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Analyzing the performance of DEA models for bankruptcy prediction in the energy sector: with emphasis on Dynamic DEA approach

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ABSTRACT

Predicting bankruptcy risk is one of the most critical issues in corporate financial decision-making. Investors always try to predict the bankruptcy of a firm to reduce the risk of losing their assets, so they are looking for ways by which they can predict the risk of bankruptcy. We predict the position of companies active in the oil and gas industry based on their financial health in the 2020 ranking of S&P global up to three years before 2020. This study uses three data envelopment analysis models (CCR, BCC, and DDEA) and the traditional Altman model for forecasting. We have shown that dynamic data envelopment analysis is a powerful tool for predicting bankruptcy risk.

1 Introduction

Bankruptcy prediction is a common topic in business analytics because of the significance of accurate and timely strategic business decisions. In a prediction model, in addition to the accuracy, the understandability and transportability of the model are also crucial factors. Accurate bankruptcy prediction has been a critical concern for shareholders, creditors and company managers [1].

The thought of risk assessments includes a long history. More than 2400 years ago, the Athenians demonstrated their ability to assess risk [2]. The term "risk" has a French etymon and states a situation involving exposure to danger. Risk is a crucial concept in various fields, but there is no agreement on how to define it [3]. Some definitions are composed of expected values, others on probabilities, some on uncertainty, and others on objectives [4]. One of the important descriptions of risk was presented by Lowrance [5] who said "Risk is the measure of probability and the weight of undesired consequences". According to Fetisovová et al. [6], for any financial conclusion, it is essential to pay attention to its expected return and its related risk. Campbell [7] introduced the following concept of risk: "Risk equals expected damage."

Business failure is a characteristic of any economy. Accordingly, it is crucial to predict the risk of business failure [8]. Investing provides means to obtain value maximization in terms of dividend payment or capital gains. Therefore investors are always looking to predict and assess the company's bankruptcy risk to manage their investment risk. Evaluation of bankruptcy risk and factors that affect it, is of main importance for creditors, management, employees, and society [9].

Financial insolvency has often been used in the asset pricing literature to explain the unusual pattern of stock returns [10]. Financial distress includes not only the incapability to pay obligatory payments but also a condition of negative net asset value, which means that a company's accounting for its total liabilities is more than its total assets. When a company faces financial distress, there are serious consequences for many internal and external economic factors occur, such as shareholders, lenders, customers, suppliers, employees, and managers [11].

A company is called insolvent when it cannot pay its obligations to creditors. A company's debts may be used to finance its operations. But doing so puts you at greater risk of facing insolvency. Therefore, if the company's insolvency does not improve, it will lead to bankruptcy [12]. The difference between insolvency and bankruptcy is in such a way that, whenever the rate of return realized for the capital employed in the firm is considerably and consistently lower than the rate of return demanded; insolvency has occurred. Bankruptcy, on the other hand, is a legal situation that occurs for a firm in insolvency. The company may face insolvency in the long term, but because there is no legal prohibition; that company does not face bankruptcy [13].

One way to help make the most of investment opportunities and better resource allocation is to assess insolvency. Through the assessment of insolvency, the financial situation of companies is clarified and their insolvency is examined so that shareholders and managers can find a way to prevent insolvency or change in the structure of the company, and perhaps by taking appropriate decisions, prevent them from bankruptcy [14].

The first studies in the field of bankruptcy were conducted in the early 1930s by Winakor and Smith [15], Fitzpatrick [16], and Merwin [17] to investigate the application of financial ratios in bankruptcy. Beaver [18] developed a univariate model with 30 financial ratios that can distinguish bankrupt companies from non-bankrupt ones. The main types of models used to predict bankruptcy are statistical models, artificial intelligence, and non-parametric models. Common statistical approaches developed for bankruptcy prediction models are univariate analysis [18], multivariate analysis [19, 20], logistic regression [21], and factor analysis [22]. In recent years, artificial intelligence and non-parametric methods have been used more to predict bankruptcy. Methods such as data envelopment analysis [8], artificial neural network method [23, 24], support vector machines [25, 26] are widely and well used in the financial area.

Compared to statistical methods, DEA is a relatively recent, non-parametric methodology, which represents one of the major possible approaches to evaluating a company's financial health and it's risk of bankruptcy [27].

This article makes various contributions to the literature. First, applying the dynamic model of the DEA to predict the bankruptcy of energy companies. Second, introducing the intermediate variable of the ratio of retained earnings to long-term debt as a useful intermediate variable in the dynamic DEA model for predicting the bankruptcy of energy companies. Third, comparing the accuracy of dynamic DEA with basic data envelopment analysis models, including CCR and BCC models, and also with the Altman model, which is one of the most widely used bankruptcy prediction models.

This article is organized as follows. Section 2 reviews prior studies on different bankruptcy prediction models. Section 3 explains data and methodologies. Data analysis and model prediction accuracy are covered in Section 4, and our conclusions are covered in Section 5.

2 Literature Review

Bankruptcy risk is one of the most popular concepts in business and finance literature due to its consequences for stakeholders' decisions [28]. There are various types of financial risks associated with a business: market risks will be obtained by the fluctuates in financial instruments prices like stocks and commodities, foreign exchange risks and interest rate risks have interdependence on each other, the interdependence being identified when managers are planning risk management systems [29]. Bankruptcy risk represents the possibility that a firm will not be able to face its obligations. One of the main topics for investors and managers to implement funding and investments is bankruptcy risk assessment. It is also an important topic for bankers, rating agencies, and even distressed firms themselves [30].

Several models have been used to predict business bankruptcy. All of these patterns have certain pros and cons, strengths and weaknesses; therefore, picking out the most appropriate one is not aboveboard [31]. Statistical and dynamic models can be used to predict bankruptcy risk according to the variables used [32].

