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A Critical Analysis of the Origin and Nature of Classical Mathematical Economics: Why Classical Economists Did Not Use Mathematics?

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Abstract

The role of mathematics in economic analysis is not yet a settled question. Smith, Ricardo, Mill and other eminent classical economists did not use mathematics in their economic theorizations. We have defined classical mathematical economics as the whole body of literature in mathematical treatment of economics originating mainly from the contributions of Cournot, Jevons and Walras. There are a number of different explanations for the origin of classical mathematical economics suggested by different authors, which may also explain the lack of general interests among classical economists in using mathematical methods in economic analysis. This paper attempts to examine critically the views put forward by Debreu, Cournot, Walras and von Neumann and Morgenstern on the origin of mathematics in economics. Using historical evidences through direct references to their original works, we have demonstrated that none of their views are convincing. It is also shown that the tradition of classical mathematical economics did not have any significant impact on the process of economic theorization within the framework of classical economics.

Keywords: Mathematical Economics, Cournot, Jevons, Walras, von Neumann and Morgenstern.

JEL Classification:B13, B14, B16.

1. Introduction

The application of mathematical methods in economic analysis has always been a controversial issue. Two basic attitudes in mathematical treatment of economics can be identified as follows. *i*) The prime objective of applying mathematics in economic analysis has been merely to rank economics as a branch of science such as physics. *ii*) The application of mathematics in economic analysis provides results of value

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which could not be obtained otherwise. Using a historical approach, this paper attempts to highlight the underlying issues in these two conflicting attitudes by an examination of the nature and origin of classical mathematical economics as well as the general disbelief on the instrumentality of mathematics in economic theorization by eminent classical economists.

We define classical mathematical economics as the whole body of literature marked by the contributions of Cournot, Jevons, Walras and their followers. Let us start the analysis of the origin of classical mathematical economics by presenting the following questions. Why Cournot (1838) is unanimously known as the birth of mathematical economics while it was totally ignored by classical economists for more than 30 years until Jevons (1871) revived it? Why is the importance and significance of the application of mathematics to economics unsettled whereas physical sciences can hardly do without mathematics?

We have first examined the historical background and the analytical framework of classical mathematical economics in section 2. The views expressed by Debreu, Cournot, Walras and von Neumann-Morgenstern on the origin and limitations of classical mathematical economics are examined in sections 3 to 6, respectively. We have shown that these view can neither satisfactorily explain the origin of classical mathematical economics nor the lack of interests amongst eminent classical economists in applying mathematical methods in economic analysis. And finally the summary and concluding remarks are the subject matter of section 7.

2. Classical Mathematical Economics: Historical Background and the Analytical Framework

It is now generally agreed that Civa (1642-1734), an Italian mathematician, is the first author to apply mathematical methods to economic problems. His work on money, written in 1711, is the first true example of mathematical treatment of economic issues, in which the mathematical ideas such as definition, postulate, remark, proposition, theorem and corollary are used in the analysis of money. This work, however, was completely ignored until 1871 when it appeared in Jevons's *List of Mathematico-Economic Books, Memoirs, and Other Published Writings.*

127 years after Civa's work, Cournot, professor of mathematics at Lyon and the Rector of the Academy of Grenoble, published his epoch-making contribution to economics under the title *Recherches sur les Principes*

Mathématiques de la Théorie des Richesses in 1838. Economists today unanimously agree that the symbolic birth of mathematical economics is the year in which Cournot published his book. The first key question is that why Cournot (1838) and not Civa (1711) or any other work among the 38 research works published before Cournot on mathematical economics[1], is not considered to be the pioneering work in this field? What has made Cournot's work to be recognized as an epoch-making contribution? Has Cournot's mathematical excellence been responsible for this success or has it been realized later that this work can be regarded as a turning point for a new current of thoughts in political economy?

Cournot's book received little or no attention at the time: "For several years not a single copy of the book was sold. In 1863 the author tried to overcome the indifference of the public by recasting the work and omitting the algebraic formulae. This time the book was called *Principes de la Theorie des Richesses.* In 1876 he published it again in a still more elementary form and under the title of *Revue Sommaire des Doctrines Economiques* but with the same result".[2] J. B. Cherriman, a Canadian mathematician, published a ten page review on Cournot in 1857. This was the only published recognition of Cournot's book.

