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Financial Distress of Companies Listed on the Tehran Stock Exchange using the Dynamic Worst Practice Frontier-based DEA Model

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ABSTRACT

One of the main concerns of financial institutions for investing in companies is to evaluate financial performance and, most importantly, the financial distress of organizations applying for investment. Therefore, various approaches and techniques are used in this evaluation. Financial decision-making has always been associated with the risk of uncertainty. One way to help investors is to provide forecasting models for the overall corporate prospect. It is noteworthy that in all these approaches, various criteria are used to identify corporate financial distress. In this study, a dynamic worst-practice-frontier DEA model was used to identify financially distressed decision-making units over several time-periods. Another feature of the model presented in this study was to provide some improvement solutions for financially distressed decision-making units. Finally, a new ranking approach was introduced to evaluate companies based on the inefficiency trend over several time-periods. The study's approach provides decision-makers with the ability to evaluate the inefficient DMUs during each time-period according to the relationships between these time-periods. The efficiency slope can also be evaluated over time-periods, and companies can be ranked based on this slope. Finally, it is suggested to use this model to dynamically predict financial distress in various industries, including metals, rubber, automobiles, etc., so that companies are informed of their financial distress promptly and take appropriate measures to prevent bankruptcy.

1 Introduction

Financial distress often leads to the bankruptcy or death of a firm. A new set of financial crises has made companies increasingly wary of this risk [7]. On the other hand, capital and money market actors need knowledge about existing companies' financial conditions and efficiency. Given the country's current economic situation, the number of distressed companies and the importance of financial distress are increasing. Even auditors who have a good knowledge of the corporate financial situation cannot make an accurate judgment about the corporate continuity (going concern). Corporate financial distress assessment has always been very important for stakeholders due to high direct and indirect costs [2]. The use of financial ratios to assess corporate financial distress has always been considered by creditors, shareholders, and financial analysts. Proper evaluation and forecasting can help decision-makers find

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the optimal solution and prevent financial distress [21]. Therefore, forecasting financial distress is very important for financial institutions, creditors, and investors [2]. In the last century, forecasting corporate financial distress and bankruptcy has been a category that has attracted the attention of many academic and business communities [27, 38]. These predictions are important because they prevent material and immaterial damages by promptly sending warning signals and dealing with the situation correctly and logically [27]. Financial decision-making has always been associated with the risk of uncertainty. One way to help investors is to provide forecasting models for the overall corporate prospect. The closer the predictions are to reality, the more accurate the decisions will be [33]. Various models have been used for financial distress assessment [28]. These models are widely used in the financial market activists' decision-making. Efforts have always been made to increase the accuracy of prediction and evaluation of these models using more advanced methods [2]. While research is still ongoing, interestingly, a single and clear theory of how and why large companies fail has not yet been developed.
Due to lacking a conceptual framework, some classification techniques and models were used and evaluated according to their efficiency. Almost universally, the decision-making criterion used to evaluate the usefulness of models has been how accurately they classify a company as bankrupt or non-bankrupt compared to its actual state, known as the post-reality situation [13]. Financial ratios can be actively changed with fundamental changes in corporate finances and changes in the global economic environment [29]. It is essential to develop a revolutionary approach in the face of dynamic financial environments in the future. On the other hand, due to the high personal, economic, and social expenses imposed by financial distress, it is essential to address this issue and conduct research to prevent companies from distress and bankruptcy and avoid wasting national resources and wealth. Therefore, in the present study, WPF-DEA's novel approach is proposed to assess corporate financial distress. DEA has been vastly utilized in financial distress studies such as in Izadikhah study that is for financial assessment of banks and financial institudes [18]. Given the importance of financial distress forecasting, scholars have made great efforts to develop high-precision financial distress prediction models. Numerous studies have been conducted on this subject both at home and abroad. By referring to the studies on financial distress, despite extensive research in this field, in terms of the subject novelty, to date, no research has been conducted on the application of the worst practice frontier-based DEA (WPF-DEA) in the dynamic prediction of financial distress both at home and abroad. Therefore, given the above, this study aims to develop a WPF-DEA model for the dynamic prediction of financial distress. This study's main question includes: How is the WPF-DEA model designed to predict corporate financial distress dynamically? Therefore, thematically, the present study is novel research. This study is innovative in both mathematical modeling and its application in Tehran Stock Exchange. These innovations generally include:

- Develop dynamic WPF-DEA model
- Use the dynamic WPF-DEA model to predict financial distress.
- Use the WPF-DEA to predict the financial distress of companies listed on the Tehran Stock Exchange.
- Use the WPF-DEA to dynamically predict the financial distress of companies listed on the Tehran Stock Exchange.

2 Literature Review

In recent decades, globalization, technological change, and a competitive atmosphere have increased uncertainty in financial environments. In such circumstances, economic growth certainly depends on correct decision-making and optimal allocation of resources. It can be done by introducing appropriate tools and models for assessing corporate financial conditions, including financial distress and bankruptcy [2]. Financial distress or bankruptcy is a procedure that greatly affects management, shareholders, employees, creditors, customers, and other stakeholders. Hence, financial distress and bankruptcy challenge the country both socially and economically [39]. Corporate financial distress and bankruptcy lead to wasted resources and underutilization of investment opportunities. Financial distress forecasting can inform companies about financial distress and bankruptcy by developing appropriate indicators and models [2]. One way to take advantage of investment opportunities and better allocate resources is to predict financial distress or bankruptcy. Thus, by providing the necessary warnings, companies can be alerted to the occurrence of financial distress to take appropriate action following these warnings. Second, investors and creditors distinguish favourable investment opportunities from unfavourable ones and invest their resources in appropriate opportunities [23].

Distress and bankruptcy prediction is one of the tools for estimating the future status of companies. Investors and creditors have a strong tendency to predict firm distress and bankruptcy because they face high costs in the event of distress [39]. Bankruptcy is the final stage of financial distress. In other words, financial distress is one of the stages of bankruptcy in which companies do not have their solvency [2]. In their study, Jebelli et al. implemented SVM and Naïve Baysian algorithms for bankruptcy prediction [19]. using financial ratios, Ramezanzadeh Zeidi and Faghani Makrani predicted financial distress [36]. In one of his first studies on financial distress theory, Gordon [16] defined it as a reduction in a firm's profitability, which increases the likelihood of inability to pay service debt. Economically, the financial distress can be interpreted as the company's loss-making, which has failed [32]. Therefore, the above indicates that companies' health and success have always been a major concern of policymakers, managers, investors, and industry participants, which can be considered an indicator of the economy's development and strength.