The development of bankruptcy models began with the Beaver [18, 33] univariate analysis model for ratios that were well predicted [32]. The practical use of financial ratios began in 1870, when the banks demanded financial statements to lend to companies. In the 1890s, various financial ratios developed. According to Horrigan [34] the current ratio is one of the oldest financial ratios that has the most significant impact on the analysis of financial statements. It also indicates the ability to pay short-term debts [35].

In the 1920s, various financial ratios were developed by commercial institutions. In the 1930s, the development and use of financial ratios intensified with the formation of the US Securities and Exchange Commission. Studies over the decade have shown that there is a considerable difference between the financial ratios of bankrupt firms and the financial ratios of non-bankrupt firms [36].

In subsequent research, multivariate models were considered. Because these statistical models are unlike univariate models, which consider only one variable at a time, they also include interactions between variables. In this case, the possibility of wrong classification in these methods is less than in the study of univariate methods [14]. Altman [19] for the first time studied the effect of different combinations of financial ratios to predict corporate financial insolvency. Altman used discriminant analysis in this study. The model he developed, known as the Z-score, is still used as an index of corporate financial health. Altman's main theory was that his bankruptcy prediction model, which consists of five financial ratios, could be used to separate bankrupt companies from non-bankrupt ones. He suggested that his model be used to evaluate commercial lending, internal control processes, and explore investment options.

Altman et al. [37] developed a model by modifying the accuracy of accounting information. In the new model, they used a quadratic relation for classification, and the new model of analysis was called ZETA. The results of this study showed that the ZETA model is a better predictor than the Z -Score model, up to five years before the bankruptcy.

Logistic regression has many advantages in contrast to models based on discriminate analysis. Due to its high predictive power and its application does not need to comply with assumptions that could confine it. Altman has demonstrated the validity of the Z-score model with a lot of data and globally. He also compared his model to logit models, which performed similarly or better [30]. Zmijewski [38] used the probit analysis method to build his model and used liquidity, performance, and leverage ratios in his model. He tested the model on 40 bankrupt companies and 800 non-bankrupt companies, which achieved 78% correct predictions.

One of the most practical and useful methods for data classification is nonparametric methods such as decision tree which are still widely used. It was first used for the prediction of bankruptcy by Frydman et al. [39].

One of the usable machine learning topics in this area is the Artificial Neural Network [40]. Neural networks have been indicated to be effective at solving various problems. Odom and Sharda [41] were the persons who made the initial effort to apply synthetic neural networks (ANNs) for bankruptcy prediction. Iturriaga and Sanz [42] combined multilayer perceptrons and self-organizing maps to prepare a model that indicates the chance of insolvency up to three years before bankruptcy to investigate the bankruptcy of U.S. banks.

Some studies have stated that machine learning models are not widely applied in the field of business for two key reasons. First, the prediction accuracy does not far exceed the statistical models and second, the results are uninterpretable [43].

DEA is considered more efficient than other analyzing methods such as ratio analysis and multi-criteria evaluation approaches [44]. DEA has been known since its inception as one of the most accurate methods of predicting bankruptcy risk and has developed well in this area [45]. Charnes et al. [46] used a model that combined multiple inputs and outputs to calculate business performance. The attitude of researchers demonstrates a two-stage productivity calculation. The enterprises on the production frontier line are among the best. After determining the distance of enterprises from the production frontier, their efficiency score is computed. Simak [47] was the first to use the DEA concept in predicting bankruptcy and compare the results with the Altman model. Horváthová and Mokrišová [8] used the CCR model to predict bankruptcy risk in 25 Slovak hotels and compared the results with the Altman model. The results of their study showed the high accuracy of the DEA model in predicting bankruptcy. The term “Dynamic DEA” refers to the application of the DEA model to obtain relative performance in multi-period sets in which there are different relationships between periods [48].

Moshabbaki et al. [49] evaluated the bankruptcy of 110 companies admitted to the Tehran Stock Exchange using the methods of diagnostic analysis and data coverage analysis. They concluded that the accuracy of the incremental data envelopment analysis model is higher than the diagnostic model. Li et al. [50] develop the cross-sectional DEA models for time varying Malmquist DEA, since dynamic predictive models allow one to incorporate changes over time. This decision support system can intelligently adjust the efficiency frontier over time and generate robust forecasts. The sample of this study includes 742 Chinese listed firms over a 10-year period. They concluded that Malmquist DEA offers insights into the competitive situation of a company in addition to accurate financial distress predictions based on the DEA efficiency measures.

Štefko et al. [51] applied the additive DEA model to predict bankruptcy. Their sample includes 343 companies active in the heating sector in Slovakia, which has a stable situation. In order to check the DEA model used in their research, they finally compared it with the logit model. The results of their research were that DEA predicts bankrupt firms with a higher accuracy than the logit model; and as a result, they introduced DEA as a suitable alternative for predicting bankruptcy. In their research, they stated that the DEA method does not consider the primary situations of bankruptcy, but its results are based on the values obtained from financial indicators, and unlike the logit model, the results of the DEA model are independent of any assumptions.

Setiawan [52] compared the DEA model with the Altman model in predicting the bankruptcy of 7 Indonesian steel and iron companies. The results of his research show that the DEA approach is more accurate than the Altman model in predicting bankruptcy. Rahimi et al. [53] used a dynamic worst practice frontier DEA model to distinguish financially distressed decision making units throughout several time periods. They also offer some development solutions for financially distressed decision making units.

3 Data and Methods

The research sample consists of 20 global energy companies active in the oil and gas industry that have been gotten from S&P global ranking. We want to predict the position of these companies in the 2020 ranking of S&P global[1].

