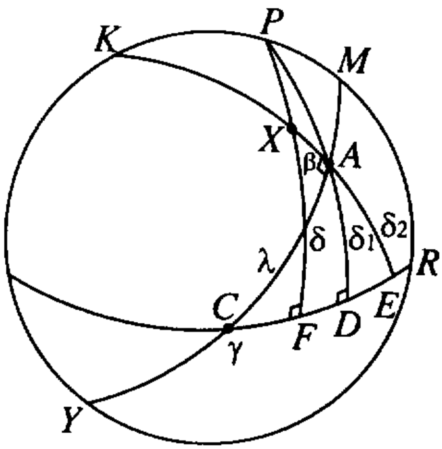

Figure 2

the tables of both declinations δ_1 and δ_2 for arcs from 0° to 90° in his zīj.

III- Declination of other points (194r:21-27)

The distance (*bu*ʿ*d*) or the present day declination δ of a point X on the celestial sphere from the celestial equator is defined to be an arc of the declination circle, issuing from X and perpendicular to the celestial equator (figure 2).

To determine this distance $\delta = XF$, Naṣīr al-Dīn says (lines 22-24) find the algebraic sum of β, the celestial latitude, and δ_2, the second declination of its celestial longitude λ. This sum, $\beta + \delta_2$, is called the argument (***hissah***) of the declination, its dirction being that of the algebraic sum. Then he determines the first declination δ_1 corresponding to λ, and calls it the *inverse declination* of the point X. Finally, to determine δ, he prescribes the following formula (line 25);

$$Sin(\beta + \delta_2) \cdot \frac{Cos\delta_1}{R} = Sin\delta$$

from which δ can be determined using the inverse sine function. Alternatively, (lines 26-27) in our symbols, he suggests the following formula:

$$\frac{Sin(\beta + \delta_2)Cos\epsilon}{Cos\delta_2} = Sin\delta$$

from which again δ can be determined.

to obtain δ_1. The formula he gives (lines 14-15), in our notation, is the following:

$$Sin\lambda \cdot \frac{Sin\epsilon}{R} = Sin\delta_1, \qquad (1)$$

or

$$\delta_1 = Sin^{-1}[Sin\lambda \cdot (Sin\epsilon)/R]$$

which can be read from the sine table of the zīj.

Given the same values, Naṣīr al-Dīn uses the right spherical triangle EAC and the following formula (line 16)

$$Sin\lambda \cdot \frac{Tan\epsilon}{R} = Tan\delta_2 \qquad (2)$$

to obtain δ_2 as

$$\delta_2 = Tan^{-1}[Sin\lambda \cdot (Tan\epsilon)/R]$$

Then, using the same triangle, he first obtains the angle $\widehat{AEC}$ (line 17-18) from

$$Cos\lambda \cdot \frac{Sin\epsilon}{R} = Cos\widehat{AEC}$$

and, having calculated $Cos\ \widehat{AEC}$, he uses it in the following formula to obtain $Cos\delta_2$ as

$$Cos\delta_2 = \frac{Cos\widehat{AEC}}{(Cos\epsilon)/R}$$

Then

$$\delta_2 = Cos^{-1}[(RCos\widehat{AEC})/Cos\epsilon]$$

The last two lines of the third section of our text state that if a table is available for the right ascensions of an ecliptic point λ, which we denote by $A_0(\lambda)$, to be defined later in the text, then λ is found for each value of $A_0(\lambda)$ from this table. Then δ_1 and δ_2 are determined by the above rules. To facilitate the determination of the declinations of the ecliptic arc and the corresponding inverse operations, Naṣīr al-Dīn has tabulated

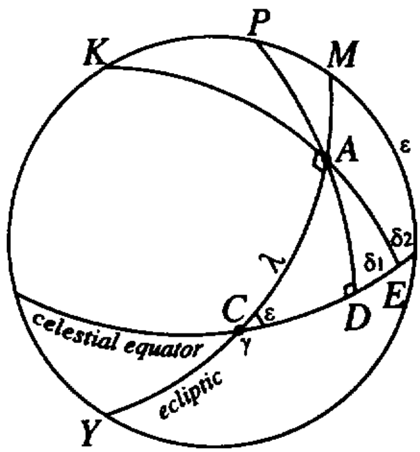

Figure 1

mayl-i awwal) is defined as the great circle are issuing from A and perpendicular to the celestial equator (fig. 1). This circle is called the *declination circle* and it passes through the north pole P.

