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Research Paper

Profitability evaluation of dynamic investment projects by using ordered fuzzy numbers

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ABSTRACT
The purpose of this paper is to provide a new approach to incorporating uncertainty into assessing the profitability of investment projects. In the real world, the capital budgeting problem is accompanied by uncertainty and risk associated dealing with imprecise data. The major contribution of this research is the development of a novel approach to evaluating the profitability of an investment project in uncertainty condition. At first, we presented a new discount method that can be used by investors when they wants to be able to make an investment decision. That is, we developed a new method to evaluate the profitability of investment projects by ordered fuzzy net present value (OFNPV). In addition, ordered fuzzy numbers (OFN) are used to describe the dynamics of changes of the defined investment parameters in the assumed time horizon. By using ordered fuzzy numbers, we develop an effective tool for assessing the profitability of investment projects. This assessment tool not only enables decision-makers to decide under uncertainty conditions whether or not a given investment project should be carried out or rejected, but also facilitates selecting the most effective project, e.g. a project with the most expected probability of success.

1 Introduction

An investment project is to incorporate capital, human and monetary resources besides infrastructures, for the purpose of acquiring future, financial or non-financial, benefits. An investment implies an expenditure of resources, but it doesn't necessarily mean that those resources are our own resources. Nowadays, the importance of investment and its particular role in economics of current world increasingly reflects the necessity of different models and techniques in this field, so that various methods and techniques have been used by many scholars. Application of mathematics [1], decision making techniques and scientific management can help the management of an organization or a company to reach its predetermined objectives [2]. Due to their complex nature, investment projects are burdened with a certain degree of uncertainty and risk [3, 4] .Deciding whether a given investment project is to be carried out necessitates careful planning and predicting potential outcomes that could have an impact whether positive or negative on the given investment. This is accompanied by uncertainty and risk which in general stem from the lack of access to exact data [5].Trying to describe the construction project, it often happens that the values for some variables are approximate and subjective. According to possibility theory along with fuzzy set theory and fuzzy systems provide a new window to deal with impreciseness in decision making problems. Recently, several studies highlighting the potential applications of possibility theory in decision making using possibility moments of adaptive fuzzy numbers have been conducted, leading to development of a model for fuzzy net present value (FNPV) of future cash flows. As we know well, construction projects are often described in very vague terms. This is expressed in such statements as “substantially”, “good”, “almost”, etc. [6] Therefore; the future conditions for the execution of an investment project are difficult to predict and to define in a clear-cut manner. This specific context in which a given investment project is implemented is named fuzzy project environment. It must be emphasized that investment projects are characterized by high level of uncertainty at the beginning of their execution. This is due to a key feature of each project, and long-term projects in particular providing something that is innovative and in consequence, burdened with a degree of uncertainty. The level of knowledge associated with a project thus starts at virtually zero, increasing as the project progresses, to reach the stipulated level of one hundred percent towards the end of the project. Only at that point all the effects and benefits, as well as cost become fully known [7].

In practice, the mentioned issues highlights the inability to predict the behavior of the market during the time-frame of the project’s execution, including: weather conditions, the level of prices and costs, availability of resources, exchange rates, interest rates, behavior of competition, changes in the demand/supply level for a given product or service, etc. Traditionally, these investment parameters are assumed to be deterministic. As we know the capital budgeting problem is accompanied by uncertainty and risk which in general, derives from the lack of access to imprecise data [8, 9]. Therefore several authors began to use fuzzy sets theory to help solve capital budgeting problem in uncertainty environment [10, 11]. The aim of this study is to extend the work done by [12] to present the application of OFN in evaluation of investment projects under uncertainty conditions. In this research, the analyzed problem can be framed as follows. Given is a company which has to decide whether or not to execute a potential investment project in a specified time horizon. It is assumed that the initial conditions of the execution of the investment project, including the initial costs, are given. Also known is the duration of the execution of the investment. What is not known, however, is the complete and unambiguous information about the market capitalization rate or the future inflows and outflows related to the execution of the project in the considered time horizon. These quantities are determined by experts based on their knowledge and experience. The analyzed problem comes down to finding out whether the investigated investment project is profitable for the investor under the specified conditions, taking into account the associated uncertainty and risk.

The remainder of this paper is divided into seven main sections. The second section provides the relevant literature. The third section outline the methodology employed in the research with particular attention given to principal component analysis i.e. OFN. The forth section provides a description of the resulting measures, a discussion on the validity and reliability of the measures, as well as a presentation of key findings. We will consider Investment project decision based on the ordered fuzzy numbers in section 5.The section 6 includes discussion and recommendations. And finally in section 7, we will provide concussion of the paper.