This industry has been chosen because oil became the world's vital source of energy since the mid-1950s. Oil and gas are also important for the number of jobs they provide. Failure of these companies may lead to a major crisis. Therefore, our purpose was to study the applicability of DEA models on a sample of oil and gas companies to predict the financial health rank of these companies.

3.1 Selection Financial Indicators

According to Horváthová and Mokrišová [8] which introduced a method to study financial health and predict bankruptcy risk, we selected creditors payment period, cost ratio, equity ratio as inputs and return on assets and total liquidity as outputs. We also used retained earnings to long term debt ratio as intermediate variable for dynamic DEA. Table 1 shows the method used to calculate these indicators.

Table 1: Method Used to Compute Financial Indicators

	Symbol	Indicator	Computation Method
Inputs	CPP	creditors payment period	(current liabilities / sales)*360
	CR	cost ratio	total assets / total revenues
	ER	equity ratio	equity / total assets
Outputs	ROA	return on assets	(EBIT / assets)*100
	TL	total liquidity	current assets / current liabilities
Intermediate		retained earnings to long term debt	retained earnings / long term debt

We used retained earnings to long-term debt ratio as an intermediate variable because in different periods there was a significant relationship between this variable and input and output variables. On the other hand, if the ratio of retained earnings to long-term debt ratio be higher in a company, it will be more able to face future debts and crises. The relationship between this intermediate variable and one of the inputs and outputs for 2019 is given below.

[1] https://www.spglobal.com/platts/top250/rankings

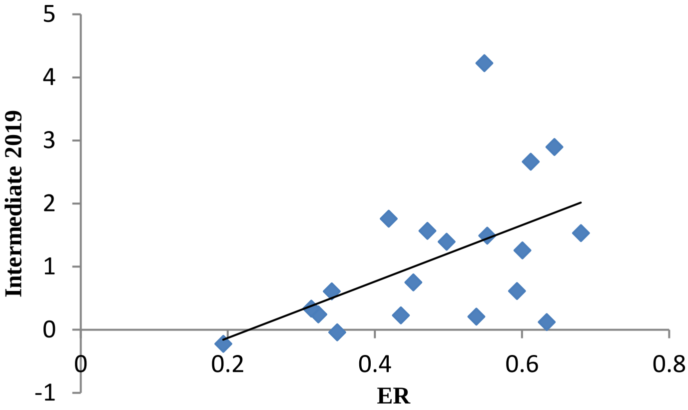

Fig. 2: The Relationship between Intermediate 2019 as Output Variable and Equity Ratio (ER) as Input

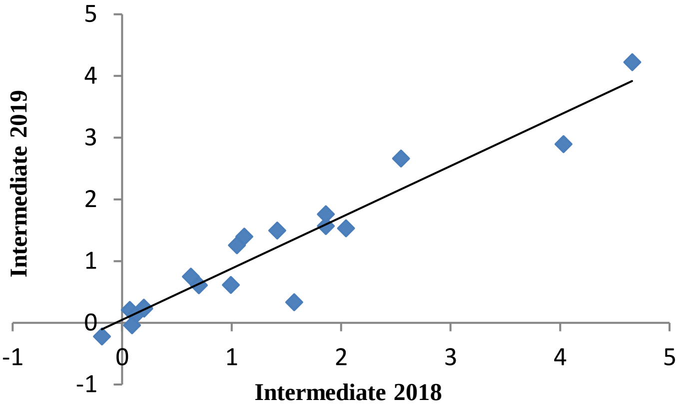

Fig. 1: The Relationship between Intermediate 2019 as Output Variable and Intermediate 2019 as Input Variable

3.2 Altman method

Altman [19] presented a model for predicting bankruptcy called the Z-score model or multiple discriminant analysis model (MOA). The Z-score model is not appropriate for all kinds of companies because of the usage of market value in its calculations [30]. The Z-score model is a weighted combination of 5 financial ratios. It is a very useful formula that has gained wide acceptance with various stakeholders like investors, financial analysis, bankers, auditors, management accountants, financial institutions, courts, and database systems. The feature of the Altman model is to differentiate between firms that are financially distressed and those that are not financially distressed. The variables of the model have been gotten from financial statements for analysis. The usage of this model is appropriate for predicting the bankruptcy of the firms in at most two years.
The model presented as follows:

$$Z = 1.2 * x_1 + 1.4 * x_2 + 3.3 * x_3 + 0.6 * x_4 + 0.99 * x_5 \tag{1}$$

where x_1 is working capital / total assets, x_2 is retained earnings / total assets, x_3 is earnings before interest and tax / total assets, x_4 is market value of equity / total liabilities, x_5 is sales / total assets

3.3 DEA Models

One of the most familiar, suitable, and useful methods for calculating the relative performance and arranging DMUs is called DEA. In addition, there are several different branches including medical care, transportation, education, banking, and the insurance industry which DEA is broadly used [54].

In terms of returns to scales, there are two types of basic DEA models: constant returns to scale (CRS) and variable returns to scale (VRS). DEA models can also be classified as input-oriented or output-oriented.

Assume that there are "n" DMUs, each using "m" inputs to produce "s" outputs. By using the following DEA model, the variables for all DMUs should be positive [46].

$$Max\ Z_0 = \frac{\sum_{r=1}^{s} u_r\, y_{r_0}}{\sum_{i=1}^{m} v_i\ x_{i_0}}$$

St: (2)

$$\frac{\sum_{r=1}^{s} u_r\, y_{rj}}{\sum_{i=1}^{m} v_i\ x_{ij}} \le 1$$

$$u_r\,, v_i \ge\ 0$$

The CCR is the first and simplest DEA model. This model evaluates the performance of each DMU by using input and output data.