On the other hand, high personal, economic, and social expenses incurred by companies facing failure or bankruptcy have led to making efforts to understand better and predict corporate financial distress and bankruptcy. Therefore, the early detection of such conditions can prevent potential disasters and high costs for these companies by making appropriate decisions [1, 40]. In their study, "dynamic prediction of financial distress using Malmquist DEA," Li et al. [21] extended the cross-sectional DEA models to time-varying Malmquist DEA. Their results based on a sample of 742 Chinese listed companies observed over ten years indicated that Malmquist DEA offers insights into a company's competitive position and accurate financial distress predictions based on the DEA efficiency measures. In another study, “a comparative analysis of two-stage distress prediction models,” Mousavi et al [28] extended the expert system application in credit scoring and distress prediction using different DEA models to compute corporate market efficiency. They also provided a comprehensive comparison between two-stage distress prediction models by estimating various DEA efficiency measures in the first stage and employing static and dynamic classifiers in the second stage. Based on their experimental results, guidelines were provided to help practitioners develop two-stage distress prediction models.

Khajavi and Ghadirian-Arani [20] used a support vector machine (SVM) model for corporate financial distress prediction. They compared the results of the SVM model with the MVA and LR and the Back Propagation Neural Network (BPNN). The results showed that the SVM model for training and experimental data with 88.01 and 83.06, respectively, was more accurate than other models. Heydary Farahany et al [17] examined corporate financial distress prediction using genetic algorithm and Multiple Discriminant Analysis (MDA). The results showed an increase in the prediction accuracy of the genetic algorithm model compared to the MDA model. Megginson et al [25] used the SVM model for the forecasting process to compare the results with the artificial network model. This study also reported greater generalizability and overall accuracy of the SVM model than the neural network. In a study, Mirarab Baygi et al. [26] made a dynamic prediction of financial distress treatment of Tehran Stock

Exchange companies using a Malmquist Index (MI). The results of this study indicated the high power of this index in predicting corporate financial distress and eliminating the inefficiency of previous methods. Rahimi et al [34] also explained the financial variables affecting corporate financial distress prediction. In this study, 106 companies were selected through simple random sampling. The financial data of these companies were extracted between 2007 and 2020. The relationship between variables was investigated using the Pearson correlation test.

Out of 34 financial ratios, 24 ratios that had a significant correlation were selected. Nevertheless, to what extent can financial distress be predicted? Due to its importance for corporate managers and stakeholders, the answer to this question has led them to constantly seek to find the best solution for predicting organizational performance. This rationalist approach to the decision-making process over time has led finance scholars to use a wide range of methods to address financial distress. In the interim, the use of new and high-precision methods in achieving forecasting goals that include accuracy, precision, and timeliness has become increasingly important. Thus, by providing the necessary warnings, companies can be alerted to the occurrence of financial distress to take appropriate action following these warnings. Second, investors and creditors distinguish favourable investment opportunities from unfavourable ones and invest their resources in appropriate opportunities [32, 27]. Various methods have yet been used to predict and assess financial distress and bankruptcy. Traditional statistical methods, including multiple discriminant analysis (MDA), Logit analysis, and Probit analysis, could make good predictions about the corporate financial distress or bankruptcy [30]. However, some of these models' restrictive assumptions, including the linearity, normality, and independence of predictor variables, might affect this method [28]. Therefore, other methods, including MCDM and artificial intelligence techniques, were progressively introduced to address some or all of these limitations and improve the efficiency of predictions [13]. Data envelopment analysis (DEA), as a non-parametric and multi-criteria evaluation technique with its unique features, has made the concept of efficiency highly accurate in financial management [12]. DEA is one of the most successful techniques used in research activities related to banks and financial institutions' performance evaluation. Extensive research into financial performance appraisal offers several models and methods. However, all of these models aim to provide a way to select the best decision-making unit (DMU) in an efficiency maximization scenario and to develop a way to identify the worst DMU to prevent further activity.

The DEA models are all best-practice frontier-based DEA (BPF-DEA) models. Cooper et al. [9] noted that in optimizing BPF-DEA models, weights assigned to DMUs are the most appropriate weights to maximize the DMU's efficiency under study. Although the BPF-DEA models can typically detect even the worst DMUs, in the real world, it does not seem appropriate to measure the inefficiency of the DMUs in an efficiency maximization scenario (by assigning the best weights maximizing the DMUs' efficiency). Therefore, Liu and Chen [22] introduced a model to identify and assess investment risks and forecast bankruptcies. They believed that it would make more sense to design a model to evaluate and rank DMUs to identify the worst practice in an efficiency minimization scenario. It is known as the worst practice frontier- based DEA (WPF-DEA) model. However, WPF-DEA models still ignore the fact that corporate efficiency has a temporal dimension, and in fact, they are static models [6, 8]. Corporate efficiency throughout their life is interconnected as a chain. Therefore, their performance evaluation over several time-periods is essential and provides better information for managers. In other words, there is a need for multi-period evaluation, but conventional DEA models are built for one single period [23]. Given the above and since the financial distress forecasting is one of the most important issues facing companies, in this study, a new approach, WPF-DEA, has been proposed to predict corporate financial distress. Then, the results obtained by this method are compared with those of traditional methods. Therefore, this study's main problem is to develop a WPF-DEA model for dynamic financial distress prediction.

3 Data Envelopment Analysis

Farrell [15] first proposed non-parametric methods for estimating efficiency. Instead of estimating the production function, the values of DMUs' inputs and outputs were observed. A frontier, called the efficient frontier, was defined for these DMUs. It was used as a criterion for efficiency measurement. Farrell's study played an important role in Charnes, Cooper, and Rhodes's (CCR) [4] fundamental research and served as a starting point for DEA. In the CCR's study, linear programming was generalized to measure a DMU's efficiency in multiple inputs and outputs. After the CCR model, the Russell measure was proposed by Färe and Lovell [14] for difficult calculations. Banker et al. [3] also introduced the variable returns to scale version of the CCR model and named it Banker, Charnes, and Cooper (BCC). The Free Disposal Hull (FDH) model was also introduced by Deprins and Simar [11]. Then, Charnes et al. [5] proposed an additive model. In addition to the above models, Cooper et al. [9] also introduced the range-adjusted measure (RAM).
Another non-radial model for evaluating the efficiency of DMUs was proposed by Tone [37], known as the Slacks-based Measure (SBM) model. Besides, other basic models such as Andersen and Petersen (AP), cross efficiency (CE), and a common set of weight (CSW) were introduced [24]. According to Cooper et al. [10], several articles and books have been written on the subject since the DEA's inception. However, standard DEA models have many drawbacks. The WPF-DEA concept was first discussed by Parkan and Wang [31] and Paradi et al. [30]. They showed how WPF-DEA could evaluate the worst efficiencies by identifying the worst efficient companies. Parkan and Wang [31] proposed the WPF-DEA model to identify poor efficiencies or predict bankruptcies in an efficiency minimization scenario. To detect the worst efficient DMUs, they introduced model (1), known as the WPF-CCR fractional model. They argued that designing a model for evaluating and ranking DMUs in an efficiency minimization scenario would make more sense to identify the worst efficiencies.

