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Research Paper

The Sensitivity Analysis and Stability Radius of the Cost Efficiency in Interval Data Envelopment Analysis: A Case Study from Tehran Stocks

Esmaeil Mombini[a], Mohsen Rostamy-Malkhalifeh[b,] *, Mansor Saraj[c]

[a]*Department of mathematics, Ahvaz Branch, Islamic Azad University, Ahvaz, Iran*
[b]*Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran*
[c]*Department of Mathematics, Faculty of Mathematical Sciences and Computer, Shahid Chamran University of Ahvaz, Ahvaz, Iran*

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ABSTRACT

Interval Data Envelopment Analysis (Interval DEA) is a methodology to assess the efficiency of decision-making units (DMUs) in the presence of interval data. Sensitivity analysis and stability evaluation of decision- making units are as the most important concerns of Decision Makers (DM). In the past decades, many scholars have been attracted to the stability evaluation of DMUs from different perspectives. This study focuses on the sensitivity analysis in DEA and proposes an approach to determine the stability radius of the cost efficiency of units with interval data. Potential application of our proposed methods is illustrated by a numerical example in the literature review.

1 Introduction

Data Envelopment Analysis is a mathematical programming method to assess the efficiency of decision-making units with multiple inputs and multiple outputs (Charnes et al. [7], Banker et al. [5]). The original DEA models suppose that all inputs and outputs have certain values. However, this assumption may be violated by the existence of uncertain data. Several scholars have been attracted to the problem of the evaluation of units with imprecise data. For example, Cooper et al. [12] developed Imprecise Data Envelopment Analysis (IDEA) method to the situation where there exist both imprecise and exactly-known data. Kim et al. [32] proposed a method to incorporate partial data into DEA. Lee et al. [36] suggested approaches to measure the inefficiency of units in IDEA. Despotits and Smirlis [15] showed that the units with imprecise data do not have constant efficiency scores and proposed a method to determine the upper and lower bounds for the efficiency score of units in the case of data uncertainty. See Cooper et [14], Entani et al. [17], Zhu [59] Jahanshahloo et al. [28], Wang et al. [57], Smirlis [47],

* Corresponding author. Tel: *+989122992645*
E-mail address: *mohsen_rostamy@yahoo.com*

Park [40, 41], Toloo et al. [52], Kao and Liu [31], Esmaeili [19], Hatami Marbini et al. [25] and Sun et al. [50] for more studies about IDEA models. Kordrostami and Jahani Sayyad Noveiri [33] presented an approach to measure the optimistic and pessimistic efficiency scores of units with fuzzy data. Amirteimoori et al. [2] proposed a method to obtain the interval efficiency scores of units in the presence of interval data. Azizi et al. [4] presented an approach to determine the upper and lower bounds for the efficiency scores of units with imprecise data. Jiang et al. [30] developed a model to evaluate the scale efficiency of DMUs with imprecise data. Toloo et al. [53] proposed models to obtain the interval efficiency scores of units based on the optimistic and pessimistic perspectives. DEA can be used to evaluate the different types of efficiency of DMUs, such as cost efficiency, revenue efficiency and profit efficiency of units. The cost efficiency (CE) can be interpreted as the ability of each decision-making unit with multiple inputs and multiple outputs to produce the current outputs at minimal cost. Farrell [23] introduced the concept of CE in the situation that the input and output values and input prices are known exactly. Färe [21] proposed methods that present empirical implementations of the cost efficiency measures in DEA. The problem of the measuring the cost efficiency of units has attracted attentions of several scholars. See, Cooper et al. [13], Sueyoshi [49], Tone [54], Tone and Sahoo ([55], [56]), Maniadakis and Thanassoulis [37], Sengupta and Sahoo [44], Sahoo et al. [42], Mirdehghan et al. [38], Ghiyasi [24], Tohidnia and Tohidi [51] among others.