Cournot's significant contribution to mathematical economics was finally revived by Jevons (1871). On page 26 in the preface, Jevons stated that "This work must occupy a remarkable position in the history of the subject. It is strange that it should have remained for me among Englishmen to discuss its value"[3].

According to Fisher (1891, p.109), "The introduction of mathematical method marks a stage of growth [in economic analysis]- perhaps it is not extravagant to say, the entrance of political economy on a scientific era ... Before Jevons all the many attempts at mathematical treatment fell flat. Every writer suffered complete oblivion until Jevons unearthed their volume in his bibliography". Despite the importance of Jevons in reviving the whole body of literature on mathematical treatment of economics developed before him, the key question remains as why Cournot's significant contribution together with the previous work on mathematical

[1]. For a list of 38 works before Cournot, i.e. during the period 1711-1838 and 62 works from Cournot to Jevons, i.e. 1838-1871, published on mathematical economics, see Jevons's *List of Mathematico-Economic Books, Memoirs and Other Published Writings,* pp. 322-339, in his *Theory of Political Economy,* 1871.
[2]. See Charles Gide and Charles Rist (1909, 1948), p. 499.
[3]. All references to Jevons (1871) are from its 4th edition, London: Macmillan, 1911, 339 pages.

economics were completely ignored, or at least were not taken seriously, by classical economists?

3. Debreu's Claim: The Incidental Developments in Mathematical Economics

The view that mathematical economics has emerged from nowhere and has grown with no aims while being independent of any current of economic thoughts has received supports from a number of economic historians and even from mathematical economists. Debreu (1986, p. 1259) regarded the emergence of mathematical economics simply as an historical coincidence: "[The early progress of mathematical economics] is marked by several major scientific accidents. One of them occurred in 1838 ... with the publication of Augustin Cournot's [book] ... The University of Lausanne was responsible for two others of those accidents. When Leon Walras delivered his first professional lecture there on 16 December 1870, he had held no previous academic appointments; he had published a novel and short story[1] but nothing on economic theory and he was exactly 36 ... For Vilfredo Pareto, who succeeded Walras in his chair in 1893, it was also a first academic appointment; like his predecessor he had not published anything on economic theory before; and he was 45"[2].

Debreu's argument on the incidental developments in classical mathematical economics is not convincing at all. We now refer to the following historical facts, which clearly invalidate Debreu's view particularly on the "accidental scientific developments" in mathematical economics in Lausanne school.

1- According to Jaffe (1954, the translator's forward to Walras 1874, pp. 5-6), "Walras, unhappy with his engineering studies at *Ecole des Mines* and dissatisfied with literature and journalism as his second academic challenge, was persuaded by his father, an economist, to study economics at the age of 24 to continue his father's research on mathematical economics. It was after 12 years of hard work that this self-taught economist was offered the new chair of political economy at the University of Lausanne."

2- The reason that Walras had held no previous academic position was his lack of any officially recognized educational credentials in economics. (*ibid*)

[1]. Debreu refers to *Francis Sauveur*, published by Walras in 1858, Paris: E. Dentu

[2]. See, also, Debreu (1987. p. 399).

3- Walras presented a paper on Taxation in 1860 in an international conference on taxation in Lausanne, which remarkably impressed the audience. (*ibid*)

4- During 1859 to 1862, when Walras was working as a journalist for the *Journal des Economistes* and *La Presse*, he published *l'Economie Politique et le Justice*, Paris: Guiilaunin, 1860, in which he strongly attacked the normative economic doctrines of P. J. Proudhon, [See Donald A. Walker (1987), p. 852].