The linear programming model of Eq. 2 is shown below (input-oriented primary model): [55]

$$\text{Max}\ Z_0\ = \sum_{r=1}^{s} u_r\, y_{r\,j_0}$$

St: (3)

$$\sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} \le 0$$

$$\sum_{i=1}^{m} v_i\, x_{ij_0}\ = 1$$

$$u_r\,, v_i \ge 0$$

The BCC model changed the constant return to scale (CRS) to variable return to scale (VRS). It is obtained that changes in inputs do not result in relative changes in outputs when the DMU controls under VRS. The input-oriented primary model of BCC is shown below:

$$\text{Max}\ Z_0\ = \sum_{r=1}^{s} u_r\, y_{r\,j_0} + w$$

St: (4)

$$\sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} + w \le 0$$

$$\sum_{i=1}^{m} v_i\, x_{i_0}\ = 1$$

$$u_r\,, v_i \ge 0$$

3.3.1 Dynamic DEA

Sometimes the efficiency of a DMU, in addition to the inputs and outputs of a particular period, also depends on the products and resources of previous periods. Dynamic DEA models evaluate DMUs when there are affiliations between different time periods of data [56]. In this study, the structure of the dynamic method shown as fig. 1 is used, which combined of n DMUs. Suppose DMU_j; j = 1,2,..,n has m inputs at period t as $X_{ij}^{(t)}$: i= 1,2,…,m; s final outputs $Y_{rj}^{(t)}$: r = 1,2,…,s; and g links $Z_{fj}^{(t)}$: f = 1,2,..,g. The total amount of i^{th}input and r^{th}output over P periods are indicated by $X_{ij}=\sum_{t=1}^{P} X_{ij}^{(t)}$, $Y_{rj}=\sum_{t=1}^{P} Y_{rj}^{(t)}$ respectively [57].

Kao and Hwang [58] considered a relational model for series systems that can be adopted for formulation. If the two quantities ignored by Kao and Hwang are restored, it then becomes:

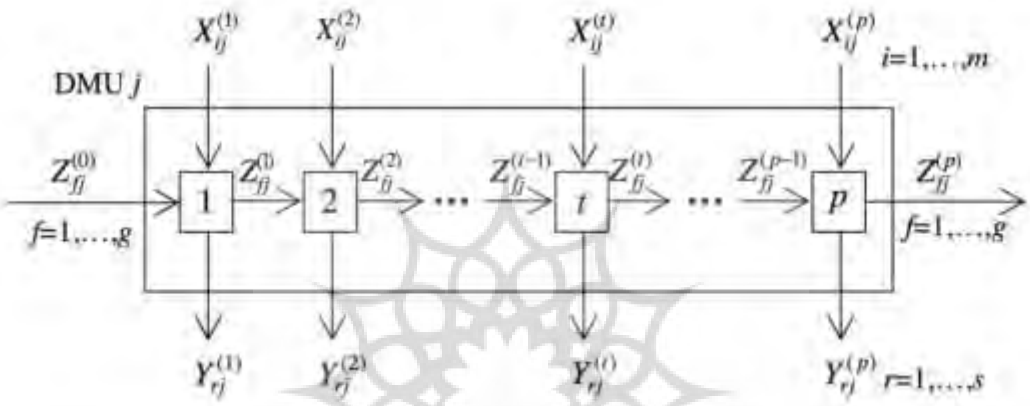

Fig. 3: Dynamic System with Flows Connecting Two Consecutive Periods

$$\frac{1}{E_K^R}=\min\sum_{i=1}^{m} v_i X_{ik}+\sum_{f=1}^{g} w_f Z_{fk}^{(0)}$$

St:

$$\sum_{r=1}^{s} u_r Y_{rk}+\sum_{f=1}^{g} w_f Z_{fk}^{(p)}=1 \tag{5}$$

$$\left(\sum_{i=1}^{m} v_i X_{ij}+\sum_{f=1}^{g} w_f Z_{fj}^{(0)}\right)-\left(\sum_{r=1}^{s} u_r Y_{rj}+\sum_{f=1}^{g} w_f Z_{fj}^{(p)}\right)\geq 0\,,\qquad j=1,\ldots,n$$

$$\left(\sum_{i=1}^{m} v_i X_{ij}^{(t)}+\sum_{f=1}^{g} w_f Z_{fj}^{(t-1)}\right)-\left(\sum_{r=1}^{s} u_r Y_{rj}^{(t)}+\sum_{f=1}^{g} w_f Z_{fj}^{(t)}\right)\geq 0\,,$$

$$j=1,\ldots,n;\quad t=1,\ldots,p$$

$$u_r, v_i, w_f \geq \mathcal{E}, \qquad r = 1, \dots, s; \quad i = 1, \dots, m; \quad f = 1, \dots, g$$

The following equation is used to calculate the DDEA for each period.

$$E_k^{(t)} = \frac{\sum_{r=1}^{s} u_r^* \, Y_{rk}^{(t)} + \sum_{f=1}^{g} w_f^* \, z_{fk}^{(t)}}{\sum_{i=1}^{m} v_i^* \; X_{ik}^{(t)} + \sum_{f=1}^{g} w_f^* \, z_{fk}^{(t-1)}} \tag{6}$$

4 Empirical Results

This section presents the results of our research. In this research, we attempt to predict the exact position of companies in the 2020 corporate financial health ranking. We have tried to predict the financial health of companies for 3 years up to 2020. For this purpose, we first rank the results of different models, then place the ranking of the existing results with the S&P global ranking inside a graph and examine the results. We have arranged the DMUs based on the S&P global ranking. Obviously, if the slope of the chart is closer to 1, the ranking is more accurate. The results of the Altman model are shown in table 2.

