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Approach of Maximal Overlap Discrete Wavelet Transforms to Stock Return in The Iran Capital Market

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ABSTRACT

The primary objective of this study is to determine the genuine and dynamic relationship between returns and stock market fluctuations across various time horizons. The key outcomes of this study aim to assess the predictive power of returns and stock market fluctuations in determining investors' level of economic activity. Accurately measuring these relationships assists investors in predicting future stock market movements and developing, planning, and implementing their investment strategies within each time horizon. In this research, the Maximal Overlap Discrete Wavelet Transform has been employed to examine the relationships between industry-specific return fluctuations and the main stock indices within different time horizons from 2011 to 2020. The results indicate that the wavelet variance of return varies among different industries. The return of investment companies exhibits similarity to the banking industry's investment return across various time scales. Furthermore, the variance of the total index is lower across different time scales compared to the value of the cash index.

1 Introduction

Studying the behavior of returns and fluctuations in securities requires identifying a behavioral pattern of stock returns. This understanding enables investors to make informed decisions regarding holding, selling, or replacing stocks. It assumes temporal stability and linearity in the relationships between variables over time, even when changes occur within the company such as management changes, alterations in production lines, variations in input composition, and other factors unrelated to economic conditions, tastes, government policies, or increased competition in the industry. While such changes are expected throughout a company's lifespan, determining the process of temporal changes in return rate and return variance can lead to more accurate estimations and, consequently, enhance investors' decision-making. Considering these issues, it becomes necessary to employ more relevant and adaptable methods for accurate calculations of financial variables.

One of the most precise approaches to predict variables involves assessing them across different time scales and comparing the results. Thus, the objective of this research is to study variables within various

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time scales using a novel and innovative tool called wavelet analysis. This approach provides a clearer understanding of the relationships between variables, identifies the optimal time period for holding stocks in the Tehran Stock Exchange, and determines the trend of changes in economic decisions made by investors in the stock market. In financial analysis and investment management, the calculation of return rates and return variance based on a single time scale often leads to errors in professional and scientific decision-making and analyses.

Previous research encountered challenges in recognizing the real and dynamic relationship between returns and stock market fluctuations in different industries due to limited time scales. Therefore, the main focus of this research is to determine the genuine and dynamic relationship between returns and stock market fluctuations across different time horizons. The key results of this study can be utilized to gauge the predictive power of returns and stock market fluctuations in determining the level of economic activity for investors. Accurately measuring these relationships assists investors in predicting future stock market movements.

2 Literature Review

A look at scholarships offered by various countries shows that most of the successful investors are people who have formed a diversified portfolio to protect themselves against all kinds of investment risks. Considering that the Iran Stock Exchange consists of several industries working in an assortment of fields, how these industries relate to each other both in the middle and long term can pave the way for starting a portfolio. Conventional correlation measurement methods such as covariance are calculated assuming that the series are stable, and in case they are unstable, the main data should be manipulated. Thus, using methods that work with the main data, such as a wavelet, is preferable [9].

Overall, the longer it takes to reach a financial goal, the more risk an investor may take. The basic premise is that higher risk assets may have higher return. On the other hand, potential losses will be compensated over a longer period of time. The shorter this period, the less risk the investor may take. Of course, Investors should never take more risks than they can handle; this is known as risk tolerance or risk taking. Ultimately, the time horizon will determine how long an investor can use the power of money mixture to their advantage [15]. Discrete Wavelet Analysis with Maximum Overlap is an improved version of Discrete Wavelet Analysis. Discrete wavelet analysis with maximum overlap can accommodate any sample size compared to discrete wavelet analysis; It is also not very sensitive to filtering data from the starting point of the timeline [12]. In practice, transforming discrete wavelet is implemented as a filter bank, that is, they Acts as a sequence of low-pass and high-pass filters. To transform a discrete wavelet into a signal, we first start with small scales; Small scales correspond to high frequencies. Thus, we start with analyzing high scales.

Afterwards, we increase the scale by a factor of two (we decrease the frequency by a factor of two) and in this case, we analyze the behavior around half of the maximum frequency. In the third step, we consider the scale factor equal to 4 and we analyze the frequency behavior around the quarter of the maximum frequency. This process continues in the same way to reach the maximum level of decomposition. To understand the concept of Maximum Decomposition Level, given that at each successive stage of decomposition the number of samples in the signal decreases by a factor of two, at lower frequency values, fewer samples are required therefore no more samples need to be stored in the signal, because it increases the computational cost [4]. The discrete wavelet analysis filtering method with maximum overlap describes the financial time series movements far beyond the existing methods and can decompose a financial time series into different time scales and can detect structural failures and fluctuations. Also, discrete wavelet analysis filters with maximum overlap can easily solve the correlation structure of a process on time scales. Studies show that the average allocation to growth stocks increases significantly with increasing investment horizon. It needs to be mentioned that for stock investors growth stocks are less risky than value stocks in the longer-term investment horizon. However, by combining treasury bills and bonds in the model, conservative investors with long-term horizons do not hold stocks in their portfolios considering the fact that growth stocks are empirically not hedged well against the uncertainty of loan returns. Jurek and Viceira [10] found that as the investment horizon increases, the allocation to growth stocks increases dramatically while Munas and others using the self-regression vector error correctness, found that in the long run, that is, with increasing the time horizon, the allocation of assets to value stocks increases. Anyways, Croce [3] and Sousa [21] Showed that the deviation of stock returns standard in the short-term investment horizon is due to the return of different average stock returns from the long-term investment horizon. This indicates that stocks are safer in the long run and suggests that the long-term investment horizon factor has more demand for stocks. The main results of the empirical analysis show that the success of following the value strategy (selling borrowed growth stocks and staying on value stocks in the long run) is affected by the approach used to classify the returns of growth and value stocks.

A study entitled BRICS Stock Markets, Oil and Natural Gas: Evidence of Systematic and Moving Risks in Time-Frequency has been conducted by Menci et al. [15]. In this study, the wavelet analysis method has been implemented to investigate the movement in five stock markets of Brazil, Russia, India, China and South Africa and the oil and natural gas markets. The results show that there is a correlation between oil prices and stock market returns on a lower or long-term scale. Coherence between oil and stock markets peaked during the 2008 financial crisis, followed by a slow economic recovery since 2009. a sharp movement occurs between the crude oil price markets and the stock market returns of the mentioned countries after eliminating the effects of natural gas price returns; also, the results depict a strong correlation between stock market returns and natural gas prices relative to crude oil price returns. In this study, the highest level of risk is related to the short-term scale.

Osu et al. [17] In a study, examined the wavelet analysis of international financial markets. In this study, they analyzed the joint movements of the Nigerian stock market with the stock markets of 10 countries: Bangladesh, Egypt, Indonesia, Iran, Mexico, Pakistan, Philippines, Turkey, South Korea and Vietnam; Nigeria and these 10 countries are also known as N11. Moreover, they also examined the effect of stock return fluctuations on market dynamics and used wavelet-based analysis methods to analyze market dynamics. The results showed that there is a very strong joint movement between the mentioned countries and the presence of noise has almost the same effect in terms of time frequency on those countries. It can be said that most of the market dynamics of countries at lower scales (high frequency) are due to sharp fluctuations, while the dynamics of higher scales (low frequency) are driven by market principles. Deviation and cortisone can also be used to determine the energy distribution in wavelet decomposition.