Camanho and Dyson [6] and Fang and Li [20] evaluated the cost efficiency of units in the presence of data uncertainty. Kuosmanen and Post ([34, 35]) proposed models to determine the cost efficiency of units in the situation that the input prices are uncertain. Toloo et al. [52] considered the cost efficiency of units in the presence of interval data. Cherchye et al. [10] considered the cost efficiency analysis of research programs in economics and business management faculties. Mostafaee and Saljooghi [39] considered two scenarios for assessing the cost efficiency of DMUs. The first scenario evaluated the cost efficiency of units in the presence of data uncertainty and the second scenario assessed the cost efficiency of DMUs in the situation that both data and input prices were uncertain. However, few papers concern the evaluation of the cost efficiency in the presence of imprecise data. See Jahanshahloo et al. [29], Kuosmanen and Post ([34, 35]) and Camanho and Dyson [6] for more studies about the cost efficiency evaluation of units in the case of data uncertainty. The sensitivity analysis is one of the most important problems in DEA. The sensitivity analysis considers the possible changes in the data of DMUs in such a way that the efficiency of units does not change. Therefore, we have to find a region called the stability region with the mentioned feature. Mathematical methods, algorithmic and metric methods have been used by some scholars to perform sensitivity analysis. The first DEA sensitivity analysis method by Charnes et al. [8] examined the changes in a single output and showed that the sensitivity analysis methods in linear programming problems were not suitable in DEA. For more studies about the sensitivity analysis methods in DEA, see Charnes et al. [9], Seiford and Zhu [43], Allahyar and Rostamy-Malkhalifeh [1] and He et al. [26].

Given the importance of the sensitivity analysis in DEA, specially, in Interval DEA, this paper focuses on the sensitivity and stability analysis and proposes some models to determine the stability radius of the cost efficiency of DMUs with interval data. The rest of this paper is organized as follows: section 2 reviews the interval DEA and the cost efficiency evaluation preliminaries. Section 3 suggests models to determine the stability radius of the cost efficiency of DMUs with interval data. A numerical example and a case study reported in Mostafaee and Saljooghi [39] are applied to illustrate the potential application of our proposed methods. Section 5 concludes the paper.

2 Preliminaries

This section reviews some basic preliminaries for the cost efficiency evaluation in DEA and the efficiency assessment in interval DEA.

2.1 The Cost efficiency evaluation in DEA

Assume that we deal with a set of DMUs denoted by $DMU_j, j = 1, \dots, n,$ where each DMU consumes m different inputs to produce s different outputs. x_{ij} and y_{rj} are the i^{th} input and r^{th} output for $\mathrm{DMU_j}$, respectively, for $i = 1, \dots, m$ and $r = 1, \dots, s$. Also, Suppose that $c_o = (c_{1o}, \dots, c_{mo})$ is the unit cost vector. Färe et al. [21] proposed model (1) to obtain a measure of cost efficiency, when the input and output data are known exactly.

$$
\begin{aligned}
&\min c_o x \\
&\text{s.t.} \\
&\textstyle\sum_{j=1}^{n} \lambda_j x_{ij} \le x, && i = 1, \dots, m, \\
&\textstyle\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, && r = 1, \dots, s, \\
&\lambda_j \ge 0 && j = 1, \dots, n.
\end{aligned} \tag{1}
$$

Assume that x^* is an optimal solution for model (2), the cost efficiency of DMU_o was defined as follows by Färe et al. [21]:

$$
CE_o = \frac{c_o x^*}{c_o x_o} \tag{2}
$$

2.2 The efficiency evaluation in interval DEA

Consider a system of n DMUs, $DMU_j, j = 1, \dots, n,$ with the input vector $x_j = (x_{1j}, \dots, x_{mj})$ and the output vector $y_j = (y_{1j}, \dots, y_{sj})$. Also, suppose that the input and output values are not deterministic for all units and $x_{ij} \in [x_{ij}^L, x_{ij}^U]$ and $y_{rj} \in [y_{rj}^L, y_{rj}^U]$ in which the lower and upper bounds are positive and finite values. Wang et al. [57] formulated models (3a) and (3b) to evaluate the lower and upper bounds for the efficiency of DMU_o.