2 Literature Review

Evaluation of project effectiveness is a crucial step in the decision-making process which investment projects are selected for execution in projects. Evaluating an investment project before its commencement is time-devouring and requires the utilization of appropriate knowledge, sophisticated modeling and forecasting methods, as well well-defined scientific methods. The results of an investment project assessment determine the subsequent stages of the investment, the analysis and allocation of resources, scheduling, budgeting and the control framework. In the literature, we can find a variety of methods used in capital budgeting [4, 13] [14]. The main methods are: net present value method (NPV), profitability index (PI), and internal rate of return (IRR). Based on conducted literature review, we can state that the classical forms of these methods do not take into account the uncertainty and risk which may be associated with information used to estimate them. This information includes: future cash inflows, cash outflows and available investment capital, the required rate of return of the investment or cost of capital, the duration of the project. Despite the multitude of methods and tools for assessing the profitability of investment projects, investment decision-making remains a challenging task [8, 9]. Classical methods do not take into account the uncertainty that arise from the lack of access to reliable, precise information – these issues need to be addressed in novel, effective computing methods. Literature review suggests that attempts have been made at considering the fuzzy environment of investments, taking into account both certain and uncertain data. In the literature, we observe an increasing interest in the theory of fuzzy sets, which lays the foundations for describing events of uncertain nature, cf. the papers on capital budgeting by [5, 8, 9, 15-17]. These studies generally use fuzzy numbers instead of crisp numbers in the known formulas. Further works take into consideration the uncertainty using fuzzy modeling for assessing investment projects, e.g., [18-22]. In their presented developed fuzzy equivalent annual worth and fuzzy benefit/cost analyses, Gharanfoli and Valmohammadi, 2019 [23] have applied a Fuzzy Decision Making and Trial Evaluation Laboratory (FDEMATEL) and Fuzzy Analytic Network Process (FANP) techniques as a framework to dynamically identify and prioritize risks in construction investment projects. Recent publications have demonstrated that the increased precision of operations and the possibility of solving equations in the set of ordered fuzzy numbers (OFN) are valuable for their application [12, 24, 25]. Regarding the facts that the precision of calculations drops as the number of performed operations increases and even linear equations cannot be solved in the set of fuzzy number, Prokopowicz, Ślęzak and Kosiński [26] developed a model based on the concept of ordered fuzzy number (OFN). Zhou and Xu [27] proposed a hesitant fuzzy linguistic portfolio model based on the max-score rule and the hesitant fuzzy linguistic element with variable risk appetite (HFLE-RA). They defined the HFLE-RA which can be used to describe the investor's risk appetite and the qualitative evaluation information and provided a linguistic portfolio model to calculate the optimal investment ratios under the HFLE-RA environment.

According to Mellichamp [28] , the profit criterion proposed essentially moves away from the minimum short-term risk condition (where ROIBT and NPV% are maxima) toward the maximum long-term profit (where NPV project is maximum), but not past the point where further increase in investment capital (project expense) is unjustified. Wu et al. [29], used Triangular intuitionistic fuzzy numbers to describe the uncertainties. They used PROMETHEE II to utilize and sort the LSR-PV project alternatives. Table 1 shows the most recent and relevant studies performed to evaluate the projects/manufacturing process in terms of profit/risk. A comparison has been conducted between them and our paper. As it is observed in our study we try to present a fuzzy-based NPV model to capture uncertainty in all parameters used in calculating NPV and help investors to decide which projects are more efficient.

Table 1: Comparison Between the Most Relevant Works in Literature with this Study

Author(s)	**Year**	**The defined problem**	**Model with uncertainty**	**Measure**
Kosinski et al.	2013	Project evaluation	Yes	IRR (with uncertain cash flow)
Rudnik and Kacprzak	2017	Manufacturing process	Yes	Operational costs and benefits
Wu et al.	2018	Project evaluation	Yes	NPV (with certain discount rate)
Zhou and Xu	2019	Portfolio selection	Yes	Risk appetite of investors
Mellichamp	2019	Project evaluation	No	Risk, NPV%,NPV
This study	**2020**	**Project evaluation**	**Yes**	**Modified NPV (with uncertainty in all parameters)**

In other words, the main contribution of this study is to evaluate the profitability of investment projects in uncertainty condition. Therefore, we tried to provide ordered fuzzy numbers to evaluate the profitability of investment projects in imprecise and ambiguous situation. The approach based on ordered fuzzy numbers is proposed. And new discount method is developed that can be used by an investor when the decision-maker wants to be able to make an investment decision. The ordered fuzzy numbers were used for taking the dynamics of changes of the defined investment parameters in the assumed time horizon into account.

3 The Concept of Ordered Fuzzy Numbers

The introduction of the concepts of fuzzy sets and fuzzy numbers was propelled by the need to mathematically describe imprecise and ambiguous phenomena. The above concepts were described in the attempts of Zadeh (1965) as a generalization of the classical set theory [30]. Classical fuzzy sets are convenient as far as a simple interpretation in the set-theoretical language is concerned. However, we could ask: how can we imagine fuzzy information, say X, in such a way that by adding it to the fuzzy information A the fuzzy information C will be obtained? In the classical approach for numerical handling of fuzzy quantities the extension principle is of fundamental importance. The commonly accepted theory of fuzzy numbers is that set up by [31]. However, if one wants to stay within their class of (L, R)–numbers while following the extension principle, approximations of fuzzy functions (and operations) are needed. They may lead to large computational errors that cannot be further controlled when applying them repeatedly.