$$\begin{aligned}
& \min Z_0 = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\
& \text{s.t.} \\
& \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \geq 1, \qquad (j = 1, \dots, s), \\
& u_r \geq 0, \qquad (r = 1, \dots, s), \\
& v_i \geq 0, \qquad (i = 1, \dots, m).
\end{aligned} \tag{1}$$

$$\begin{aligned}
& \min Z_0 = \sum_{r=1}^{s} u_r y_{ro} \\
& \text{s.t.} \\
& \sum_{i=1}^{m} v_i x_{io} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \geq 0, \qquad (j = 1, \dots, s), \\
& u_r \geq 0, \qquad (r = 1, \dots, s), \\
& v_i \geq 0, \qquad (i = 1, \dots, m).
\end{aligned} \tag{2}$$

Model (2) calculates the weights of inputs and outputs to consider the lowest possible relative efficiency score for DMUs. The first constraint of this model ensures that the weighted sum of the outputs to the weighted sum of the inputs for any DMUs is not less than 1. The inputs and outputs' weights will be greater than or equal to 1 using the second and third constraints. Note that in this study, "BPF-efficient" DMUs are known to be efficient using the BPF model, and "WPF-efficient" DMUs are known to be efficient using the WPF model. Using Charnes-Cooper transformations, model (1) is linearized as Model (2). The result of Model (2) includes an efficiency score of 1 for the DMUs identified by the WPF-based model as the worst DMUs. This model also sets an efficiency score greater than 1 for DMUs that have failed to present themselves as weak DMUs. Since a constraint must be written for each DMU, a linear programming model will be obtained whose constraints are greater than its variables. Also, since the volume of operations depends more on the number of constraints than variables, solving the above model's secondary problem will require fewer operations. In the secondary problem, if the variable corresponding to the constraint $\sum_{i=1}^{m} v_i x_{i0} = 1$ is expressed by θ, and the variables corresponding to the constraint $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \geq 0$ are expressed by λ_j, the second model of Model (2) will be as follows:

$$\begin{aligned} & Max\ y_0 = \theta, && && (3)\\ & s.t. \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \leq y_{r0}, && (r = 1, 2, \dots, s), \\ & \sum_{j=1}^{n} \lambda_j x_{ij} \geq \theta x_{i0}, && (i = 1, 2, \dots, m), \\ & \theta\ free, \quad \lambda_j \geq 0, && (j = 1, 2, \dots, n). \end{aligned}$$

However, the radial WPF-CCR and WPF-BCC models have a fundamental problem: they cannot calculate SBMs directly and require a more complex indirect method to calculate SBMs. Therefore, Liu and Chen [8] presented the WPF-SBM model based on the SBM model's production possibility set. The worst inefficiencies of the WPF-SBM is obtained using the following fractional model.

$$\begin{aligned} & [WPF - SBM]\rho^* = Max\ \rho = 1 + \frac{1}{m}\sum_{i=1}^{m}\left(\frac{s_i^+}{x_{io}}\right) \Big/ 1 - \frac{1}{s}\sum_{r=1}^{s}\left(\frac{s_r^-}{y_{ro}}\right), \\ & s.t. \\ & \sum_{j=1}^{n} \lambda_j y_{rj} + s_r^- = y_{ro}, \qquad (r = 1, 2, \dots, s), \qquad\qquad (4) \\ & \sum_{j=1}^{n} \lambda_j xy_{ij} - s_i^+ = x_{io}, \qquad (i = 1, 2, \dots, m), \\ & \lambda \geq 0, s^- \geq 0, s^+ \geq 0. \end{aligned}$$

The fractional model (4) is converted into a linear model (5) by applying the Charnes-Cooper transformations. Note that $t > 0$ means that the conversion is reversible. Consider the optimal solution of the above model (τ^*, t^*,Λ*, S-*, S+*); then we have the optimal solution (WPF-SBMt) which is defined as follows:

$\rho^* = \tau^*, \lambda^* = \Lambda^*/t^*, S^{-*} = S^{-*}/t^*, s^{+*} = S^{+*}/t^*$

In most BPF-DEA models, BPF-efficient DMUs or DMUs on the efficient frontier are specified with an efficiency value of 1. Usually, a set of multiple DMUs has this position. The WPF-DEA models may also include a set of WPF-efficient or inefficient frontier DMUs. It seems interesting to differentiate between these DMUs.

$$
\begin{aligned}
&[LP-WPF-SBM]\tau^* = Max\ \tau = t + \frac{1}{m}\sum_{i=1}^{m}\frac{S_i^+}{x_{io}},\\
&s.t.\\
&1 = t - \frac{1}{s}\sum_{r=1}^{s}\frac{S_r^-}{y_{ro}},\\
&\sum_{j=1}^{n}\Lambda_j y_{rj} + S_r^- = y_{ro}, \qquad (r = 1,2,\dots,s),\\
&\sum_{j=1}^{n}\Lambda_j xy_{ij} - S_i^+ = x_{io}, \qquad (i = 1,2,\dots,m),\\
&\Lambda \geq 0, S^- \geq 0, S^+ \geq 0, t > 0.
\end{aligned} \tag{5}
$$