The Lower Efficiency Score:

$$
\begin{aligned}
&E_{oo}^L = \max \textstyle\sum_{r=1}^{s} \mu_r y_{ro}^L \\
&\text{s.t.} \\
&\textstyle\sum_{i=1}^{m} w_i x_{io}^U = 1, \\
&\textstyle\sum_{r=1}^{s} \mu_r y_{rj}^U - \sum_{i=1}^{m} w_i x_{ij}^L \le 0, && \forall j, \\
&w_i, \mu_r \ge \varepsilon, && \forall i, r.
\end{aligned} \tag{3a}
$$

The Upper Efficiency Score:

$$
\begin{aligned}
&E_{oo}^U = \max \textstyle\sum_{r=1}^{s} \mu_r y_{ro}^U \\
&\text{s.t.} \\
&\textstyle\sum_{i=1}^{m} w_i x_{io}^L = 1, \\
&\textstyle\sum_{r=1}^{s} \mu_r y_{rj}^U - \sum_{i=1}^{m} w_i x_{ij}^L \le 0, && \forall j, \\
&w_i, \mu_r \ge \varepsilon, && \forall i, r.
\end{aligned} \tag{3b}
$$

where ε is a non-Archimedean.

3 The stability radius of the cost efficiency of units with interval data

This section proposes some models to determine the stability radius of the cost efficiency of units with interval data. We deal with a set of DMUs, $DMU_j, j = 1, \dots, n,$ with m inputs $x_{ij} \in [x_{ij}^L, x_{ij}^U], i = 1, \dots, m,$ and s outputs $y_{rj} \in [y_{rj}^L, y_{rj}^U], r = 1, \dots, s$. Also, Suppose that $c_o = (c_{1o}, \dots, c_{mo})$ is the unit cost vector. In the following, we determine the minimal cost of DMU_o to produce its current outputs in the absence of DMU_o. For this purpose, based on the idea of Anderson and Peterson [3], we eliminate the unit under evaluation, DMU_o, from the set of the observed units with interval data and reformulate models (4) to obtain the minimal cost of DMU_o to produce its current outputs in the absence of DMU_o.

$$
\begin{aligned}
& z^* = \min c_o x \\
& s.t. \\
& \textstyle\sum_{j=1, j\neq o}^{n} \lambda_j x_{ij} \le x_i, \qquad i = 1, \dots, m \\
& \textstyle\sum_{j=1, j\neq o}^{n} \lambda_j y_{rj} \ge y_{ro}, \quad r = 1, \dots, s \\
& x \le x_o, x \ge 0, \\
& \lambda_j \ge 0, \qquad\qquad\qquad j = 1, \dots, n,
\end{aligned} \tag{4}
$$

where $x_{ij} \in [x_{ij}^L, x_{ij}^U], i = 1, \dots, m$ and $y_{rj} \in [y_{rj}^L, y_{rj}^U], r = 1, \dots, s$. It is clear that the units with imprecise data may not have constant minimal cost for generation the outputs. In the other word, the optimal value of model (4) may not be unique for all possible values of $x_{ij} (i = 1, \dots, m)$ and $y_{rj} (r = 1, \dots, s)$. Therefore, we propose two models (5a) and (5b) to determine the upper and lower bounds for the minimal cost of DMU_o to generate its current outputs in the absence of DMU_o.

The Lower Bound for the Minimal Cost:

$$
\begin{aligned}
& z^L = \min c_o x^l \\
& s.t. \\
& \sum_{j=1, j\neq o}^{n} \lambda_j x_{ij}^L \le x_i^l, \qquad i = 1, \dots, m \\
& \sum_{j=1, j\neq o}^{n} \lambda_j y_{rj}^U \ge y_{ro}^L, \quad r = 1, \dots, s \\
& x^l \le x_o^U, x^l \ge 0, \\
& \lambda_j \ge 0, \qquad\qquad\qquad j = 1, \dots, n.
\end{aligned} \tag{5a}
$$

