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What's behind meaning?*

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Abstract

The paper addresses the main questions to be dealt with by any semantic theory which is committed to provide an explanation of how meaning is possible. On one side the paper argues that the resources provided by the development of mathematical logic, theoretical computer science, cognitive psychology, and general linguistics in the 20th Century, however indispensable to investigate the structure of language, rely on the existence of end products in the morphogenesis of meaning. On the other, the paper argues that philosophy of language, which, either in the analytic or the structuralist or the hermeneutical tradition, made little use of such resources (when they are not simply rejected). Left the main question unanswered. Though phenomenology intended to focus on the constitutive process, it ended up mostly with philology. Cognitive semantics paved the way to focus on patterns of bodily interaction within the natural environment out of which basic schemes emerge and are metaphorically "lifted" to any universe of discourse. The explanatory commitment is thus endorsed through two hypotheses: (1) these schemes, of topological and kinaesthetic structure, determine the range of forms of atomic sentences of any natural language, and (2) the category-theoretic notion of universality allows for a proper analysis of how such schemes are "lifted".

Keywords: Cognitive Semantics, Naturalised epistemology, Embodied Mind, Emergence, Metaphor Schemes, Category Theory, Phenomenology.

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1. from the end-product to the patterns of its genesis

Just as Aristotle claimed about being, meaning also is spoken of in many ways. The different facets of its study reflect the diverse interests and priorities of linguists, logicians, philosophers and cognitive psychologists. If the theory of meaning is to form a unified scientific inquiry, it should cover all these aspects and offer some account of their interconnection. However, most lines of research in semantics have oscillated between an obsession with taxonomy or exercises in modelling only those aspects of meaning which lend themselves to formalization within a chosen theoretical framework – with respect to which other aspects are dismissed as negligible or incidental.

I want to propose a different focus of inquiry: what makes meanings possible in the first place? What if semantic inquiry aimed to explain the processes operating in the constitution of meanings rather than to analyse meanings as already constituted and wholly determinate "entities"? Such a shift of focus suggests that the study of the constitutive grounds of meaning can no longer consist in listing its multiple components and selecting those which best allow of being modelled within a pre-existing framework for semantic theory, but rather in trying to identify and clarify the principles which govern meaning-formation. I also intend to show how such a refocused investigation of meaning might be connected with a refocused conception of the foundations of mathematics.

Such a re-oriented perspective has broadly guided my research since my 1974 Thesis at the University of Florence. At the outset, this tie between the sources of semantic capacity and the sources of mathematical thought was simply a working hypothesis, aimed at a renovation of phenomenology. Later it developed into a more substantial and autonomous perspective centered on making explicit the connection between the "embodied mind" approach to cognitive semantics and the category-theoretic view of the foundations of mathematics. That connection lies in those perception/action patterns which underpin the morphogenesis of basic mathematical notions.

The motivations for such an inquiry into the bodily roots of formal constructions were already present within the phenomenological tradition, although they were largely ignored both by cognitive scientists and by mathematicians addressing foundational questions from the standpoint of the axiomatic method as usually understood and applied. On the other hand, with the exception of a few original investigations (such as those by Hermann Weyl and, many years later, by Robert Tragesser),[1] phenomenologically oriented studies became little more than an exercise in Husserlian philology, which

made little or no contribution to understanding the architecture of formal thought other than through mereology.

Filling this gap is in this respect far more than an eclectic enterprise. First, the naturalistic orientation of any appeal to the notion of "embodiment" is at odds not only with many achievements of logic-inspired semantics, but also with much of semiotics and with the established ways in which analytic philosophers have thought about meaning in terms of a descriptive metaphysics. Secondly, the category-theoretic perspective on foundations of mathematical thought is likewise at odds with an equally strong tradition in the identification and interpretation of the principles of mathematics. Thirdly, it is not clear how these two alternatives to the "received view" are linked in a fruitful way.

2. Tools for understanding

"This is no longer semantics. It is just mathematics in disguise" and "This is not semantics but rather some odd sort of formal psychology, one in which different aspects of meaning are confused in a vain attempt to bypass the effort needed to achieve a rigorous semantics on the lines already established in the analytic tradition". These were typical of the reactions to such a program linking the study of patterns of meaning-formation and category-theoretic research into the foundations of mathematics.

To counter these reactions, three principal sources are relevant: gestalt theory in psychology, topos theory in mathematics, and cognitive grammar in linguistics. I suppose the present audience is familiar with gestalt theory. As for topos theory, here I confine myself to saying it is a branch of category theory in which algebraic geometry and logic met in a deep and fruitful way, and out of this meeting, a new viewpoint on foundations emerged.[2] Turning to cognitive grammar: one milestone was the analysis of metaphor as a cognitive process which transfers meanings out of bodily experience into and across other domains, thereby revealing the motion-laden nature of mind and providing a clue to the way in which the structure of any thought is manifested in language.

In light of the evidence gathered by cognitive grammarians, the task of semantics differs from to the way traditionally conceived in logic and linguistics, since priority is assigned to the sensorimotor patterns behind the meaning of any sentence. Thus, both the idea of formal semantics as a refuge for metaphysical nostalgia and the idea of philosophy as conceptual analysis underpinned by the clarification of meanings – through the logical investigation and/or regimentation of language – are seen to rest on flawed assumptions. But at the same time, the pragmatist and

hermeneutical repudiation of formal semantics is also rejected. That repudiation is revealed as driven by a kind of compulsion to return to the womb (the magical open sesame permitting re-entry to which was the appeal to the context-relativity of all meanings) – a compulsion born of lack of an explanatory framework for intentionality, joined with rejection of the very attempt at a theory.

In semantics, both the taxonomy of basic patterns of meaning across natural languages and the range of possible conventions about logical form to model linguistic evidence were superseded by "cognitive grammar", which finally assumes the explanatory task by bridging the gap between the data collected by comparative linguistics and the cognitive gestalts instantiated by basic, logic-free sentences in any natural language or any fragment of it used in the metalanguage of a formalised language. The underlying project was usually associated with the rejection of formal semantics. But formal semantics as such is of little help (as saying that astronomy benefits from equations: which ones?) until it becomes model theory, but model theory assumes an extensive range of structures as given. Thus I was led to rethink the cognitive grounds of model theory, rather than dismissing it.[3]

"Cognitive grammar" is a name for a species of cognitive semantics. Now, sticking to the genus, suppose cognitive grammar shows the inadequacy of any formal/computational approach and any purely referential/causal determination of meaning, the question "What lies behind meaning?" is still in need of an answer. You could claim that behind meaning there are (unconscious) cultural constructs not fixed by causal chains, actual reference, rules of logical syntax or algorithms. But you could also claim there are selective patterns residing at some level of the structure and/or functioning of the brain.