Table 2: Results of Altman Z-score Model

Altman			
DMU \ YEAR	**2019**	**2018**	**2017**
DMU1	6.3	7.3	8.3
DMU2	3.5	3.9	3.8
DMU3	3	2.9	2.1
DMU4	3.9	4.3	3.9
DMU5	4.8	5.1	5
DMU6	5.6	6.4	5.4
DMU7	5.1	5.4	5.4
DMU8	3.3	3.1	2.9
DMU9	5.2	4.2	4.6
DMU10	7.9	8.6	9.4
DMU11	1.9	2.1	2
DMU12	7.7	8.6	8.7
DMU13	1.7	1.9	1.5
DMU14	3.2	4.8	4.1
DMU15	6.7	7.3	6.3
DMU16	0.9	2.8	2.7
DMU17	1.8	2.0	1.8
DMU18	1.4	1.5	3.7
DMU19	3.1	4.2	3.6
DMU20	3.8	4.4	4.2

In the Altman model, if the Z-score is higher, the unit has better financial health. The power of the model to predict financial health is shown in Fig. 4.

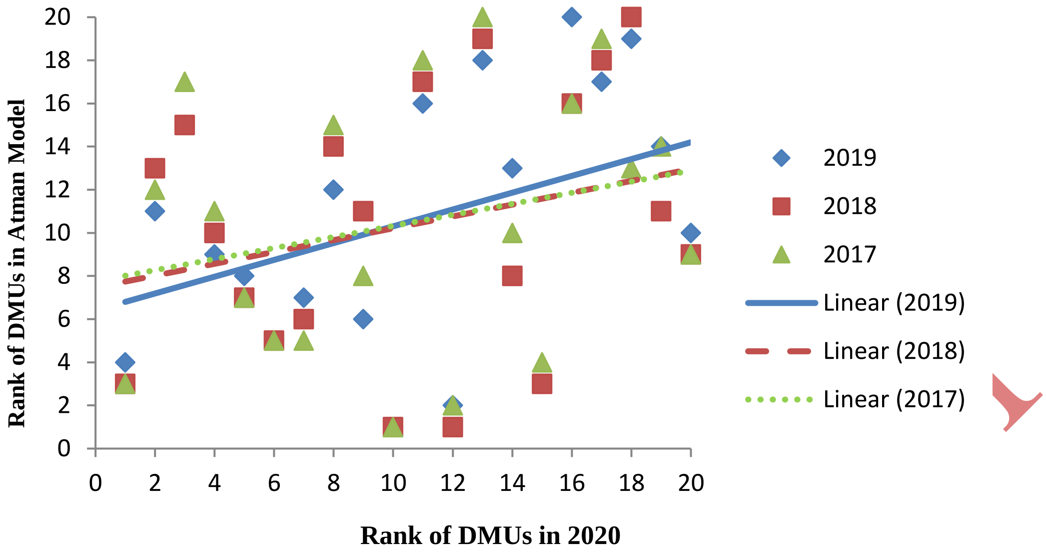

Fig. 4: Comparing the Ranking of Units by Altman Model with The Main Ranking

Table 3: Details of The Accuracy of The Altman Model

Year	Linear Equation	R^2	Accuracy	Total Accuracy
2019	y = 0.3895x + 6.4105	0.1517	0.3895	0.3067
2018	y = 0.2744x + 7.4684	0.0721	0.2744	
2017	y = 0.2564x + 7.7579	0.0648	0.2564	

The results show, the Altman model has higher predictive power in the years close to the target year. The accuracy of the model in predicting the company's financial health one year, two years, and three years before the target year was 0.38, 0.27, and 0.26, respectively. In general, it can be said that the average accuracy of the model is about 0.3.

The results of DEA models (CCR input-oriented, BCC input-oriented and dynamic DEA) are shown in table3. According to Horváthová and Mokrišová [8], we left out data of data of DMU14 that reached a negative value of ER. If the output of the model is closer to 1, the unit is more efficient and has more financially health.