5- Vilfredo Pareto, graduated in mathematical and physical sciences in 1867 and engineering in 1870, started to write and publish articles, as early as 1872, on commerce, the state of Italian industry, railways, advantages and disadvantages of public and private use of the railway system and support of free trades to prevent any form of state interventions in economic activity. Pareto was one of the founders of the Adam Smith Society, which spread and upheld the doctrine of economic liberalism. In October 1891, Pareto published his controversial article "L'ltalie Economique" which was followed by another critical work in April 1892 on Italian Government economic policies. In 1890, Maffeo Pantaleoni, the famous Italian economist, advised him to study the work of Walras on mathematical economics, and Pareto met Walras himself on September 1890, before accepting the chair of Walras in political economy in 1893. [See G. Busino (1987), p. 800].

We now examine Debreu's claim regarding Cournot's contributions in mathematical economics. To invalidate Debreu's claim we refer to the first paragraph of the preface in Cournot (1838). He has clearly admired and appreciated the one hundred years of developments in political economy before him, but at the same time has urged the necessity of a *positive economics* due to the fact that the public has become so *tired* of theories of different economic *systems and doctrines:* "The science known as Political Economy, which for a century has so much interested thinkers, is to-day more generally diffused than ever before … [and attracted] the attention of the great journals, which are to-day the most important means of spreading information; but the public is so tired of theories and systems that now the demand is for so-called "positive" matters … such as will throw the light

of experience on the important questions which are being agitated before the country and which so greatly interest all classes of society"[1].

Despite the fact that Cournot's prime objective was to support an econometric type of analysis, the shortage of organized data and the lack of appropriate statistical methods of estimation, forced him to concentrate on pure theorization of economic concepts towards building up a positive economics. The second paragraph in his preface explains this point: "I will only observe that *Theory* ought not to be confounded with systems ... and that, to a man of my position in particular, more than to any other, it should be permissible to consider from an exclusively theoretical standpoint, a subject of general interest which has so many different sides".

In summary, we may classify the pre-Cournot's mathematical economics, starting from Civa, (1711), as purely academic exercises in which economic concepts were being translated into mathematical symbols and operations. These works all lacked any sense of direction. On the contrary, pioneers in mathematical economics in the 19th century, i.e. Cournot (1838), Jevons (1871), Walras (1874), Marshall (1890), Fisher (1891) and Pareto (1896), were all completely aware of their backgrounds, their current positions and, most important, their aims.

The facts presented above disqualify Debreu's argument that classical mathematical economics has come from nowhere and has developed with no clear aims. However, the question remains why the early mathematical economists failed to achieve their objectives? Or equivalently, why eminent classical economists did not employ mathematical methods in their economic analysis? In this regard, we examine Cournot's explanation on the general lack of interests amongst eminent classical economists in applying mathematical methods in their economic analysis.

4. Cournot's Claim: The Prevailing Erroneous Presentations and Poor Mathematical Knowledge

According to Cournot (1838), the inaccurate early writings in mathematical economics and their weak economic contents together with the fact that classical economists were not well-equipped with mathematical knowledge, were the significant factors which hindered the pace of developments in mathematical economics. This claim received

[1]. A11 references to Cournot (1838) made in this paper are from its English translation by Nathaniel T. Bacon: *Researches into the Mathematical Principles of the Theory of Wealth*, New York: Macmillan, 1897, reprinted 1927.

supports from a number of economists including Fisher (1891). Cournot (1838, preface) maintained that "The attempts which have been made in this direction have remained very little known and I have been able to learn only the titles of them, except one, *Les Principes de l'Economie Politique* by Canard, a small work published in 1801 … These pretended principles are so radically at fault and the application of them is so erroneous, that the approval of a distinguished body of men was unable to preserve the work from oblivion. It is easy to see why essays of this nature should not incline such economists as Say and Ricardo to algebra."