The Upper Bound for the Minimal cost:

$$
\begin{aligned}
& z^L = \min c_o x^l \\
& s.t. \\
& \textstyle\sum_{j=1, j\neq o}^{n} \lambda_j x_{ij}^L \le x_i^l, \qquad i = 1, \dots, m \\
& \textstyle\sum_{j=1, j\neq o}^{n} \lambda_j y_{rj}^U \ge y_{ro}^L, \quad r = 1, \dots, s \\
& x^l \le x_o^U, x^l \ge 0, \\
& \lambda_j \ge 0, \qquad\qquad\qquad j = 1, \dots, n.
\end{aligned} \tag{5b}
$$

In model (5a) the levels of inputs and outputs are adjusted in unfavourably situation of the evaluated unit, DMU_o, and in favourably situation of the other units. Therefore, model (5a) determines the lower bound for the minimal cost of DMU_o. In model (5b) the levels of inputs and outputs are adjusted in favourably situation of the evaluated unit, DMU_o, and in unfavourably situation of the other units.

Hence, model (5b) determines the upper bound for the minimal cost of DMU_o. Therefore, $z^* \in [z^L, z^U]$.

3.1 The proposed model to measure the stability radius

In this section, we propose two models to determine the interval stability radiuses of the cost efficiency of units with interval data by using the optimal solutions of models (5a) and (5b). For this purpose, we consider the pre-determined vectors and propose models to determine the maximum possible movement along these directions such that the lower bound and upper bounds for the minimal cost of units do not change. Hence, the movement vectors $g_1 = \begin{pmatrix} -x_o \\ 0 \end{pmatrix}$, $g_2 = \begin{pmatrix} 0 \\ y_o \end{pmatrix}$ and $g_3 = \begin{pmatrix} -x_o \\ y_o \end{pmatrix}$ are defined and the inputs and the outputs of DMU_o are disturbed along these directions for determining the stability radius of the minimal cost of DMU_o. In the following, we consider the direction vectors $g_i = \begin{pmatrix} g_x \\ g_y \end{pmatrix}, i = 1,2,3,$ and formulates models (6a) and (6b) to determine the maximum possible movement along these direction vectors such that the lower bound and upper bounds for the minimal cost of DMU_o do not change.

The Stability Radius of the Lower Bound of the Minimal Cost of DMU_o:

$$\begin{aligned}
&\theta^L = \max \theta \\
&s.t. \\
&\textstyle\sum_{j=1, j\neq o}^{n} \lambda_j x_{ij}^L \leq x_{io}^U + \theta g_{ix}, && i = 1, \dots, m \\
&\textstyle\sum_{j=1, j\neq o}^{n} \lambda_j y_{rj}^U \geq y_{ro}^L + \theta g_{ry}, && r = 1, \dots, s \\
&C(x_o^U + \theta g_x) \leq C x^{l*}, \\
&\lambda_j \geq 0, && j = 1, \dots, n.
\end{aligned} \tag{6a}$$

The Stability Radius of the Upper Bound of the Minimal Cost of DMU_o:

$$\begin{aligned}
&\theta^L = \max \theta \\
&s.t. && (6b) \\
&\textstyle\sum_{j=1, j\neq o}^{n} \lambda_j x_{ij}^U \leq x_{io}^L + \theta g_{ix}, && i = 1, \dots, m \\
&\textstyle\sum_{j=1, j\neq o}^{n} \lambda_j y_{rj}^L \geq y_{ro}^U + \theta g_{ry}, && r = 1, \dots, s \\
&C(x_o^L + \theta g_x) \leq C x^{u*}, \\
&\lambda_j \geq 0, && j = 1, \dots, n,
\end{aligned} \tag{6b}$$