Indeed, meaning is said in many ways. Without further expanding the list of alternatives, it is already difficult to see how one could reconcile even these first two. Such a difficulty is increased by the moral which is often drawn from the failure of the "received view" – a relativistic and skeptic moral that is due to two flawed implications.

3. **Flawed implications**

First, does cognitive semantics really imply the dismissal of the very concept of objective reality? This concept had historically taken various shapes, the negation cannot concern just that naive form of "objectivism" as a position which blends traditional tenets (in both a metaphysical and an epistemological key) of rationalism, appeal to brute facts, Platonism and naïve realism. Moreover, if the bodily roots of meaning count as objective, and the

corresponding phenomenological alternative to objectivism is consistent, the attack on objectivity is self-refuting: If reality itself, inclusive of our body, is socially constructed, the objectivity of both the literal and the metaphorical collapses, but then the appeal to the bodily roots of meaning is also deprived of any explanatory value. Why be worried by the notion of the bodily roots of meaning if the body itself is not real?

The second implication concerns the rejection, in keeping with Husserl's legacy, of any use of mathematics within cognitive semantics, on the grounds that since mathematics itself becomes the object of linguistic and psychological investigation, to exploit mathematical concepts, particularly in analyzing the meaning of mathematical sentences, is just straightforwardly circular. Is one then to say that cosmology is circular because it is done by living beings whose very existence is made possible by what they are trying to understand?

Opposition to the above argument does not come from denying the need to consider subjective experience and self-reference in talking about reality. This is the position I argued in developing the viewpoint named "entwined naturalism".[5] On this view, we can recover (so to speak) "second order" objectivity from the recognition that the cognitive/linguistic resources of the knowing/speaking subject themselves form a dynamical system (or a structure/object acting on other structures/objects), which in common with any dynamical system is itself acted on by other structure/objects arising in nature.

It is just this recognition which makes category theory an appropriate tool for describing the dynamical architecture of meaning-formation, since category theory focuses on invariance under structure-transfer and looks for cross-domain principles rather than for ontological homogeneity. Thus, in the study of cognition, it allows for a direct formulation of the patterns of meaning-formation and in this way meaning becomes the subject matter of a richer natural science, rather than a subject whose principles are beyond formalisation (the "ineffability of semantic content" which has often been appealed to as an argument against physicalism or any causal closure of the world).

4. Wisdom and deception

Once the two flawed implications are set aside, such a renewed phenomenological perspective admits the relevance of logic and set theory in semantics just as was suggested within analytic philosophy, with the aim of eliminating loose and ambiguous talk about the notions of truth and meaning.

But is also enables us to see why the promptings of the analytic turn were at the same time both wise and deceptive.

In what sense deceptive? The moral drawn from the disappointments and frustrations encountered using the tools of classical logic, set theory, Tarskian semantics, possible world semantics and, later, programs for AI or even constructive type theory for semantics of programming languages, was that access to meaning has nothing to do with toolboxes of any formal description. The deceptiveness encountered in the choice of toolkit made blinded some to the need of mathematics in a scientific theory of cognition.

The resort to mathematical logic at the origins of analytic philosophy was indeed wisdom. It was well-motivated and led to great advances in the understanding the structure of language. And just as the early analytic philosophers had sound motives for making use of logic in the analysis of both natural and formal languages, so there are sound motives for exploring topics relevant to the theory of meaning with the aid of notions drawn from the category-theoretic presentation of linguistic and inferential structure.[6]

One century ago the motives for the "linguistic turn" did not spring from the conviction that the mathematicians had made available some kind of miraculous oracle or key to the universe, and today the categorical approach to semantics should remind philosophers to be on guard against the risk that what they say about the structure of language or the architecture of the mind may often be unwittingly influenced by paleo-ontology – buried metaphysical presuppositions and assumptions suitably reassembled to build up a new version of metaphysics, be it Aristotelian or Platonic, monistic or pluralistic, Being-oriented or Becoming oriented, materialistic or anti-materialistic, because any version makes use of the same array of semantic resources which are literally understood only through our bodily experience. It is this array of resources and theory patterns of combination which are under investigation. If these give rise to a system, we are obliged to make its structure explicit. But this structure has a schematic nature and it is not monolithic: it has many components (each admitting many instances) which can be combined in many ways. If such components are organized into sufficiently independent substructures which formally compose with each other, describing the resulting architecture requires the language of mathematics. While it is an illusion that pre-packaged recipes work in philosophy (as, indeed, anywhere else) - be they provided by a successful scientific model, an all-around methodology, a beautiful mathematical theory or a merely therapeutic analysis of language - no solution to semantic problems is obtained

by claiming that meaning is beyond any systematic, principled, precisely formulated description.

Both in logic and in linguistics the central defect of semantics was a fixation on the end-product, ignoring the process of meaning-formation. The meaning of terms, predicates and sentences (truth-conditions) is defined by recursion starting from the respective inductive base, i.e., primitive terms and predicates giving rise to atomic sentences, but the identification of what they respectively stand for is simplicity assumed. Both philosophers of language who typically thought of meanings as complete and absolutely determinate, and philosophers who denied this, nonetheless shared such a fixation. Causal theories of reference were an attempt to remedy this defect but they did not go beyond the setting of already established natural languages. Moreover, in cognitive psychology the attention paid to the logical aspects of the understanding of meanings generally stopped at the syllogistic level. In computer science there was reason for this focus in the end-product, since programming languages and their semantics are designed from scratch in terms of "procedures", to be executed on data provided by whatsoever input device. There are also theorems which inform us that there are inherent limits to computability, but they do not concern the window of accessibility to data. On the one hand, meanings are supposed to be in the head of the programmer in already complete determinate form, on the other hand AI systems for language understanding still rely on a disembodied syntax, or else, in connectionist approaches, bypass syntax altogether.

5. How to overcome the obstacles?

The obstacles met so far can be overcome by taking jointly into consideration the following three kinds of constraints:

D1) the system dynamics which allows for the emergence of gestalts – and in particular the role of topological ur-gestalts in establishing primal reference to identifiable places, objects, object-states and actions;

D2) the way in which logical properties are intrinsically tied to the cohesive and variable structure of the objects composing the universe of discourse;

D3) the spatial roots of any syncategorematic expression, and thus of all notions through which syntax is built up.