Zho et al . [25] studied the effect of transmission in stock markets (in Asia, Europe and America) under different time frequencies using decomposing complete noisy group empirical wavelet analysis. The results show that shocks caused by irregular events and severe accidents can be transmitted between different stock markets. In addition, shocks due to irregular events can pose a sudden and short-term risk to stock returns, and shocks due to severe accidents can pose a positive and lasting risk to stock returns. Tiwari et al. [22] exploited Wavelet analysis to examine the relationship between inflation and stock returns over a long period of time (1790 to 2017) and at different frequencies. The results are also compared with those in the United States and two developing countries (in India and South Africa) , Overall, the results of this study showed that while the relationship between inflation and stock returns varies widely at different frequencies and time intervals, there is no evidence that stock returns are a cover for inflation. Such a conclusion was true both in the two developed countries, the United Kingdom and the United States, and in the two developing countries of India and South Africa. Ramzi radchoobeh et al. [18] investigated the interaction between risk, ambiguity and return . Results revealed the existence of ambiguity in tehran stock exchange, which affects the Asset pricing negatively , also the effect of ambiguity was negative and highly significant in all the tests that they employed and the effect of risk was generally positive, which was consistent with risk aversion but its significance depended on the risk measure that they use results have suggested negative impact of ambiguity and positive impact of risk on asset pricing. Boubaker and Raza [2], conducted a study entitled Wavelet Analysis of Fluctuations and Return's Overflow between the stock markets of Brazil, Russia, India, China and South Africa, and oil which showed that oil prices and stock market prices are directly affected by the news and fluctuations of their own market and are indirectly affected by fluctuations in other prices wavelets. Overflow effects and fluctuations and averages are also affected by many overflows except in different time dimensions based on heterogeneous investors and market participants.

Allahyaribeik et al.[1] presented A Quantum Model for the Stock Market. The obtained results show that the quantum potential behaves in the same manner for P/E and price return, also confines the variations of the P/E and price return into a specific domain. Furthermore, a joint quantum potential as a function of return and P/E is derived by the probability distribution function (PDF) constructed by the real data of a given market. It serves as a suitable instrument to investigate the relationship between these variables.The resultant PDF and the corresponding joint quantum potential illustrate that where we have light points in joint quantum potential chart, the probability of those amount of P/E and price return are more than other points. In addition, because of the rectangular shape of the joint quantum potential chart we can say that these two variables behave as two independent variables in the Market. Yilmaz and Unal [23], implemented Wavelet analysis to examine the movement of Asian stock markets based on the financial indicators of the top 100 stocks on the London Stock Exchange **1**and the top 500 stocks on the New York and Nasdaq stock markets**2**. The results of wavelet analysis showed that there are internal relationships between these stock markets and these relationships have changed at different time intervals and frequencies. At the same time, it was found that the stock markets of countries with advanced economies have had a great impact on the stock markets of Asian countries; However, the degree of dependence of Asian stock markets on strong global markets varies from country to country, and these differences can be seen in different time periods. Fallahi et al. [7], have taken a new approach in which they break down the returns of a particular investment strategy into multiple investment horizons using the wavelet analysis method.

The results of their research show that in investment companies with medium and low risk aversion, when there is an increasing investment horizon, the amount of investment in growth stocks is low and the amount of investment in value stocks is increased, while in the market portfolio, the weight of growth stocks and Values does not differ significantly. Ehteshami et al.[6] Forecasted Stock Trend by Data Mining Algorithm, Results indicated that algorithms are able to forecast negative stock return. However, random forest algorithm is more powerful than decision tree algorithm. In addition, stock return from last three years and selling growth are the main variables of negative stock return forecasting. Rostami et al. [19] conducted a study entitled " Study of the Co-movement of the Returns of Indices of Different Industries in Tehran Stock Exchange with the Returns of the Oil, Gold, Dollar and Euro Markets" using wavelet analysis. The results of their research show that there is a significant relationship between the returns of various industries in the Tehran Stock Exchange with the returns of oil, gold, dollar and euro markets.

[1]- FTSE 100 (Financial Times Stock Exchange 100 Index)
[2]- S&P 500 (Standars and Poors Index)

Also there is a stronger relationship between independent and dependent variables at shorter intervals. moreover, according to the total beta coefficients of the independent variables in different time periods and different industries, it is determined that the price return variables of oil, gold, dollar and euro have the highest power to explain the index of different industries, respectively.
Masih and Majid [14] have done a study called the Co-movement of Selected International Stock Exchange Indices: A Continuous Wavelet Analysis and Cross-sectional Wavelet Analysis. The results of this study can be an important tool in decision making for different types of investors. Kazemzadeh et al. [11], using discrete wavelet transform with maximum overlap and threshold vector auto regression pattern, have studied the mutual effects of inflation and government budget deficit in the two modes of total deficit and operating deficit on the Iranian economy during the years(1990-2017) from new angles. The results based on the wavelet converter show that in the horizons longer than 8 years, the causal relationship between both types of budget deficit and inflation is mutual. According to the results of the threshold vector auto regression pattern estimate, in seasonal inflation less than 6.28%, the total budget deficit increases sharply in the face of the inflation shock. Also, the operating budget deficit before and after the threshold shows a positive reaction to the inflation impulse. It needs to be noted that at seasonal inflation rates above 2.54%, the reaction intensity of this variable will be higher. in other words, the tensile effect is always stronger than the anti-tensile effect. In previous studies, many have been hamstrung due to a limited time scale in recognizing the real and dynamic relation between the return and fluctuations of the stock market. Thus, the main issue in this study is to determine the real and dynamic relationship between returns and stock market fluctuations in different time horizons so that the results can be used to measure the predictive power of returns and stock market fluctuations for determining the level of economic activity of investors. Accurate measurement of these relationships helps investors predict stock market movements in the future and at each time horizon, formulate, plan and implement their specific investment strategies based on the results of this research.
In previous researches, they have mostly studied and analyzed the movement of stock markets in different countries or the reason of economic shocks and crises affecting capital markets or the relationship between a series of economic variables and stock indices have been evaluated and in none of the researches In the past, stock market fluctuations in different time horizons had not been studied; Therefore, this study specifically examines and analyzes the fluctuations and wavelet variances of different indicators of the Tehran Stock Exchange in different time horizons to identify the economic activity of investors in variety of times Be able to present specific strategies of different traders.
Hypotheses:
The hypotheses of the present study are as follows:
1- The wavelet variance of returns varies in different industries.
2- The wavelet variance of returns in the banking industry is more than investment companies.
3- The wavelet variance of the total index is more than the cash index.