Table 4: Results of DEA Models

		Inputs			Outputs		Intermediate	DEA		
Years	**DMUs**	**CPP**	**CR**	**ER**	**ROA**	**TL**	**Retained Earnings / Long Term Debt**	**θ (CCR)**	**θ (BCC)**	**θ (DDEA)**
2017	**DMU1**	94.1	1.33	0.49	5.15	1.2	1.67	0.61	0.82	0.88
	DMU2	87.69	1.47	0.56	5.53	0.82	4.3	0.44	0.74	1
	DMU3	103.82	2.25	0.42	-1.53	1.76	0.89	0.86	0.88	0.89
	DMU4	42.41	0.53	0.46	7.33	1.74	1.8	1	1	1
	DMU5	104.73	2.93	0.32	3.84	1.89	0.16	1	1	0.92
	DMU6	108.39	2.11	0.41	13.71	1.38	0.1	1	1	0.85
	DMU7	97	1.15	0.36	3.11	1.16	0.68	0.74	1	0.7
	DMU8	74.14	1.88	0.59	3.75	1.03	2.27	0.43	0.73	0.87
	DMU9	92.24	2.36	0.49	6.81	1.13	0.24	0.6	0.82	0.44
	DMU10	122.47	4.93	0.74	7.72	3.17	1.98	0.96	1	1
	DMU11	153.45	4.55	0.36	3.17	0.94	0.48	0.5	0.86	0.64
	DMU12	76.02	1.34	0.7	8.3	1.56	3.75	0.67	0.69	0.96
	DMU13	162.01	5.03	0.53	-0.35	1.3	0.81	0.43	0.62	0.5
	DMU14	49.34	2.77	-0.06	14.69	1.52	-0.08	-	-	-
	DMU15	94.37	6.39	0.63	6.75	0.97	0.85	0.49	0.67	0.46
	DMU16	212.98	3.36	0.49	3.13	1.12	1.56	0.44	0.63	0.38
	DMU17	321.38	5.36	0.58	5.86	1.39	1.55	0.5	0.53	0.63
	DMU18	172.49	6.45	0.72	6.92	0.62	-0.02	0.31	0.52	0.23
	DMU19	405.73	4.69	0.25	8.38	0.88	-0.19	1	1	0.87
	DMU20	146.29	2	0.3	5.98	1.19	0.06	0.88	1	0.76
2018	**DMU1**	72.13	1.03	0.51	9.59	1.25	1.86	0.69	0.81	0.94
	DMU2	73.64	1.24	0.57	9.16	0.84	4.66	0.56	0.72	0.99
	DMU3	68.74	1.81	0.46	15.78	1.79	1.11	1	1	0.87
	DMU4	32.99	0.43	0.45	9.37	1.65	1.86	1	1	1
	DMU5	112.63	2.77	0.33	8.92	1.48	0.2	1	1	0.91
	DMU6	93.54	1.82	0.46	18.46	1.52	0.2	1	1	1
	DMU7	82.23	0.94	0.36	6.63	1.05	0.7	0.76	1	0.67
	DMU8	61.56	1.6	0.61	8.4	1.25	2.55	0.55	0.71	0.95
	DMU9	43.76	1.64	0.5	-9.69	1.23	0.07	0.67	0.89	0.55
	DMU10	119.21	4.56	0.74	7.39	1.52	2.04	0.52	0.54	0.62
	DMU11	106.06	3.25	0.42	10.39	1.02	0.63	0.66	0.84	0.64
	DMU12	81.93	1.29	0.69	8.54	1.53	4.03	0.6	0.62	0.96
	DMU13	111.75	3.61	0.57	8.01	1.59	1.05	0.69	0.69	0.5
	DMU14	42.29	2.23	-0.07	24	1.16	-0.03	-	-	-
	DMU15	71.79	5.17	0.68	8.41	2.31	0.99	0.91	1	0.46
	DMU16	149.7	2.46	0.49	12.92	1.34	1.57	0.73	0.78	0.38
	DMU17	147.27	4.55	0.6	9	1.94	1.42	0.77	0.87	0.63
	DMU18	168.67	9.92	0.66	4.85	0.91	0.12	0.33	0.53	0.23
	DMU19	132.45	2.85	0.28	18.42	0.45	-0.19	1	1	0.87
	DMU20	147.83	2.28	0.36	12.72	1.16	0.09	0.89	0.95	0.76
2019	**DMU1**	83.12	1.17	0.47	7.25	1.16	1.57	0.64	0.81	0.72
	DMU2	90.13	1.42	0.55	5.76	0.78	4.23	0.41	0.7	0.82
	DMU3	69.14	1.92	0.5	15.17	2.4	1.4	1	1	1
	DMU4	43.74	0.5	0.42	7.31	1.44	1.76	1	1	0.91
	DMU5	138.34	3.06	0.32	8.01	0.97	0.24	0.7	0.92	0.69
	DMU6	109.49	1.87	0.43	16.15	1.07	0.23	1	1	0.85
	DMU7	95.17	1.06	0.34	3.75	1.12	0.61	0.74	1	0.72
	DMU8	68.29	1.7	0.61	2.67	1.07	2.66	0.43	0.68	0.89
	DMU9	42.5	1.67	0.54	5.28	1.3	0.21	0.79	1	0.55
	DMU10	113.73	3.82	0.68	12.96	1.05	1.44	0.59	0.66	0.49
	DMU11	103.84	3.4	0.45	7.99	0.9	0.75	0.51	0.79	0.7

	DMU12	91.66	1.41	0.64	3.85	1.42	2.9	0.56	0.63	0.64
	DMU13	124.08	4	0.6	3.32	1.22	1.26	0.35	0.61	0.59

Table 4: Results of DEA Models (Continue)

		Inputs			**Outputs**		**Intermediate**	**DEA**		
Years	**DMUs**	**CPP**	**CR**	**ER**	**ROA**	**TL**	**Retained Earnings / Long Term Debt**	**ϴ (CCR)**	**ϴ (BCC)**	**ϴ (DDEA)**
2019	**DMU14**	135.64	2.85	-0.26	19.18	0.6	-0.2	-	-	-
	DMU15	83.55	6.02	0.59	3.72	4.11	0.62	1	1	1
	DMU16	263.9	5.36	0.31	1.15	1.25	0.34	0.58	0.91	0.58
	DMU17	213.55	3.8	0.55	10.04	1.45	1.5	0.56	0.64	0.69
	DMU18	114.7	5.94	0.63	6.31	0.69	0.12	0.3	0.6	0.26
	DMU19	184.5	3.65	0.19	10.86	0.42	-0.22	1	1	0.95
	DMU20	167.04	2.32	0.35	3.3	0.98	-0.04	0.49	0.87	0.49

As mentioned before in the DEA models, DMU14 has been removed due to having negative ER. The results of the model are very different from the Altman method. In the CCR model, DMUs 4, 6, and 19 were efficient every 3 years.

