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# Increasing the Mathematical Literacy Based on the Problem Solving Teaching and Realistic Mathematics 

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#### Abstract

Purpose: The main of this research was to calculate the efficiency of 9th grade high school female students as well as educational process development to increase the math literacy. Methodology: This educational process is done by combining realistic mathematics education and the process of solving mathematical problems by quasi-experimental method. The statistical population of the study was 242 ninth grade students in district 19 of Tehran province in the 98-99 academic year; they were divided into two groups of 122 students as experimental group (A, B) and 122 students as control group (C, D). The data analysis method was based on Data Envelopment Analysis (DEA). The output-oriented Charles, Cooper, and Rhodes (CCR) model was used for determining the efficiency of the students. Findings: Each student was considered as a decision-making unit (DMU) in which inputs were mathematical pre-test scores and outputs were the mathematical post-test scores. $91.185 \%$ of class A students and $93.6 \%$ of class B students from the experimental group were efficient, which has much better offspring than the control group. Conclusion: The results showed that the students involved in problem solving and realistic math instruction were efficient and the Pisa test performed very well. In addition, data envelopment analysis was a suitable method for evaluating and analyzing students' annual performance.


## Keywords:

Data envelopment analysis (DEA), Mathematical literacy, ProblemSolving math (PSM), Realistic mathematics education (RME).

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## 1. Introduction

It can be said that educational activities in each country can be considered as an investment of one generation for another generation. In other words, the goals of the educational system are growth and development and empowerment of potential human talents. There are various factors that are involved in the process or giving and learning. Some of these factors determine the methods or giving and learning and textbooks and methods of evaluation and training programs. Increasing mathematical literacy is one of the most important goals of the educational systems of each society (Milinković, 2019).

Mathematics is a science that has internal order and adaptation, and teaching mathematics in schools should be done with the aim of developing mathematical literacy and increasing the ability of each student to use mathematical knowledge to solve life problems or understand real life conditions. Due to the realistic teaching of mathematics, real world problems are used as a source or starting point for learning and developing mathematical concepts. Developing students' math problem solving skills is an important goal in learning mathematics (Lavonen \& Mikkola, 2020).

To avoid the approach of teaching mechanical mathematics, new teaching methods should replace the old and traditional methods of teaching and learning mathematics. The role of mathematics in a national program is very important in any country, which is justified by the desired skills: "Development of concentration, art of economic life, power of expression, self-confidence, approach to discovery, and understanding of resources "Common, hard work quality." We use math operations every day (Stinson, 2020). We need math skills for all activities. Most normal children have problems and do not succeed in doing so. Students with learning difficulties are exposed to math problems. In this case, the role of the teacher is very important. The teacher must combine sufficient strategies for each student with interactive learning strategies. The role of the teacher is to create a learning environment in which there are many opportunities for active student participation, providing valuable information, and teaching specific skills and strategies (Vázquez-Cano, et al, 2020).

Among these, the role of reality-oriented mathematics and the mathematical relationship of the real world can be mentioned. One of the most important educational methods in teaching mathematics cuurently is the modeling of different concepts for the ease of learning and increasing problem-solving skills in students (Treffers, 1987), Presented an effective mathematical model for better mathematical learning: Given the importance and regarding the above-mentioned problems, students' mathematical literacy needs to be expanded (Stinson, 2020). In this mathematical model, a set of educational, research and service activities was considered as an input and output indicator. Due to significance of the indicators in the performance set, the ratio of the sum of the balanced outputs to the sum of the inputs was calculated as the efficiency of the decision maker unit (Charnes, et al, 1978).

Moreover, the total weighted ratio of outputs to inputs was calculated as the efficiency of decisionmaker unit (Sağlam, 2017). The issue of computing efficiency in training performance and human resources section is relatively new because the efficiency measurements have been studied so far (private, military, economic, factories). Emrouznejad, et al (2008) used DEA to analyze the statistical data in emotional intelligence as a method for evaluation in behavioral sciences. They used the Super Efficiency model to clean up students who demonstrated a good performance (Dickinson, et al, 2009). In this research, a new evaluation method (data envelopment analysis) has been tried to be used to assess students. Developing math problem-solving skills of students is an important goal in mathematics learning. New studies use realistic mathematical methods to improve mathematical abilities. Putri \&Lavenia Ulandari has improved student selfefficacy in their study by using a realistic math process (Putri, et al, 2019).