where x^{l*} and x^{u*} are the optimal solutions of models (5a) and (5b), respectively. Model (6a) is solved for three detection vectors $g_1 = \begin{pmatrix} -x_o \\ 0 \end{pmatrix}, g_2 = \begin{pmatrix} 0 \\ y_o \end{pmatrix}$ and $g_3 = \begin{pmatrix} -x_o \\ y_o \end{pmatrix}$ and the minimum amount of θ^L, obtained by considering these direction vectors, are introduced as the stability radius of the lower bound for the minimal cost of DMU_o. Hence, model (6a) determines the step length θ^L such that the lower bound for the minimal cost of DMU_o does not change along the directions $g_i = \begin{pmatrix} g_x \\ g_y \end{pmatrix}, i = 1, 2,3.$ Similarly, model (6b) is solved for three detection vectors $g_1 = \begin{pmatrix} -x_o \\ 0 \end{pmatrix}, g_2 = \begin{pmatrix} 0 \\ y_o \end{pmatrix}$ and $g_3 = \begin{pmatrix} -x_o \\ y_o \end{pmatrix}$ and the minimum amount of θ^U, obtained by considering these direction vectors, are introduced as the stability radius of the upper bound for the minimal cost of DMU_o. Hence, model (6b) determines the step length θ^U such that the upper bound for the minimal cost of DMU_o does not change along the directions $g_i = \begin{pmatrix} g_x \\ g_y \end{pmatrix}, i = 1, 2,3.$

4 Numerical example

In this section, the proposed models are illustrated in a numerical example with five DMUs and a case study, reported in Mostafaee and Saljooghi [39], with 20 DMUs.

Example 1: Consider five decision making units reported in Table 1. Each DMU consumes two inputs to produce two outputs. The last two columns of Table 1 show the input prices for all DMUs. In this example, the vector of input prices is not the same for all DMUs.

Table 1: The Data of Five DMUs in Example 1.

DMU	x_1^L	x_1^U	x_2^L	x_2^U	y_1^L	y_1^U	y_2^L	y_2^U	c_1	c_2
1	12	15	0.21	0.48	138	144	21	22	100	50
2	10	17	0.1	0.7	143	159	28	35	110	40
3	4	5	0.16	0.35	157	198	21	29	105	42
4	19	22	0.12	0.19	158	181	21	25	107	50
5	14	15	0.06	0.09	157	180	28	40	111	47

Now, we apply the proposed approaches to determine the stability radius of the lower bound and upper bounds for the minimal cost of units in the presence of interval data. Hence, models (5a) and (6a) are solved to obtain the lower bound of the minimal cost of units and the stability radius of it.

Table 2: The Results of Models (5a) and (6a): Example 1

DMU	x_1^l	x_2^l	z^L	θ^L
1	2.90	0.12	295.45	0.6348
2	3.86	0.15	431.01	0.6298
3	9.87	0.10	1040.94	0.0000
4	3.19	0.13	347.92	0.4138
5	3.86	0.15	435.95	0.0000

Table 3: The Results of Models (5b) and (6b)): Example 1

DMU	x_1^u	x_2^u	z^U	θ^U
1	5.24	0.37	542.14	0.0850
2	8.33	0.58	940.00	0.0000
3	18.92	0.11	1991.07	0.0000
4	5.95	0.42	657.74	0.0519
5	9.52	0.67	1088.48	0.0000

The results are reported in Table 2. The second and the third columns of Table 2 show the first and the second components of the input vector obtained by model (5a), respectively. The fourth column of this table reports the optimal value of model (5a) and the fifth column of Table 2 shows the stability radius of the lower bound for the minimal cost of DMUs in the presence of interval data. Similarly, models (5b) and (6b) are solved to obtain the upper bound for the minimal cost of units and the stability radius of it. The results are reported in Table 3. The second and the third columns of Table 6 show the first and the second components of the input vector obtained by model (5b), respectively. The fourth column of this table reports the optimal value of model (5b) and the fifth column of Table 3 shows the stability radius of the upper bound for the minimal cost of DMUs with interval data.

Example 2: In this example, the results of applying our proposed method to the dataset in Mostafaee and Saljooghi [39] are presented. This dataset has 20 decision making units with three inputs to produce five outputs. The data of the input ranges and the input prices for these units have been listed in Table 4. The data of the output ranges for these units have been listed in Table 5. Now, we apply our proposed method to determine the stability radius for the cost efficiency for this data set. Hence, models (5a) and (6a) are solved and the results are reported in Table 6. The second, the third and the fourth columns of Table 6 show the first, the second and the third components of the input vector obtained by model (5a), respectively.

Table 4: The Input Ranges and the Input Prices of Twenty DMUs in Example 2.

