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Stock Option Pricing by Augmented Monte-Carlo Simulation Models

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ABSTRACT

Studying stock options is still a pristine area of research in the Iranian capital market. This is due to the lack of data as well as the complexity of valu-ation methodologies. In the present paper, using the Monte-Carlo simulation, we have estimated the value of stock options traded on Tehran Stock Exchange and examined whether the use of a control variate or antithetic variate augmented methods can lower the standard error of estimates. Furthermore, the estimated values of the three models under consideration, including of crude Monte-Carlo, control variates augmented Monte-Carlo, and antithetic variates augmented Mon-te-Carlo are compared with each other and with options market prices. The results show that the standard error of the antithetic variate method is less than the crude method and control variate method. However, control variate augmented Monte-Carlo model is more powerful than the crude Monte-Carlo and antithetic variate augmented Monte-Carlo method. Therefore, we can conclude that the control variate augmented Monte-Carlo model has a better performance in estimating the value of trading stock options and its estimated values are closer to the market prices.

1 Introduction

In the present paper, we focus on the listed stock option on the Tehran Stock Exchange (TSE). After launching these contracts on TSE as of 2018 their trading volume has been low, mostly because of the complications in their valuation. To solve this problem, TSE provided its own option price calculator, based on Black-Scholes methodology, on its website. However, there are several alternative methods for valuating stock options that could be helpful to reach a more accurate value.in the present paper, we investigate the utilization of Monte-Carlo simulation models in valuating stock options. We also compare the strength of Control Variates Augmented Monte-Carlo Simulation Model with the strength of the Monte-Carlo Simulation Model to see which one has a smaller standard error. Monte-Carlo simulation model is based on the hypothesis that stock prices follow a Brownian motion [13].
In this method, numerous stock price possible trajectories are produced through simulation and the distribution of the stock price at the expiration date is estimated based on those. After that, based on the stock price distribution the option price is estimated. Listed stock options on TSE are of European type and only exercisable at maturity. Hence, in the present paper, we focused on European stock options. Moreover, as the price of call options and put options are related to each other through the put-call parity and we can use this parity to drive the price of put options from the price of call options, we exclusively focused on call options so we can thoroughly explain our methodology. The present paper is comprised of four main sections. Literature review and theoretical background of option pricing are

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explained in the second section. The main methods of option pricing including the Monte-Carlo models are presented in this section. In the third section, we explained the methodology of our study. And in the fourth section, we demonstrated the results of our tests, and finally, in the fifth section, we discussed the results and inference regarding them.

2 Literature Review

An option is a financial contract that gives the right (not the obligation) to its holder to buy (or sell) a certain amount of an underlying asset (stocks in our case) by a predetermined price (exercise price) at a predetermined date (expiration date). If the right is to buy the underlying asset, then the option is called a call option and if the right is to sell the underlying asset then it is called a put option [3]. Moreover, there are three types of options regarding exercise time. In European options, you can only exercise the option at the expiration date. In American options, you can exercise the options at any time until the expiration date. In Bermuda options, you can exercise your options at specific times until maturity [22]. Stock option valuation has been a challenging topic in finance. These challenges exist in both the theory and implementation of valuating options. Researchers have proposed several methods for valuating options over the years. One of the main proposed methods is the well-known Black-Scholes model that has been introduced 40 years ago [6-19]. However, the assumption of this model regarding interest rate, lognormal distribution, exercising mechanism, trading costs, etc. required a lot of justification in the model before it could be implemented properly for pricing the exchange-traded stock options. For example, Wu incorporated Fuzzy logic into the model [27]. One of the alternative models introduced in recent years to cover some of the deficiencies of Black-Scholes model is Cox-Ross-Rubinstein model or simply the Binomial model [9-10].