Liu and Chen [20] called this "hypo-efficiency" and considered it an efficiency that is worse than the worst efficiency, and developed it based on Tone's [37] super-efficiency model to rank the worst efficiencies by the WPF-SBM. Henceforth, we consider hypo-efficiency as super-efficiency as defined by Liu and Chen [22]. Liu and Chen [22] presented the Hypo SBM model based on the Super SBM model's production possibility set' The worst WPF-SBM super-efficiencies are obtained using the following fraction model:

$$
\begin{aligned}
&[Hypo\ SBM]\delta^* = Max\ \delta = \frac{1}{m}\sum_{i=1}^{m}\bar{x}_i/x_{io}\bigg/\frac{1}{s}\sum_{r=1}^{s}\bar{y}_r/y_{ro},\\
&s.t.\\
&\bar{\boldsymbol{x}} \leq \sum_{j=1,\neq o}^{n}\lambda_j\,\boldsymbol{x}_j,\\
&\bar{y} \geq \sum_{j=1,\neq o}^{n}\lambda_j\,y_j,\\
&\boldsymbol{0} \leq \bar{\boldsymbol{x}} \leq \boldsymbol{x}_o,\\
&\bar{\boldsymbol{y}} \geq y_o,\\
&\boldsymbol{\lambda} \geq \boldsymbol{0}.
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
&[LP-Hypo\ SBM]\tau^* = Max\ \tau = \frac{1}{m}\sum_{i=1}^{m}\tilde{x}_i/x_{io},\\
&s.t.\\
&\frac{1}{s}\sum_{r=1}^{s}\tilde{y}_r/y_{ro} = 1\\
&\tilde{x}_i \leq \sum_{j=1,\neq o}^{n}\Lambda_j\,\boldsymbol{x}_j,\\
&\tilde{y}_r \geq \sum_{j=1,\neq o}^{n}\Lambda_j\,y_j,\\
&\boldsymbol{0} \leq \tilde{\boldsymbol{x}} \leq \boldsymbol{x}_o t\ ,\\
&\tilde{\boldsymbol{y}} \geq t y_o,\\
&\boldsymbol{\Lambda} \geq \boldsymbol{0}, t > 0.
\end{aligned} \tag{7}
$$

The fractional model (6) is converted into a linear model (7) by applying the Charnes-Cooper transformations. Note that $t > 0$ means that the conversion is reversible. Consider the optimal solution of the above model ($\tau^*, t^*, \Lambda^*, S-^*, S+^*$); then we have the optimal solution (Hypo-SBMt) which is defined as follows:

$$\rho^* = \tau^*, \lambda^* = \Lambda^*/t^*, S^{-*} = S^{-*}/t^*, s^{+*} = S^{+*}/t^*$$

As mentioned earlier, the above WPF-DEA models ignore the temporal dimension, and they are, in fact, static models [6, 8]. However, the corporate efficiency throughout their life is interconnected as a chain. Therefore, this study has developed a WPF-DEA model for the dynamic prediction of financial distress.

4 Proposed Model

In this section, the inefficient-dynamic DEA model is presented. DMUs are used to describe the model, which means the companies being evaluated. Here, the possible space is *n* DMUs ($n = 1, ..., N$), during which *t* period ($t = 1, \ldots, T$) the DMUs are evaluated. In each period, the DMUs have m input($i = 1, \ldots, p$), *s* output($i = 1, \ldots, s$), and *r* fixed uncontrollable output($i = 1, \ldots, r$). Also, x_{ijt} is a controllable input ($i = 1, \ldots, m$); $x_{ijt}^{fix}$is a fixed or uncontrollable input ($i = 1, \ldots, p$); y_{ijt} is a controllable output ($i = 1, \ldots, s$); and y_{ijt}^{fix} is an uncontrollable output related to the DMUs in period *t* ($i = 1, \ldots, r$). Besides, four types of relations are displayed with $z^{bad}, z^{free}, z^{fix}$, and z^{good}. For example, symbols like z_{ijt}^{good} where $(i = 1, \ldots, n_{good}), (j = 1, \ldots, n),$ and $(t = 1, \ldots, T)$ indicate good relations. $z^{bad}$is used for bad relation, z^{free} for free relation, and z^{fix} for fixed relation.

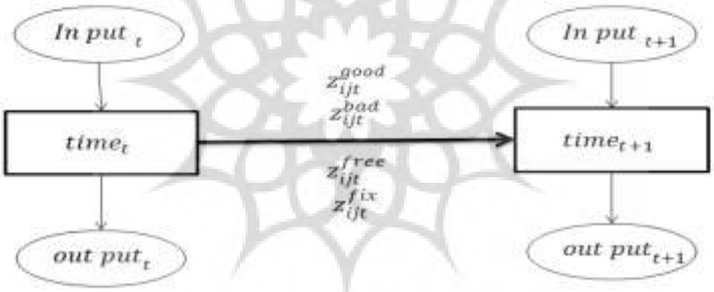

Fig. 1: Dynamic network topology

The production possibility set for these variables is as follows:

$$x_{it} \le \sum_{j=1}^{n} x_{ijt}\lambda_j^t \;\; (i = 1, \ldots, m\,; t = 1, \ldots, T) \tag{8}$$

$$x_{it}^{fix} = \sum_{j=1}^{n} x_{ijt}^{fix}\lambda_j^t \;\; (i = 1, \ldots, p\,; t = 1, \ldots, T)$$

$$y_{it} \ge \sum_{j=1}^{n} y_{ijt}\lambda_j^t \;\; (i = 1, \ldots, s\,; t = 1, \ldots, T)$$

$$y_{it}^{fix} = \sum_{j=1}^{n} y_{ijt}^{fix}\lambda_j^t \;\; (i = 1, \ldots, r\,; t = 1, \ldots, T)$$

$$z_{it}^{good} \geq \sum_{j=1}^{n} z_{ijt}^{good} \lambda_j^t \ (i = 1, \dots, n\ good\ ; t = 1, \dots, T)$$
$$z_{it}^{bad} \leq \sum_{j=1}^{n} z_{ijt}^{bad} \lambda_j^t \ (i = 1, \dots, n\ bad\ ; t = 1, \dots, T)$$
$$z_{it}^{free} : free(i = 1\ , \dots, n\ free : t = 1, \dots, T)$$
$$z_{it}^{fix} = \sum_{j=1}^{n} z_{ijt}^{fix} \lambda_j^t \ (i = 1, \dots, n\ fix\ ; t = 1, \dots, T)$$
$$\lambda_j^t \leq 0\ (j = 1, \dots, n : t = 1, \dots, T)$$
$$\sum_{j=1}^{n} \lambda_j^t = 1\ (t = 1, \dots, T)$$

