i firefly to j fire is,

$$X_i^{k+1} = X_i^k + \beta_0 e^{-\gamma r^2} (X_j^k - X_i^k) + \alpha \xi_i \quad (18)$$

In relation (18), second sentence represent attraction of i firefly to j firefly and third sentence is accidently moving in attraction process which specify rate of jump, ξ also is a random vector achieved from Gaussian distribution or uniform distribution [32].

Pseudo-code FA.

```
Begin
    Insert the objective function f(x), x=(x1, x2… xd) T
    Initialize the fireflies population xi, i=1, 2… n
    Determine the brightness intensity Ii at xi using f (xi)
     Set brightness absorption coefficient γ, randomize coefficient α
     While (t < Max Generation)
          For i=1: n   all n fireflies
            For j=1: n
              If (Ii < Ij), Move firefly I toward j; end if
                Vary attractiveness with distance r via exp [-γr^2]
                Evaluate new solutions and update brightness intensity
            End for j
          End for i
        Rank the fireflies and find the current global best
      End while
End.
```

3.2. Imperialist Competitive Algorithm (ICA)

Imperialist- competitive algorithm (ICA) was inspired from a social process in 2007 by Smaeil Atash Gargari et al. In Imperialist-competitive Algorithm (ICA), with the purpose of solving optimization problem, N country is chosen and each one will show with a vector and it shows a point in n-dimension space. In fact, countries are possible answers of problem. All countries are categorized in two sets: Imperialist and colony.

Colonial countries attract colony countries to themselves with attraction policy (assimilation) in order to different optimization orientations. Imperialistic competition aside assimilation policy forms the main core of this algorithm and leads to moving countries to minimal absolute side of the function.

As mentioned, countries are variables of optimization problem which must be optimized. In the one optimization problem N_{var}-dimension, a country is an array of $1 \times N_{var}$.

$$Country_i = [P_1, P_2, P_3 \dots P_{Nvar}] \tag{19}$$

Costs of a country can find with evaluate of f function in $(P_1, P_2, P_3 \dots P_{Nvar})$ variables, therefore,

$$cost_i = f(country_i) = f(P_1, P_2, P_3 \dots P_{Nvar}) \tag{20}$$

For start of algorithm, $N_{country}$ of initial country will be built. N_{imp} s will be chosen as the best member of this population(countries with the least amount of cost function) as Imperialist and N_{col} residual of countries are colonies which each one belongs to one Empire.

In the purpose of dividing colonies between Imperials, we give some colonies according to power. For this purpose, cost of all Imperials, their normalized cost is,

$$C_n = c_n - \max\{c_i\} \tag{21}$$

Where, c_n is cost of n Imperils, $\max\{c_i\}$ is the most expense between Imperials, and C_n is normalized cost of this Imperials. The more cost for each Imperils, the less normalizing. If we have normalized cost, relative power of normalized of any imperialist is as, (in which colony countries are divided ween Imperialists),

$$P_n = \left| \frac{C_n}{\sum_{i=1}^{N_{imp}} C_i} \right| \tag{22}$$

From another view, normalized power of an Imperial is colonial ratio that is controlled by that Imperial. Therefore, initial number of colony of an Imperial is equal of:

$$N.C_n = round\{P_n.(N_{col})\} \quad (23)$$

Where, $N.C_n$ id initial number of colonies of an Empire and N_{col} is number of al l existed colony countries in primitive countries population. Round is function of the nearest integer to a decimal number. Due to $N.C_n$ for each Empire, we select those randomly and give n t Imperial. If we have all primitive of Imperials, Imperialist-competitive Algorithm (ICA) will start. Evolutionary process is in one ring that is stopped with providing one condition.

Assimilation policy was done in purpose of culture and social structure analysis of colonies as aimed to socio-politics dimension close together. Moving of colony country to colonialist with d distance is as,

$$x \sim U(0, \beta \times d) \quad (24)$$

Where, β is bigger than 1, and close to2. This coefficient causes each colonial country, during moving to colonialist country, close to it from different sides.

In purpose of increasing searching area around colonialist, an angel deviance θ is adhere to random uniform distribution which is a stock portfolio-optimization to main vector:

$$\theta \sim U(-\gamma, \gamma) \quad (25)$$

γ is a parameter that control angel deviance area and it is considered close to $\frac{\pi}{4}$ in experiments.