The Binomial model is more easily understandable and it is easier to implement it on real-world data, however, it also has some determinative assumptions such as discrete-time assumption, discrete scenarios assumptions, and no-arbitrage assumption. These constraints encouraged researchers to develop more efficient models. Hence, Boyle proposed an alternative option pricing model by using Monte-Carlo simulations for option pricing [8]. Monte-Carlo method of options pricing is more flexible and it is capable of valuating options with different assumptions. Monte Carlo simulation is one alternative for analysing options markets when the assumptions of simpler analytical models are violated [11]. For example, it can be used for pricing options, which are written on a stock whose dynamics follow a jump-diffusion [5]. This model is based on the idea that underlying assets price distribution at the expiration date depends on the price movements from its inception to its expiration date. We can simulate these price movements via computer programs and use these simulated alternative price movements to form the price distribution on the expiration date. Then we can use the formed price distribution to estimate the option value. There are some ways to reduce the variance of Monte-Carlo simulation estimates. One way is to use control variates. Control variate aims at reducing the Monte-Carlo estimator's variance using a correcting factor that depends on the distance between a control variate and its expectation [18]. Researchers such as Agarwal et al, [1] and Jia et al, [16] used this method to reduce the standard error. Mostly because of the lack of data, there has not been that much scientific work on the subject of options in Iran. However, Vajargah et al, [26] studied valuating Asian options through Monet-Carle simulation with control variates but they didn't test their model on options traded on Iran capital market. Nabavi et al, [21] also did not use real trading data of options. Other studies such as Rafiei and Absossamadi [23], Shabani and Baharvandi [25] and Mohebbi and Gholizade Pasha [20] only studied the legal and Islamic aspects of options and did not focus on option valuation models. Another approach to reducing the standard error of Monte-Carlo simulated estimates is antithetic variate method [17]. The antithetic variate approach uses pairs of negatively correlated random numbers that in turn tend to produce pairs of negatively correlated simulation results. If the results from each pair are averaged, the simulation results should be less variable than those produced by ordinary random sampling [14]. This method has been used by researchers such as Bouaziz et al, [7] and Alzubaidi [2] to estimate options value. Again, we could not find any evidence of using antithetic method for valuating stock options traded on TSE.

3 Methodology

Generally speaking, the Monte-Carlo method provides the expected value of the option as the average of the estimates of the values over the iterations of simulation [12]. More specifically, in risk-neutral option pricing we aim to estimate the expected value of a function *g(x)* under a random variable x as:

$$E\big(g(x)\big) = \int_{-\infty}^{+\infty} dx g(x)\, \varphi(x) \tag{1}$$

Where $\varphi(x)$ is the probability density function of x. Generally, it is difficult to derive an analytical formula for (1). Therefore, to find the expected value of the mentioned function through Monte-Carlo simulation consider a random sample $\{x_1, x_2, \dots, x_n\}$ generated based on the $\varphi(x)$. Then, the estimates of the mean and variance of *g(x)* are as follow:

$$m = \frac{1}{n}\sum_{i=1}^{n} g(x_i) \ \text{ and } \ s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(g(x_i) - m)^2 \tag{2}$$

According to the central limit theorem, the random variable defined as (3) tends to follow a standard normal distribution with increasing sample size *n* and is irrespective of the distribution of *g(x).*

$$\frac{m - E\big(g(x)\big)}{\frac{s}{\sqrt{n}}} \tag{3}$$

Thus, the sample average m approaches a normal distribution with mean *E(g(x))* and standard deviation $\frac{s}{\sqrt{n}}$. On this basis, we can obtain the confidence interval in the estimation of *E(g(x))* from the formula (4).

$$E\big(g(x)\big) = m \pm z\frac{s}{\sqrt{n}} \tag{4}$$

The term $\frac{s}{\sqrt{n}}$ in formula (4) is referred to as the standard error in the estimation of *E(g(x)).* To reduce the standard error by a factor of 10, the sample size has to be increased one hundredfold. Therefore, this method, which is called the crude Monte-Carlo model, is not efficient in reducing standard error.

Therefore, we have to incorporate some techniques to reduce the standard error. One of these techniques is called control variate, which focuses on a similar, but simpler problem to improve the persistency of estimates. To understand this technique, suppose that the expected value *E(h(x))* can be evaluated analytically as H. In relation to the original function *g(x),* we can define a new function through the control variate *h(x)* as:

$$\tilde{g}(x) = g(x) - h(x) \tag{5}$$

So now, we can rewrite the formula (1) as follow:

$$E\big(g(x)\big) = H + \int_{-\infty}^{+\infty} dx\tilde{g}(x)\varphi(x) \tag{6}$$