Let us now assume the validity of Cournot's claim. It follows therefore that Cournot's work, being a concise and original work on mathematical treatment of economics, should have attracted the attention of economists of his time; but we know that his work was absolutely ignored by economists for more than 30 years until Jevons (1871) revived it. The subsequent developments in classical mathematical economics also provide useful evidences to invalidate Cournot's argument. For example, the concise and mathematically elaborated contribution of Walras, i.e. *Elements d'Economie Politique Pure* was hardly noticed, even in France, during the twenty-five years after its publication in 1874. Interestingly, Alfred Marshall, a mathematician and an economist, has only mentioned Walras in the briefest of comments in his *Principles of Economics* (1890) and did not take Walras's general equilibrium seriously at all. It was about eighty years after Walras that the eminent mathematical economists of the 20th century, e.g. Abraham Wald, John von Neumann, John Hicks, Frank Hahn, Oscar Lange, Paul Samuelson, Lionel McKenzie, Gerad Debreu, Kenneth Arrow and Michio Morishima acknowledged Walras's contribution and paid attention towards further developments in Walrasian general equilibrium analysis.[1]

Let us now examine the idea of "poor or inadequate mathematical knowledge" among classical economists, put forward by von Neumann, as an explanation of the lack of interest to apply mathematical methods in economic analysis. This argument is not convincing either. Economists, in a rare unanimous agreement, would select Jevons (1871), familiar with mathematics and logic and educated in chemistry but self-taught in economics, as the first economist who made known to the world the remarkable position of Cournot in the history of economics. However,

[1]. See Weitraub (1986) for extensions of Walras's general equilibrium.

Jevons himself confessed that he was not able to understand mathematically all parts of Cournot's book. On page *xxx* in the preface to the 2nd edition of his book (1879), Jevons maintains that "Even now I have by no means mastered all parts of it, my mathematical power being insufficient to enable me to follow Cournot in all parts of his analysis".

It should be noted that Jevons and Walras were aiming to design a *scientific economics* which was characterized mainly by its mathematical nature. This is exactly what Cournot had in mind. This explains why Jevons appraised Cournot without fully understanding him. In fact, it was the *compatibility* of Cournot's aim and attitude with those of Jevons and Walras which, after all, brought him recognition after 30 years.

5. Walras's Claim: The Narrowness of Ideas

According to Walras[1], the dichotomy between deduction and induction or between pure reasoning and experience which had separated sciences from arts was the main reason that classical economists disregarded the use of mathematics in their work. "If nineteenth century ... has completely ignored [mathematical economics], the fault lies in the idea, so bourgeois in its narrowness, of dividing education into two separate compartments: one turning out calculators with no knowledge whatsoever of sociology, philosophy, history, or economics; and the other cultivating men of letters devoid of any notion of mathematics" [*Elements of Pure Economics* (1900), 4th edition, p. 48]. Walras claimed that by employing both deductive and inductive reasoning, mathematical economics can be ranked with sciences such as astronomy and mechanics: "The twentieth century, which is not far off, will feel the need, even ... of entrusting the social sciences to men of general culture who are accustomed to thinking both inductively and deductively and who are familiar with reason as well as experience. The mathematical economics will rank with the sciences of astronomy and mechanics; and on that day justice will be done to our work", (*ibid*. p. 48).

The question arises as why Walras did not simply add the experimental dimension (quantitative analysis and measurements) to classical economics? In other words, if according to Walras, *the familiarity of economists with reason as well as experience* would have ranked economics with the acknowledged physical sciences, why instead of completely ignoring the well-established classical economics, did he not

[1]. A11 references to Walras (1874) are from its English translation by William Jaffe: *Elements of Pure Economics, or the Theory of Social Wealth*, London: George Allen and Unwin, 1954, 620 pages.

make an effort to represent classical economics mathematically for the purpose of quantitative analysis and empirical measurements?

The above question is of prime theoretical significance because a number of mathematicians in the early nineteenth century tried to present mathematically the classical economics. For example, William Whewell, the Cambridge mathematician, represented mathematically some doctrines of political economy in general and Ricardo's system in particular[1]. However, these works were completely ignored by Walras.

An examination of the *Elements of Pure Economics* shows clearly that Walras did not make any significant contribution either towards inductive thinking in economics or in the measurement of economic relations. His work, instead of advancing classical economics one step towards experimentations, completely erased the empirical contents of classical political economy. In summary, Walras's actual contributions to economics did not follow his injunctions on the objectives of economic studies.