λ represents the patterns of inefficient DMUs. These patterns are repeated for a number of time periods where $(t = 1، \dots ، T)$ $\lambda^t \in R^n$ represents the intensity vector in period t. *n fix*, *n bad*, and *n free* are the number of fixed, bad, and free relations, respectively. The last constraint represents variable returns to scale. If this constraint is removed, a model with fixed returns to scale is obtained. Note that on the right side of the above model, $x_{ijt}, x_{ijt}^{fix}, y_{ijt}, y_{ijt}^{fix}, z_{ijt}^{good}, z_{ijt}^{bad}$, and z_{ijt}^{fix} are positive, while $x_{it}, x_{it}^{fix}, y_{it}, y_{it}^{fix}, z_{it}^{good}, z_{it}^{bad}, z_{it}^{fix}$, and z_{it}^{free} are on the left side of the model, which are connected by λ_{jt}. The continuity of the relationship between period t and t + 1 is guaranteed using the following conditions:

$$\sum_{j=1}^{n} x_{ijt}^{\propto} \lambda_j^t = \sum_{j=1}^{n} z_{ijt}^{\propto} \lambda_j^{t+1} \qquad (\forall i; t = 1، \dots ، T-1) \tag{9}$$

Here the words *good*, *free*, *bad*, and *fix* can be used instead of α, and they are also repeated in each time-period. These constraints are very important in the inefficient-dynamic model because they relate period *t* to period t + 1. B this formula, the DMU in question can be written as a set of constraints (10):

(10)

$$x_{iot} = \sum_{j=1}^{n} x_{ijt} \lambda_j^t - s_{it}^- \ (i = 1، \dots ، m\ ; t = 1، \dots ، T)$$
$$x_{iot}^{fix} = \sum_{j=1}^{n} x_{ijt}^{fix} \lambda_j^t \ (i = 1، \dots ، p\ ; t = 1، \dots ، T)$$
$$y_{iot} = \sum_{j=1}^{n} y_{ijt} \lambda_j^t + s_{it}^+ \ (i = 1، \dots ، s\ ; t = 1، \dots ، T)$$
$$y_{iot}^{fix} = \sum_{j=1}^{n} y_{ijt}^{fix} \lambda_j^t \ (i = 1، \dots ، r\ ; t = 1، \dots ، T)$$
$$z_{iot}^{good} = \sum_{j=1}^{n} z_{ijt}^{good} \lambda_j^t + s_{it}^{good} \qquad (i = 1، \dots ، n\ good\ ; t = 1، \dots ، T)$$
$$z_{iot}^{bad} = \sum_{j=1}^{n} z_{ijt}^{bad} \lambda_j^t - s_{it}^{bad} \qquad (i = 1، \dots ، n\ bad\ ; t = 1، \dots ، T)$$

$$z_{iot}^{free} = \sum_{j=1}^{n} z_{ijt}^{free} \lambda_j^t - s_{it}^{free} \qquad (i = 1، \dots ،n\, free\, ; t = 1، \dots ،T)$$

$$z_{it}^{fix} = \sum_{j=1}^{n} z_{ijt}^{fix} \lambda_j^t \ (i = 1، \dots ،n\, fix\, ; t = 1، \dots ،T)$$

$$\sum_{j=1}^{n} \lambda_j^t = 1 \ (t = 1، \dots ، T)$$

$$s_{it}^{free} : free\ (\forall i،t) \text{ و } s_{it}^{bad} \geq 0 ،s_{it}^{good} \geq 0 ، s_{it}^{+} \geq 0 ، s_{it}^{-} \geq 0 ، \lambda_j^t \geq 0$$

s is a variable that standardizes constraints. $s_{it}^{-}, s_{it}^{+}, s_{it}^{good}, s_{it}^{bad}$, and s_{it}^{free} are SBM variables that represent input surplus, output shortages, lack of good relations, bad relation surplus, and relation deviation, respectively.

4.1 Objective and Efficiency Functions

The overall efficiency of each DMU is input-oriented. The input-oriented models deal with reduced inputs, and they try to maintain the number of current outputs. THE dynamic SBM (D-SBM) model maximizes input SBM variables and SBM variables related to bad relations. In output-oriented models, we try to maximize the outputs and maintain the inputs. The D-SBM model simultaneously increases the output variables. Differences in the nature of the models affect the objective function, which is presented below.

4.1.1 Input-Oriented-WPF Model

The input-oriented model, θ_0^*, with overall input-oriented WPF is defined as follows.

$$\theta_0^* = \text{MAX} \sum_{t=1}^{T} w^t \left[1 - \frac{1}{m + n\, bad} \left(\sum_{i=1}^{m} \frac{w_i^- s_{it}^-}{x_{iot}} + \sum_{i=1}^{n\, bad} \frac{s_{it}^{bad}}{z_{iot}^{bad}} \right) \right] \tag{11}$$

The objective function (11), with constraints (9) and (10), represents the dynamic input-oriented WPF-SBM model, in which w_i^- and w^t are the weights of *ith* input and period *t*, respectively. It will be selected in terms of its importance, which will be discussed below.

$$\sum_{t=1}^{T} w^t = T \text{ و} \sum_{i=1}^{m} w_i^- = m \tag{12}$$

If all weights are equal, w_i^- and w^t can be considered 1. This objective function is based on an input-oriented non-radial model. It not only deals with surplus inputs but also considers bad relations. In model (12), the inefficiency value is 1, which means inefficient DMU being evaluated $(1 \leq \theta^*)$.

Note that s_{it}^{bad} like s_{it}^{-} is included in the objective function because they have common features, i.e., the less it is, the more efficient the DMU will be. However, bad relations are not inputs. They act as a link between time-periods. In model (11), each period inside the parentheses represents the inefficiency of period *t*. If all SBM variables are zero, the value in parentheses is 1. Hence, model (11) is the weighted average of time-period inefficiencies in all periods, which is more than 1. If its value equals 1, it means DMU is inefficient$(1 \leq \theta^*)$. If we denote the optimal value by *, the input-oriented WPF of period *t* is defined as follows:

$$\theta_{ot}^{*} = 1 - \frac{1}{m + n\,bad}\left(\sum_{i=1}^{m}\frac{w_i^- s_{it}^-}{x_{iot}} + \sum_{i=1}^{n\,bad}\frac{s_{it}^{bad}}{z_{iot}^{bad}}\right) \quad , \qquad (t = 1 ، \ldots ، T) \tag{13}$$

A model is provided to obtain the appropriate weight for each period considering each time-period's importance. It is noteworthy that managers can select these weights. However, to eliminate the human factor's influence in presenting the results, the following model is provided for selecting input-oriented weights. Note that W_p weights are presented to show the relative importance of each period compared to the entire period. One method for determining W_p is the total volume of resources allocated to each period p during all periods, indicating the relative importance of that period. In particular, formula w_1 is described to calculate the weight of the first period.