3 Research Methodology

This research is practical in terms of its target, quantitative in terms of the nature of its data and descriptive-correlational in terms of its data collection method. For this purpose, the information related to the stock market indices of Tehran Stock Exchange was collected daily from 2011-2020 and then, time series were broken into different time intervals using wavelet analysis followed by a study on the relationship and effects of stock market fluctuations on the levels of economic activity in different time

periods. In this research, maximum overlap discrete wavelet transformation3 has been used through MATLAB software to investigate the relationships between variables in the Iranian capital market at different times. The MODWT is a filtering operation that transforms a series into coefficients related to variations over a set of scales. It is similar to the DWT[4] in that both are linear filtering operations producing a set of time-dependent wavelet and scaling coefficients. Both are suitable for analysis of variance and multi-resolution analysis. The MODWT differs from the DWT in that it is a highly redundant, nonorthogonal transform. The MODWT retains downsampled values at each level of the decomposition that would be otherwise discarded by the DWT. The MODWT is well-defined for all sample sizes N, whereas for a complete decomposition of J levels the DWT requires N to be a multiple of 2^k .
The MODWT offers several advantages over the DWT. The redundancy of the MODWT facilitates alignment of the decomposed wavelet and scaling coefficients at each level with the original time series, thus enabling a ready comparison between the series and its decomposition. analysis of variance derived using the MODWT are not influenced by circular shifting of the input time series, whereas values derived using the DWT depend upon the starting point of the series. Finally, the redundancy of the MODWT wavelet coefficients modestly increases the effective degrees of freedom on each scale and thus decreases the variance of certain wavelet-based statistical estimates. Since the MODWT is energy conserving, it is well suited for analyzing the scale dependence of variability in analysis of variance studies. Wavelettes are generated in groups using a scale function (φ) and a wavelet function (ψ) [5]
The integral of the scale function is 1 and the integral of the wavelet function is zero. The scale function is used to identify the smooth and low frequency characteristics of the signal or time series, and the wavelet functions are used to identify the details of the high frequency characteristics of the signal [4]
In the first stage of maximum overlap discrete wavelet transform, the main signal passes through two low-pass and high-pass filters, the output of the low-pass filter is a scale function, and the output of the overpass filter is a wavelet function and when time series are decomposed into different time layers, Approximation coefficients are generated from the scale function and details are generated from the wavelet function. Approximation coefficients represent long-term patterns of fluctuations (general trend of fluctuations - slow changes - slow dynamics) and detail coefficients indicate short-term patterns of fluctuations (noise - details of high frequency frequencies - dynamics and rapid changes).
The main goal in this transformation is to find the method that has the most detection and discretization of the desired features. If we are looking for a general trend and behavior, we should look for it in low frequency components, but if the goal is to detect exceptions, fractures and severe fluctuations, it should be searched for in high frequency components [25]. In general, the main wavelet function, which is called the mother wavelet function, is expressed as follows:

$$\boldsymbol{\varphi_{a,b}(t) = \frac{1}{\sqrt{a}}\,\varphi(\frac{t-b}{a})} \quad (1)$$

a: Scale / periodicity / time range / wavelet life / duration
Scale means how long the wavelet function appears and indicates the duration or tension and compression of the wavelet and results in giving scales.
b: Location / moment of occurrence
Positioning means where the wavelet function appears and causes a time shift.
Finally, in wavelet transform, the goal is where, for how long and by what coefficient the wavelet function appears.

3_ MODWT

4 _ Discrete Wavelet Transformation

The method of calculating the similarity coefficient or wavelet coefficient of a child (girl) in forming a function f (t) is as follows:

$$\boldsymbol{C_{f,\varphi}(a,b) = \int_{-\infty}^{+\infty} f(t)\, \varphi_{a,b}(t)^{*} d(t)} \tag{2}$$
$$\boldsymbol{= \int_{-\infty}^{+\infty} f(t) \frac{1}{\sqrt{a}} \varphi^{*}(\frac{t-b}{a}) d(t)}$$

Given that we are faced with infinite dimensions in continuous wavelet transform and calculating the wavelet coefficient has its own complexity, thus It is necessary to use it in a limited series of points to reduce the computational complexity. One way is wavelet discretization, which is achieved by exponentializing a changes. Thus a is doubled at each step of the signal decomposition, which is expressed mathematically as follows:

$$\boldsymbol{a_j = 2^j} \tag{3}$$

And due to the inverse relationship between scale and frequency, the frequency is halved at each stage. Analysis of time series wavelet f (t) is decomposes as follows:

$$\boldsymbol{f(t) = A_{n,t} + D_{n,t} + D_{n-1,t} + \cdots + D_{2,t} + D_{1,t}} \tag{4}$$

Approximation coefficients(A) and detail coefficients (D) are constructed by shifting and scaling from the scale function φ(t) and the wavelet function ψ(t) and based on their degree of similarity to the main signal [17] .

$$\boldsymbol{\varphi_{a,b}(t) = \frac{1}{\sqrt{a}} \varphi(\frac{t-b}{a})} \tag{5}$$
$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi(\frac{t-b}{a}) \tag{6}$$

Various techniques are traditionally used for analyzing such time series. In the time domain, averaging the series using a fixed smoothing window permits the decomposition of a signal into smooth and fluctuating components for successive time segments. In the frequency domain, Fourier analysis is widely used to decompose a series into components at different frequencies. While the former is a localized decomposition in time but not frequency, the latter is a global fit of decomposition by frequency based on the assumption of stationarity. Both lack the ability to extract localized events occurring over multiple time scales. Although time windowing and Fourier analysis techniques are widely used in researches , due to the dynamics of financial time series and the need to consider both time and frequency factors for better analysis, wavelet transform is used to examine the results more accurately. according to the discrete nature of financial time series, in this study, discrete wavelet transform with maximum overlap (MODWT) has been used to analyze the data.
Given that the time series trends of stock returns fluctuations have many dynamics and irregularities, it is necessary to use a method and tool to analyze them to have the necessary frequencies to express their trends more accurately and can change the number of frequencies intelligently according to the intensity of the fluctuations and observe the required dynamics. Fourier transform and previous approaches of correlation analysis, due to considering a limited time scale for correlation analysis between variables, could not describe the dynamics of stock market fluctuations well, so a tool is needed to consider the dynamic nature of stock return fluctuations and this analysis tool is performed by maximum overlap discrete wavelet transform with to simultaneously examine the behavior of stock returns fluctuations in general and in detail at different time horizons.

4 Research Results

Given that the first hypothesis of the study is about the difference between the wavelet variance of the rate of return in different industries, we first consider the wavelet variance between the returns of different industries and then examine and analyze them.

4.1 Wavelet Variance Analysis of All Industries Returns

To analyze data related to the variance of industry returns using wavelet, first the relevant data are decomposed into 12 layers of approximation coefficients and 12 layers of detail coefficients. long-term behavioral patterns are extracted using diagrams and coefficients of approximation layers and short-term behavioral patterns are extracted by implementing diagrams and coefficients of detail layers. The number of layers is determined by the complexity of the time series; The more complex the time series under consideration, the more layers are needed to describe the data; Therefore, the analysis of layers proceeds until the coefficients of approximation related to the last layer becomes zero and the coefficient of detail of the last layer becomes a simple wavelet pattern. The approximation coefficients are denoted by the symbol and the detail coefficients are denoted by the symbol d. The algorithm of decomposition of the main function into coefficients of approximation and coefficients of detail is such that in the first layer the main function is decomposed into a1 and d1 and in the second layer a1 is decomposed into a2 and d2 and this signal decomposition continues until the twelfth layer. The Diagrams For The Coefficients Of These 12 Layers Are Shown In The Figure Below:

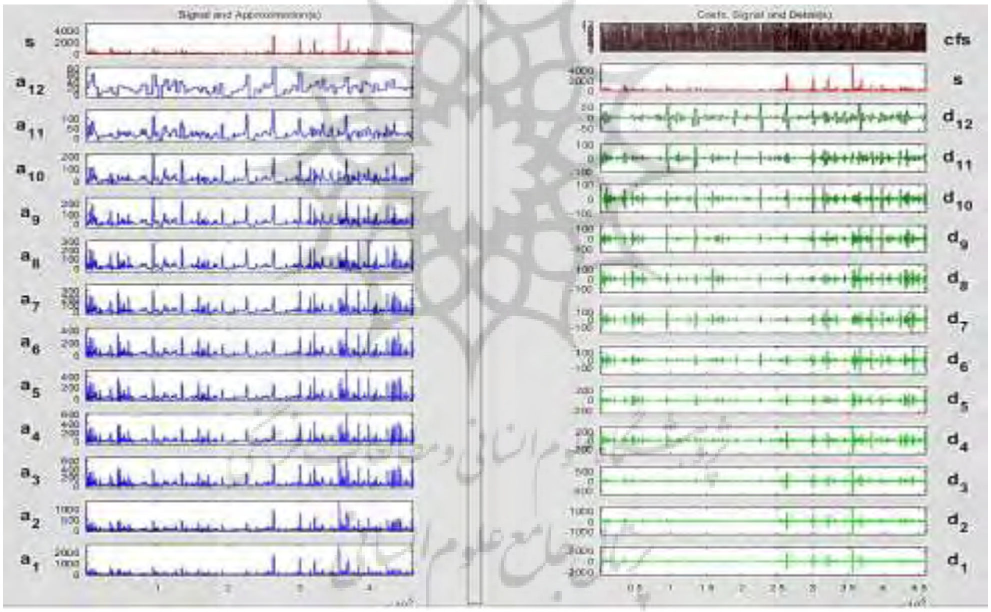

Fig. 1: Diagrams Related to Approximation Coefficients And Detail Coefficients In 12 Layers

According to the diagrams above, the initial layers represent the details of the high frequency of the signal and the final layers describe the low frequencies of the signal. As we move from the initial layers to the final layers, the signal is displayed longer; In general, approximation coefficients show the trend of fluctuations longer than detail coefficients; In fact, the fastest dynamic corresponds to d1 and the slowest dynamic corresponds to last layer. The detail coefficients examine the signal in a very short time. As it can be seen from the above figures, long-term regressions are approximated by layers of coefficients of approximation and short-term regressions are approximated by layers of coefficients of detail. The following table shows the standard deviation and variance of each layer among all industries.

Table 1: Standard Deviation And Co-Movement Variance Of The Industries At Different Time Scales

Layer number	Standard deviation of approximation coefficients	The variance of the approximation coefficients	Layer number	Standard deviation of detail coefficients	Variance of detail coefficients
D1	57/61	3318/80	A1	39.41	1553.20
D2	50.36	2536.10	A2	27.98	782.70
D3	46.62	2173.60	A3	19.04	362.47
D4	44.88	2014.00	A4	12.63	159.56
D5	43.92	1929.20	A5	9.21	84.84
D6	43.03	1851.70	A6	8.80	77.50
D7	41.47	1719.60	A7	11.49	132.07
D8	39.40	1552.00	A8	12.95	167.61
D9	36.20	1303.10	A9	15.78	248.91
D10	31.06	964.70	A10	18.36	337.14
D11	23.85	568.88	A11	19.91	396.27
D12	17.06	291.02	A12	16.67	277.86

Based on the table above, as we move from the initial layers to the middle layers, the wavelet variance and investment risk is reduced and again in the final layers, the wavelet variance is increases with some deviation. The amount of standard deviation and variance of each industry separately is as follows:

Table 2: Standard Deviation And Variance Of Industry Wavelet Returns By Industry

automobile manufacturing	69.80	Standard deviation
	4872	variance
chemicals	52.473	Standard deviation
	2767.5	variance
coal	72.34	Standard deviation
	5233	variance
paper	70.365	Standard deviation
	4951.233	variance
basic metals	68.568	Standard deviation
	4701.57	variance
oil products	53.416	Standard deviation
	2853.27	variance
Metal ores	55.482	Standard deviation
	3078.252	variance
Multi String	62.86	Standard deviation
	3952.385	variance

According to Table 1, different industries have had lower wavelet variance and mobility during long-term scales, but during short-term scales, there were more co-movement between industries as well as a higher variance of wavelet efficiency.

Given the comparison of wavelet variances between different layers, the middle layers seem to offer a better approximation of the signal and are more suitable for investment, so the sixth layer is suggested as the optimal time layer for stock holding decisions. According to Table 2, the variance of return rate varies in different industries; the variance of the coal industry wavelet efficiency is the highest and the chemical industry wavelet efficiency variance is the lowest. Thus, corroborating the first hypothesis of the research.

4.2 Test results of the second hypothesis

4.2.1 Wavelet Variance Analysis of Investment Companies' Returns

To analyze data on the variance of returns of investment companies using wavelets, First, the relevant data are decomposed into 12 layers of approximation coefficients and 12 layers of detail coefficients. The first one is exploited to extract long-term behavioral patterns and the second one is used to extract short-term behavioral patterns. The diagrams for the coefficients of these 12 layers are shown in the figure below:

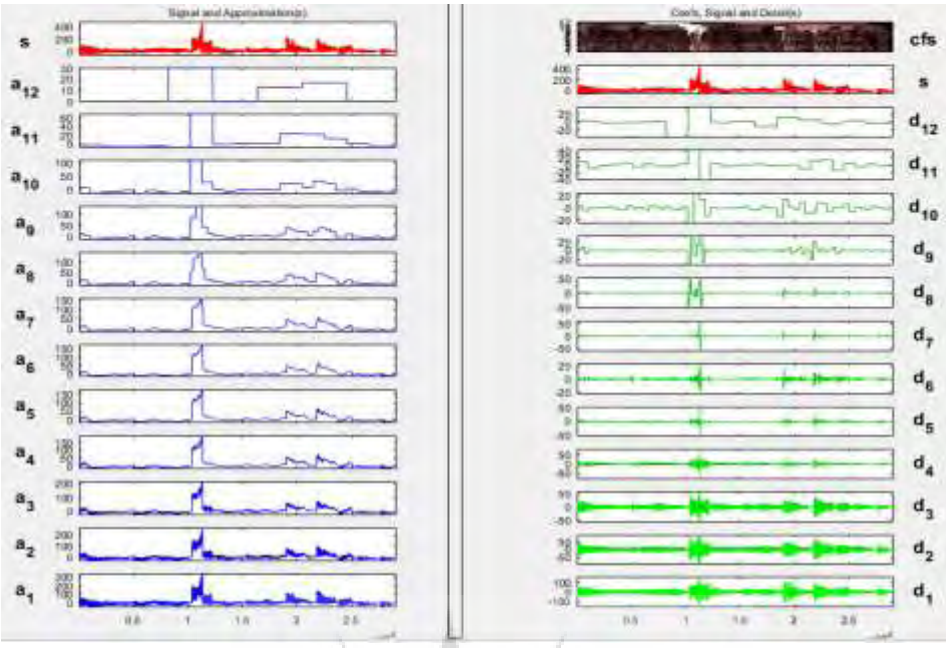

Fig. 2 : Diagrams Of Approximation Coefficients And Detail Coefficients In 12 Layers

According to the diagram above, the initial layers represent the details of the high frequency of the signal and the final layers describe the low frequencies of the signal. As we move from the initial layers to the final layers, the signal is displayed longer; In general, approximation coefficients show the trend of fluctuations longer than detail coefficients; In fact, the fastest dynamic corresponds to d1 and the slowest dynamic corresponds to last layer. The detail coefficients examine the signal in a very short time. As the above figures illustrate, long-term regressions are approximated by the coefficient layers and short-term regressions are approximated by the coefficient layers; The following table shows the standard deviation and variance of each layer:

Table 3- Standard Deviation And Co-Movement Variance Of Investment Companies At Different Time Scales