Within this integrated perspective we can recognize the limitations of such static pictures as those associated with the earlier frameworks for the study of meaning discussed above and we are led rather to consider the genesis of conceptual structure. Such a search is directed not so much – or at least not only – at a uniform solution to the philosophical problems related to language

(such uniformity might still in principle be the consequence of a heavenly realm of pure Platonic Forms of Meanings). It is rather a search for the kind of uniformity exhibited by basic interaction patterns of an organism (as opening/closing, joining/dividing, grasping/leaving) whose very existence is made possible by conditions subject to the natural constraints. With this shift of focus, we are led out of the old statics centered on the study of semantic content as an end-product and towards a dynamics of the formation of concept-patterns which also compose with each other in a finite number of ways but produce a potentially infinite range of propositions.

This research project does not to start from scratch, since different theoretical guidelines and methods already converge on it. One that deserves particular attention has been pursued for many years by Jean Petitot, exploiting the resources of differential topology, after the pioneering intuitions of René Thom, as for example when he looks at predication as capture, according to the prey-predator dynamics.[7] But much remains to be done especially as concerns the cognitive genesis of logical structure. We should not forget that if a semantic theory re-focused on meaning formation marks a step forward from previous study of static, timelessly reified meanings, it is thanks to the prior achievements in logical analysis of language and particularly of the relationships between syntax and semantics. It was this which led 20th Century mathematical logic to the creation of model theory, one of its greatest achievements. One could say the analysis of language led out of itself – not towards a jungle of context-bound usage, unstructured by principles, but rather towards a geometry of meaning-patterns (in a wide sense, since no specific notion if distance is presupposed).

The small group of researchers who have pioneered this field over the last 30 years or so know from experience how resistant philosophers in both the analytic and the so-called "continental" tradition remain to such ideas. I hope the present audience is not disposed to view cognitive semantics as a marginal field of inquiry which leaves the core of philosophy of language untouched, or to think it is marginal because of the mathematical notions to which it appeals, i.e., those of category theory. My claim is that this research centrally affects all the various aspects of the study of meaning. To argue this I now survey[8] the main steps which led to my present view of "what lies behind meaning".

6. from disembodied intentionality to embodied intentionality

The idea of using categories within semantic theory (other than the category of sets in its membership-as-primitive-based guise) can be seen in retrospect as a natural option. It provided the most general and rigorous framework for

the study of the relationship – and notably the irreducibility – of sense to reference and intension to extension. This idea should not be confused with a proposal for an intensional ontology. Rather, it allows us to recognise the fallacy implicit in all such proposals hitherto.

That fallacy consists in the "instantaneous" treatment of a map as an object (in particular of a function as a set) and thereby fails to take account of the objectification process which permits the faithful representation of different maps, $f,g : A \rightarrow B$, into different "elements" of a function space object BA. But, when I suggested at the end of my 1974 thesis that category theory might provide a natural means to distinguish intension and extension within the same universe of discourse, it was not considered a "natural" option at all.

If intensionality is bound up with intentionality – that is to say if the sense, as distinct from the reference, of an expression is the outcome of the very constitutive process whereby the human mind exploits patterns of objects and actions, then the language of categorical language becomes a natural resource also for phenomenologists, provided two Husserlian tenets are set aside: 1) that phenomenology is a sort of pre-science, and 2) that such a pre-science could only be of a purely descriptive character. But if the noema, as the phenomenological counterpart of meaning, is subject to composition and transformation laws, why could these laws not be presented according a formal, even axiomatic, treatment?

It was with the aim of exploring that possibility that I presented, at Ettore Casari's "Saturday Seminar" in 1975, an axiomatic treatment of the notion of noema. What was missing in that proposal was a characterisation of the ambient category for interpretation maps. In fact the needed universe of discourse already existed and was already properly defined. It was provided by Bill Lawvere's functorial semantics and its use in topos theory, the logic-related aspects of which had been extensively developed in the wake of Lawvere's path-breaking discoveries. At the time I was unaware of this and still believed that any language, typed or untyped, could have a semantics only within the universe of sets.

Moreover, since set-theoretic semantics for just a small fragment of natural language was beset with so many obstacles in trying to model contextual variation of meaning, one could jump to the conclusion that some aspects of linguistic praxis were beyond formalisation and that the variety of usage precluded any systematic general theory of meaning. It seemed that philosophical analysis could at most aim at de-misting different portions of the mirror of language, without hope of an overall understanding of the principles behind the patchwork. The missing link was what I summed up as

the depth of surface, namely the essential role of spatial structure in the capacity to refer-to, as the identification of surfaces is required to discerning the reference of basic expressions. Furthermore, the topological description of continua provides the background out of which the discrete characteristics of language emerge, rather than the converse. Without a recognition of this link, functions are identified as sets of ordered pairs and we are ready for a medieval ontology of substance and accidents (or both), now in a relational guise, or we may choose an ontology of relations out of which individuals and classes are identified, by passing over both the processes which allow for the constitution of objects and maps and over the constraints acting on such processes.

This impasse was bound up with the dogma that any rigorous formal semantics must rest on a set-theoretic framework. Refinements involving modal notions required for the intensional aspects did not affect the core dogma and in particular the idea that the interpretation map is an arbitrary (unconstrained) function from one set to another. From this dogma follow Hilary Putnam's arguments about the limits of model-theoretic semantics directly applied to natural language, starting from the non-categoricity of reference (Löwenheim-Skolem Theorem).

Some have thought that the information-processing model of the mind holds out the prospect of an intensional solution to "the problem of meaning". On this view conceptual content can in principle be identified with the algorithmic procedures governing the symbols of a hypothetical inner language called "Mentalese". But, aside from the difficulties of completing such a program, there is a prior objection to the assimilation of an algorithmic procedure to a special kind of function, when functions are supposed to be sets: if a procedure requires the set of all its values to be given in order to be defined, it cannot correspond to the understanding of any meaning by a mind – other than an omniscient one.

Without necessarily accepting Richard Rorty's prognosis of the hermeneutical destiny of analytic philosophy, one can recognise how far-ranging was the impact of the so-called "linguistic turn" on the information processing model of the mind. It led to the view that if there is such inner language of thought, then meaning is confined within the enclosed Cartesian theatre of inner representations – say, as Mentalese propositions. But then the "symbolic fallacy" lies in wait. If the meaning of an expression of Language[4] is in turn just an expression of a deeper-level Language 2, and so on recursively, semantics becomes syntax. How then can we enter or exit the theatre in which meanings are supposedly to be found? Once meaning is just a "role" in this inner theatre, the fact that language hooks on to the world

becomes either a miracle or an illusion.[9] Moreover, if the grammar and lexicon of inner representations with which Thought is here identified is a closed system X of recursive procedures which can be applied to each other, then its semantics cannot reside in the universe of classical set theory (with sets as objects and functions as maps). For Cantor's Theorem, applied to such a closed system X, would imply that it is a singleton (since XX =X). Whereas it seems we can think more than one thought.[10] Thus, typing is necessary, and yet self-reference remains a risky game.