As during moving colonies to new colonialist, it may moves to better colonialist (reach to cost function that produces less cost in compare of amount of cost function in Imperials position), so, colonialist and colonial change their position with together and its algorithm is found about new position and that time, new Imperial country does assimilation policy to its colonials.

Power of Empire is equal of power of colonialist country a stock portfolio-optimization with percentages of power of all colonies. Therefore, whole cost in Imperial is:

$$T.C_n = Cost(imperialist_n) + \xi mean\{Cost(colonies\ empire_n)\} \quad (26)$$

Where, $T.C_n$ is total cost of Empire n and ξ is a positive number between 0-1 and close to zero. $\xi = 0.05$ Has the desire answers in most of the performances.

If each Empire can't increase its power and lose its competition power, it will be omitted in Imperialists competitions flows. It means, powerless imperialists lose their colonies over the time and stronger Imperial takeover this colonies and raise its power. In the purpose of modeling competition between Imperials for takeover this colony, first we will calculate takeover possibility of each Imperial due to total cost of Imperial. So, total normalized cost calculated from total imperial cost.

$$N.T.C_n = \max\{T.C_i\} - T.C_n \quad (27)$$

Where, $T.C_n$ is total cost of n Imperial, $N.T.C_n$ total cost of normalized of that Imperial.

If we have total cost of normalized, possibility of colony takeover competition by Imperial is as,

$$P_{Pn} = \left| \frac{N.T.C_n}{\sum_{i=1}^{N_{imp}} N.T.C_i} \right| \quad (28)$$

If we have probable of each Imperial, because of having corresponding colonial randomly but possibility of dependent possible to probable of each imperial, vector P will be,

$$P = [P_{P_1}, P_{P_2}, P_{P_3} \ldots P_{P_{N_{imp}}}] \quad (29)$$

Then vector R will be formed as size as vector P. Arrays f this vector are random numbers with normal distribution in [1, 0]:

$$R = [r_1, r_2, r_3 \ldots r_{N_{imp}}] \quad (30)$$

$$r_1, r_2, r_3 \ldots r_{N_{imp}} \sim U(0,1)$$

Then vector D s formed as:

$$D = P - R = \left[D_1, D_2, D_3 \ldots D_{N_{imp}}\right] = \left[P_{P_1} - r_1, P_{P_2} - r_2, P_{P_3} - r_3 \ldots P_{P_{N_{imp}}} - r_{N_{imp}}\right] \quad (31)$$

With given vector D, mentioned colonies will be given to Imperial that its index in vector D is bigger than the others. Then, in Imperialists flow, weak Imperials collapse eventually and their colonies are takeovered by the powerful one. That algorithm will continue up to one independent Imperial and positions, costs of colonies are equal to the cost of Imperial country [3].

Pseudo-code of ICA

1) Select some random points on the function and initialize the empires

2) Move the colonies toward their relevant imperialist (Assimilating)

3) If there is a colony in an empire which has lower cost than that of imperialism, exchange the position of that colony and the imperialist

4) Compute the total cost of all empires (Related to the power of both imperialist and its colonies)

5) Pick the weakest colony (colonies) from the weakest empire and give it (them) to the empire that has the most likelihood to possess it (imperialistic competition)

6) Eliminate the powerless empires.

7) If there is just one empire, stop if not go to 2.

4. Conclusion and Policy Implication

In this article, daily stock prices of 25 companies, super in Fourth quarter of year in 2015 have been used. Calculating of stock returns of companies is essential estimating of Cardinality Constrained Mean – Semi -Variance (CCMSV), Cardinality Constrained Mean – Variance (CCMV), and specifying optimal stock basket. In this article, calculating of daily sock returns is done by relation (32) and information of the final daily prices of stock securities transaction companies.

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \quad (32)$$

Where, R_t is daily returns of stock, i and P_t is the final stock price in t day and P_{t-1} is the final stock price in t-1 day.

Risk will explain as possible negative fluctuations of economic returns and will explore as standard deviation and semi-variance.

In a stock portfolio-optimization to,

Efficient portfolio means that combination of optimal securities transaction in a basket as if its risk is in exchange for minimum returns and maximum risk of given level. Efficient frontier indicates baskets which has maximum returns in given risk and minimum risk in given returns. We searched about selecting daily basket using daily price information of stock companies. In this way, we investigated different investments that an investor can consider them in the purpose of forming his investment basket. Risk and returns of 25 companies in the securities transaction stock market and inputs of algorithm were in this research. As mentioned above, there are two models for investigation about ability of suggested algorithms: Cardinality Constrained Mean – Semi -Variance (CCMSV) and Cardinality Constrained Mean – Variance (CCMV). One can be said, in current paper, integer constraint and investors prefers have been considered based on risk rate as coefficient of target function. We got acceptable results for both of the models after 200 frequencies. We drew efficient frontier which achieved from two functions in separated charts.

