



## Evaluating the Performance of Forecasting Models for Portfolio Allocation Purposes with Generalized GRACH Method

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### ABSTRACT

Portfolio theory assumes that investors accept risk. This means that in the equal rate of return on the two assets, the assets were chosen that have a lower risk level. Modern portfolio theory is accepted by investors who believe that they are not cope with the market. So they keep many different types of securities in order to access the optimum efficiency rate that is close to the rate of return on market. One way to control investment risk is establishing the portfolio shares. There are many ways to choose the optimal portfolio shares. Among these methods in this study we use loss functions. For this, we choose all firms from the year 2011 to the end of 2015 that had been a member in the Tehran Stock Exchange. The results of this research show that the likelihood functions have the best performance in Forecasting the optimal portfolio allocation problem.

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## 1. Introduction

A special feature of economic forecasting compared to general economic modelling is that we can measure a model's performance by comparing its forecasts to the outcomes when they become available. Generally, several forecasting models are available for the same variable and forecasting performances are evaluated by means of a loss function (Elliott and Timmermann [9]). Model selection generally involves the evaluation of forecasts of volatility within loss functions, which are classified as either direct or indirect by Patton and Sheppard [15]. Direct loss functions are measures of the forecast accuracy based on traditional statistical measures of precision. Although it would appear that

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direct loss functions should be easy to implement and interpret, the fact that volatility is unobservable, thereby necessitating the use of an observable proxy for volatility, confounds the issue. Indeed, Hansen and Lunde [13] and Patton [16] demonstrate that noise in the volatility proxy renders certain direct loss functions incapable of ranking forecasts consistently in the univariate setting.

Subsequent studies by Laurent et al. [14] and Patton and Sheppard [15] have reported equivalent results in the multivariate setting. On the other hand, indirect measures of the volatility forecasting performance evaluate forecast efficacy in the context of the application for which the forecast is required, for example the portfolio allocation problem. One appealing attribute of this type of evaluation is that it is specifically related to the economic decision from which the forecast derives its value (Elliott & Timmermann, [9]). Danielsson [6] argues that forecasts should be evaluated and selected on the basis of their intended application. Many studies have used indirect measures to evaluate volatility forecasting models. For example, Engle and Colacito [8] evaluate the forecasting performance in terms of portfolio return variance, while Fleming et al. [10, 11] apply a quadratic utility function that values one forecast relative to another.

Despite the strong economic appeal of measures that combine risk and return, especially those that report a measure of relative economic value, it is easy to show that these measures can favour incorrect forecasts of volatility. One notable exception is the portfolio variance, which does not display this problem. Engle and Colacito [8] and Patton and Sheppard [15] have demonstrated that the portfolio variance is minimised when the correct forecast is applied; a result that links the portfolio variance with robust statistical loss functions (Berker [4]).

## 2. Literature review

Provide an excellent survey on the state of the art of forecasting in economics. Details on volatility and correlation forecasting can be found in Andersen et al. [2]. The evaluation of volatility forecasts raises the problem that the variable of interest is latent. This problem can be solved by replacing the latent conditional variance by a proxy; see e.g. the realized variance estimator of Andersen and Bollerslev [1]. However, as noted by Andersen et al. [2], the evaluation of volatility forecasts using a proxy may not lead, asymptotically, to the same ordering that would be obtained if the true volatility was observed. In a general framework, Hansen and Lunde [13] show that when the evaluation is based on a target observed with error, the choice of the loss function becomes critical in order to avoid a distorted outcome. They also provide conditions on the functional form of the loss function which ensure consistency of the proxy based ordering. For univariate volatility, Patton [16] derives a class of loss functions which are able to order consistently in the presence of noise. Building on these results, Patton and Sheppard [15] give a direct multivariate analogue but without providing any general ex-

pression. The above results have important implications on testing procedures for superior predictive ability, see among others Diebold and Mariano [7], Clark and McCracken [5] and the contributions of Hansen and Lunde [13] with the superior predictive ability test and Hansen et al. [12] with the model confidence set test.