The early version of type theory, with just one ground-type (for individuals or "logical particulars") is of no help here, since it provides no way of understanding how the individuals of that type – whatever they are – could be identified and hence acquire the phenomenological salience to become referents for constants (0-ary singular terms) as proper names in natural language are usually taken to be. By hypothesis the bearers of such names should be naked, whereas they are in fact spatially qualified.[11] Nor does the admission of many ground-types for different primitive kinds, as mooted in Richard Montague's Intensional Logic and present-day dependent/polymorphic type theory, provide a way out of the difficulty. Although taking into account the (no less primitive) kinds of maps whereby the ground-types are related to each other was certainly an advance, the cognitive genesis of ground types (and maps) calls for more than a formal typed hierarchy.

The study of the real semantics of a real language used by real speakers in the real world cannot ignore a consideration of the cycles of perception and action which underlay the emergence of meaning either in its extensional or intensional aspects. Consideration of such cycles leads us out of the fly bottle of a disembodied semantics, by suggesting three necessary conditions for the emergence of meaning:

i) principle-governed stability of reference to objects undergoing continuous changes,

ii) principle-governed stability of the senses of expressions across distinct domains,

iii) principle-governed constructive reasoning about variable objects in variable domains.

The import and mutual relationship of these principles has already been made precise thanks to the categorical analysis of logic in topos theory(Peruzzi, 2011) and the resulting analysis is of deep consequence for the transition from a "static" view of semantic contents as finished products to a

dynamic view of cognitive patterns operating in the constitution of meanings. This transition turns on the generative power of meaning-patterns, and it is here that the cognitive role of metaphor enters the scene.

7. Three philosophical turns

The limitations of formal semantic theories for natural language, resting on either first-order logic favoured by analytic philosophers or the algorithmic form favoured by computational linguists, had already been detected and classified in great detail by the early 1980's. Meantime, a great step towards understanding the source of such limitations, and towards a unified alternative picture, came from the work of Jeffrey Gruber, Leonard Talmy, George Lakoff, Ray Jackendoff, Ron Langacker and other linguists who provided evidence for the role of spatial metaphors as the trace of gestaltic constraints, for the fuzzy and radial character of concepts and for what could in general be termed "positional" thought. These investigations at the core of cognitive grammar have demonstrated how pervasive is topological structure in the perception-action cycles underlying the constitution of meaning. They have also showed the schematic character of basic action-patterns which, linguistically expressed, act as meaning generators.

By the second half of 1980's I was asking myself whether

H1) the diagrammatic reasoning whose resurgence is so marked in category theory can be related to such spatial, schematic and generative structure of meaning,

and whether

H2) far from being "abstract nonsense", the category-theoretic description of universal constructions can provide the way to make precise the conditions i)-iii) for the emergence of meaning.

Could the turn in cognitive science away from the disembodied view of mind and rationality converge with the turn from set-theoretic to category-theoretic semantics?

My conviction as to this convergence was strengthened by the presentation of the conceptual architecture of mathematics articulated by Saunders Mac Lane – an articulation based on an atlas of maps depicting the disseminated sources of meaning within mathematics and the ways mathematical structures are linked and transformed.[12] This viewpoint suggests that no single ultimate ingredient of definition provides an absolutely prior or ontologically privileged point of departure, from which all further definition must proceed. The "vertical model "of knowledge as a building, in which each floor depends on the previous floors, or as a tree, in which any node depends on the previous

nodes, until the foundations, or the roots, are reached, and charged with supreme responsibility, is indeed an efficient metaphor, but not the only one, and even though there are good reasons for its widespread use, what we say by adopting the model is not to be taken literally. Taking it literally is related to the tendency to see in the "lifted content" of patterns an ontologically primordial factor – a tendency more pronounced in connection with symbolic, as distinct from diagrammatic, representations of structure. The reconceptualisation of semantics by cognitive grammar frees us from such tendency: a "consistent" city planning does not reduce to a set of solid ("consistent") buildings, and since it has horizontal rather than vertical structure, it cannot be thought of as a "supersolid" mother-building, generating any other building of the city. Both kinds of structures matter and neither absorbs or excludes the other. The same holds for the manifold roots of meaning into bodily patterns and the bond patterns between meanings. But the path to realising these facts was not short.

The "linguistic turn" in philosophy owed its success to the novissimum organum: mathematical logic. The "cognitive turn" in linguistics and philosophy of mind likewise owed its success to a byproduct of that organum, namely computability theory and its applications in computer science. The above-mentioned limitations of both logic and computability theory in representing meaning independently of its formation process, led many philosophers to conclude that there are aspects of meaning outrunning any formal means of representation, a parallel to Gentile's and Husserl's conclusion that something essential is lost when the knowing subject is itself turned into an object of knowledge.

This premature conclusion can be seen as a further instance of what Putnam termed the recurrent tidal effect in the history of philosophy. A certain arrogant confidence in the near-magical virtues of formalisation clashed with a sense of the glamourous attractions of "mysterianism" – such as that of the "hermeneutic circle" whereby meaning emerged from the swamp of brute matter in a fashion akin to Baron Munchausen's dragging himself up from the waters by his own topknot – or that of a phenomenological discourse which proclaimed itself super-scientific by being anti-scientific. That Form collides with Life was already a slogan widespread within early 20th Century German Philosophy. Now it was rediscovered in a new guise, not uncongenial to trends already set in motion by mid-century: on one side "cold" logic plus algorithms, science reduced to techne, one-dimensional white-collar workers, deified methodology, on the other freedom of imagination, primacy of intuition, the "warmth" of life and … the lawlessness of meaning in public language games, irreducible to any formal or naturalistic explanation. When computational

models of mind entered the scene, there were also the rights of the first person to be defended. As an illustration, think of the burgeoning literature on mental qualia, the import of which can be summed up by the antique cartographical tag: Hic sunt leones.

Now if language is all-embracing, the perceived opposition between the linguistic and the cognitive turn melts away. From the logical structure of (either natural or formal) language to the algorithmic structure of the supposed language of thought, the two turns appeared to confirm the idea that philosophy is to be identified with the analysis of language (be it ordinary, formal or programming language). Thus, that very identification allows the description of what lies beyond language only in terms of a further language ... or else to admit explanation has given out. Hic sunt leones. The semiotic dogma according to which everything is a sign simply pushes the indefinite regress one step further in locating ubi leones sunt.