We are going to interoperate and analyze Meta- Heuristic techniques. it can be said, suggested algorithms of article was done by MATLAB R2016b and finally its result will be presented as chart and table. As mentioned, charts (1) and (2), represent Firefly-Algorithm (FA) parameters and Imperialist-competitive Algorithm (ICA) parameters which fit with portfolio problem.

Table (1): Firefly-Algorithm (FA) fitted with portfolio problem

Parameters	Amount
Number of Fireflies	60
gamma	1
Initial Beta	2
alpha	0.2
Maximum Iteration	200

Table (2): Imperialist-competitive Algorithm (ICA) parameters fitted with u problem

Parameters	Amount
Number of Countries	60
Number of Imperialists	5
Assimilation Coefficient (β)	0.9
Assimilation Angle Coefficient (Revolution Rate)	0.05
γ	0.1
Maximum Iteration	200

Source: author

The results of Firefly-Algorithm (FA) and Imperialist-competitive Algorithm (ICA) using Cardinality Constrained Mean – Semi -Variance (CCMSV) and Cardinality Constrained Mean – Variance (CCMV) are shown in (1) and (4) charts, respectively. Each of figures are different in target function and decreasing risk aversion. Charts are based on percent. Horizontal vector is risk and vertical vector is returns.

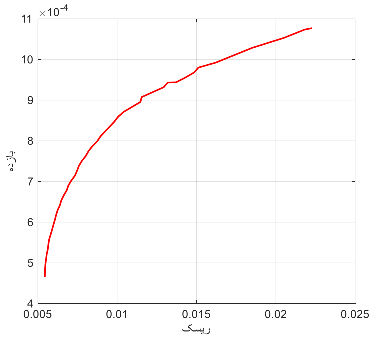

Chart (1) efferent frontiers using of CCMV model-FA algorithm

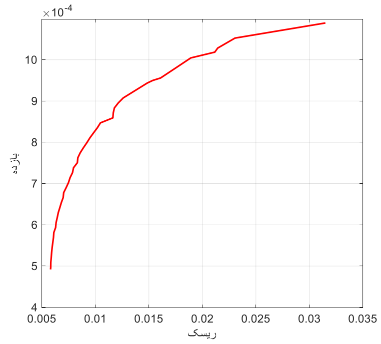

Chart (2): efferent frontiers using of CCMSV model-FA algorithm

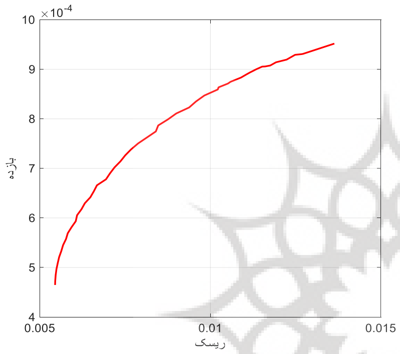

Chart (3): efferent frontiers using of CCMV model-ICA algorithm

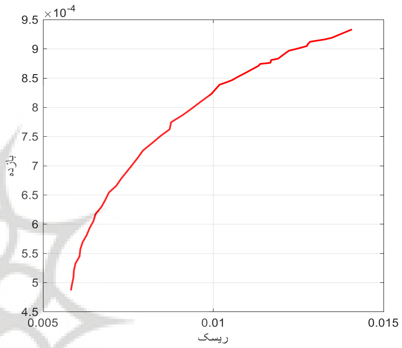

Chart (4) efferent frontiers using of CCMSV model-ICA algorithm

In the chart (1) and (3) variance and in the chart (2) and (4) semi-variance were used as risky factor. As it can be seen, both algorithms are acceptable accurate of finding optimal answers in all levels of risk and returns and could draw efficient frontier of investors well. In charts (5) and (6) achieved efficient frontier were compared with Firefly-Algorithm (FA) and Imperialist-competitive Algorithm (ICA) for Cardinality Constrained Mean – Semi -Variance (CCMSV) and Cardinality Constrained Mean – Variance (CCMV), respectively. Efficient frontier related to CCMV is specified with red line and for CCMSV is with blue stars. As variance is as general risk factor, and semi-variance is as estimated of undesirability of portfolio, it seems the CCMSV pattern specifies optimal portfolio more accurately. As Cardinality Constrained Mean – Variance (CCMV) pattern just measures undesired risk of portfolio, it shows less risk compared to Cardinality Constrained Mean – Semi -Variance (CCMSV) in same times. For that reason, the curve related Cardinality Constrained Mean – Variance (CCMV) for ach algorithm in the curve related Cardinality Constrained Mean – Semi - Variance (CCMSV).