It is interesting to note that the academic life of Pareto is a clear manifestation of the contradiction inherent in Walras's objective in economic theorization and his actual economic contributions. Recall that Pareto, who accepted Walras's chair at Lausanne in 1892, realized the weakness of pure economics after making a number of contributions to Walras's theory of general equilibrium[2]. In *Cours d' Economie Politique* (1896-7) he stated that "... pure economics shows us the general form of the phenomenon; applied economics provides a second approximation; but neither will ever be able to show us how to manage the economic life of every individual" [Busino (1987), p. 801]. In *Cours* he emphasized the importance of interrelations of economics and social phenomena. In 1905, Pareto published his *Manuale d'Economia Politica.* His words at the end of this book are clearly a departure from Walras's principles: "Whoever wants to make a scientific study of the social facts has to take account of reality and not of abstract principles and the like ... " Pareto then gave up economics and concentrated exclusively on sociology (*ibid*, p. 802).

In summary, the above-mentioned facts suggest that it is very difficult to accept Walras's claim that the nineteenth century economists ignored

1. Whewell (1829, 1831, 1850)

2. It should be noted that after accepting Walras's chair in 1892, "Pareto spent the whole of the next year writing a refutation of Marx's theory of value which was published in Paris in 1893 as the introduction to an anthology of passages by Paul Lafargue taken from Marx's *Das Kapital.* See Busino (1987), p. 801.

mathematical economics simply because of the prevailing “narrowness of ideas”, which discredited experimentations in economic analysis. On the contrary, evidences are more in favour of the argument that Walras's own contributions have further strengthened the so-called narrowness of ideas.

6. von Neumann and Morgenstern's Claim: The Unfavorable Circumstances

von Neumann and Morgenstern (1944) have examined the factors at work in the underdevelopment of mathematization of economics within a wider context. If economics is a science why, in contrary to other sciences where mathematics has been applied with great success, has its use not been highly successful? Most sciences could hardly make any progress without mathematics and yet the real contribution of mathematics to economics has remained an unsettled question. According to von Neumann and Morgenstern, the combination of the following unfavorable circumstances is the main factor at work.

6-1. Vagueness of Basic Economic Concepts

von Neumann and Morgenstern (1944, p. 4) have pointed out that "Economic problems were not formulated clearly and are often stated in such vague terms as to make mathematical treatment *a priori* appear hopeless because it is quite uncertain what the problems really are. There is no point in using exact methods where there is no clarity in the concepts and issues to which they are to be applied". This is in sharp contrast to the general view held among mathematical economists that the mathematization of economics facilitates a more concise exposition of problems and avoids the digressions of vague argumentations.

von Neumann and Morgenstern's claim implies the following contradiction: Further developments in mathematical economics as a *science* depends entirely on prior developments in *"non-scientific"* descriptive economics. To provide further supporting evidence, we refer to page 4 (*ibid*): "Consequently, the initial task is to clarify the knowledge of the matter by further careful descriptive work". This is in dispute with the established view in the profession that if economics is to be a science it must be mathematical. Moreover, von Neumann and Morgenstern have not specified the conditions under which careful advances in descriptive economics can be attained -i.e. with or without mathematics. If the latter holds, the uniqueness of mathematical economics as a science will suffer a collapse.

6-2. Limitations in Mathematical Treatment of Human Behaviour

It appears that von Neumann and Morgenstern are the first mathematical economists of reputation in the twentieth century who have acknowledged the fundamental objection that economic theory cannot be modelled in the same format as in physical sciences. A concise economic analysis necessarily requires careful examination of a number of non-economic elements such as social, political, historical, psychological and cultural factors. This implies serious limitations in mathematical formulations of human behaviour: In economic analysis, "we should attempt to utilize only some commonplace experiences concerning human behaviour which lends itself to mathematical treatment" (*ibid.* p. 5).

von Neumann and Morgenstern have made an important point that there are uncertainties about the exact mathematical methods which should be used in economic analysis. These uncertainties exist even in the process of mathematization of that class of human behaviour which lends itself to mathematical treatment. The existing tools in mathematical economics such as the calculus of variations or differential equations might not be the right instruments for economic analysis since they are mainly developed for physical sciences: "It is therefore to be expected … that mathematical discoveries of a stature comparable to that of calculus[1] will be needed in order to produce decisive success in [economic analysis] ... it is unlikely that a mere repetition of the tricks which served us so well in physics will do for the social phenomena too" (*ibid.* p. 6).