$$\sum_{j=1}^{n} x_{ijt}^{\propto}\lambda_j^t + \sum_{j=1}^{n} z_{ijt}^{\propto}\lambda_j^{t+1} \qquad (\forall i;\, t = 1 ، \ldots ، T-1) \tag{14}$$

This expression, which is at the denominator, represents the total consumption of time-periods in Fig.1. Here, $\propto$ can be bad, fixed, or even free relations. For inputs $\propto$ can also be controllable and uncontrollable inputs. Due to the nature of the relation, the expression $\sum_{j=1}^{n} x_{ioo}^{\propto}\lambda_j^t$ in the numerator indicates the sum of input weights used in the studied period and DMU. The first period certainly does not include any relation and only has controllable and uncontrollable inputs. Therefore, $\propto$ is defined as the weighted sum of the controllable and uncontrollable inputs.

$$w^1 = \frac{\sum_{j=1}^{n} x_{ioo}^{\propto}\lambda_j^t}{\sum_{j=1}^{n} x_{ijt}^{\propto}\lambda_j^t + \sum_{j=1}^{n} z_{ijt}^{\propto}\lambda_j^{t+1}} \tag{15}$$

Since in other time-periods, it is possible to have different types of relations in the inputs related to each period, the W_p model is described to calculate each period's weights. In this case, all the time-periods' inputs include bad, fixed, and free relations. $\sum_{j=1}^{n} x_{ioo}^{\propto}\lambda_j^t + \sum_{j=1}^{n} z_{ioo}^{\propto}\lambda_j^{t+1}$ indicates the weighted sum of the inputs considered for the DMU during the time-period under study. Therefore, for other input-oriented DMUs, the weights of each time-period are defined as follows:

$$w^t = \frac{\sum_{j=1}^{n} x_{ioo}^{\propto}\lambda_j^t + \sum_{j=1}^{n} z_{ioo}^{\propto}\lambda_j^{t+1}}{\sum_{j=1}^{n} x_{ijt}^{\propto}\lambda_j^t + \sum_{j=1}^{n} z_{ijt}^{\propto}\lambda_j^{t+1}} \tag{16}$$

The overall inefficiency of each DMU is based on the weighted sum of different periods' inefficiencies using the extracted weights. The time-period's WPF represents the input-oriented WPF in period *t*. The total WPF, θ_0^*, is the weighted sum of time-period's inefficiencies, θ_{ot}^*, which is as follows:

$$\theta_0^* = \sum_{t=1}^{T} w^t \theta_{ot}^* \tag{17}$$

5 Case Study

In this study, corporate statistics and data were collected through Rahavard Novin Software in a field manner by studying the companies' monthly and annual reports and searching related websites. Data were collected from secondary sources and through library studies, websites, and reputable scientific journals in the literature review section. The data required for this study were collected from the information sources of the Securities & Exchange Organization and existing data systems, including Tehran

Stock Exchange (TSE Client) software and Rahavard Novin database software, and these data were used after validation. The collected data was first stored in a database, and then by transferring this data to Excel software, the data and their results were analyzed. According to the study's model, 105 companies listed on the Tehran Stock Exchange are evaluated in this section. Experts' opinions are used to identify inputs, outputs, and relations. The inputs, outputs, and relations for evaluating these 105 companies are as follows.

Inputs:

- *Working capital to total assets (WCTA):* WCTA is a ratio that represents the share of working capital to total assets. The larger the WCTA is, the greater the company's liquidity becomes. It indicates the company's avoidance of the financial distress risk.
- *Current assets to current liabilities (CACL):* CACL is the most common measure of short-term debt solvency. The larger the CACL is, the less financially distressed the company will be.
- *Earnings before interest and taxes to total assets (EBITA):* Interest on the cost of raising capital through borrowing is the result of creditors' share of corporate profits and taxes and the government's share of corporate profits. The company's profit before these two factors indicates the corporate profitability by using the company's assets. The larger the EBITA is, the higher the corporate profitability will be. Therefore, it indicates the company's avoidance of the financial distress risk.
- *Earnings before interest and taxes to sales (EBIS):* EBIS represents the corporate profitability. The larger the EBIS is, the more profitable the company is, and the less exposed it is to financial distress.

Outputs:

- *Total debt to total equity (TDTE):* Debts and equity represent how the company is funded. If the company's assets are more secured by the liabilities, it will be more likely to be financially distressed.

Relations:

- *Bad- receivables collection period (RCP):* The average RCP is one of the activity ratios that indicates the time it will take for the company to receive its receivables from customers.
- *Good- liquidity ratio (LR):* LR is one of the financial ratios obtained by dividing cash, cash equivalents, and highly liquid securities by current liabilities. LR is a corporate liquidity test. All cash and securities traded on the market are summed and divided by the total current liabilities to calculate the LR.
- *Free- current asset turnover (CAT):* CAT shows the impact of asset turnover on the corporate revenue. It also describes how a company's assets are used to generate revenue. By comparing this ratio in previous periods, it can be argued whether the increase in assets has been effective in generating more revenue for the company or not.

Since these values are evaluated over five time-periods, they are used to evaluate the data of 105 companies during these five time-periods. Five time-periods from 2015 to 2019 are used to calculate the time-periods' WPF or inefficiency using the study model. The WPF results of these companies over five time periods are presented in Table 1. In financial distress assessment, financial ratios whose small values can cause financial distress are considered input variables, and ratios whose large values can cause financial distress are considered output variables.