DMU	x_{1j}^L	x_{1j}^U	x_{2j}^L	x_{2j}^U	x_{3j}^L	x_{3j}^U	c_1	c_2	c_3
1	8254.56	8263.56	30.26	45.09	4847	5007	12	2	11
2	3600.53	38910.53	17.69	20.78	9005	10,032	11	1	12
3	5682.21	5697.21	17.47	19.39	15,823	17,101	10	4	14
4	512.76	600.76	17.8	25.18	18,319	21,305	14	3	12
5	12495.58	12531.58	15.39	21.35	1886	1875	14	1	14
6	11189.68	13193.68	19.02	34.3	14,527	14,533	14	1	13
7	771.61	809.64	16.34	20.12	13,977	14,056	12	2	13
8	4341.02	8347.33	27.75	40.34	9224	9618	14	0.5	12
9	1457.18	1958.18	19.73	20.01	9786	9961	12	4	11
10	9092.21	9306.25	11.89	25.89	8085	8268	10	2	10
11	3155.11	4195.11	20.27	22.08	1326	1345	11	3	14
12	8625.98	12356.03	22.5	32.45	4764	5543	12	1	14
13	16278.93	16679.45	24.23	30.23	9326	11,329	12	2	12
14	5010.87	5113.68	21.62	22.62	5814	5837	13	4	10
15	4011.24	4242.24	37.36	30.65	35,310	35,563	14	3	13
16	8702.27	8831.33	23.7	30.17	226,017	226,345	12	1	10
17	6927.43	8990.65	24.72	30.17	9852	10,063	13	2	10
18	850.67	1221.67	13.43	20.43	12,691	12,736	14	1	12
19	6181.84	10875.84	28.12	32.32	17,507	18,205	11	2	14
20	1261.57	2270.57	20.81	22.11	30,253	30,916	11	4	12

Table 5: The Output Ranges of Twenty DMUs in Example 2.

DMU	y_1^L	y_1^U	y_2^L	y_2^U	y_3^L	y_3^U	y_4^L	y_4^U	y_5^L	y_5^U
1	1,262,798	1,291,506	325,071	327,038	1,092,933	1,154,312	93128.57	93246.34	7575.97	7670.33
2	302,316	332,725	38,509	41,267	66,399	66,450	20179.39	20559.37	328.52	346.22
3	652,583	661,236	123,230	123,580	1,517,439	1,517,687	78297.51	88395.69	2409.54	2412.77
4	737,317	737,547	261,702	26,232	301,968	302,573	28734.36	34286.21	304.52	317.21
5	365,134	367,007	15,612	15,786	80,153	80,893	365,134	11,996	279	305
6	537,502	567,669	51,363	51,702	229,105	435,438	21798.65	23112.45	489.53	571.73
7	205,122	206,143	54,177	54,196	757,565	759,043	47568.64	47989.7	431.85	448.45
8	243,663	247,809	264,451	264,685	728,856	734,568	55581.36	56882.25	1727.73	1745.78
9	279,091	280,974	179,083	185,632	945,771	949,551	40436.67	41200.9	445.82	449.06
10	383,585	386,578	13,135	13,164	1,464,666	1,465,112	524689.8	526284.4	90.05	92.37
11	261,142	261,829	144,716	147,218	604,120	610,986	12480.8	12595.35	1161.27	1201.35
12	401,836	402,379	61,311	61,717	151,190	151,345	16264.01	20345.15	262.08	265.18
13	569,375	578,903	456,902	498,437	275,812	276,361	47051.4	47906.2	848.44	849.34
14	261,658	262,090	220,581	221,381	735,733	737,256	19613.36	22890.38	1224.77	1235.79
15	347,687	348,762	285,715	265,945	462,277	463,478	131041.6	131732.6	1925.56	1937.06
16	433,362	455,660	80,860	82,360	304,659	332,673	186072.3	187890.4	1286.52	1311.73
17	528,743	570,965	301,168	301,464	4,146,106	4,156,223	11096.29	11245.62	4291.84	4330.22
18	396,342	425,679	177,633	177,955	32,968	33,345	9463.04	10371.8	109.15	130.79
19	537,025	537,327	328,473	357,623	1,662,874	1,663,364	62951.63	63045.46	1585.29	1711.12
20	876,301	877,402	104,341	109,004	1,207,702	1,218,342	25554.16	28095.24	1094.32	1294.32

The fifth column of this table reports the optimal value of model (5a) and the sixth column of Table 6 shows the stability radius for the lower bound of the minimal cost of units. Similarly, models (5b) and (6b) are solved and the results are reported in Table 7.