In summary, von Neumann and Morgenstern's final recommendation to mathematical economists is to wait for new discoveries in mathematical methods which are more appropriate for the analysis of social sciences. But have they proved the *existence* of such mathematical methods? The answer is no. Moreover, before the discovery of such mathematical methods, how can the real economic problems examined "scientifically" for policy recommendations? According to von Neumann and Morgenstern, economists are not entitled to examine such real economic problems simply because they are not yet qualified: "... How to stabilize employment, how to increase the national income, or how to distribute it adequately? Nobody can really answer these questions and we need not concern ourselves with the pretension that there can be scientific answers at present" (*ibid.* p. 6).

[1]. von Neumann and Morgenstern have referred to the role played by infinitesimal calculus in the creation of the discipline of mechanics.

7. Summary and Concluding Remarks

To recapitulate the main factors at work in the formation of classical mathematical economics, recall that according to Cournot (1838, p. 1), the wide varieties of different theories and doctrines had motivated the public desire for "positive" economics. However, the impact of eminent classical economist such as Smith, Say and Ricardo on the community of economists was so great that Cournot's book sank into oblivion for 30 years until Jevons revived it.

It follows therefore, that when Jevons and Walras started their campaign to popularize mathematical economics, the work of Cournot together with 38 works on mathematical economics before Cournot and 62 works from Cournot to Jevons[1], were all revived to provide an army of supportive literature for this novel mathematical or scientific approach. Even the antiquated work of Civa (1711) was needed to give more strength to this army which was about to launch an attack upon the tradition of great classical economists.

Regarding the origin and limitations of classical mathematical economics, we critically examined the arguments put forward by Debreu, Cournot, Walras and von Neumann and Morgenstern.

Debreu's claim on the incidental developments in mathematical economics was proved invalid on the basis of being inconsistent with the views clearly expressed by pioneers of classical mathematical economics like Cournot, Jevons and Walras. Moreover, we demonstrated that Debreu's argumentation in supporting his claim included a number of erroneous historical records.

Regarding why the early developments in mathematical economics were totally ignored by classical economists, Cournot's claim (1838), which is supported by Fisher (1891), is examined. According to Cournot, there are two possible explanations: *a*) erroneous presentations of mathematical economics and *b*) the poor mathematical knowledge among classical economists. We have found evidences contrary to this argument.

Walras's claim on the "narrowness of ideas", which gave much emphasis on inductive reasoning in economics failed on the ground that the revival of mathematical economics by Jevons, Walras and Pareto was not accompanied by any serious attempt in measurements and experimentations in economic analysis. We have shown that Walras's own work can be regarded as a contribution to deductive reasoning with the

[1]. See Jevons (1871), pp. 322-339, for a complete list of these publications.

effect of widening the gap between pure theorizations and economic experimentations.

According to the argument put forward by von Neumann and Morgenstern (1944), the "unfavorable circumstances" resulting from the following factors produced the general lack of interest on mathematical economics among classical economists: the vagueness of basic economic concepts, the inadequate empirical economic facts, and the limitations in mathematical treatment of human behaviour. We have shown that the lack of interest amongst the eminent classical economists in mathematical economics has had nothing to do with these factors. On the contrary, von Neumann and Morgenstern suggest that the real developments in mathematical economics necessarily require discoveries of new mathematical methods for social sciences.

Classical mathematical economics and particularly the contributions of Cournot, Jevons and Walras did not convince the community of classical economists to adopt mathematical approach in economic analysis; hence they remained faithful to their old tradition of political economy. It is generally agreed that Ramsey (1928) is the first serious attempt in reviving mathematical economics. This work, which was followed by Kantorovich (1939) and von Neumann (1928 and 1944) is regarded as the beginning of modern mathematical economics, whose analysis is beyond the scope of the present paper.

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