In fact, when the target variable is unobservable, an unfortunate choice of the loss function may deliver unintended results even when the testing procedure is formally valid. With respect to the evaluation of multivariate volatility forecasts little is known about the properties of the loss functions. This is the first paper that addresses this issue. This paper extends the previous literature by considering the role played by loss functions in ex-ante multivariate volatility model selection, where forecasts from these models will subsequently be used in mean–variance portfolio optimisation. In doing so, it will assess the ability of a range of loss functions to discriminate between volatility forecasting models where the intended use of the forecasts is a portfolio optimisation problem. While this paper focuses on mean–variance portfolio optimisation, there is nothing to prevent the consideration of higher moments of returns, estimation error or portfolio constraints. However, the main focus here is not the final application itself, but rather the way in which the loss functions perform in terms of model selection with a given application in mind. This is achieved in part by a simulation study that considers the relative powers of a range of statistical loss functions and portfolio variances. Power is important in the context of this problem because it reflects the ability of loss functions to discriminate between forecasts. A subsequent empirical study then assesses the consistency between the various loss functions and the final portfolio application. It will gauge whether the best models selected from the evaluation period continue to be the best performers in the application period, the optimal outcome in terms of model selection. This differs from the traditional forecast evaluation literature in that it considers the use of statistical measures to discriminate between models, the performances of which are then measured based on an economic criterion in a subsequent period (Berker, [4]).

### 3. Methodology

This section describes the intended portfolio application and the loss functions employed to evaluate the volatility forecasts. Begin by considering a system of  $N$  asset excess returns

$$r_t = \mu_t + v_t, \quad v_t \approx F(0, \Sigma_t)$$

Where  $r_t$  is an  $N \times 1$  vector,  $\mu_t$  is an  $N \times 1$  vector of expected excess returns and  $v_t$  is an  $N \times 1$  vector of disturbances following the multivariate distribution  $F$ . In this context, the optimization problem of an investor who seeks to minimise the variance of a portfolio of  $N$  risky assets and a risk-free asset is

$$\min_{\tilde{S}_t} \tilde{S}_t' \Sigma_t \tilde{S}_t \quad s.t. \quad \tilde{S}_t' \tilde{r}_t = \tilde{r}_0,$$

The specific loss functions employed are now described.

### Mean square error (MSE)

The MSE criterion is simply the mean squared distance between the volatility forecast,  $H_t$ , and the proxy

$$l_t^{MSE}(H_t, \hat{\Sigma}_t) = \frac{1}{N^2} t' \text{abs}(H_t - \hat{\Sigma}_t) t.$$

### Mean absolute error (MAE)

An alternative to squared errors is to measure the absolute error,

$$l_t^{MAE}(H_t, \hat{\Sigma}_t) = \frac{1}{N} \text{abs}(H_t - \hat{\Sigma}_t) t.$$

### Quasi-likelihood function (QLK)

Given a forecast of the conditional volatility,  $H_t$ , the value of the quasi log-likelihood function of the asset returns assuming a multivariate normal likelihood is

$$l_t^{QLK}(H_t, \hat{\Sigma}_t) = \ln(H_t) + t' \text{abs}(\hat{\Sigma}_t \otimes H_t^{-1}) t.$$

### Portfolio variance (MVP)

To evaluate the forecasts within the given portfolio application, the portfolio optimisation problem is solved by

$$l_t^{MVP}(\tilde{\Sigma}_t, \hat{\Sigma}_t) = \tilde{\Sigma}_t' \hat{\Sigma}_t \tilde{\Sigma}_t.$$

To examine the performance of the loss functions, a range of models for producing one-step-ahead multivariate volatility forecasts,  $H_t$ , are required. A number of models have been chosen for this purpose. While this is clearly not an exhaustive list, each model is able to generate volatility forecasts for moderately sized covariance matrices, with the quality of their forecasts being expected to vary widely.