A fuzzy set in is characterized by a membership function which associates each point in $x \in X$ with a real number in the interval [0,1], i.e. the grade of membership of in. Thus, we can write $A = \{(x, \mu_A(x)); x \in X\}$, where $\mu_A: X \to [0, 1]$ is the membership function of fuzzy set. This function assigns to each element $x \in X$ its membership degree to fuzzy set A [30]. A fuzzy set A defined on the set of real numbers, whose membership function satisfies the following conditions:

1) $\sup_{x \in R} \mu_A(x) = 1$, i.e., set A is normalized,
2) $\mu_A[\lambda x_1 + (1-\lambda)x_2] \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$, i.e., set A is convex,
3) μ_A is a piecewise continuous function

A fuzzy number and hence its membership function has two basic interpretations. It can be understood as a degree, to which x possesses a certain feature, or as a probability, with which a certain and at this point not entirely known value will assume a value x. The number A is positive, if $\mu_A(x) = 0$ for all $x < 0$. As there are infinite number of ways to characterize fuzziness, there are infinite number of ways to graphically depict the membership functions that describe this fuzziness. Generally speaking, triangular MF is one of the most encountered MF in practice. Of highly applied MFs, the triangular MFs are formed using straight lines. Triangular shapes represent fuzzy numbers, while trapezoid shapes represent fuzzy intervals [32].

A triangular fuzzy number is denoted with three real numbers $[a, b, c]$, where $a < b < c$. Its membership function assumes the following form:

$$\mu_A(x) = \begin{cases} 0 & for \quad x \leq a \\ \dfrac{x-a}{b-a} & for \quad a < x \leq b \\ \dfrac{c-x}{c-b} & for \quad b < x \leq c \\ 0 & for \quad x > c \end{cases} \tag{1}$$

If an expert generates a triangular fuzzy number as a result of assessing the distribution of possible values of certain unknown quantity, it means that the expert deems the values below a, and above c, not possible; whereas the value b is possible with a degree of 1, and the remaining values are possible to a varying degree that decreases with their distance from b. The concept of an ordered fuzzy number introduced by Prokopowicz, Ślęzak and Kosiński [26] in order to eliminate postulated deficiencies of fuzzy numbers – viz. the loss of precision increasing with the number of performed operations and the fact that even linear equations cannot be solved in the set of fuzzy numbers. The theorem formulated by Kosiński (2004) concerning the universal approximation of any nonlinear and continuous defuzzification operator [33, 34] offers new tools for the application of ordered fuzzy number to fuzzy inference and modeling, including assessing the profitability of investment projects. An ordered fuzzy number is an ordered pair of continuous functions (f, g), such that f, g: $[0, 1] \to R$.

The set of ordered fuzzy numbers is denoted by Fig (1) introduces the following notation: UP = f ([0, 1]) and DOWN=g ([0, 1]). Graphically the curves of (f, g) and (g, f) do not differ. However these pair of functions determines different ordered fuzzy numbers, they vary in so called orientation which is denoted by an arrow.

Let $A = (f_A, g_A)$, $B = (f_B, g_B)$, $C = (f_C, g_C)$ be ordered fuzzy numbers. Let the following operations: sum $C = A + B$, product $C = A \cdot B$ and quotient $C = A \div B$ be defined in the set as follows:

$$f_C = f_A \times f_B \text{ and } g_C = g_A \times g_B \tag{2}$$

Where: $*$ denotes, respectively operation: $+$, $\cdot$ and $\div$, while $A \div B$ is defined only when $f_B\,(y)$, $g_B\,(y) \neq 0$ for every $y \in [0,1]$. In the set of ordered fuzzy numbers, subtraction, exponentiation and taking a root can also be defined in the usual fashion, for example:

$$(f, g)^n = (f^n, g^n) \tag{3}$$

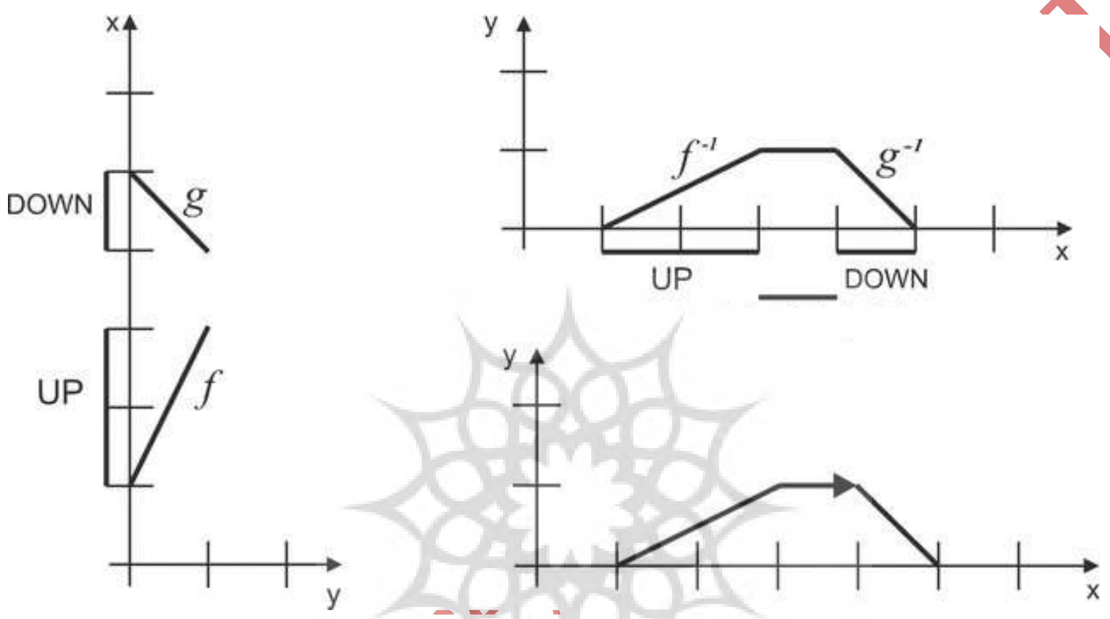

Fig. 1: Membership function for an ordered fuzzy number [21]

When considering the set $\mathcal{R}$ and the associated operations of addition and multiplication, we obtain a commutative ring with unity. By augmenting this with scalar multiplication, we obtain a linear space, i.e., algebra over real numbers. Moreover, this set constitutes a commutative Banach algebra with unity in the supremum norm in each of the factors $C[0,1]\times C[0,1]$ that are the Banach space $\mathcal{R}$. By introducing an appropriate relation of partial order, we also obtain a lattice [12].We observe that the set of pairs of continuous functions, where the first function is increasing and the second – decreasing and simultaneously, the first function always assumes values lower than the second function is a subset of the set of ordered fuzzy numbers which represents the class of all continuous convex fuzzy numbers (Fig. 1).
Notice that every ordered fuzzy number A which is a pair of affine functions is uniquely determined by a 4-D vector composed of the real numbers and the operations of addition, subtraction and multiplication by a scalar are consistent with linear operations in the space of 4-D vectors.

$$[f(0), f(1), g(1), g(0)] \tag{4}$$

The model of constructing deffuzification functional presented in [34] allows us to obtain a number of defuzzification functional whether linear or non-linear. In this work we applied the non-linear center of gravity defuzzification functional.

4 Investment Project Decision Based on Ordered Fuzzy Numbers

In economic practice, NPV is the most commonly used discount method. In essence, this method consists in assessing the present value of an investment project based on the forecasted streams of net cash flows which are the measure of investor's future benefits.

NPV is defined as a sum of net cash flows (NCFs) discounted separately for each year with a constant level of interest (discount) rate. This value expresses the updated (on the day of the assessment) value of benefits, which the undertaking in question can yield in the future. The general form of NPV can be expressed as:

$$NPV = \sum_{i=1}^{n} \frac{CF_i}{(1+k)^i} - N_0 \quad (5)$$

where:

n – number of years,

k – market capitalization rate,

CF_i – cash flow in the *i*-th year of investment,

N_0 – initial investment (outlay).

NPV allows making an investment decision having analyzed cash flows reduced by a specific outlay, and discounted by a weighted average cost of capital. Therefore, NPV allows assessing the economic value of an undertaking. The employment of a given method requires forecasting future cash flows which involve forecasting several uncertain variables, such as interest rate, prices of resources and services, exchange rate, etc. This affects the reliability and quality of forecasting future effects and outlay.

The classical of NPV does not take uncertain data into account. When considering the non-deterministic environment of an investment project, modifying NPV to take into account uncertain data is necessary. This allows taking information uncertainty into consideration and decreases the risk of making a mistake in assessing the profitability of an investment project. For the problem of defining a generalized NPV to incorporate ordered fuzzy numbers, we assume that the initial outlay N0 and the net present value NPV are real numbers, whereas cash flows CFi and the capitalization rate K are ordered fuzzy numbers. The discounted cash flows in the i-th year of investment are calculated as follows:

$$DCF_i = \frac{CF_i}{((1,1) + K)} \quad (6)$$

where: (1,1) stands for a pair of constant functions that assume a value of one, and +, / signify addition and division in a set of ordered fuzzy numbers defined through (2); exponentiation is performed according to (3), and is the center of gravity defuzzification functional defined through (5). Then, equation (6) assumes the following form:

$$NPV = \sum_{i=1}^{n} DCF_i - N_0 \quad (7)$$

5 Case Study

Here, we consider a case study of potential investment project execution in Iran. Investment decisions are made under conditions of uncertainty, since it is impossible to prepare an accurate estimation of economic and financial conditions for the output of the considered project in the future. The use of NPV with ordered fuzzy numbers (ordered fuzzy net present value method i.e. OFNPV) allows to limit the effects of uncertainty and risk. In order to define the fuzzy conditions of the execution of the investment project, the decision-making process involved an expert, who has appropriate knowledge and experience of planning and executing similar projects.
One problem related to the use of ordered fuzzy numbers was the requirement for the experts to give an opinion on individual elements of an investment project in the form of ordered fuzzy numbers, i.e., pairs of functions. In the considered case, we propose that the expert describe project parameters by means of triangular fuzzy numbers which will be subsequently converted into ordered fuzzy numbers. In practice, membership functions depend on the context and are defined subjectively by an expert. The most commonly used membership functions are piecewise linear functions (triangular, trapezoidal or rectangular), and can be defined with little information.
In the considered case, it was assumed that the initial outlay N0 will be IRR 300,000. The project is planned for the period of 5 years. The remaining project parameters remain uncertain, therefore, they were determined by the expert in the form of triangular fuzzy numbers. The triangular fuzzy capitalization rate assumes the form of K = [0.11; 0.13; 0.15]. This means that according to the expert the capitalization rate of below 11% and above 15% is not possible, whereas the value of 13% is the most probable one, and other values are probable to a different degree, the higher, the closer they are to 13%. In a similar way, the expert determined the fuzzy values of cash flows in subsequent years (Table 2).

Table 2: Fuzzy input data for the considered investment project

Investment year	Fuzzy cash flows
1	[75000,80000,85000]
2	[90000,93000,96000]
3	[105000,111000,118000]
4	[110000,120000,126000]
5	[115000,1230000,130000]

A triangular fuzzy number $A = [a, b, c]$ corresponds to an ordered fuzzy number:

$$A_{OFN} = ((b - a)x + a, (b - c)x + c) \qquad (8)$$

It is an ordered pair of linear functions. Using the above formula, we defined ordered fuzzy numbers corresponding to the values determined by the expert. For instance, the capitalization rate expressed by ordered fuzzy numbers assumes the form:

$$K_{OFN} = (0.02x + 0.11; -0.02x + 0.15) \qquad (9)$$

The values of cash flows were expressed analogously (Table 3)

Table 3: Investment input data modeling with the use of OFN

Investment year	Cash flow expressed as ordered fuzzy numbers

1	(5000x+75000, -5000x+85000)
2	(3000x+90000, -3000x+96000)
3	(6000x+105000, -7000x+118000)
4	(10000x+110000, -6000x+126000)
5	(8000x+115000, -7000x+130000)

The discounted cash flows for the remaining periods of the project were determined analogously. These values are presented in Table (3) along with the present net values for subsequent years of investment. Table (3) presents cash flows obtained using ordered fuzzy numbers. Subsequently, these values underwent defuzzification using the functional (8). In this case, NPV is equal to IRR 55646, which confirms the expected profitability of the project (Table 4).

Table 4: Investment discounted cash flows and NPV using OFN

Investment Year	Discounted cash flows	NPV
0	300000	–300000
1	70768	–229232
2	72825	–156407
3	77112	–79295
4	72750	– 6545
5	62191	55646
	NPV	55646

The presented case demonstrates that the conclusions drawn from calculations on ordered fuzzy numbers are in agreement with current knowledge and economic analysis. Moreover, owing to the elimination of issues related to using classical fuzzy numbers (such as increasing fuzziness over the subsequent operations, impossibility of solving equations, or high computational complexity), the model of ordered fuzzy numbers may prove to be an ideal tool for economic analysis and modeling. Table 4 presents cash flows obtained using ordered fuzzy numbers. Subsequently, these values underwent defuzzification using the functional (8). In this case, NPV is equal to IRR 55646, which confirms the expected profitability of the project (Table 3).

6 Analysis of Dynamics Changes with Ordered Fuzzy Numbers

In this section, we presented a case of NPV inference based on the actual values of discounted net cash flows during the execution of a given investment project. In this case, the mathematical apparatus in the form of ordered fuzzy numbers will be employed to describe the changes in the values of discounted net cash flows and the dynamics of these changes. The changes result, respectively, from increased or decreased inflows and outflows of executing an investment with respect to the baseline expected inflows and outflows for a given period. Investors are interested in how the expected net present value of an investment changes over a time period with respect to the corresponding values for the scheduled period. Information of this kind can be gleaned through the use of ordered fuzzy numbers, which enable the simultaneous presentation of planned and actual values of present cash flows. Ordered fuzzy number can be represented by four elements = ($l_A, 1_A^-, 1_A^+, p_A$) (Fig. 2).