They tried to be effective in improving problem-solving methods in research by using teacher guidance and teaching realistic mathematics (Putri, et al, 2019). In another study, researchers attempted to demonstrate the ability to represent mathematical concepts using a realistic hypothesis so that the teacher could assess the individual abilities of each learner (Widada, et al, 2019). In a research, Lailiyah examined the
increasing skill of mathematical literacy by gender to address the goals of NCTM; according to the results, male students achieved better mathematical skills and literacy. To avoid the use of mechanical mathematical education, modern education methods should be replaced by traditional methods of education and math's learning (Lailiyah, 2017). Many of these functions are studied to increase student's math literacy to increase their learning and development opportunities. Hsiao Ching She et al. used math literacy in the PISA measure to try math and literacy. Assess students scientifically and be effective in improving it (She, et al, 2018).

Due to the importance of increasing mathematical literacy by modern standards, the main purpose of the present study is using new educational methodologies in math teaching to eliminate the students' disinterest toward math. The idea of un relatedness of math's concepts with real-world issues necessitates the realistic mathematics education (Esther, et al, 2018). In this regard, we have tried to improve mathematical literacy by using new methods of mathematics education. Objectives of the Study: The main objectives of the study are to find out:1) We can use new educational methods to increase students' mathematical literacy, 2) We can use Data Envelopment Analysis for the evaluation of students' efficiency and teaching methods assessment. Research Questions: 1) What is the prediction of students' performance, that were taught with realistic mathematics education and problem solving process that challenges the students' mathematical literacy? 2) The applicability of which teaching methods problem solving mathematics (PSM), realistic mathematics education (RME), or BOTH (PSM, RME), leads to the increase of students' mathematical literacy? 3) Is Data Envelopment Analysis (DEA) appropriate for the evaluation of students' performance, teaching methodologies (PSM, RME, BOTH), and research data analysis?

Some of the basic concepts used in this research are as follow: Data Envelopment Analysis (DEA): There are different methodologies for measuring the appropriateness and applicability; Data Envelopment Anslysis (DAE) today is one of the most important and the most applicable techniques. This technique is highly precise and accurate because of using mathematical techniques and avoiding mental and subjective methods. Charactristics and capabilities of the most of the evaluation models are summarized in this technique. Thus, recently, it is used noticably for the evaluation of decision making units (DMU) (Koyuncu, et al, 2016).

The data envelopment analysis as a nonparametric method is able to measure the relative efficiency of the decision-maker unit based on the total weighted ratio of inputs to multiple outputs without any assumption regarding the production function. This methodology involves techniques for assessing efficiency or measuring the productivity of decision makers. Using this method can provide better relative conditions for comparison and performance evaluation. DEA also provides many opportunities for collaboration between the analyst and the decision maker. Assume that n is the Decision Making Unit (DMU) in the form of $\{D M U j: j=1 \ldots n\}$, each of which uses different $m$ inputs to generate $s$ outputs, and yrj and xij are respectively the ith input $(I=1 \ldots m)$ and $r$ th output $(r=1 \ldots s)$ of the $j$ th decision maker $(j=1 \ldots n)$ (Wen, et al, 2015).


Fig1. Decision Making Unit
Charles, Cooper, and Rhodes who developed data envelopment analysis, defined it as follows (Stacey et al., 2015): "Data Envelopment Analysis is a mathematical programming model used for observed data, which is a new method for experimental estimation of weight ratios, or it provides a functional boundary such as a production function that is the basis of modern economics" (Sağlam, 2017). Types of data envelopment analysis models are increasing and they are becoming more specialized. However, the basis of all different types of DEA models is models with returns to the fixed CCR scale and return to the BCC variable scale. The data envelopment analysis model can also be categorized in terms of being input-oriented or output-oriented (Emrouznejad \& Yang, 2018).