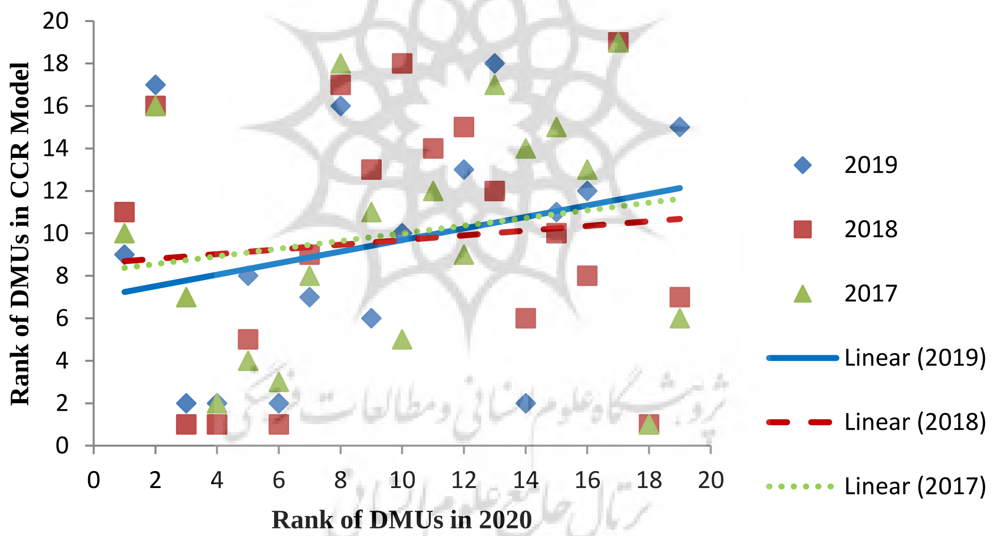

Fig. 5: Comparing The Ranking of Units by CCR Model With The Main Ranking

Table 3: Details of The Accuracy of The CCR Model

Year	**Linear Equation**	**R^2**	**Accuracy**	**Total Accuracy**
2019	y = 0.2719x + 6.9649	0.0648	0.2719	0.1877
2018	y = 0.1105x + 8.5789	0.0105	0.1105	
2017	y = 0.1807x + 8.193	0.0327	0.1807	

The accuracy of the CCR model in ranking forecasting was weaker than the Altman model. The accuracy of this model one year, two years, and three years before the target year was 0.27, 0.11, and 0.18, respectively.

In the BCC model, the number of efficient units is more than the CCR model. The BCC model is a variable return to scale and the CCR model is a constant return to scale. Units 4, 6, 7, and 19 are considered efficient every 3 years. The result of the BCC model is closer to the Atman model, but it still has lower accuracy than the Altman model. The results of the BCC model are given in Table 6.

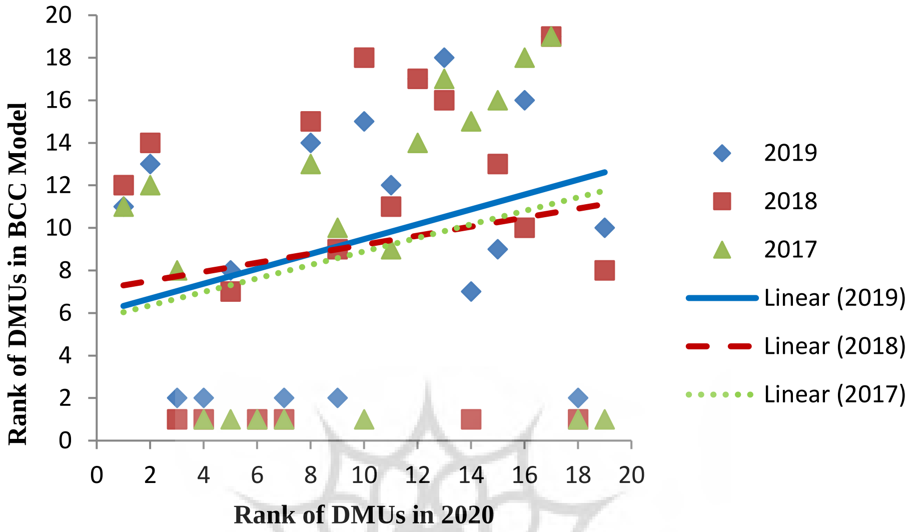

Fig. 6: Comparing The Ranking of Units by BCC Model With The Main Ranking

Table 4: Details of The Accuracy of The BCC Model

Year	Linear Equation	R^2	Accuracy	Total Accuracy
2019	y = 0.3491x + 5.9825	0.1	0.3491	0.293
2018	y = 0.2123x + 7.0877	0.0332	0.2123	
2017	y = 0.3175x + 5.7193	0.0689	0.3175	

The accuracy of the BCC model in ranking forecasting was better than the CCR model. The average accuracy of the BCC model is 0.29. The accuracy of this model one year, two years, and three years before the target year was 0.35, 0.21, and 0.32, respectively. The accuracy of the model in the first and second years has been weaker than the Altman model, but in the third year before the target year, the accuracy of the BCC model is higher than the Altman model.

One of the procedures of dynamic DEA is to absorb the time-dependent features of production into the performance evaluation [59]. Dynamic models feature carryovers to explain the interdependence of successive periods [60].

The result of the dynamic DEA model was better and more accurate than the Altman, CCR, and BCC models. The model results are shown in Table 7.

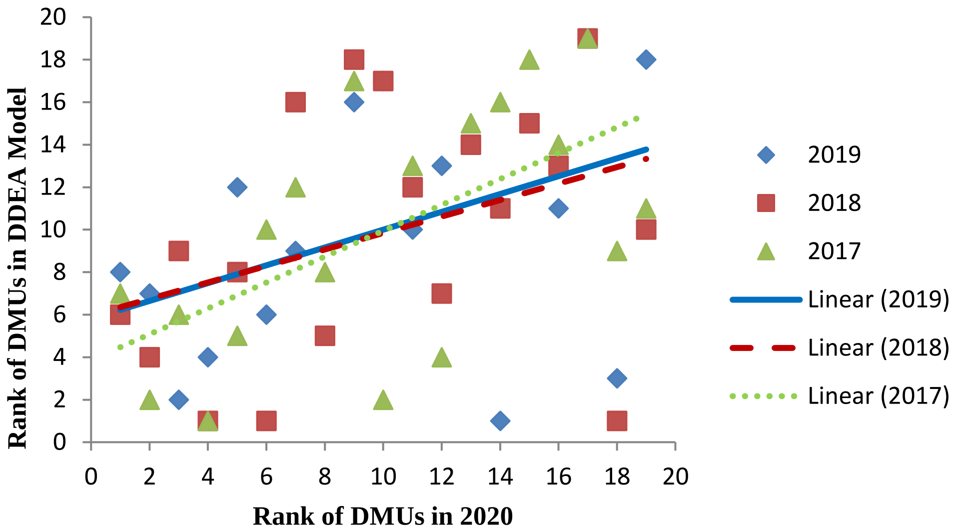

Fig. 7: Comparing The Ranking of Units by Dynamic DEA Model With the main Ranking