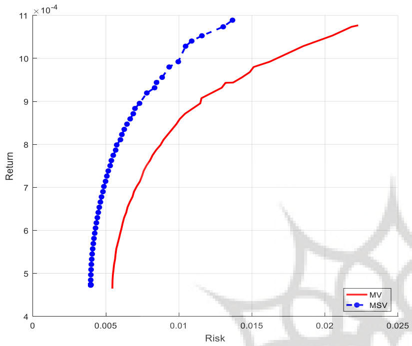

Chart (5): Comparison of efficient investors using of CCMV and CCMSV- FA

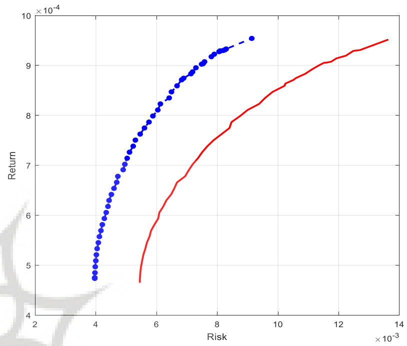

Chart (6): Comparison of efficient investors using of CCMV and CCMSV- ICA

Charts (7) and (8) shows path of function evaluation of Firefly-Algorithm (FA) and Imperialist-competitive Algorithm (ICA) with two current research suggested model, respectively, in purpose of optimal point. In these charts horizontal vectors show frequency number and vertical vectors show value of target function.

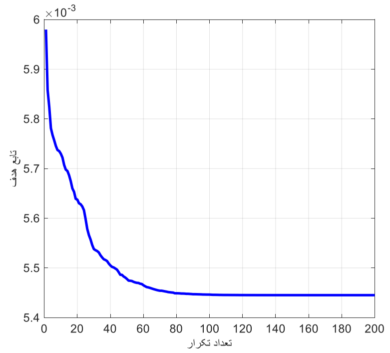

Chart (7): evaluation function path using of FA- CCMV

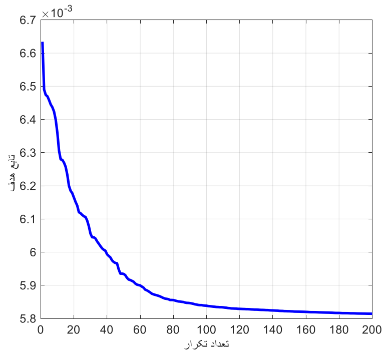

Chart (8): evaluation unction path using of FA –CCMSV

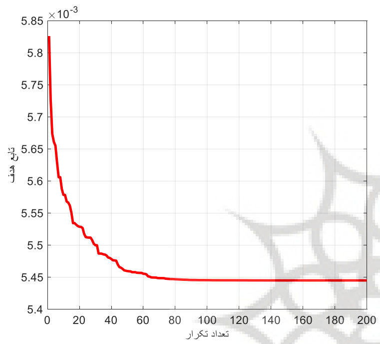

Chart (9): evaluation function path using of ICA -CCMV

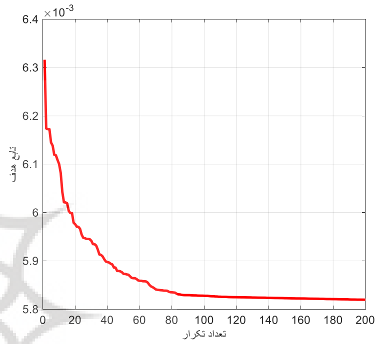

Chart (10): evaluation unction path using of ICA -CCMSV

According to population in chart (1) and chart (2), Firefly-Algorithm (FA) and Imperialist-competitive Algorithm (ICA) design 60 baskets and rating them based on risk goal and returns. Showing of several same rate in categories show that these baskets are dominant compared to each other. Therefore, selecting with priority can be according to risk preference o manager risk.