Table 6: The Results of Models (5a) and (6a)

DMU	x_1^l	x_2^l	x_3^l	z^L	θ^L
1	20127.59	128.10	8570.23	336059.72	0.0000
2	1932.23	7.08	1134.59	34876.65	0.4916
3	5440.77	18.15	4751.11	120995.88	0.0446
4	6085.75	30.38	3085.38	122316.19	0.0000
5	6834.67	10.48	5879.95	178015.13	0.0000
6	3435.40	12.59	2017.24	74332.29	0.4629
7	2241.93	6.54	2299.09	56804.43	0.0000
8	6220.47	37.04	2881.85	121687.31	0.2099
9	4291.96	24.38	2331.57	77248.32	0.0000
10	46447.76	170.27	27273.69	737555.04	0.0000
11	3632.11	13.29	2307.33	72295.71	0.0000
12	2568.30	9.42	1508.08	51942.23	0.5502
13	9925.67	63.06	4236.10	170067.37	0.0000
14	4739.83	30.39	1998.03	81719.64	0.0441
15	7911.59	41.38	4190.48	165362.74	0.0000
16	4826.27	11.03	3686.83	94794.51	0.3562
17	21410.34	137.55	8998.14	368590.96	0.0000
18	3948.07	22.78	1815.35	77079.92	0.0000
19	7674.04	44.74	4002.11	140533.55	0.1669
20	5982.48	21.85	4186.31	116130.44	0.0000

Table 10: The Results of Models (5b) and (6b)

DMU	x_1^u	x_2^u	x_3^u	z^U	θ^U
1	21568.61	93.04	15084.33	424937.09	0.0000
2	2177.30	11.88	1319.26	39793.28	0.2525
3	6216.85	27.31	5062.13	133147.65	0.0000
4	4826.40	26.34	2924.38	102741.08	0.0000
5	2401.64	13.10	1455.18	54008.63	0.0820
6	3714.74	20.27	2250.81	81287.17	0.0636
7	2594.91	9.78	2388.06	62203.28	0.0000
8	7066.08	38.03	3498.90	140930.81	0.0000
9	4931.89	24.93	3249.12	95022.70	0.0000
10	20866.54	40.39	5961.46	268360.80	0.0000
11	3791.82	19.99	2468.40	76327.50	0.0000
12	2633.11	14.37	1595.43	53947.74	0.2357
13	14361.33	75.71	4782.58	229878.27	0.0000
14	6349.85	33.51	2173.38	104416.00	0.0000
15	7717.19	39.45	4956.28	172590.70	0.0260
16	5094.07	20.06	3903.75	100186.50	0.1317
17	28861.51	151.91	9253.33	468036.65	0.0000
18	4979.33	26.45	1960.97	93268.76	0.0000
19	10930.66	55.60	4039.12	176896.07	0.0000
20	6429.77	32.73	4470.19	124500.69	0.0000

The second, the third and the fourth columns of Table 7 show the first, the second and the third components of the input vector obtained by model (5b), respectively. The fifth column of this table reports the optimal value of model (5b) and the sixth column of Table 7 shows the stability radius of the upper bound for the minimal cost of units.

5 Conclusion

This study considered the stability radius of the cost efficiency of units with interval data based on the sensitivity analysis. For this purpose, we eliminated the unit under evaluation and proposed some models to evaluate this unit. The most important feature of the proposed models is that these models can be applied to determine the stability region in which the efficiency of units does not change. Finally, we proposed some models for introducing the stability radius of cost efficiency of units in the presence of interval data. The proposed approaches can help the managers to identify the permissible changes in the data of units such that their performances remain unchanged.

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