Layer number	Standard deviation of approximation coefficients	The variance of the approximation coefficients	Layer number	Standard variation of detail coefficient	Variance of detail coefficients
D1	30.69	941.78	A1	17.33	300.39
D2	28.51	812.87	A2	11.35	128.91
D3	27.40	750.73	A3	7.88	62.14
D4	26.88	722.71	A4	5.29	28.02
D5	26.71	713.64	A5	3.01	9.06
D6	26.62	708.45	A6	2.28	5.19
D7	26.20	686.61	A7	4.67	21.84
D8	25.21	635.36	A8	7.16	51.24
D9	24.27	588.97	A9	6.81	46.32
D10	23.25	540.76	A10	6.94	48.21
D11	19.75	390.19	A11	12.27	150.58
D12	12.53	157.03	A12	15.27	233.15

Based on the table above, as we move from the initial layers to the middle layers, the wavelet variance and investment risk is reduced. Here, the variance and the wavelet correlation of the variables in different time scales is analyzed. In fact, the role of each individual frequency (time horizon) in the fluctuation of variables can be determined by looking at the variances and their difference at each part. figure 2 as well as the results of the table above presents the variance of the percentage of investment companies' return. According to those figures and the above table, the variance of approximate co-efficient of the investment companies' return increases at smaller scales but the variance is lessened when the scale enlarges. This implies that the fluctuation of return changes of investment companies while pivoting around the average, decreases from short-term to long-term. But considering the alterations in variance's details, return changes of investment companies at smaller scales (short-term) rise. However, by enlarging the scale, the variance is reduced in the medium term, while in the long term, this deviation slightly grows.

4.2.2 Wavelet Variance Analysis of Banks' Returns

To analyze the data related to the variance of banks' returns using wavelets; First, the relevant data are decomposed into 12 layers of approximation coefficients and 12 layers of detail coefficients. Long-term behavioral patterns are extracted using diagrams and coefficients of approximation layers, and short-term behavioral patterns are extracted using diagrams and layer of detail coefficients. Now, these data are analyzed using the wavelet. The graphs related to the coefficients of these 12 layers are shown in the following figure:

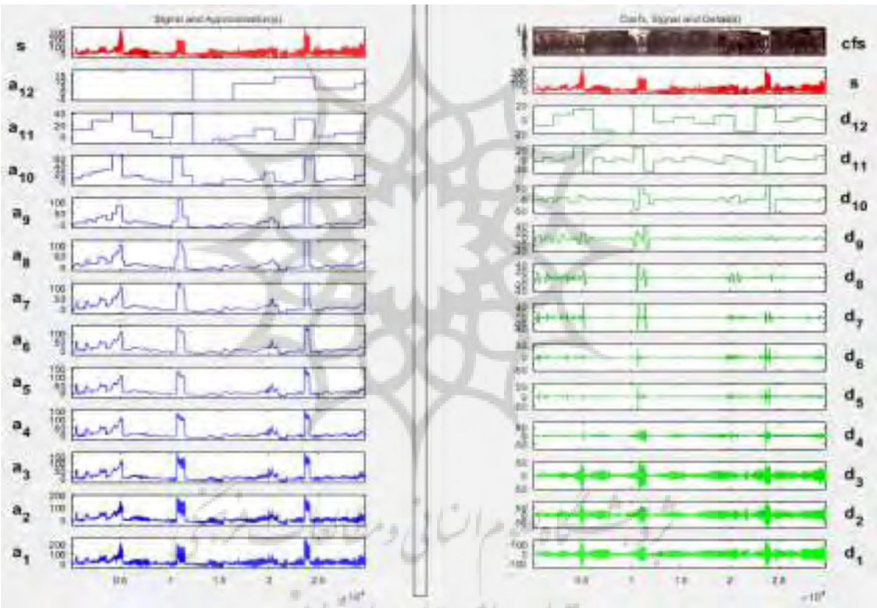

Fig. 3 : Diagrams of approximation coefficients and detail coefficients in 12 layers

According to the diagram above, the initial layers represent the details of the high frequency of the signal and the final layers describe the low frequencies of the signal. As we move from the initial layers to the final layers, the signal is displayed longer; In general, approximation coefficients show the trend of fluctuations longer than detail coefficients; In fact, the fastest dynamic corresponds to d1 and the slowest dynamic corresponds to last layer. The detail coefficients examine the signal in a very short time. As can be seen from the above figures, long-term regressions are approximated by coefficient layers and short-term regressions are approximated by detail layers. The following table shows the standard deviation and variance of each layer:

Table 4: Standard deviation and co-movement variance of the banking industry at different time scales

Layer number	Standard deviation of approximation coefficients	The variance of approximation coefficients	Layer number	Standard variation of detail coefficients	The variance of detail coefficients
1	33.49	1121.70	A1	18.47	341.30
2	31.16	970.91	A2	12.28	150.76
3	29.86	891.46	A3	8.91	79.45
4	29.33	860.35	A4	5.58	31.11
5	29.03	842.61	A5	4.21	17.74
6	28.67	821.99	A6	4.54	20.62
7	28.00	784.02	A7	6.16	37.97
8	27.20	739.87	A8	6.64	44.15
9	26.09	680.67	A9	7.69	59.20
10	20.73	429.82	A10	15.84	250.86
11	15.83	250.48	A11	13.47	181.45
12	10.21	104.23	A12	12.09	146.25

Based on the table above, as we move from the initial layers to the middle layers, the wavelet variance and investment risk is reduced. According to Figure3 as well as the results of Table 4, The ratio of approximation coefficients and the variance of detail coefficients demonstrate the banking industry returns. Variance logarithm is used to examine more thoroughly and observe the changes in variance. As shown in the mentioned figure and table, the variance of approximation coefficients of banking industry's return changes increments. However, when the scale is magnified, variance rate decreases. This suggests that the variance of banking industry's return changes, while hovering around the average, decreases from short-term to long-term. However, due to changes of detail coefficient, the return variance of investment companies on smaller scale (short term) increases. But when the scale increases, the variance decreases in the medium-term. This deviation slightly increases in the long-term.

Table 5: Comparison Of Standard Deviation And Variance Of Banking Industries And Investment Companies

Investment companies	35.24	Standard deviation
	1242.20	variance
Banking industry	38.25	Standard deviation
	1463	variance

The results of the variance return of return wavelet of investment companies and banking industry suggest that the variance of return changes of investment companies on different scales is lower than that of banking industry. Moreover, according to movement scales of each individual bank and investment industry in the above tables, the wavelet variance and co-movement are lower in long-tern scales while in the short-term scales, they increase. This indicates that in Iran return variance of investment companies are more stable than that of banking industry thus, proving the second hypotheses of this research.

4.3 Test results of the second hypothesis

4.3.1 Wavelet Variance Analysis Of total Index

To analyze the data related to the variance of the total index wavelet, first, the data related to the total index are decomposed into 11 layers of approximation coefficients and 11 layers of detail coefficients; the diagrams related to the coefficients of these 11 layers are shown in the figure below.

According to the diagram above, the initial layers represent the details of the high frequency of the signal and the final layers describe the low frequencies of the signal. As we move from the initial layers to the final layers, the signal is displayed longer; In general, approximation coefficients show the trend of fluctuations longer than detail coefficients; In fact, the fastest dynamic corresponds to d1 and the slowest dynamic corresponds to last layer. The detail coefficients examine the signal in a very short time. As can be seen from the figures, long-term regressions are approximated by layers of approximation and short-term regressions are approximated by layers of detail. The following table shows the standard deviation and variance of each layer.