As number of baskets are a lot (60 baskets), we suggest one of the optimal stock basket randomly from tables (3) and (4). Numbers of chart indicate stock which should be selected in portfolio and also show value of each stock in portfolio.

Table (3): Stock and sock ratio in portfolio based on Firefly-Algorithm (FA) and other models in current research

Company	Value of each stock in portfolio based on current research models	
	CCMV	CCMSV
Karafarin bank	0.049455	0.047471
Egtesad novin bank	0.063531	0.063828
Parsian bank	0.052209	0.046073
Sina bank	0.044832	0.039988
Pholad Khurasan	0.053256	0.0708
Pholad Mobarkie Isfahan	0.036307	0.031893
Omid investor management group	0.034232	0.030864
Mapta group1	0.07417	0.099536
Iran khodro	0.037803	0.056679
Iran khodro diesel	0.028814	0.021575
Iran Transfo	0.030163	0.027583
IRISL	0.055154	0.061713
Informatics services	0.036472	0.029982
Lising rayan Saipa	0.038492	0.037459
Gol gohar mine	0.02335	0.016504
Chader malo mine	0.036826	0.028439
National Copper Industry of Iran	0.037355	0.031523
Isfahan oil refining	0.029796	0.029675
Pars khodro	0.02126	0.023265
Khark oil refining	0.038397	0.032887
Saypa	0.023284	0.022579
National Development Group Investment	0.049566	0.049573
Rena Holding Investment	0.038839	0.042229
Pension Fund Investment	0.045719	0.04153
Mining and metals development	0.020718	0.016353

Source: author

Table (4): Stock and sock ratio in portfolio based on Firefly-Algorithm (ICA) and other models in current research

Company	Value of each stock in portfolio based on current research models	
	CCMV	CCMSV
Karafari bank	0.049511	0.047665
Egtesad novin bank	0.063436	0.063817
Parsian bank	0.052165	0.045543
Sina bank	0.044788	0.040513
Pholad Khurasan	0.053311	0.070361
Pholad Mobarkie Isfahan	0.036291	0.033474
Omid investor management group	0.03418	0.030757
Mapta group1	0.07413	0.092475
Iran khodro	0.037793	0.057093
Iran khodro diesel	0.028837	0.021196
Iran Transfo	0.030257	0.028415
IRISL	0.055232	0.061713
Informatics services	0.036515	0.031087
Lising rayan Saipa	0.03852	0.037594
Gol gohar mine	0.023406	0.016261
Chader malo misne	0.036852	0.029208
National Copper Industry of Iran	0.037484	0.031768
Isfahan oil refining	0.029744	0.030206
Pars khodro	0.02121	0.023469
Khark oil refining	0.038363	0.033465
Saypa	0.023292	0.021763
National Development Group Investment	0.049493	0.051202
Rena Holding Investment	0.038824	0.042848
Pension Fund Investment	0.04568	0.042244
Mining and metals development	0.020687	0.015864

Source: author

At all, it should be noticed that target function and similar constraints are effective for stock baskets which had discussed. The main point is that populations are selected randomly in Meta-Heuristic algorithms and also existed operators cause random answers during performing process of each algorithm and answers are different in difference performs. For that reason, when result is good or bad, it can't be talked easily that a Firefly-Algorithm (FA) has ability or no. Therefore, ever algorithm must perform more than once. In this article, after modeling and specifying optimal parameters of used algorithm, 100 different performs with 200 frequencies have been done. Meta-Heuristic technique is one of the closest to achieved answers of algorithm in different performs. The results show that there is so little difference between the resulting answers from different frequencies that indicate there is high stable of algorithm in different performs. Charts (11) and (12) show assimilation of two algorithms.