Consequently, we can determine the confidence interval in the estimation of *E(g(x))* based on the estimates of the mean and variance of $\tilde{g}(x)$ instead given by:

$$E\big(g(x)\big) = (H + \tilde{m}) \pm z\frac{\tilde{s}}{\sqrt{n}} \tag{7}$$

The variances of g(x) and $\tilde{g}(x)$ can be related as:

$$var(\tilde{g}(x)) = var(g(x)) + var(h(x)) - 2cov(g(x), h(x)) \quad (8)$$

Now, if *g(x)* and *h(x)* are similar problems, the covariance between them is positive. In (8), the variance of $\tilde{g}(x)$ will be less than the variance of *g(x)* as long as $2cov(g(x), h(x)) > \frac{1}{2}var(h(x))$. It is, therefore, possible to reduce the size of the standard error by identifying a highly correlated problem with a known analytic solution. We could also use the antithetic variate method to reduce the standard error of estimates. In the case of a standard normal variable, antithetic variate method makes use of the symmetric property around zero in the density function. Again, we can introduce a new function given by formula (9) through antithetic variate of the form *–x*:

$$\hat{g}(x) = \frac{[g(x) + g(-x)]}{2} \quad (9)$$

We can rewrite (1) using the symmetric property of the standard normal variable *x* as:

$$E(g(x)) = \int_{-\infty}^{+\infty} dx \hat{g}(x)\varphi(x) \quad (10)$$

Similarly, we can determine the confidence interval in the estimation of *E(g(x))* based on the estimates of the mean and variance of $\tilde{g}(x)$ given by:

$$E(g(x)) = \hat{m} \pm z\frac{\hat{s}}{\sqrt{n}} \quad (11)$$

The variance of $\tilde{g}(x)$ is expected to be smaller than the variance of *g(x)* because it is an average of two samples. We can show that the two variances are related as:

$$var(\hat{g}(x)) = \frac{1}{2}var(g(x)) + \frac{1}{2}cov(g(x), g(-x)) \quad (12)$$

If the covariance between *g(x)* and *g(-x)* is negative, then it is more efficient to consider the estimates for $\tilde{g}(x)$ rather than doubling the size of independent samples. In the following, we explain the implementation of these methods in option pricing.

The current price of an option can be defined based on the present value of its average maturity payoff at time *T* as:

$$f_0 = \hat{E}(e^{-rT} f_T | S_0) \quad (13)$$

Where *r* is the constant interest rate. Here, we are averaging over realized maturity payoffs of the option f_T in respect to sample asset prices generated through their risk-neutral process that initiated at current price S_0. In the stochastic model, the asset price return during the time increment from *t* to *t+Δt* is assumed to follow a random normal process as:

$$\Delta S_t / S_t = \mu\Delta t + \sigma\sqrt{\Delta t}\varepsilon(0.1) \quad (14)$$

Where μ and σ are respectively the mean rate and volatility of return. For traded assets such as stocks, the risk-neutral process is simply given by (14) with μ replaced by r in the drift term. Practically, it is convenient to consider the asset price movement based on the risk-neutral process. For constant volatility of return, it is shown to follow an iterative equation with arbitrary time duration given by:

$$s_{t+\tau} = S_t exp\left(\left(r - \frac{1}{2}\sigma^2\right)\tau + \sigma\sqrt{\tau}\varepsilon(0.1)\right) \quad (15)$$

Particularly, we have the formula (16) that generates the maturity price s_T directly from s_0.

$$s_T = S_0 exp\left(\left(r - \frac{1}{2}\sigma^2\right)\tau + \sigma\sqrt{T}\varepsilon(0.1)\right) \quad (16)$$

Now, if we consider the strike price as *k*, we can use formula (16) to generate the maturity price of the asset by a single random number ε from $\varepsilon(0,1)$. The sample maturity price can then be used to evaluate the sample maturity payoff of the option according to the function:

$$f_T(\varepsilon) = max\{S_T(\varepsilon) - k, 0\} \tag{17}$$

For variance reduction, we can adopt the maturity price S_T itself as the control variate and develop a new function (18) as:

$$\tilde{f}_T(\varepsilon) = max\{S_T(\varepsilon) - k, 0\} - S_T(\varepsilon) \tag{18}$$

Because $S_T(\varepsilon)$ follows a Brownian motion, we expect that its present value to be equal to S_0. Therefore, we can rewrite formula (13) as:

$$f_0 = S_0 + \hat{E}\left(e^{-rT}\tilde{f}_T \middle| S_0\right) \tag{19}$$

Alternatively, we can take -ε as the antithetic variate and develop a new function:

$$\hat{f}_T(\varepsilon) = \frac{1}{2}[max\{S_T(\varepsilon) - k, 0\} + max\{S_T(-\varepsilon) - k, 0\}] \tag{20}$$

Through the mentioned methodology, we can estimate the value stock options traded on TSE by crude Monte-Carlo model, control variate augmented Monte-Carlo Model and antithetic variate Augmented Monte-Carlo Model and compare the results to see which method estimates the option value more accurately and which one has the lowest standard error in practice. Moreover, we also estimate the options prices by Black-Scholes model to compare the efficiency of Monte-Carlo models whit Black-Scholes model. We also can use the Black-Scholes model to generate the implied volatilities of the underlying stock and use it as an input of Monte-Carlo models. This approach of estimating underlying stocks volatility could be more suitable than using past volatilities because price futures movements do not necessarily resemble their past behaviour [4].