The third turn, dating from the 1980s, the so-called "embodied mind" turn, towards cognitive grammar, was intended to overcome the shortcomings of the previous two. Focused on the centrality of perception, sensory-motor systems, kinaesthetics, imagery and proprioception for the study of meaning, a new horizon took shape through many books and research papers and brought back to attention the earlier seminal investigations of Charles Fillmore. The relevance of his "frame analysis" and classification of "thematic roles" in frames was clear to the cognitive grammarians who turned to a renewed study of metaphor, and its link to the study of patterns of interaction within dynamical systems theory in its biological applications is yet to be appraised (Fillmore, 1968). Further developments attempted to establish a direct connection with neuroscience. These ran the risk of ending up glorifying the (human) brain, making the rest of the body a footnote, with masters of neural microstructure becoming jury-rigged masters of philosophy. Apart from the standard practice of advertising, which is ubiquitous, and the scientific community is no exception, the advance of the neurosciences in the last decades is undoubtedly changing the traditional debate in philosophy of mind, but so far philosophy of language is much less affected: even now, classical objections against the possibility of describing a purely connectionistic architecture governing just syntax, let alone semantics, have yet to be overcome.

8. Metaphor-patterns

The extended phenomenology of metaphor-patterns studied by George Lakoff and Mark Johnson was explicitly intended to focus on the bodily roots of meaning, as organised into "image schemas" – essentially of positional and

dynamic character – which are projected from a small collection of domains of direct sensory-motor experience to any others.[13]

Taken at face value as a new frontier in cognitive science, such a view of metaphor implies that amongst the tools required to understand the grounds of semantic competence – the "grasp" of meanings, that mysterium ineffabile for Frege – are concepts belonging to algebraic and differential topology. Their direct relevance for semantics becomes clear in a category-theoretic setting. Thus one would expect investigation of metaphoric patterns as part of human nature makes use of the mathematical tools which could make such investigation part of overall natural science. But there was no such use, in perfect agreement with the two Husserlian tenets about the character of phenomenology.

After more than two millennia of Reason as pure vs the impurity of the Flesh, of the height of Form over Matter, it comes as no surprise that the guilt trip of Western philosophy affected also the third turn under examination. Rather than looking for invariants behind the variations, attention was mainly paid at the cultural relativity of metaphors. No doubt there are cultural parameters involved in the selection of one schema rather than another, but if physical bodies and their physical properties are not a cultural construction, the asymmetry between the literal and the metaphorical calls for an analysis of the specific biological constraints on specific "image schemas". Consider for example the salience of the UP-DOWN orientation. Gravity is not (pace Luce Irigary) a cultural force. The denial of the idea of knowledge as a mirror of nature is welcome to the extent that it aims to free us from a realism which is at once cunningly naïve and metaphysical. It is no longer welcome when it seeks to turn the bodily roots of meaning into a further mental construction – that would simply return us to the symbolic fallacy. In treating nature as the product of nurture, and what is culturally constructed as anterior to the motion of bodies, claims of the social construction of knowledge reveal themselves as a further edition of Idealism.

Advances by Jerome Feldman and his coworkers in the understanding of the neural architecture underlying various semantic phenomena suggest that a culture-dependent flexibility in the selection and application of "image schemas" does not preclude the existence of built-in resources (which themselves permit and constrain such flexibility) (Feldman, 2006).The study of such resources is similar to the study of the numerical invariants associated with a space, as in algebraic topology (think of Betti numbers).

If we take on the task of explaining semantic competence, we have to meet a theoretical commitment. It is not enough to recognise the myth of a pure,

exclusively formal rationality, or to reject the notion of reference as a pre-conceptual, unmediated relation between words and things-in-themselves. If we want to identify the natural grounds from which semantic content arises and semantic structure is constituted, and the way in which it is then transferred to cultural domains and, on a lower scale, to pragmatic contexts, we have to identify what it is that permits us to contextualise. Meaning is context-laden, but the ability to identify contexts is not. That is the commitment to be met.

So, for instance, we are able to use the UP-DOWN scheme in talking about positions in a corporate hierarchy or in the context of a discussion of emotional states because our body is primarily acquainted with the experience of standing up or lying down in a gravitational field, and not vice versa. It is not our experience of a hierarchy in powers of decision within an organisation, or amongst inner states that lets us understand the notion of verticality. In this one-dimensional case, as in many others of 2D or 3D configurations, there is scope for an investigation of the topology of our built-in quality spaces and, consistently with hypothesis H2), the interactional patterns which organise such a space into basins of attraction are universals in the sense of category theory, thus in a very different sense of the notion of universality from that still guided by medieval ontology.

9. Qualities and quantities

Amongst the most fundamental ingredients making the constitution of meaning possible is the cognitive capacity permitting us to discriminate the discrete from the continuous. Dating in its current manifestation at least from the arithmetisation of analysis beginning nearly 150 years ago, there has been a clear priority assigned to the discrete in the foundations of mathematical thought. That priority carried with it a conception of the interrelationship between the ontological, epistemological and logico-semantic dimensions of the notion of "what it is to be a foundation for" closely bound up with an understanding of symbolic (vs diagrammatic) representation as allied to the disembodied conception of thought and rationality criticised above, which aimed at an arithmetised ontology of essences.

It is a question for debate whether that priority is either an adequate or an inevitable option. My conviction is that, exactly as the stability of a system emerges from its dynamical interactions, the discrete emerges from the continuum. There is evidence that our apprehension of spatial structure is already at work in the very genesis of semantic content and that the understanding of basic ingredients of meaning calls for a theory which cannot be expressed in terms of point-set topology. Category theory makes manifest

the geometrical nature of logical principles[14], and in this connection the primordial character of spatiality is a claim in perfect agreement with the naturalisation of semantics and epistemology.[15]

Why then did arguments in favour of an "embodied mind" not exploit such a mathematical framework? The objection was that if every mathematical concept results from the metaphorical projection of an image schema, the resulting conceptual outcome cannot be used to explain the very source of the concept itself, on pain of an evident vicious circle. I already anticipated that this objection has a long ancestry and I mentioned Husserl who also used it against the formalistic drift he saw as manifest in mathematical physics. That drift led, he claimed, to the confusion of mathematical models with the real world, thus losing sight of the genesis of those very models in the qualitative experience of what he called Lebenswelt (our life-world), with the end result that human beings felt themselves cognitively and conceptually estranged from that world. Husserl's own reaction to that estrangement took the form of the nostalgic dream of a purely qualitative understanding of quantity. But at the same time such a reaction rested on the assumption that no mathematics of quality was possible, and this assumption contributed to the failure of the phenomenological project, whereas the 20th Century saw the growth of topology and qualitative dynamics, with manifold applications, and both kinds of notions are used in recent attempts at a "naturalised phenomenology".