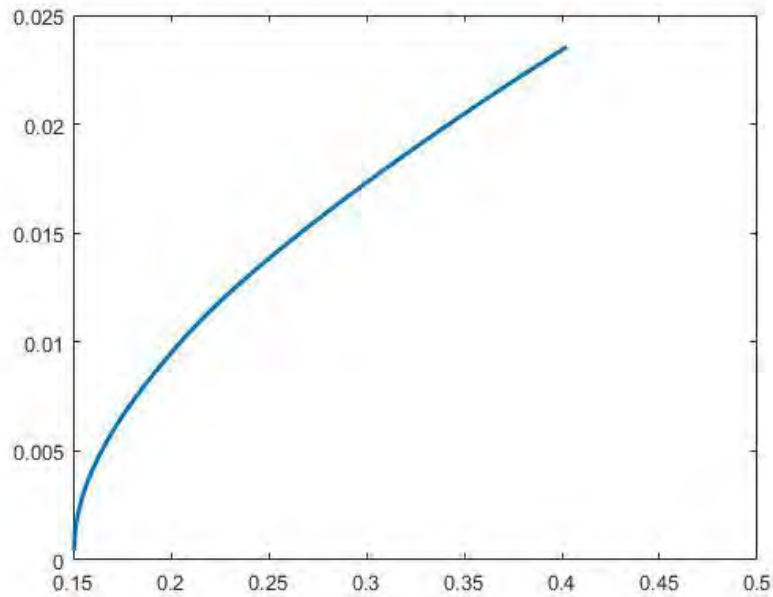
The Orthogonal GARCH (OGARCH) model of Alexander and Chibumba [3] is also considered.

To obtain the conditional covariance matrix of the original return series, the following calculation is required to reverse the transformation and standardization that was performed.

$$\text{OGARCH: } H_t = AV_t A' = \sum_{k=1}^K a_k a_k' a_{k,t}^2$$

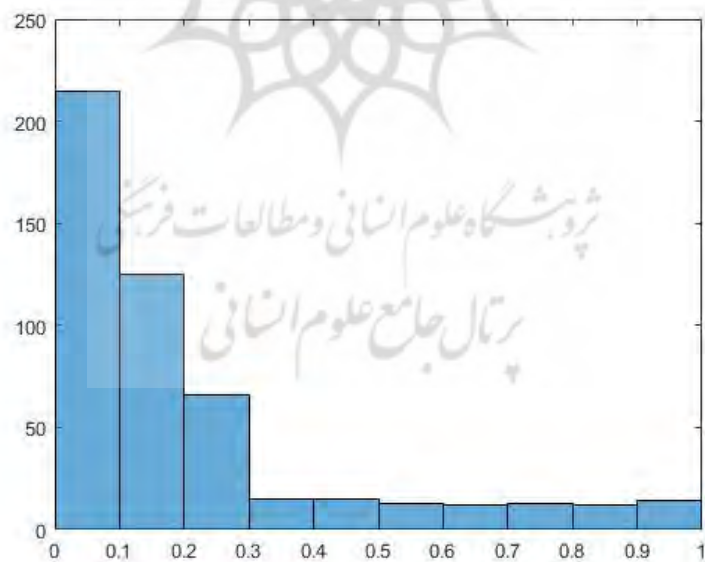
## 4. Empirical Results

By solving the portfolio application with Matlab software, the Optimize risk efficient frontier portfolio was as Fig. 1.



**Fig. 1:** Optimize risk efficient frontier

As we can see, with the increase in the minimum expected return, variance portfolio has increased. It is notable that here the variance is considered as a measure of risk. Also portfolios histograms obtained by this method are as Fig. 2.



**Fig. 2:** Portfolios histogram

The performances of MSE, MAE and QLK are examined first, and the results are presented in Table 1. These results show that QLK outperforms MSE and MAE, and MAE is not a robust loss function

for multivariate volatility forecast evaluations. They are also consistent with the univariate results of Patton [16].

**Table 1:** Obtained Results

	Evaluating the Performance			
	MSE	MAE	QLK	MVP
Value for OGRACH method	0/2499	0/2500	0/2605	0/3178

## 5. Conclusion

Techniques for evaluating univariate volatility forecasts are well understood, the literature relating to multivariate volatility forecasts is less developed. Many financial applications employ multivariate volatility forecasts, and thus it may be appealing to evaluate and select forecasting models on the basis of such an application.

This paper has investigated the performances of a range of loss functions for selecting models to be applied in a subsequent portfolio allocation problem. Two issues underlie this problem: the relative power of the loss functions, and the consistency with the final portfolio application. Simulation results demonstrate that a statistical loss function based on the multivariate normal likelihood (QLK) and has more power than any other loss function considered.

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