Black-Scholes formula to estimate options value is as follow:

$$f_0 = S_0N(d_1) - ke^{-rT}N(d_2)$$
$$d_1 = \frac{ln\left(\frac{S_0}{k}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T} \tag{21}$$

By inserting the trading price of stock options into the formula (21), we can use it to find the value of σ as implied volatility. Finally, we perform a stepwise regression to see which model's estimates are more in accordance with market prices. In stepwise regression, we repeatedly add and remove a variable among all the variables; then conduct regression analysis with the remainder, and select the variable associated with the highest value of R-squared as the explanatory power of the regression model [15]. Hence, stepwise regression is more like an algorithm through which we perform several linear regressions. Here we perform the stepwise regression with forward selection approach. Our general regression model is as follows:

$$MP_i = c + \alpha_1\beta_1 CMC_i + \alpha_2\beta_2 CVMC_i + \alpha_3\beta_3 AVMC_i \tag{22}$$

Where:

i = {1,…,35} number of sample options

MP: market price of each sample option

CMC: options values estimated by Crude Monte-Carlo

CVMC: options values estimated by Control Variate Monte-Carlo

AVMC: options values estimated by Antithetic Variate Monte-Carlo

β: a binary variable the only takes 0 and 1 and $\beta_1 + \beta_2 + \beta_2 = 1$

In this procedure, we set the value of one β to 1 and set the value of other βs accordingly and then perform the regression to reach to the highest value of R-squared.

4 Empirical Results

In order to select the sample, it should be noted that following the approval of the final instructions of stock options trading on December 5, 2016, by the Board of Directors of the Securities and Exchange Organization, the trading of this instrument was officially launched in TSE on January 5, 2016. Options online trading platform and risk management system for options contract also became operative since October 23, 2017. Hence, available data include contracts launched from the second half of the year 2017. We can say that the reason for the lack of scientific research on this instrument is that it has just begun trading on TSE. The selected example includes call options that have not yet expired and can be traded on TSE. Therefore, the options with maturities of 2019-2020 are selected. Another reason for choosing the latest tradable contracts is that as time went by, traders have become more familiar with these instruments and as a result, the volume of transactions and market efficiency has increased. Moreover, if a company has not yet determined the cash dividend of its recent fiscal year and there is a probability of holding a general meeting and determining the cash dividend before the maturity of options, the company's earnings and its dividend pay-out ratio should be forecasted. Since earnings prediction is out of the focus of this paper, so we chose call options that do not have an annual general meeting until the maturity date of the contract. Thus, 35 call options are selected. The descriptive statistics of the selected sample are as follows:

Table 1: Sample Descriptive Statistics

	maximum	minimum	average
Maturity (Days)	184	45	93
Open Positions	170,741	290	18,397
Moneyness	31% in the money	69% out of the money	0% at the money

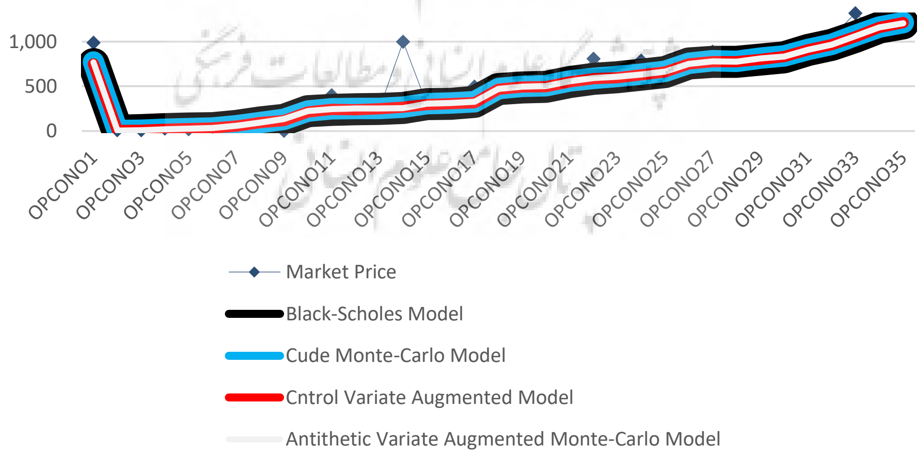

Fig.1: Call Options Values Estimated by Our Four Alternative Models, Along with Their Market Prices