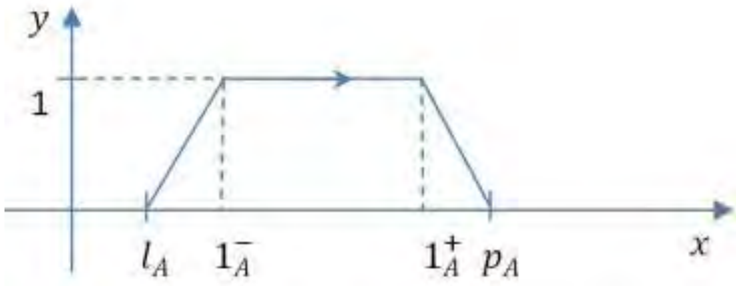

Fig. 2: Membership function for an ordered fuzzy number with A with characteristic points [35]

Ordered fuzzy numbers by virtue of being ordered can be used to elaborate the evolution of actual discounted cash flows in relation to the expected ones. Moreover, the width of the support reveals the magnitude of the change of discounted cash flows. We set out to facilitate the interpretation of the change of the value of discounted cash flows by introducing an auxiliary quantity that characterizes this change: the change dynamics indicator. The change dynamics indicator of discounted cash flows will be represented by the following ordered fuzzy numbers. Insight into the change dynamics of discounted cash flows is often crucial for evaluating the profitability of a given investment. By expressing the width of the support flows in relation to the expected ones, with a positive value corresponding to an increase and a negative value – to a decrease in the value of discounted cash flows. Fig. 3 serves as an illustration how the particularities of the change in the values of discounted cash flows in time can now be interpreted through inspection. Growth trends correspond to positive ordering whereas downward trends – to negative ordering. If the value of discounted cash flows remains unchanged, both graphs assume the same form corresponding to a constant.

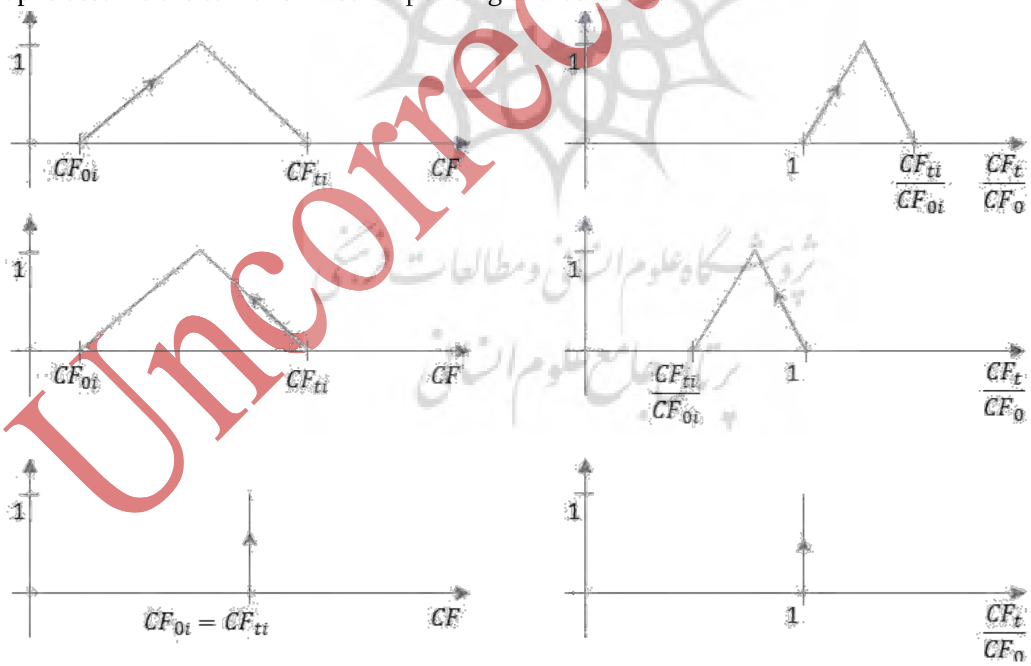

Fig. 3: Change in value of discounted cash flows and their dynamics illustrated using ordered fuzzy numbers

A graphical illustration of ordered fuzzy numbers that mirrors the percent change of discounted cash flows in time measured by the support width enables rapid evaluation and ordering of individual periods in the execution of an investment project, e.g. from the cash flow change that is the most unprofitable to the investor (the largest decrease in discounted cash flow), to the most profitable (corresponding to the highest increase). It becomes possible to illustrate change dynamics in one chart which facilitates evaluating how individual discounted cash flows change in time. However, a long planning horizon determined by the project execution period, together with the fact that discounted cash flows can decrease, increase or remain unchanged over subsequent periods, hinder the evaluation of the resultant impact of the changes on the values of discounted cash flows and on their dynamics. Ordered fuzzy numbers facilitate information mining both for individual discounted cash flows and for the sum of discounted cash flows of a given investment.We expound on the benefits of ordered fuzzy numbers by applying them to the execution of case investment project. presents the expected and actual discounted cash flows expressed in IRR. We verify NPV after the three years of project execution to check the reliability of the results obtained in previous planning phase. Present discounted cash flows, expressed as ordered fuzzy numbers and their change dynamics indices (also as ordered fuzzy numbers) are given in Table (5). According to Table (4), the discounted cash flow in the first year of investment project execution decreased from the expected level of IRR 70768 to IRR 65,00 i.e., by almost 8%. During the second and third year, however, an increase in the discounted cash flows was observed in relation to the expected flows, respectively, by 10% (an increase of IRR 7676) and 12% (IRR 9588) (Fig. 4).