Table 1: the inefficiency results of 105 DMUs evaluated during five time-periods

2019	2018	2017	2016	2015	DMU	2019	2018	2017	2016	2015	DMU	2019	2018	2017	2016	2015	DMU
4.46	4.32	4.5	4.04	3.71	71	2.87	2.71	2.62	2.55	2.38	36	4.56	4.40	4.37	4.18	3.91	1
4.87	4.72	4.53	4.41	4.05	72	1	1.14	1	1.07	1	37	2.47	2.39	2.37	2.27	2.12	2
2.02	1.96	1.88	1.83	1.68	73	3.17	2.99	2.90	2.81	2.63	38	3.12	3.00	2.98	2.86	2.67	3
2.04	1.98	1.9	1.85	1.7	74	1.21	1	1.10	1.07	1	39	2.49	2.40	2.38	2.28	2.13	4
5.96	5.69	4.55	3.75	2.45	75	2.67	2.51	2.44	2.36	2.21	40	1.17	1.13	1	1.07	1	5
4.10	3.97	3.81	3.72	3.41	76	4.32	4.07	3.95	3.83	3.58	41	2.49	2.40	2.38	2.28	2.13	6
1.20	1.17	1.12	1.09	1	77	2.15	2.02	1.96	1.90	1.78	42	1.95	1.88	1.87	1.79	1.67	7
1.20	1	1	1.09	1	78	2.81	2.65	2.57	2.49	2.33	43	2.95	2.85	2.83	2.71	2.53	8
3.37	3.26	3.13	3.05	2.8	79	1.21	1.14	1	1	1	44	1.17	1.13	1.12	1.07	1	9
4.38	4.24	4.07	3.97	3.64	80	2.75	2.59	2.51	2.44	2.28	45	2.45	2.36	2.35	2.25	2.1	10
3.76	3.65	3.5	3.41	3.13	81	2.94	2.77	2.69	2.61	2.44	46	4.18	4.03	4.00	3.83	358	11
3.01	2.91	2.8	2.73	2.5	82	2.58	2.43	2.36	2.29	2.14	47	1.17	1.13	1.12	1.07	1	12
3.80	3.68	3.53	3.44	3.16	83	3.19	3.00	2.91	2.82	2.64	48	1	1.02	1.81	2.26	2.51	13
3.98	3.86	3.7	3.61	3.31	84	1.21	1.14	1.10	1.07	1	49	2.00	1.92	1.91	1.83	1.71	14
1.20	1.17	1.12	1.09	1	85	2.59	2.44	2.37	2.30	2.15	50	2.86	2.76	2.74	2.62	2.45	15
2.97	2.88	2.76	2.69	2.47	86	2.08	1.96	1.90	1.84	1.72	51	1	1	1.12	1.07	1	16
2.72	2.63	2.53	2.46	2.26	87	2.97	2.80	2.71	2.63	2.46	52	5.65	5.45	5.41	5.18	4.84	17
2.87	2.79	2.67	2.61	2.39	88	3.69	3.48	3.37	3.27	3.06	53	5.69	5.49	5.45	5.22	4.88	18
2.54	2.46	2.36	2.3	2.11	89	4.56	4.30	4.17	4.04	3.78	54	14.34	13.83	13.74	13.15	12.29	19
2.78	2.69	2.58	2.52	2.31	90	1.21	1.14	1.10	1.07	1	55	12.27	11.84	11.76	11.26	10.52	20
1.20	1.17	1.12	1.09	1	91	2.91	2.74	2.66	2.58	2.41	56	6.23	6.01	5.97	5.71	5.34	21
1	1.17	1	1.09	1	92	1	1	1	1	1	57	1	1.13	1	1.07	1	22
3.66	3.54	3.4	3.31	3.04	93	2.70	2.55	2.47	2.40	2.24	58	3.45	3.33	3.31	3.17	2.96	23
3.56	3.45	3.31	3.23	2.96	94	6.00	5.65	5.48	5.32	4.97	59	2.86	2.76	2.74	2.62	2.45	24
10.63	10.30	9.88	9.64	8.84	95	1.21	1.14	1.10	1.07	1	60	2.60	2.51	2.49	2.39	2.23	25
4.75	4.6	4.42	4.31	3.95	96	3.97	3.74	3.63	3.52	3.29	61	10.58	10.21	10.14	9.70	9.07	26
3.08	2.98	2.86	2.79	2.56	97	2.09	1.97	1.91	1.85	1.73	62	4.17	4.02	3.99	3.82	3.57	27
4.25	4.11	3.95	3.85	3.53	98	6.70	6.31	6.12	5.94	5.55	63	3.93	3.79	3.77	3.61	3.37	28
1.20	1.17	1.12	1.09	1	99	2.98	2.81	2.72	2.64	2.47	64	4.04	3.89	3.87	3.70	3.46	29
1	1.17	1	1.09	1	100	1.21	1	1	1.07	1	65	3.14	3.03	3.01	2.88	2.69	30
5.77	5.6	5.37	5.23	4.8	101	2.04	1.92	1.86	1.81	1.69	66	4.21	4.06	4.04	3.86	3.61	31
3.75	3.64	3.49	3.4	3.12	102	3.96	3.73	3.62	3.51	3.28	67	3.36	3.24	3.22	3.08	2.88	32
3.04	2.95	2.83	2.76	2.53	103	2.67	2.51	2.44	2.36	2.21	68	3.51	3.39	3.36	3.22	3.01	33
3.64	3.53	3.39	3.3	3.03	104	3.25	3.06	2.97	2.88	2.69	69	3.35	3.23	3.21	3.07	2.87	34
3.19	3.09	2.96	2.89	2.65	105	5.60	5.27	5.11	4.96	4.64	70	2.47	2.39	2.37	2.27	2.12	35

This input-output classification determines the distress threshold. Companies with financial distress tend to have a financial distress score of 1. The connection of these companies creates a distress threshold, based on which the corporate financial distress can be assessed. Thus, in Table 1, the efficiency of companies with financial inefficiency and distress is shown with "1". These companies are introduced as inefficient DMUs, and these DMUs are presented as candidates for financial distress. As shown in Table 1, over five years, DMU No. 57 has financial distress of 1 during all years. This DMU is predicted as a financially distressed unit in the future period. Based on the study's model, an improved solution is provided for this DMU. Also, DMUs No. 5, 16, 22, 37, 44, 65, 78, 92, and 100 are only financially distressed, equal to 1 in some years. In some companies, a growing trend can be seen during time-periods. For example, DMUs No. 9, 12, 49, 60, 77, 91, and 99 were considered inefficient units in the first time-period evaluated but increased their efficiency in pairwise comparisons over subsequent periods. The overall inefficiencies (WPFs) of these 105 companies over five time periods are presented in Table 2.