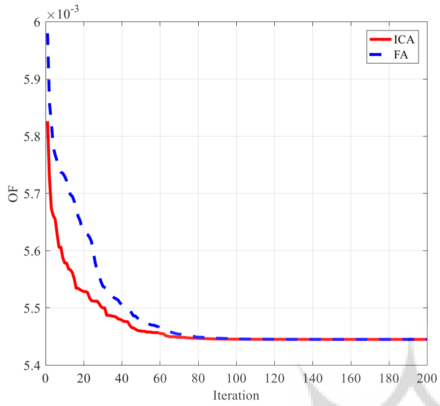

Chart (11): assimilation of FA7 ICA –CCMV

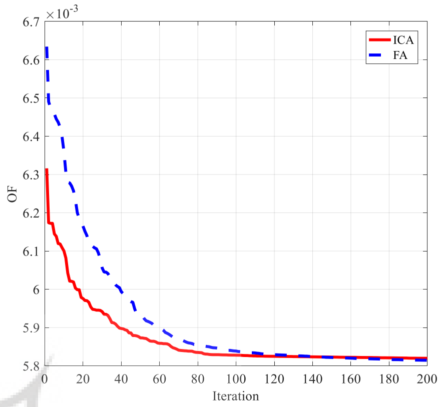

Chart (12): assimilation of FA&ICA –CCMSV

At all, criteria which were used in specifying of an algorithm compared to other algorithms in stock basket optimization process are: quality and accuracy of answers and elapsed time in terms of achieving optimal answer. The quality of answer means that how much corresponding algorithm can help to optimization and accuracy and the mean of the accuracy of the provided answer is that in order to all Meta-Heuristic ways depend on initial population, it often produces randomly, how much used algorithm could act in assimilation to optimal answer particularly in the number of frequencies. According to charts (11) and (12), used algorithm in this article, had acceptable accuracy in all of risk and returns.

In conclusion, the results show that performance of Firefly-Algorithm (FA) and Imperialist-competitive Algorithm (ICA) in optimization of stock basket due to real market constraints and different definition is risky. From quality view, it can be said that both of them were good and Imperialist-competitive Algorithm (ICA) was faster and better than

Firefly-Algorithm (FA) related to elapsed time to achieving optimal answer. As specifying value of optimal parameters is one of the Meta-Heuristic properties, so parameters have the main role in reaching algorithm to answer. The less number of parameters in an algorithm, the easier suitable number of these parameters. Firefly-Algorithm (FA) is more suitable than Imperialist-competitive Algorithm (ICA) because this algorithm has 3 parameters and the value can set in Yang (2008) suggested values. In fact, time for drawing efficient frontier for designed algorithm is more than Imperialist-competitive Algorithm (ICA).

In terms of comparison and evaluation of accuracy in stock basket optimization form two criteria time and variance) for each of the models and applied algorithm have been shown in table (5).

Table (5): comparison of suggested algorithm in used models of current article using of time and variance criterions

Firefly-Algorithm (FA)			
MV		MSV	
Time	Variance	Time	Variance
27994.72	4.49-E10	60460.96	1.25-E09
Imperialist-competitive Algorithm (ICA)			
MV		MSV	
Time	Variance	Time	Variance
1038.42	1.80-E09	2171.63	4.94-E09

Source: author

According to economics theory, the smaller value of the best variance and also the shorter assimilation speed and available time, the more accuracy rate in predicted model. According to results of current article from 25 stocks, the best variance between 100 frequencies for Imperialist-competitive Algorithm (ICA) is smaller than for Firefly-Algorithm (FA) with difference applied models. Also, in this algorithm, Cardinality Constrained Mean – Variance (CCMV) model has smaller variance compared to Cardinality Constrained Mean – Semi -Variance (CCMSV).

Also, the highest assimilation speed is related to Imperialist-competitive Algorithm (ICA), in fact this algorithm can achieve solution in shorter times and in this algorithm, Cardinality Constrained Mean – Semi -Variance (CCMSV) gets in shorter time to optimal answers compared to Cardinality Constrained Mean – Variance (CCMV).

According to above reasons, both of the algorithms can achieve global optimal which it seems Imperialist-competitive Algorithm (ICA) is faster than Firefly-Algorithm (FA) I assimilation and Imperialist-competitive Algorithm (ICA) has higher accuracy optimization in compared to Firefly-Algorithm (FA). Also in this algorithm, Cardinality Constrained Mean – Variance (CCMV) has higher accuracy compared to Cardinality Constrained Mean – Semi -Variance (CCMSV). These results are according to literature of current research and most of the research in the field of stock basket optimization. For the future researches the following areas researches can be suggested:

1. This research has been done on stock basket optimization using Meta-Heuristic algorithm. In the future researches can be studied on profit estimation modeling of each stock in Tehran stock using synthesis of artificial neural networks and Meta-Heuristic algorithm.
2. In this article, supplied stock in stock exchange, as existed properties, has been used. For future research can be suggested that a complex of mixed properties of basket and other properties like investment in banks, contribution papers, currency, gold, and earth will be considered used algorithm can be done.

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