10. Self-organisation is natural

The same objection against the use of mathematical language is behind the alleged vicious circularity of any attempt at a formal theory of meaning, on the grounds that the very meaning of formal notions is in need of explanation. If the circularity were really as vicious as alleged, how could we assert a non-circular understanding of the preconditions for the existence of speakers?[16] Planets do not use calculus to determine their orbits. We use it. Breathing is a precondition of our thinking, not vice versa, yet we clearly had to do a lot of thinking to arrive at an understanding of lung function, and lung function is not explained simply by the fact we breathe.

A clear and detailed understanding of how the capacity labelled "concept-formation" emerges from the self-organisation of matter in organisms is still a very long way off. But to renounce the search for (already formed!) concepts in terms of which to try to understand it would be self-destructive. A mathematical theory which turns out to be helpful in understanding the patterns of concept-formation, including the formation of mathematical concepts, no more implies vicious circularity than does the use of calculus in studying planetary orbits.

Loops in self-organisation are involved in explaining how basic, schematic, patterns of meaning compose with each other in specific ways. Here too we meet cycles of perception-action, described in terms of dynamic systems and their attractors, as summarised by the slogan "mind as motion" (Port and van Gelder, 1995). Behind this slogan lies the claim that the way minds work can be described in terms of dynamical systems (state variables, phase spaces, trajectories, stability, bifurcation) and that their "object" is rather a further set of dynamical systems with which we, as embodied minds, interact. So mind is not an entity of its own with such and such properties, although we usually talk of it in such terms: much as we say that rain is falling, i.e. through a subject-predicate sentence – whereas rainfall is in fact a relational phenomenon due to a very large number of dynamical states. Raindrops are themselves dynamical states of H2O molecules in relation with the Earth's gravitational field. The stability of any raindrop, as a feature on top of the atomic layer of H and O, is a simple illustration of emergence, and emergence is always constrained and, a fortiori, relational.

What I have labelled "entwined naturalism" is clearly a view which points at a theory of emergence and, differently from other similar views, it is one which does not support the strong thesis of "supervenience" in philosophy of mind, precisely because constraints on self-organisation assume primary importance and they are not secreted by lower levels of structure. Whether and to what extent a dynamical system is sensitive to boundary conditions may lead to very different outcomes in its state space. Thus the emergence implied in talk of the "naturalisation of semantics" does not place minds as embodied systems, patterns of meaning formation and culturally constrained contents on a par. The state spaces of physical and biological systems basically constrain the spectrum of outcomes, not vice versa, although some of these outcomes can generate relevant feedback.

Those who see a danger in the very possibility of a geometry (and physics) of cognition might recall that Maxwell's reduction of optics to electromagnetism led to the detection of many new features of optical phenomena – it deepened and enriched, and in no way impoverished, our understanding of light. Those who point out the defects of a purely formal approach to semantics, in either its logical or computational version, might equally benefit from the realisation that the context-dependence of meaning is part of the explanandum, not the explanans of the subject. Pragmatics usually takes for granted what is already an end product – namely the very existence of organisms able to use such a fantastically powerful cognitive resource as language. If the specific aspects of the natural environment and of paired, co-evolving perception-action systems such as the hand and the eye are simply

taken as given, we ignore a crucial point of purchase on the problems related to the roots of semantic competence.

11. Universal constraints

The criticism (mainly associated with Wittgenstein) of the traditional idea of meanings as Platonic essences can be endorsed, but it does not imply that the principles which permit and constrain variability of language use cannot be identified. A precise guide to their identification is provided by the notion of universality mentioned in § 4. This is not to be confused with the property of just having some very general or even unrestricted range of application – rather it requires a special and powerful kind of construction that also leads to the notion of "generic model" for a theory of a certain form[17]. That notion of universality can be used to make the Kantian sense of "transcendental" precise, and to re-animate a method which was too quickly dismissed with the logical empiricist repudiation of most aspects of the Kantian legacy (Peruzzi 1989).

Another confusion to be avoided is to think that category theory should be interpreted as a sort of holistic mathematics, and in particular as representative of that influential variant of holism labelled "structuralism". Exactly 20 years ago[18] I set out a criticism of the most common forms of epistemic and semantic holism. Structuralism is one of them. In the light of that criticism the claim that category theory is to be recognised as the main formal tool in articulating the structure of meaning (and the patterns involved in its constitution) is not to be read as a further variant of holism, nor as sharing its core assumptions.

Since there is no time here to go over the arguments I developed concerning holism, I can only point out that the entwined naturalist view of the relationship of language and world, mind and matter, subject and object, rests on interlocking principles the import of which is neither atomistic nor holistic in itself. Indeed their import is rather to rethink in a dialectic way the opposition between atomism and holism as fixed overall positions, whether in semantics, epistemology or metaphysics.

To illustrate this, two such interlocking principles may be cited here. The first is the PIRP or Principle of Invariance of Referential Potential, in order to explain the use of language aimed at the description of possible states of matter (e.g. in fictive and abstract contexts), differing from actual/known/real situations by balancing the variation in the actual reference (in such contexts) of expression A with the preservation of the actual reference of expression B within the same context (typically when A and B occur in one and the same sentence). The second is the Sheaf Condition that should be satisfied by any model of the mind in order to ensure the consistency and overall integration

of overlapping information extracted via different sensory "modules" and the consequent unity of emergent, cross-modular, self-consciousness.[19]

Once the absolute opposition of atomism and holism is abandoned or qualified in the way suggested by these two principles, it can be seen that intensional and extensional factors in the determination of semantic content are no longer in tension as appears to be the case when semantics is confined to the universe of sets (as in Tarski- or Kripke-style models). In the enriched setting provided by category theory, the "elements" and "parts" of wholes which have to be taken into account in studying extensionality depend on the kinds of objects and the kinds of maps to be considered. Such maps typically detect the cohesion and variation characteristic of evolving wholes as fibered over a parameter space: the degree of cohesion and the threshold between what varies and what remains constant is reflected in the conditions satisfied by their fibers and local sections. Consequently, some basically topological constraints on what is an element or a part of a variable whole cannot be ignored and such constraints make the interpretation map much less free than it is in the case of sets.

12. Steps into the geometry of meaning

The picture outlined so far of what lies behind meaning also implies notable changes in the conceptual tools needed for a systematic account of what lies in front of it – namely for a theory of meaning itself. The acknowledgement of the significance of spatial patterns within logical and algebraic structure is not a new idea. But previous appeals to spatial "intuition" had two defects. The concept of space was overdetermined, being referred to metric structure, and (after the crisis produced by non-Euclidean geometry and general relativity theory) it bounced to an undetermined background, admitting of indefinite plasticity. Though spatiality has a vast range of aspects (e.g., connectedness, homotopy, homology, projective and conformal structure) that can even be dealt with separately from each other at a certain extent, the basic kinds of actions through which we experience space are strongly constrained by which aspects of these various kinds are present.