It should also be noted that the price of the option is stated on a per-share basis, and then the option price can be multiplied by the contract size to calculate the total value of the contract. Therefore, in estimating the value of options we don't need the size of contracts. We now calculate the implied volatility of each option, which is the volatility perceived by traders, using the market price of each call option and the Black-Scholes formula. We then calculate the value of each call option in the sample, using the Black-Scholes model, Crude Monte-Carlo model, Control Variate Augmented Monte-Carlo Model, and Antithetic Variate Augmented Monte-Carlo Simulation Model. We do this by running 10,000 simulations. Fig. 1 shows the estimated values generated by all four models, along with the market prices of the call options. As shown in Fig. 1, the estimated values are mostly consistent with each other, so it is possible to use all four models as alternatives. We performed a correlation analysis between option values estimated by models under consideration to study the consistency between their outputs more specifically.

Table 2: Correlation Analysis of Estimated Option Values Generated by Alternative Models Under Consideration

		Crude Monte-Carlo	Control Variate Augmented Monte-Carlo	Antithetic Variate Augmented Monte-Carlo	Black-Scholes
Crude Monte-Carlo	Correlation t-Statistic Probability	1.000000 ----- -----			
Control Variate Augmented Monte-Carlo	Correlation t-Statistic Probability	0.999895 396.7952 0.0000	1.000000 ----- -----		
Antithetic Variate Augmented Monte-Carlo	Correlation t-Statistic Probability	0.999982 946.0563 0.0000	0.999869 355.3542 0.0000	1.000000 ----- -----	
Black-Scholes	Correlation t-Statistic Probability	0.999899 403.1812 0.0000	0.999931 489.3644 0.0000	0.999910 427.5074 0.0000	1.000000 ----- -----

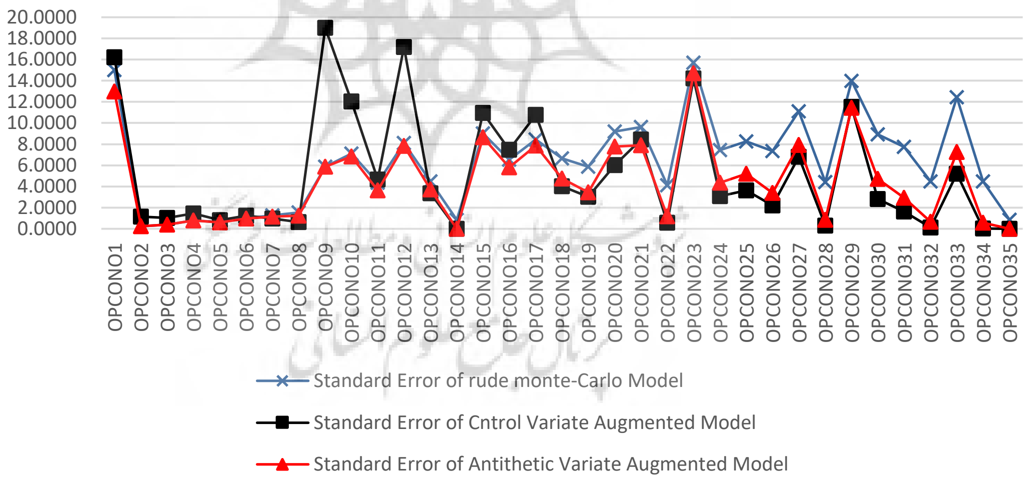

Fig. 2: Standard Errors Crude, Control Variate Augmented, and Antithetic Variate Augmented Models

As the above correlation analysis table shows, while all three Monte-Carlo models have a very high and significant correlation with the Black-Scholes model, the control variate augmented Monte-Carlo model is more correlated to the Black-Scholes model. However, we also have to consider their standard errors to see which model is more efficient. To do so, we consider the standard errors of the estimated values Crude, Control variate augmented, and antithetic Variate augmented Monte-Carlo models. Fig. 2 comparatively illustrates this. The standard error of the antithetic variate augmented Monte-Carlo model on average is 4.51, which is lower than the average of standard errors of the Crude Monte-Carlo model (6.24), and control variate augmented Monte-Carlo model (5.22). Therefore, it can be said that, since their estimated values are really close to each other, the efficiency of the antithetic variate augmented Monte-Carlo model is better than the base model. However, in order to accurately investigate this finding, assuming that the market is efficient and the market prices are close to the intrinsic value of the option values, we performed a stepwise regression to investigate which of the three versions of Monte-Carlo models can better explain market prices. This approach identifies the variable that is most capable of explaining the independent variable variations. Table 3 presents the results of the stepwise regression:

Table 3: Stepwise Regression Between Three Versions of Monte-Carlo Models and Actual Market Prices

Dependent Variable: Market Prices				
Method: Stepwise Regression				
Included observations: 35 after adjustments				
Number of always included regressors: 1				
Number of search regressors: 3				
Selection method: Stepwise forwards				
Stopping criterion: p-value forwards/backwards = 0.05/0.05				
Stopping criterion: Number of search regressors = 1				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	50.35909	43.04239	1.169988	0.2504
control variate augmented Monte-Carlo	1.015495	0.071997	14.10465	0.0000
R-squared	0.857723	Mean dependent var		542.0857
Adjusted R-squared	0.853411	S.D. dependent var		390.0627
S.E. of regression	149.3430	Akaike info criterion		12.90581
Sum squared resid.	736010.0	Schwarz criterion		12.99469
Log likelihood	-223.8517	Hannan-Quinn criter.		12.93649
F-statistic	198.9412	Durbin-Watson stat		2.210777
Prob(F-statistic)	0.000000			
Selection Summary: Added Control Variate Augmented Monte-Carlo				

As the stepwise regression table shows, the control variate augmented Monte-Carlo model has the most efficiency to explain market prices. Therefore, it can be used as a model whose estimated values are closer to real market prices. In general, any form of temporary deviation in price from its fundamental value is called noise [24]. Noise can be related to a variety of factors, one of which is market liquidity, which is usually measured by the volume of transactions. Moreover, longer maturity may lead to more uncertainty about stock's future volatility and that, in return, may lead to more deviation from intrinsic values. Therefore, we continue with the same stepwise regression methodology to determine whether the gap between market prices and values estimated by control variate augmented Monte-Carlo model is due to differences in the liquidity of different stock options or due to their longer maturity. As table 4 shows, noise in the stock options market is mainly due to their liquidity. More liquid stock options showed less noise.

Dependent Variable: NOISE
Method: Stepwise Regression
Included observations: 35
Number of always included regressors: 1
Number of search regressors: 2
Selection method: Stepwise forwards
Stopping criterion: p-value forwards/backwards = 0.05/0.05

Table 4: Factors Affecting The Noise In The Stocks Options Market

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
C	5.774026	2.339513	2.468046	0.0189
LOG(VOLUME)	-1.126121	0.519920	-2.165952	0.0376
R-squared	0.124468	Mean dependent var		1.201280
Adjusted R-squared	0.097936	S.D. dependent var		6.279150
S.E. of regression	5.963750	Akaike info criterion		6.464721
Sum squared resid	1173.688	Schwarz criterion		6.553598
Log likelihood	-111.1326	Hannan-Quinn criter.		6.495402
F-statistic	4.691349	Durbin-Watson stat		1.976912
Prob(F-statistic)	0.037639			
Selection Summary: Added LOG(VOLUME)				

5 Conclusions

In the present paper, after estimating the value of stock options by three versions of Monte-Carlo models, we first investigated to find out which model had the least standard error. The results showed that antithetic variate Monte-Carlo model has a lower standard error compared to the control variate Monte-Carlo model and crude Monte-Carlo mode. Moreover, we compared the estimated values of the three models to each other and market prices. The results of the study showed that the estimated values of the three models are very similar to each other and therefore it is possible to use all three models as alternatives for estimating the value of stock options listed on Tehran Stock Exchange. On the other hand, using stepwise regression, we showed that control variate Monte-Carlo model explanatory power is higher than other models and its estimated values are closer to market prices. Therefore, according to the results of our study, we recommend that analysts use control variate augmented Monte-Carlo simulation model to improve their valuation accuracy. Also, given that the estimated values of all three models are close to each other, analysts can use all three models as alternative valuation models to ensure their estimation accuracy. Moreover, more liquid stock options tend to have less noisy prices.

On the other hand, one of the major limitations of the present study is that due to the low depth, options market is possibly noisy and probably inefficient [24]. As a result, in order to reach to a more reliable conclusion about the relationship between options market prices and their estimated values, the volume of transactions should increase over time.

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