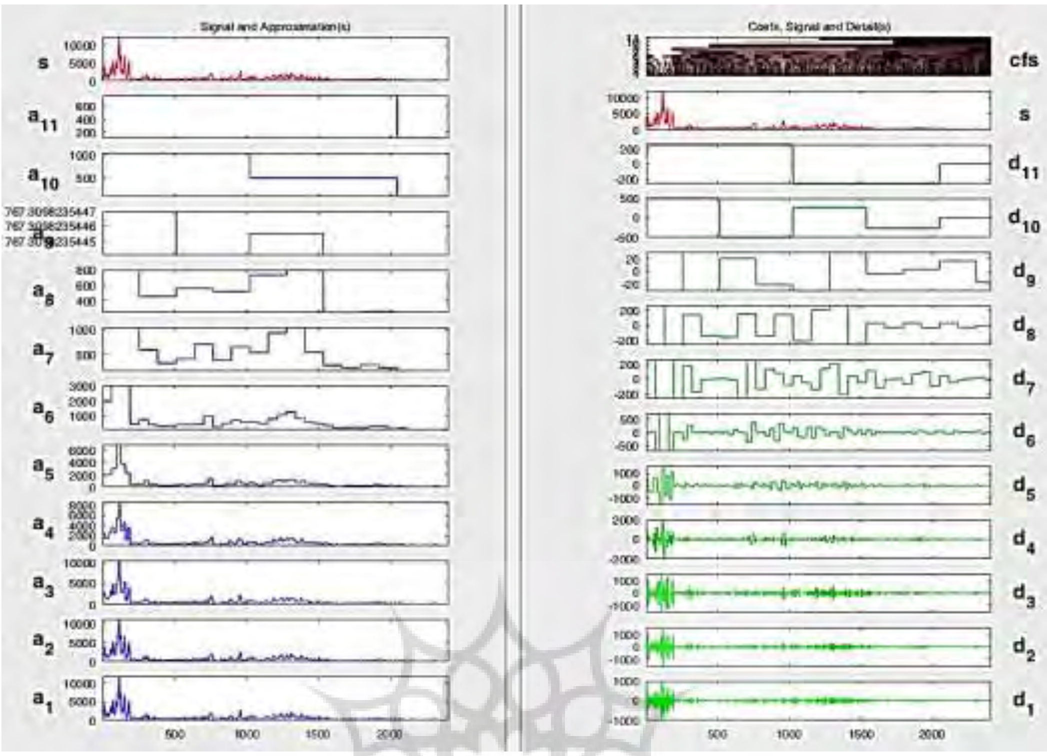

Fig.4 : Graphs Related To Approximation Coefficients And Detail Coefficients In 11 Layers

Table 6: Standard Deviation And Covariance Variance Of The Total Index At Different Time Scales

Layer number	Standard deviation of approximation coefficient	Approximation coefficient variance	Layer number	Standard deviation of detail coefficient	Detail coefficient variance
1	54.362	2955.227	A1	38.721	1499.316
2	51.593	2661.838	A2	26.767	716.472
3	48.219	2325.072	A3	20.038	401.521
4	43.264	1871.774	A4	16.361	267.682
5	42.128	1871.774	A5	11.211	125.687
6	40.302	1624.251	A6	8.034	64.545
7	30.254	915.305	A7	11.4922	132.071
8	26.214	687.174	A8	12.9464	167.609
9	23.582	556.111	A9	10.769	115.971
10	18.470	341.141	A10	9.364	87.684
11	16.881	284.968	A11	8.065	65.044

Based on the table above, as we move from the initial layers to the middle layers, the wavelet variance and investment risk is reduced. According to Figure 4 as well as the results of Table 6, The variance of approximation coefficients and the variance of detail coefficients show the details of the total index. Logarithm of variance has been exploited for a better examination and observation of changes in the variance. As it can be seen, on smaller scales and in the short time, the variance of approximate coefficients of the total index changes is large, meaning that the variance is more intense. However, as the scale increases, the variance decreases. This implies that the fluctuation of return changes of total index descends as it moves from short-term to long-term while hovering around the average. However, in the variance of the detail coefficient which shows fluctuations of a longer time, changes of total Index on smaller scales (short-term) increase. still, by enlarging the scale, the variance of detail coefficients decreases in the medium-term but slightly increases in the long-term.

4.3.2 Wavelet Variance Analysis of Cash Index

Now, the data related to the cash index are analyzed using the wavelets. First, the decomposition tree diagram of this data is shown in 9 layers of approximation coefficients and 9 layers of detail coefficients. The diagrams for the coefficients of these 11 layers are shown in the figure below

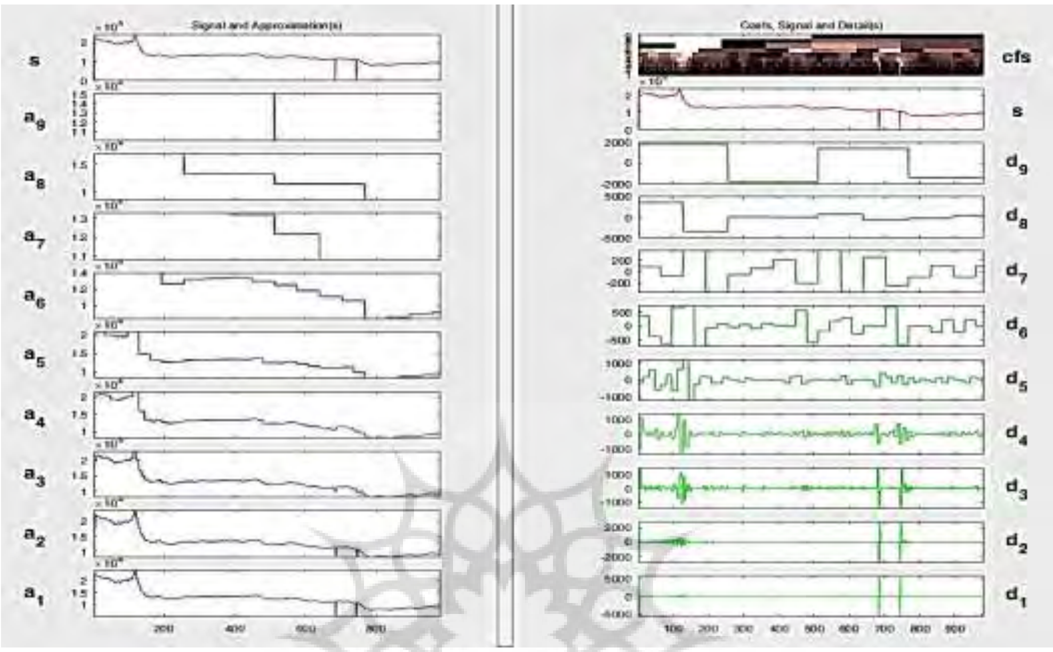

Fig. 5: Diagrams Of Approximation Coefficients And Detail Coefficients In 9 Layers

According to the diagram above, the initial layers represent the details of the high frequency of the signal and the final layers describe the low frequencies of the signal. As we move from the initial layers to the final layers, the signal is displayed longer; In general, approximation coefficients show the trend of fluctuations longer than detail coefficients; In fact, the fastest dynamic corresponds to d1 and the slowest dynamic corresponds to last layer.