Moreover, spatial structure is more than positional grammar for constant configurations in a motionless world. What language at bottom expresses – and rests on – are positions and changes of position (hence, by a recurrent metaphor, states and changes of state). The intuitive understanding of space as a unitary concept and the way that understanding is linguistically manifested stands in need of an analysis of each single aspect of the notion of space. This analysis allows us to pass from a logic of positions and pro-positions to a logic of pre-positions, in accordance with a principle-governed extraction of

patterns of meaning-formation out of each bodily scheme. The resulting approach has already been articulated in different forms of "naturalised phenomenology" wherein topological dynamics is explicitly recognised as central rather than (as common) implicitly exploited.

In consequence, the so-called "symbolic fallacy" encountered in connection with both model-theoretic and computational approaches to semantics (a fallacy inherent to any strategy of explaining symbols by means of other symbols) can be seen in a new light: while on the one hand the trace of kinaesthetic resources that allowed for the emergence of meaning is lost in those approaches, that trace is already at work when we speak of syntactic structure. Just try to eliminate all appeal to spatial ingredients from an understanding of the use of pre-positions, or to action verbs in giving an account of the rules governing the use of symbols in any natural or formal language. Far from achieving a completely pure account of syntax, you will experience the loss of syntactic understanding altogether.

But how do we arrive at an understanding of logical form starting from spatial patterns of object-and-action as these are at work in our bodily experience? Some indication of the answer may lie in the recognition that amongst the primary ingredients in our basic experience of space are the following:

– the identification of connected components,
– the detection of figures against a background,
– the relativisation of point-like entities to context,
– the paths from one position to another,
– the boundaries of regions and the possible holes in them,
– the kinds of actions which can be performed on objects located in physical space – where such actions can compose, as in a monoid or a group – in such a way to preserve or destroy the cohesiveness of the objects.

All these ingredients are related to each other and they jointly suggest that geometry (lato sensu) and algebraic topology is of primary importance in arriving at such an understanding and that only in its terms, rather than those of set-theoretic semantics, can we start to understand the object-and-action patterns involved in the emergence of meaning.

13. The freezing of concepts

Contrary to the traditional empiricist view of raw sense data as admitting an arbitrary segmentation into an unconstrained hierarchy of types, the use of nouns in real languages refers to at least relatively stable configurations against a background (even in the case of clouds and winds, where the

variability of the configuration is evident, and there is no inbuilt principle of division of reference of the kind required for count nouns). The very use of nouns, adjectives and verbs thus presupposes invariance conditions – which may be mild or strict but are in no sense arbitrary.

Together with object-patterns there are also action-patterns, which imply maps (from one configuration to another) preserving relevant properties under change of context, as in the passage from "grasping an apple" to "grasping a concept". Action-patterns are already implicit in the etymology of "concept", which comes from the Latin cum-captum, i.e. held or taken together, and whose root is the same as the word "capture". The capture-pattern (which can also be aimed at protection) is one fundamental and recurrent cognitive resource exploited by the human mind in thinking about many different aspects of the Lebenswelt. Though relevance can be context-sensitive, the underlying scheme (in this example of a processual, action-pattern type) is cross-contextual and thus endowed with a specific universality.

The language of category theory is able to express the variety of structural links between spaces, algebras, languages – formal or natural – and logic. But what is even more important is that it allows us to identify universal constructions and thereby provides the key to a unifying account of the patterns operating in the formation of meaning, across the great variability of contexts of language use. The richness (and the au fond geometrical character) of the structure already condensed into a logically "atomic" sentence is something to which logic – in conventional guise – was blind. And yet the trace of space is present in the intrinsic logic of a multi-sorted universe of discourse such as a topos of sheaves over a "base space", since the logical axioms correspond to the topology of the base.

The analysis of atomic sentences equally benefits from the adoption of such a perspective, because here pre-positional structure reigns, and it was precisely a class of logically "atomic" sentences expressing basic topological configurations that suggested to me the enormous gain in understanding which category theory provides in the study of the "geometric roots of semantics". By recognising the systematic lifting of meaning from bodily patterns associated with perisomatic space, to any other cognitive subject-matter, the traces of spatial structuration in that subject matter, even in its most abstract "transfers", are revealed. The traditional formal semantics which drew on set theory is just a limit case, corresponding to a derived universe of entities treated as point-like, discrete, with decidable identity, frozen in one state and immune from change.

From this viewpoint, in passing from atomic sentences to compound ones, we might speak of classical logic not so much as the ultimate yes-or-no logic but as a cryo-logic. The recognition of the way logical principles, in common with every ingredient of cognition, are ultimately responsible to spatial structuration, and the reconceptualisation of logical principles which this implies, means that categorial logic, which in the view presented here embodies this recognition, cannot be treated as just a further particular "style" of doing logic, possibly leading to add some new (formal) plants to a (no less formal) garden. It rather marks a basic new point of departure, in addition to covering semantic domains other than those governed by set-theoretic principles.

14. The space-scheme behind an atomic proposition

Some have objected that the category-theoretic approach to semantics of natural language is too complicated. But the use of any unfamiliar tool, language, set of notions looks as complicated at start. As for this one, it makes the structure of meaning-patterns formally explicit and paves the way to a theory of both literal and metaphorical meaning. It also reveals how the construction of meanings displays two distinct layers. In the first we find a base-structure in which we detect patterns of objects in physical space together with their perceptually salient qualities and patterns of motions and actions. In short anything which can be pointed at, or more generally indicated by localizers, as typically by indexicals, in our direct bodily experience.[20] In the second layer we find a lifting, over this base, of ground schemes of perception/action to any other universe of discourse. Whatever the way the objects in such a universe are identified, their relationships – the maps between such objects – are cognitively constituted by a transfer of such basic patterns.

For example, once we are able to identify pro-positions and we have symbols representing them, as objects occupying positions in a virtual space, we can use pre-positional schemes to express consequence relations between them. We can say that proposition B is provable STARTING FROM proposition A, intending that there is a logical path which goes FROM position A TO position B. But the root-meaning of FROM-TO lies in the experience of paths in physical space. Similarly we can speak of someone passing FROM joy TO sadness or, in describing the state space of physical systems, say a system passed FROM an ordered TO a disordered state. When the path is null, we speak of the state as staying in or remaining at. In spatialising time, Galileo simply made mathematically precise an inbuilt cognitive resource already manifested by such expressions as FROM Monday TO Saturday. In each case the transfer of the FROM-TO scheme allows for moulding the way we understand what is "going on".