Table 5: Details of The Accuracy of The Dynamic DEA Model

Year	Linear Equation	R^2	Accuracy	Total Accuracy
2019	y = 0.4193x + 5.807	0.1758	0.4193	0.4439
2018	y = 0.3877x + 5.9649	0.1385	0.3877	
2017	y = 0.6088x + 3.8596	0.3611	0.6088	

The accuracy of the dynamic DEA model is higher than all the above models. We did not use the average for the total accuracy here. We implemented the DDEA model for 3 years and used the results with total accuracy. The total accuracy of the DDEA model in ranking forecasting is 0.44. The accuracy of the DDEA model a year, two, and three years before to the target year was 0.5, 0.54, and 0.6 respectively. Results show that, in the years farther from the target year, the accuracy of the model increased and it is better in all years than in all previous methods.

We used the Altman model for comparison because it is a common model for predicting bankruptcy risk. The results of the DEA models are similar to the Altman model in some cases and different in others. DMU15 is one of the most efficient units in all models. DMU12 in Altman is considered an efficient unit, while in DEA models it did not perform well. The results of DEA models are similar in most cases, but there are differences in some cases. DMU8 is one of the efficient units in the dynamic DEA model, while it has poorer performance in the CCR and BCC models. Unit 8 is one of the efficient units in the dynamic model, while it has poorer performance in the CCR and BCC models.

In the following, we will discuss the accuracy of the models proposed in this research for the ranking of companies in 2020. In a year before bankruptcy, the most accurate is the dynamic DEA model with an accuracy of 50% and then the Altman model with an accuracy of 39%. The model with the lowest accuracy level is the CCR model with an accuracy of 27%. In the second year before bankruptcy, the dynamic DEA model has the highest accuracy (54%), and the Altman model has 27% accuracy. The minimum accuracy is 11% for the CCR model. In the third year before bankruptcy, the highest accuracy is for the dynamic DEA model (61%), and after the BCC model has 32%. The minimum accuracy is 18% for the CCR model. The results represent that, the dynamic DEA model has been more accurate than the Altman and the other DEA models in all years. The overall ranking of the models is as follows: DDEA > Altman > BCC > CCR.

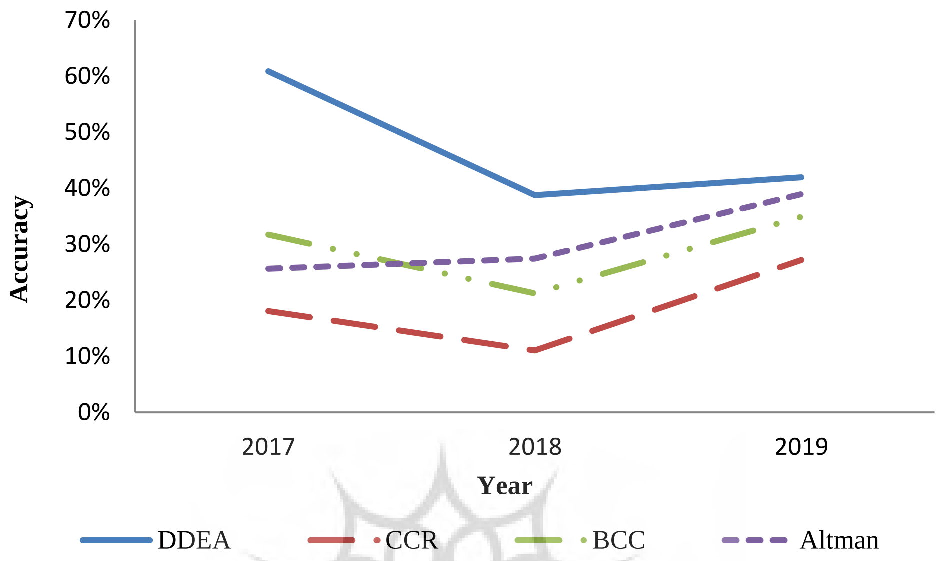

Fig. 8: Comparing Accuracy of Different Models

5 Conclusion

For many centuries, the energy sector (especially oil and gas) has been an important pillar of each economy on a global scale. Oil and gas are known as the world's main fuel sources; therefore, they are major industries in the development of economies. Therefore, the financial health of companies operating in the oil and gas field is particularly important.

With the spread of financial crises in recent years, bankruptcy risk prediction has become very important. Many models have been introduced to predict bankruptcy risk. Investors are looking for the most accurate ways to predict bankruptcy. One of the most recently investigated methods is the method of DEA. In this research, we have concluded that these models can predict the company's financial health position in the 2020 S&P global ranking up to three years before the purpose year. The accuracy of these models is significantly different from the accuracy level of Altman's classic model when we seek to predict a company's financial health more than one year before the target year. Among the DEA models, we found that the dynamic DEA model is more accurate. The reason is that, unlike the typical models in the field of bankruptcy prediction, which only consider the company's financial indicators for a specific year for modeling and forecasting, the dynamic model of DEA can consider some financial indicators (intermediate variable) in future periods of the company's activity. According to the arguments presented in this research, the ratio of retained earnings to long-term debt has been introduced as a suitable intermediate variable for the dynamic DEA model in predicting the bankruptcy of oil and gas companies.

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