Table 7: Standard Deviation And Co-Movement Variance Of Cash Index At Different Time Scales

Layer number	Standard deviation of approximation coefficient	Approximation coefficient variance	Layer number	Standard deviation of detail coefficient	Detail coefficient variance
1	81.507	6643.391	A1	44.3334	1965.450
2	77.096	5943.793	A2	32.3794	1048.426
3	71.364	5092.820	A3	25.6504	657.943
4	66.409	4410.155	A4	21.9734	482.830
5	65.273	4260.565	A5	16.8234	283.027
6	63.447	4025.522	A6	13.6464	186.224
7	53.399	2851.453	A7	17.1046	292.567
8	49.359	2436.311	A8	18.5588	344.429
9	46.727	2183.413	A9	16.3814	268.350
10	41.615	1731.808	A10	14.9764	224.293
11	40.026	1602.081	A11	13.6774	187.071
12	35.719	1275.847	A12	12.2214	149.363

The detail coefficients examine the signal in a very short time. Based on the above figures, long-term regressions are approximated by layers of approximation coefficients and short-term regressions are approximated by layers of detail approximation. Table 7 shows the standard deviation and variance of each layer. Based on the table above, as we move from the initial layers to the middle layers, the wavelet variance and investment risk is reduced. According to Figure 5, as well as the results of Table 7, the variance of approximation coefficients and variance of detail coefficients yield details of the cash index changes. Logarithm of variance has been exploited for a better examination and observation of changes in the variance. As it can be seen, on smaller scales and in the short time, the variance of approximate coefficients of the cash index changes is large, meaning that the variance is more intense. However, as the scale increases, the variance decreases. This suggests that the fluctuation of return changes of cash index lessens as it moves from short-term to long-term while hovering around the average, that is, there will be less fluctuations as time elapse. But in the variance of the detail coefficients which shows fluctuations of a longer time, the changes of cash Index on smaller scales (short-term) goes up. still, by enlarging the scale, the variance of the detail coefficients decreases in the medium-term but slightly increases in the long-term.

Table 8: Comparison Of Standard Deviation And Variance Of Total Index And Cash Index

Total index	102.120	Standard deviation
	10428.425	variance
Cash index	3593.30	Standard deviation
	12912000.00	variance

It can be gathered from the results of the wavelet variance return of the cash and the total index that the variance changing of the total index is less than that of the cash index. Also, according to the co-movement scales of cash and total index in tables 6 and 7, the wavelet variance depletes and the co-movement grows through long-term scales. Conversely, in short term scales, the co-movement diminishes and the variance of the return wavelet was increases. This points to the fact that in Iran, alterations and fluctuations of the total index has more stability than the cash index. Therefore, the third hypothesis of the research is rejected.

5 Conclusion and Discussion

Considering the first hypothesis of the study, which is about the difference of the wavelet variance of investment companies' return rate at different time scales, the results of the research show that the variance of return rate is dissimilar in different industries. From all the various industries, coal industry has the highest return variance while the chemical industry has the lowest variance. In general, different industries have less wavelet variance and less co-movement during long-term scales, but during short-term scales, there is more co-movement between industries and the variance of wavelet efficiency also increases. Thus, the first hypothesis is verified. The results of this hypothesis are in line with the results of Yilmaz and Unal [23] and Boubaker and Raza [2].

Based on the results of the first hypothesis and comparison of wavelet variances in different time layers, the optimal time horizon, the sixth layer is selected to decide on the fluctuation trend and determine the optimal investment strategy to hold stocks and because of the wavelet variances are greater in the short-term scales than the long-term scales, it is suggested that traders in the short term, their strategies with the aim of fluctuating and selecting appropriate entry and exit points to determine the trade based on technical analysis and use stocks that have more wavelet variance in the short term and have more volatility for this purpose. In the long run, investors and analysts should define their strategies with the aim of investing in the Sixth Layer timeframe by allocating more weight to industries that have less wavelet variance than other industries; Beginner traders are also advised to choose their portfolio stocks based on industries with less wavelet variance during stock boom periods and to hold stocks Then implement their sales strategy, which can be the full or partial sale of shares. It can be a full sale or part of a stock, and professional traders can also combine stocks with the aim of swinging (choosing stocks with more wavelet variance) and investing (choosing stocks with less wavelet variance).

The second hypothesis of the research maintained that the wavelet variance is higher in the banking industry than investment companies. The results of the variance of the return wavelet of investment companies and banking industries showed that on different time scales, the return changes' variance of investment companies is less than that of banking industries. Also, according to the movement scales, each banking industry and investment company had a low variance and co-movement on long-term scales. Whereas on short-term scales, both the variance and co-movement was intensified. This explains that in Iran return changes of investment companies are more stable than that of banking industries, thus proving the second hypothesis. Based on the results of the second hypothesis and comparison of wavelet variances related to coefficients of approximation of investment companies and banking groups, the fifth and sixth layers are proposed as optimal time horizons for investment to hold stocks and because the wavelet variance of returns of investment companies is less than the wavelet variance of returns of the Banks Group. It is suggested that investors allocate a higher percentage of their capital to investment companies for investment and due to the lower wavelet variance in the fifth and sixth layers, their optimal time horizon for Investment, consider the fifth- and sixth-time layers. Also, due to the fact that the wavelet variances in the intermediate levels are less than the short-term and long-term levels, it is recommended that traders gain short-term trades and fluctuations based on choosing the appropriate entry point, and they Consider Your optimal time to hold stocks for investment in the medium term and accordingly determine their time strategy. in the long run, given that the wavelet variance of returns of investment companies and banking groups have slightly increased, long-term investment has no justification. Therefore, considering the periods of boom and bust in the Iranian capital market, it is suggested that at the end of the recession, observing the signs of trend change, stocks with less wavelet variances be kept in the optimal time period obtained and then observing the signs of Changing trends left the market.

The third hypothesis of this inquiry which assumed that the variance of total index wavelet is higher than that of the cash index was rejected by the research which showed that, on different scales, the variance of changes in the total index was lower than that of the cash index. However, during short time scales, there was more co-movement as well as a higher variance of the wavelet efficiency. This implicates that in Iran, changes and fluctuations of total index are more static than cash index's fluctuations and therefore rebukes the third hypothesis. The results of this hypothesis are in line with the results of the research of Masih, and Majid [14] Based on the results of testing the third hypothesis and comparing the wavelet variances related to the coefficients of approximation of the total index and the cash index, the sixth layer is proposed as the optimal time horizons for investment to hold stocks. Therefore, considering that the changes and fluctuations of the total index are more stable and have less fluctuations than the cash index, it is suggested that if the investor's goal is investing, can identify stocks according to the optimal entry point based on technical analysis. Gives to maintain the optimal time period that is related to the sixth layer and then decide to sell based on its own sales strategy.

Based on the results of this study, it is suggested that traders base their short-term trading strategies on the coal and automotive industries due to their lower risk of investing in the long run and adjust their medium and long-term investment portfolio by allocating more weight to stocks in the petroleum and chemical products industries. Also due to the lower risk of investing in investment companies compared to the banking industry and the high co-movement of each banking industry and investment company, it is advised that in the short term, traders allocate a larger share of their short-term investment portfolio to shares of investment companies than to banks. Also, according to the high co-movement between each banking industry and investment company in the short-term, it appears that the difference in the existing stocks of banking industries and investment companies is insignificant and it requires, in the medium and long term, a more detailed comparison and analysis based on the analytic indices which are specific to each subsidiary company in each group. It is also suggested that traders formulate their strategy based on investing with medium- and long-term time horizons in index-making stocks and in the short term, plan and implement their relevant strategy with the aim of trading and gathering fluctuation rates. It is worth mentioning that owing to the high co-movement rate and the low wavelet variance between total and cash indices on long-term scale on one hand, and a low co-movement as well as an increase in the return variance of the wavelets between them in the short-term, it seems that medium- and long-term investment can be a more suitable option for beginners and non-professionals while professional traders would be better advised to choose a combination of short-term and long-term investments.