The path-scheme "forgets" various aspects of the original layer – the speed and other features of the specific manner of going FROM-TO do not matter. This "forgetful" character is essential to any pattern-transfer. All that matters is the transition from one position to the other (or the remaining in stasis), in conjunction with another scheme, the IN-OUT scheme, a conjunction manifested in expressions such as enter into, exit, come in, go out (and indeed the more recent further metaphorical transfer of outing) which scheme is likewise "forgetful" of e.g. concrete features of the container. In this case all that matters is the existence of a boundary which can be crossed or not.

The study of metaphor by cognitive linguists was very important in bringing to light the phenomenology of many other schemes beyond the path one. I have built on their work in my search for the underlying patterns of a topological-algebraic nature and in my remarks on how that nature is relevant to the understanding of logical notions. What remains to be developed is a full theoretical account of these patterns which also deals with the set of thematic roles (that could also be labelled "archetypical"). For, in every scheme

1) there is a subject as FIGURE emerging as a cohesive unity out of a BACKGROUND and the subject goes FROM a SOURCE position TO a TARGET position;
2) there is an OPERATOR which ACTS on an OBJECT by means of a TOOL.
3) the change (or stasis) of position in 1) is associated with the action in 2).[21]

Any logically "atomic" proposition instantiates this form, though one or more of the place-holders remain implicit or are simply "suspended". The polymorphic character of each slot, for a specific positional role, should not mislead us. In order to have a proper theory of meaning we must take account of the two layers: the Base and the Lifting. The very concepts of "composition" and "identity", needed to give an account of connectives and quantifiers, is present in hidden form within an "atomic" proposition. As a consequence of this, we see that logical syntax of itself cannot help us in accounting for this two-layered character of meaning-construction. And the reason for this should by now be clear: logical form is a byproduct of the richness of mathematical structure already present in the "logical atoms" at the basic layer.

From this recognition, we are led to realise why the notion of sense invoked by Frege and that of intension to which Carnap appealed – and which was later elaborated in the luxuriant framework of possible worlds semantics – are not ultimately explanatory. The understanding of both notions (and also of modalities) already presupposes the whole constitutive process of reference via embodied patterns of object and action.

The fundamental contrast in such a refocused semantics is not between the extensional and the intensional, but rather between content as an end-product and its emergence within a many-layered structure. As we start paying attention to meaning-constitution – thus to the becoming of meaning – we are led to realise that the principle of extensionality has many forms, depending on which kinds of "parts" are proper to a whole (which are homogeneous, so to speak, with respect to its structure). The wholes which can be reduced to that dust of points we name "sets" are to be recognised as a particular (and untypical) outcome of our experience with wholes which are continua, which lend themselves to constrained discretisation but which may yet satisfy their own appropriate form of extensionality condition.

It is time to conclude and the best conclusion I can offer is an invitation to the collaborative task of developing a more adequate systematic theory of the geometric roots of semantics.

Notes:

1. Text of a lecture held on May 16, 2013, at the University of Cagliari, Italy. I thank Pier Luigi Lecis for helpful remarks during conversations in Cagliari. Mike Wright edited the English translation and John Bell suggested some improvements.
2. See, for example, (Weyl, 1926) and (Tragesser, 1977).
3. For a survey, see (Lawvere, 2000). For a general introduction to category theory, see (Lawvere and Rosebrugh, 2003).
4. The "experientialist" view he later introduced together with Mark Johnson was not intended simply as a rejection of positions associated with the "received view" in the camps of analytic philosophy and generative grammar: it also aimed at a general explanatory framework, see (Lakoff and Johnson, 1980) and (Lakoff, 1987).
5. In the essay titled *From Kant to entwined naturalism*, which in 1989 circulated in preprint version and was submitted to the annals of the Department of Philosophy of the University of Florence but was accepted for publication only four years later and appeared as an appendix to the following volume, see (Peruzzi, 1994).
6. Among such topics, the first ones in which I made use of categories were indirect discourse and definite descriptions. See (Peruzzi, 1988).
7. See (Thom 1980). The width of Petitot's research on what's behind meaning is witnessed by (Petitot, 2011).
8. As I did already in a joint lecture with Lakoff given at the *Mind and Language Conference* held at the University of Bologna in 2003. An expanded version of my part of that lecture appeared in Italian as (Peruzzi, 2004).
9. As far as the world is not taken as a book. When Galileo claimed that the book of nature is written in mathematical language, he was using a metaphor, indeed an old one. If the metaphorical aspect is forgotten, the outcome is odd: is the tree in front of me a page? Then there are pages which are trees and pages which are pages. Any text is a linearly ordered and discrete set of words, while the physical world is many-dimensional, and, if the use of standard Calculus in Mechanics is taken at face-value, the space-time background is a continuum.
10. Some set-theorists might say that such closure rather leads to size-considerations.

11. This issue is technically dealt with in papers aimed to apply topos-theoretic semantics within the analysis of count and mass nouns in natural language, see (Macnamara and Reyes, 1994).

12. See (Mac Lane, 1986). Two years later I completed the Italian translation of this book for Boringhieri. Though it didn't appear, the discussions I had at the time with Mac Lane on various topics were extremely encouraging.

13. See the already mentioned investigations by (Lakoff and Johnson, 1980).

14. "Geometry" has here a more general sense than a theory of one metric space or a theory of the category of all metric spaces.

15. Twenty years ago, when I tried to express this agreement, I was not able to let the reader fully realise that in passing *FROM* Kant *TO* entwined naturalism, a dynamic systems perspective on the mind was at stake, rather than just a rejection of Kantian idealism and the sweetenings it had in 20th Century's philosophy of science. See the essays collected in (Peruzzi, 2004a).

16. The assertibility in question is intended to be framed within any consistent view of the world as a cohesive unity within which lies a hierarchy of layers of organisation, each layer of structure carrying within it preconditions for the emergence of further layers without determining any particular one of them.

17. I am thinking of what in categorical logic is a "geometric theory". Typically, universality manifests itself when a pair of adjoint functors is at hand. An extensive analysis of the correlation between logical and categorial concepts is already provided by (Bell, 1986) in the area of topos theory.

18. See (Peruzzi, 1993) Since holism took many shapes, the arguments in its support as well the objections varied widely. The relative literature is large, but it leaves a whole range of forms of holism unexplored. Of course, the forms which are more than logical exercises reduce to very few, and so the objections can concentrate on them.

19. Both these principles were formulated in (Peruzzi, 1994).

20. What is "basic" here, is different from the nominalists' naked particulars or the abstract *ur-elemente* of set-theorists. Rather, it concerns the salient units of reference in bodily experience, in the sense I described in a contributed paper to (Macnamara and Reyes, 1994).

21.The operator/agent, the object(s) and the tool are different "figures" which can also refer to one and the same entity. This occurs when a system acts on itself and, as a very particular kind of action, when we have self-reference. The background as well as the action can also become "figures" in their turn.

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