The *second declination* (δ_2, *mayl-i thānī*) of the ecliptic point A is the great circle arc indicated as such oh figure 1, issuing from A, as before, but now passing through the poles of the ecliptic. This circle is called the *latitude circle*, since the celestial latitude (*ᶜarḍ*) of a star on the celestial sphere is defined as the distanc between it and the ecliptic along such a circle.

According to our text (lines 10-11), some medieval astronomers called the first declination the *absolute* declination, and the second declination the *latitude* of the ecliptic point A.

In our notation, the text says (lines 11-13) that

$$\delta(\lambda) = \delta(180^\circ - \lambda) = -\delta(\lambda + 180^\circ) = -\delta(-\lambda)$$

As examples, the beginnings of Taurus, Pisces, Virgo, and scorpio have the same declinations except for a plus or minus sign. The declinations of Taurus and virgo are northern; those of the other two are southern.

We note that the above equatins suffice to determine only the declinations of the points of the first quadrant of the ecliptic which is pointed out in line 13 of our text.

With known λ and ϵ, Naṣīr al-Dīn uses the right spherical triangle ADC

of the Shrine (Āstan Quds) Library, Mashhad.
Section 2 below presents a description of the contents of the third section of Treatise III in modern notation. It contains the definitions of the declination functions for points on the ecliptic and some trigonometric methds for their determination, Section 3 of this paper describes the material in the fourth section of the text which defines the declination function for arbitary points on the celestial sphere and gives two formulas for its determination based on spherical trigonometry. We note that the term declination (*mayl*) was restricted to points on the ecliptic by the medieval astronomers, and they used the word distance (*buᶜd*) for arbitrary points.
The symbols used in the present paper are the standard astronomical symbols, where applicable, and defined as they are encountered. There are no figures in the original persian text, but we have made them in order to understand the material. To indicate the folios and the corresponding lines of the microfilm copy, we have used unmbers inside parentheses. For example, (194r:2-20) means folio 194r, lines 2-20 of the manuscript.
The medieval trigonometric functions were not identical with their modern counterparts, and are written here with initial capitals to distinguish them from the latter. The relation between the two is, e.g., $Sin\theta = Rsin\theta$, where R is the radius of the defining circle, usually 60, not unity. This explains the appearance of the R in equations below.

II- Declinations of the ecliptic points (194r: 2-20)

The obliquity of the ecliptic (ϵ, *ghayat-i mayl*) is defined to be an arc of the great circle that passes through the four poles. These are the north and south poles of the ecliptic and those of the celestial equator. The arc between the *summer solstice* point M and the equator is northern, and the one between the *winter solstice* point Y and the equator is southern. (See Figure 1 below.)

The obliquity of the ecliptic ϵ is determined by observation (line 4). Also, accordint to our text, ptolemy's value is 23;51; most Islamic astronomers give $\epsilon = 23;35°$, and some of their successors have given $\epsilon = 23;33°$. Naṣīr al-Dīn prefers his own value, given as $\epsilon = 23;30°$, which he adopts. Partial declinations are defined for an arbitrary point A on the ecliptic, other than the solstial points (lines 6-10). The *first declination* (δ_1,

Naṣir Al-Dīn al-Tusi on determination of the Declination Function

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I- Introduction

A commentary on the spherical astronomy in the *Zīj-i īlkhānī*[1], written by the famous Naṣīr al-Din al-Ṭusi (c. 1270) would be an important contribution to the history of medieval astronomy. But we postpone this and attempt here to describe the contents of the third and fourth sections of Treatise III of this *zīj*. We hope that it will serve to elucidate a small part of this work, which has remained unexploited. I have made a preliminary english translation of the entire *Zīj-j īlkhānī*, which awaits final revision and preparation for publication.
The rules explained verbally and without proof by Naṣīr al-Dīn in the sections under consideration are similar to the ones described by Kāshī in his *Zīj-i Khāqānī* and analysed by E. S. Kennedy[2] Therefore, we do not present our proofs of the rules, but refer the reader to this article.
The microfilm copy of Manuscript 1418/2 of the Central Library of Tehran University has been used as the main text. It seems to be a careful job, as compared with the other copies available to us: MS 684 of Sipah Salar (now Muṭahari) Library, Tehran, and MSS 5331 and 5332

1. Naṣīr al-Dīn al - Ṭusi, *Zij-i īlkhānī*. Microfilm 1418/2, Tehran University Central Library. (Other copies consulted are: MS684, Mutahari Library, Tehran, and MSS 5331 and 5332 of the Shrine Library, Mashhad.)
2. Kennedy, E. S. Spherical Astronomy in Kāshī's Khāqānī Zīj. *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften*, 2, (1985) 1-46.