Table 2: Overall inefficiencies over five time periods

Overall WPF	DMU	Overall WPF	DMU	Overall WPF	DMU
4.136	71	2.626	36	2.626	1
4.516	72	1.042	37	1.042	2
1.874	73	2.9	38	2.9	3
1.894	74	1.076	39	1.076	4
4.48	75	2.438	40	2.438	5
3.802	76	3.95	41	3.95	6
1.116	77	1.962	42	1.962	7
1.058	78	2.57	43	2.57	8
3.122	79	1.07	44	1.07	9
4.06	80	2.514	45	2.514	10
3.49	81	2.69	46	2.69	11
2.79	82	2.36	47	2.36	12
3.522	83	2.912	48	2.912	13
3.692	84	1.104	49	1.104	14
1.116	85	2.37	50	2.37	15
2.754	86	1.9	51	1.9	16
2.52	87	2.714	52	2.714	17
2.666	88	3.374	53	3.374	18
2.354	89	4.17	54	4.17	19
2.576	90	1.104	55	1.104	20
1.116	91	2.66	56	2.66	21
1.052	92	1	57	1	22
3.39	93	2.472	58	2.472	23
3.302	94	5.484	59	5.484	24
9.858	95	1.104	60	1.104	25
4.406	96	3.63	61	3.63	26
2.854	97	1.91	62	1.91	27
3.938	98	6.124	63	6.124	28
1.116	99	2.724	64	2.724	29
1.052	100	1.056	65	1.056	30
5.354	101	1.864	66	1.864	31
3.48	102	3.62	67	3.62	32
2.822	103	2.438	68	2.438	33
3.378	104	2.97	69	2.97	34
2.956	105	5.116	70	5.116	35

This table shows the average inefficiency of each company during these five time periods. Based on these five time periods, DMUs 22 and 57 can be considered as absolutely inefficient units. Both DMUs were introduced as inefficient and distressed companies in comparison with other companies during all periods. It is also possible to rank other companies based on the inefficiency obtained. Based on the results obtained in Table 2, the companies listed in the Exchange and Securities Organization can be evaluated jointly over several time-periods. It means that the inefficiency of these companies can be identified during the assessed time-periods. This inefficiency indicates that these companies have been performing poorly over five time periods.

6 Discussion and Conclusion

Assessing and predicting corporate financial distress and bankruptcy is one of the most important topics that has always been considered by accounting and financial researchers. This issue is one of the primary research in finance. Activist investors typically use these models to make managerial, investment, and credit decisions. By predicting corporate financial distress, the financial distress cause and its solution can be found, and corporate bankruptcy can be prevented. This study evaluated and predicted the financial distress of companies listed on the Tehran Stock Exchange using the WPF-DEA technique. For this purpose, eight financial variables related to 105 manufacturing companies listed on the Tehran Stock Exchange were collected, and the model was estimated. The results were presented in the previous section. In this study, the DMUs' inefficiency was evaluated over different periods, and DMUs or companies that were candidates for financial distress in the future time-period were identified.

Liu and Chen's model ignores the temporal dimension, and it is, in fact, a static model, but the corporate performance throughout their life cycle is interconnected as a chain. In this study, to address this deficiency, the WPF-DEA model was presented to predict corporate financial distress dynamically. Li et al [21], Mousavi et al [28] and Rahimi et al [35] models are dynamic and have high power in predicting financial distress dynamically. However, these models are developed based on best practices and efficiency. Though BPF-DEA models are somehow capable of detecting the worst DMUs, measuring the DMUs' inefficiency in performance maximization scenario (by assigning the best DMU performance maximizing weights) does not seem very appropriate in the real world. Therefore, the study's model is dynamic for evaluating and ranking DMUs to identify best practices in a performance minimization scenario. The model would be more logical than the models presented by the above researchers. In general, this dynamic model is a variable for the desired periods. It allows us to consider changes over time, rank companies, and predict financial distress according to corporate inefficiencies. This model is also suitable to support managers, investors, and shareholders' decision-making regarding the future financial situation of their companies. It can adjust the inefficiency threshold over periods to predict corporate financial distress. In addition to identifying inefficient DMUs as financially distressed DMUs over time-periods and relationships between these time-periods, one of this study's contributions is to provide improved solutions for distressed DMUs that non-distressed DMUs can be used as a model for distressed DMUs. Another notable contribution of this study is that each of the distressed DMUs had over different time-periods. The following are two examples of these results.

Fig. 2: The inefficiency of Company No. 75 over five time periods

As shown in Fig.2, Company No. 75 had a growing efficiency during these five time periods. Fig.3 also shows the efficiency of Company No. 13.

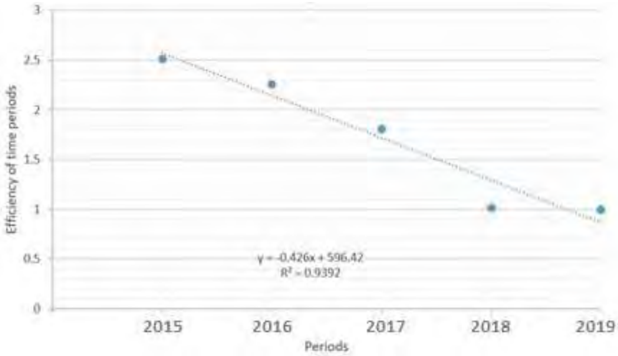

Fig. 3: The inefficiency of Company No. 13 over five time periods

As shown in Fig.3, the company is declining during these five time periods as it was introduced as a financially distressed company in 2019. The study's approach provides decision-makers with the ability to evaluate the inefficient DMUs during each time-period according to the relationships between these time-periods. The efficiency slope can also be evaluated over time-periods, and companies can be ranked based on this slope. This feature enables decision-makers to evaluate their DMU's efficiency in the future time-periods and invest in that DMU.

Using the presented method, by early detection of financial distress symptoms, companies can avoid bankruptcy and irreparable losses and minimize the risk of bankruptcy. On the other hand, stakeholders and other individuals and organizations interacting with companies can also make better decisions. Therefore, the companies listed on the stock exchange, banks, financial and credit institutions, mutual funds, and stock market investors are advised to use this technique. Companies can use this technique to anticipate financial distress early and take timely action. This method is beneficial for providing financial facilities and services and the credit rating systems of banks and financial and credit institutions. Investing companies and mutual funds can also use this method in portfolio management and advise their clients. Finally, this method could be used for stock market investors to make appropriate decisions about stock trading and optimize companies' portfolios. The results of this study indicated that financial ratios could be a good assessor for financial distress, and financial market participants can apply the financial ratios used in this study to predict corporate financial distress and decision-making processes. Finally, according to the study results, some suggestions can be made for future researchers:

- Use a more diverse and more extensive data set to predict financial distress.
- Use other data sources as inputs and outputs for DEA models
- Apply a variety of dynamic DEA models and dynamic network models in future research
- Given the importance of corporate efficiency, in addition to evaluating corporate financial efficiency, other characteristics, such as management and market efficiency, can be considered.
- Use this model to dynamically predict distress for manufacturing, service, and commercial companies and in more detail for various industries, including metals, tires, automobiles, etc.
- The approach proposed for inefficient DEA can be developed based on the network approach, and each DMU can be divided into sub-DMUs.
- Develop the basic model presented in this study based on a double frontier approach

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