Table 5: Expected and actual discounted cash flows for the considered investment project.

Investment year	Expected discounted cash flow	Actual discounted cash flow
1	70768	65300
2	72825	80501
3	77112	86700
4	72750	no data
5	62191	no data

Table 6: Present discounted cash flows and their change dynamics indices for subsequent investment year.

Investment year	Present discounted cash flow	Change dynamics index
1	[70768; 68034; 68034; 65300]	[1; 0.961; 0.961; 0.923]
2	[72825; 76663; 76663; 80501]	[1; 1.053; 1.053; 1.105]
3	[77112; 81906; 81906; 86700]	[1; 1.06; 1.06; 1.12]

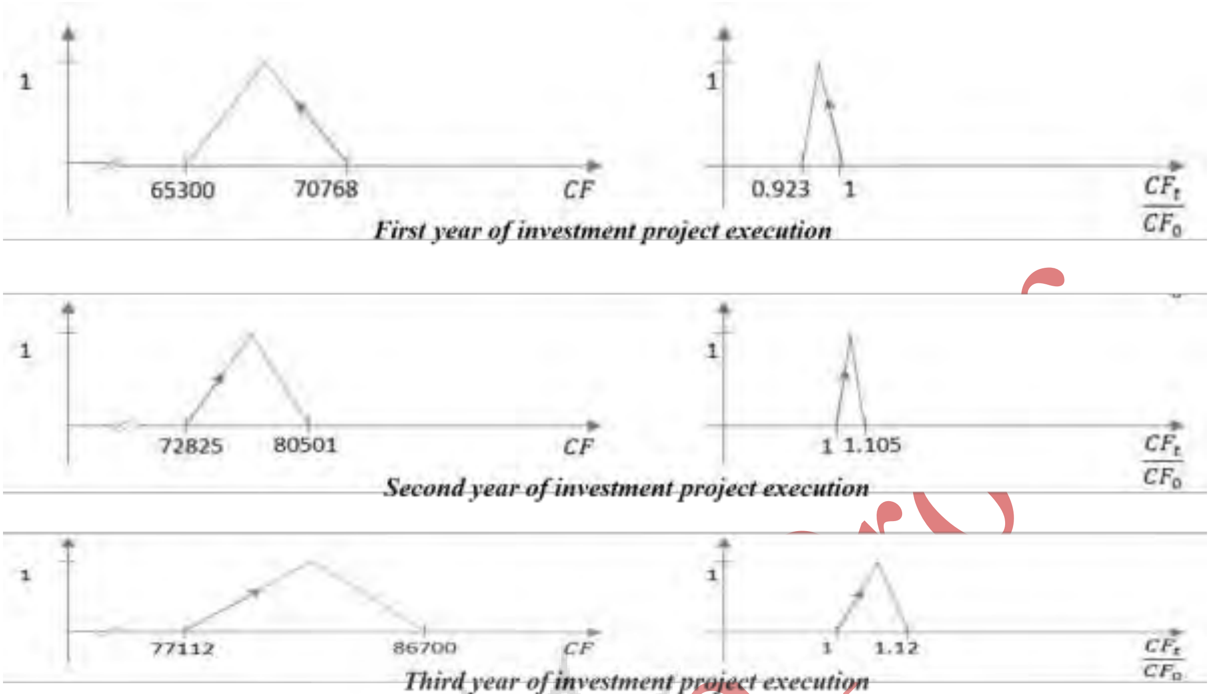

Fig. 4: Present discounted cash flows and their change dynamics for the considered investment

The change dynamics of discounted cash flows over the course of three years is expressed by means of an ordered fuzzy number as the arithmetic mean of change dynamics over three subsequent years of project execution, yielding [1; 1.025; 1.025; 1.049].

The next step in estimating the present cash flow in the fourth year of investment is the multiplication of the mean change dynamics by the scalar baseline for the fourth year of the investment (72750 a.m.u.), which yields an ordered fuzzy number [72,750; 74568.75; 74582.75; 76314.75]. Using forward inference from data obtained over the first three years of project execution, we can estimate the present cash flow for the fourth year of investment as 76314.75 a.m.u. The corresponding present cash flow for the fifth year is 65238.36 a.m.u.