References

[1] Allahyaribeik, N., Nikoomaram, H., Allahyaribeik, S., Rahnamay rood-posht, F., *A Quantum Model for the Stock Market*, Advances in mathematical finance & applications, 2022, **7**(1), P. 217-228.
Doi: 10.22034/AMFA.2020.1882182.1323

[2] Boubaker, H., Raza, S. A., *A wavelet analysis of mean and volatility spillovers between oil and BRICS stock markets*, Energy Economic , 2017, **64**, P.105-117, Doi: 10.1016/j.eneco.2017.01.026 .

[3] Croce, M. M., *Long-run productivity risk: A new hope for production-based asset pricing?* Journal of Monetary Economics, 2014 , **66**, P.13-31M , Doi:10.1016/j.jmoneco.2014.04.001 .

[4] Das, D., Bhowmik , P., Jana, R.K., *A Multiscale Analysis of Stock Return Co-movements*, Statistical Mechanics and Its Applications , 2018, **502**, P. 379-393, Doi: 10.1016/j.physa.2018.02.143 .

[5] Dinarzehi, K., Shahiki Tash, M., Zamanian, G., *Investigating the concurrence of Tehran stock market and global macroeconomic indicators using frequency analysis*, Quarterly Journal of Financial Engineering and Securities Management, 2021,**48**, P.223-252, Doi:20.1001.1.22519165.1400.12.48.10.0.

[6] Ehteshami , S., Hamidian , M., Hajiha , Z., and Shokrollahi , S., *Forecasting Stock Trend by Data Mining Algorithm*, Advances in mathematical finance& applications, 2018, **3**(1), P. 97-105.
Doi : 10.22034/amfa.2018.539138

[7] Fallahi, F., Poorabadollah, M., Sadeghi, S., Shokri, T., *Study of the Relationship between Economic Growth and Environmental Quality in Iran: New Evidence Based on Continuous Wavelet Transformation*, Journal of Economic Growth and Development Research, 2019, Doi: 10.30473/egdr.2020.49586.5499.

[8] Hoevenaars, R. P., Molenaar, R. D., Schotman, P. C., and Steenkamp, T., *Strategic Asset Allocation for Long-Term Investors: Parameter Uncertainty and Prior Information*, Applied Econometrics, 2014, **29**(3), P. 353-376, Doi: 10.1002/jae.2331.

[9] Hossein Kurd, M., Sharifi, A., *Presenting an improved version of the local linear neural-fuzzy model based on discrete wavelet transform to predict financial time series*, International Conference on Computer Engineering and Information Technology, 2016.

[10] Jurek, J. W., Viceira, L. M., *Optimal value and growth tilts in long-horizon portfolios*, Review of Finance, 2010, **15**(1), P. 29-74, Doi:10.1093/rof/rfq013

[11] Kazemzadeh, A. Karimi Petanlar, S., Jafari Samimi, A., *Analysis of the tensile and anti-tensile effect on the Iranian economy: Discrete wavelet converter approach and threshold vector autoregression model*. Journal of Macroeconomics, 2020, **15**(29), P.13-37, Doi: 10.22080/iejm.2020.16858.1701.

[12] Gallegati , M ., *Wavelet analysis of stock returns and aggregate economic activity*, Computational Statistics & Data Analysis , 2008, **52**(6), P. 3061-3074 , Doi: 10.1016/j.csda.2007.07.019 .

[13] Ismail , M , Audu , B and Tumala , M ., *Volatility forecasting with the wavelet transformation algorithm GARCH model: Evidence from African stock markets*, Finance and Data Science, 2016, **2**(2), P.125-135, Doi:10.1016/j.jfds.2016.09.002 .

[14] Masih, M., Majid, H. A., *Comovement of Selected International Stock Market Indices: A Continuous Wavelet Transformation and Cross Wavelet Transformation Analysis*, 2013.

[15] Mensi, W., Rehman, M., Maitra, D., Hamed Al, K., and Xuan Vinh Vo, Y., *Oil, Natural Gas and Brics Stock Markets: Evidence of Systemic Risks and Co- Movements in The Time –Frequency Domain*, Resource Policy, 2021, **72**, Doi: 10.1016/j.resourpol.2021.102062.

[16] Nadami, Y., Khochiani, R., *Co-movement in the Stock, Currency and Gold Markets of Iran: An Econophysical Analysis*, Financial Engineering and Securities Management, 2017, 31, P. 149-166, Doi: 20.1001.1.22519165.1396.8.31.7.5.

[17] Osu, B.O., Okonkwo, C.U., Uzoma, P.U., and Akpanibah., E.E., *Wavelet analysis of the international markets: A look at the next eleven (N11)*, Scientific African, 2020, **7**, P.1-16, Doi: 10.1016/j.sciaf. 2020.e00319.

[18] Ramzi Radchoobeh, Z., ezazade, A., Kazemi, H., *Ambiguity Theory and Asset Pricing: Empirical Evidence from Tehran Stock Exchange*, Advances in mathematical finance & applications, 2018, **3**(4), P. 101-114, Doi:10.22034/AMFA.2019.579558.1143.

[19] Rostami, M. R., Benjar, M., and Nouri Jafarabad, D., *comparing the correlation between the returns of various industries' index in the Tehran Stock Exchange with the returns of oil, gold, dollar and euro markets using wavelet analysis*. Investment Knowledge, 2016, **5** (17), P. 227-251.

[20] Saiti, B, Bacha, O. I., and Masih, M., *Testing the conventional and Islamic financial market*, Emerging Markets Finance and Trade, 2016, **52**(8) , P.1832-1849 , Doi:10.1080/1540496X.2015.1087784

[21]Sousa, R. M., *What is the impact of wealth shocks on asset allocation?*, Quantitative Finance, 2015, **15**(3), P.493-508 , Doi:10.1080/14697688.2011.647053 .

[22] Tiwari, A. K., Cunado, J., Gupta, R., and Wohar, M. E., *Are Stock Returns an Inflation Hedge for the UK? Evidence from a Wavelet Analysis Using Over Three Centuries of Data*, Operational Research, 2017, **187**(3), P.1380-1401, Doi: 10.1515/snde-2017-0049.

[23] Yilmaz, A., Unal, G., *Co-movement analysis of Asian stock markets against FTSE100 and S&P 500: Wavelet-based approach*, International Journal of Financial Engineering, 2016, **3**(04), Doi:10.1142/S242478631650033.

[24] Nasr, N., Farhadi Sartangi, M., Madahi, Z., *A Fuzzy Random Walk Technique to Forecasting Volatility of Iran Stock Exchange Index*, Advances in Mathematical Finance and Applications, 2019, **4**(1), P.15-30. Doi:10.22034/amfa.2019.583911.1172

[25] Zhu , L , Wang ,Y and Fan , Q., *Modwt-Arma Model for Time Series Prediction*, Applied Mathematical Modelling, 2014, 38, P.1859-1865, Doi:10.1016/j.apm.2013.10.002.