The analysis of data on the progress of the given investment indicates that the value of discounted cash flows changes. Deviations from expected values will be taken into account when estimating the net present value of the investment. For this investment project current NPV, determined from the data recorded during project execution and earlier prognoses, amounts to 74054.11 a.m.u. Therefore, in this case the increase in the predicted NPV is 33.08% (cf. Table 6). An increase in cash flows observed in the second and third year significantly influenced the predicted NPV. The recorded values of cash flows reassure the investor that the correct decision had been made. The values predicted for the project's fourth year, based on the actual data, allow inferring the real value of project's NPV.

Table 7: Investment discounted cash flows and NPV using OFN

Investment year	Expected cash flows	Expected NPV	Present Cash Flows	Current NPV
0	300,000.00	–300,000.00	300,000.00	–300,000.00
1	70,768.00	–229,232.00	65,300.00	–234,700.00

Investment year	Expected cash flows	Expected NPV	Present Cash Flows	Current NPV
2	72,825.00	–156,407.00	80,501.00	–154,199.00
3	77,112.00	–79,295.00	86,700.00	–67,499.00
4	72,750.00	–6,545.00	76,314.75	8,815.75
5	62,191.00	55,646.00	65,238.36	74,094.11
	NPV	**55,646.00**	–	**74,054.11**

It is necessary to verify NPVs after the project's fourth year to check the reliability of the results. Signals in the form of actual cash flow values are a valuable source of information for the investor. In the presented case, except for the first year of investment, the investor witnesses a significant increase in cash flows. In economic practice, however, we can find investments, whose expected cash flows significantly deviate from the actual values *in minus*. In such cases it becomes necessary to diligently analyze the underlying causes of decreasing cash flows, and to make correct decisions – including, as a last resort, halting the execution of the project.

7 Results and Discussion

The presented approach to the profitability analysis of investment projects can be viewed as an early warning system, whose aim is to analyze signals from the environment and to interpret them correctly. The case application, we presented can serve as a tool for detecting potential opportunities and risks in the life cycle of an investment project. This tool could constitute a valuable source of knowledge for investors involved in decision processes.
In this paper, we presented demonstrates how conclusions drawn from calculations on ordered fuzzy numbers are in line with current economic knowledge and analysis. Moreover, by eliminating issues plaguing classical fuzzy numbers, such as: the increase in fuzziness associated with subsequent calculations, the impossibility of solving equations, and high computational complexity, models using ordered fuzzy numbers emerge as ideal tools for economic analysis and modeling. Ordered fuzzy numbers answer objections commonly raised against classical fuzzy numbers, such as: the increase in fuzziness during subsequent calculations, the impossibility of backward inference, and high computational complexity. Ordered fuzzy numbers retain precision over any number of calculations. Through the use of defuzzification operators ordered fuzzy numbers can be mapped to real numbers, which allows solving problems that simultaneously involve ordered fuzzy numbers and real numbers. As with the arithmetic of real numbers, analogous operations on ordered fuzzy numbers are commutative, associative, and multiplication is distributive over addition. This enables solving equations that cannot be solved for classical fuzzy numbers.

8 Conclusion

The paper addresses the problem of assessing investment project profitability through the use of ordered fuzzy numbers. The main reasons for the complexity of the problem are: operating under conditions of uncertainty and the multi-criteria and multi-level nature of the decisions to be made. The problem of assessing the profitability of investment projects is widely discussed in today's literature. Yet more often than not opinions vary, not only on the understanding (and, in consequence, defining basic concepts), but also on elaborating the procedures for assessing the effectiveness of economic investments.

To handle the uncertainty of net cash flows due to a lack of knowledge, this paper uses the concept of ordered fuzzy numbers. Ordered fuzzy numbers presents an imprecise data by means of a subjective possibility measure associated with judgmental uncertainty. The authors applied new approach to obtain profitability of investment project in fuzzy environments. The presented approach constitutes a new perspective on this common economic problem and constitutes an effective tool for assessing the profitability of investment projects. The results of the assessment can be utilized by decision-makers to decide whether or not a given investment project ought to be carried out or rejected, but they can also facilitate selecting the most effective project, the one that is deemed the most successful, or most favorable considering a different criterion from among several proposed options.

9 Implications for Further Research

The ability to make rational decisions under conditions of uncertainty is vital, when the cost of the investment is known, yet the expected inflows remains only hypothetical. How to assess project effectiveness based on different profit rates in order to get as close as possible to the state of nature, which is meant to be achieved in the future, remains an open question

Ordered fuzzy numbers may be used to illustrate the information about cash flows and capitalization rate. They offer a clear, simultaneous representation of several pieces of information, with well-defined arithmetic operations on ordered fuzzy numbers enabling them to be aggregated. By using ordered fuzzy numbers, experts are not only able to assess to what degree they recognize the considered phenomenon as accurate and true to life, but also to express their assessment of its dynamics, which is key to the problem of assessing the profitability of investment projects. Every investor is interested in how the values of present cash flows and capitalization rates can change compared to their corresponding baseline. Ordered fuzzy numbers will allow to simultaneously present the values of present cash flows and capitalization rate over the period under study, as well as at the baseline.

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