- Return to fixed scale: input-oriented - output-oriented
- Return to variable scale: input-oriented - output-oriented

According to these definitions, the output-oriented CCR model was used for the assessment of the effective units. Output-oriented CCR model: The name of this model (CCR) is an acronym derived from the first letters of its developers: Charles, Cooper, and Rhodes (Sağlam, 2017). This model has a fixed-scale return Output-oriented models increase or maximize outputs when inputs are constant or decreasing. In fact, the goal of this model is to maximize the output, without incurring the number of inputs or incremental resource. This model is shown in Equation 1:

$$
\begin{aligned}
& \operatorname{Max} \varphi \\
& \sum_{j=1}^{n} \gamma_{j} x_{i j} \leq x_{i o}, i=1, \ldots, n \\
& \sum_{j=1}^{n} \gamma_{j} y_{r j} \geq \varphi y_{r o}, r=1, \ldots, s \\
& \gamma_{j} \geq 0 j=1, \ldots, n .
\end{aligned}
$$

The model is always valid and the optimal solution is confirmed if $1 \leq \varphi^{*}$. If $\varphi^{*}=1$, then DMU is a technical efficient in the output. If $\varphi^{*}<1$, then DMU is inefficient in the output. If this model is used, the data envelopment analysis can be used to calculate the efficiency of the units, which can produce more than one unit of the highest efficiency factor of 1 and can become effective (Prakash, 2019). Efficiency: The ratio of outputs to data, the ratio of the production or final services to the used resources. Efficiency is a general concept and refers to the degree of satisfaction, and the amount of achievement of the desired goals. Efficiency means the least amount of time or energy consumed for most of the work done (Prakash, 2019). Enhancing efficiency will effectively help to achieve organizational goals i.e., how well an organization uses its resources to achieve the best performance in a period. Efficient units are the units that their efficiency score equals "one". In this research, the three mentioned educational methodologies (PSM, RME, and BOTH) were examined. Then, their efficiency was compared to provide the educational recommendations.

Problem-solving Math (PSM): 1945 can be considered a milestone in the history of problem solving. In this year, George Polya's great work entitled "How to Solve the Problem" was published, which gave birth to a huge transformation in mathematical education, especially in general education courses around the world (Lasak, 2017). It is speculated that Socrates is the first person to teach mathematics based on a problemsolving approach (Nurkaeti \& Indonesia, 2018).

According to Polya, due to the influence of Socrates on solving mathematical problems, this teaching approach was also called "Socratic method". The problem-solving strategy from the perspective of George Polya takes place in four stages (Ryshke, 2011): a) Understanding the Problem (What is the question is asking for)? b) Deeper understanding of the problem and designing the map (Stacey et al., 2015) (How the components of the problem are connected and how these components relate to data)? c) Accomplishments of the map to help solve the problem (To prepare a map at this stage, the first and second steps must be done correctly and completed. If the student makes a mistake, $s / h e$ can repeat the path several times to reach a correct plan to solve the problem) (Lasak, 2017). d) Review and return (Controlling the correct implementation of the map (Nurkaeti \& Indonesia, 2018).

Schoenfeld defines a problem as an activity in which a student is engaged for finding a solution, and has no mathematical means available to reach the goal (Schoenfeld, 1992). In 1985, Schoenfeld concluded that the problem of problem-solving was to focus on four categories of knowledge: resources, approaches, control, belief systems (Lasak, 2017). The model selected in this research has been proposed by using and combining the solution of Polya's mathematical problem and Shewhart problem solving cycle, which has the following components: Definition D, Assessment A, P plan, Implementation I, Communication C (Sumirattana, et al, 2017). These components act like a cycle and have no starting or ending point, and no
special order and arrangement is required to use each component. One step can be omitted or repeated several times to complete the problem-solving cycle (Nurkaeti \& Indonesia, 2018).

Realistic Mathematics Education (RME): In the late 70s, explained the theory of mathematical education (new mathematics) in reforming the education and learning process in protest to the American movement and the mathematical education approach in the Netherlands (Heuvel-Panhuize, et al, 2000). The underlying philosophy of real-life mathematical education is that a learner needs to develop a mathematical understanding through working on the fields that he/she considers meaningful. According to, mathematics must have a true value, be close to the learner, and be relevant to the problems of society (Espinoza, et al, 2016). In fact, Bishop, et al (2017) emphasized mathematics as a human activity instead of thinking that it as a matter to be transmitted. He believed that training should provide a conducted opportunity for the learner to recreate mathematics through the modeling. Furthermore, in teaching mathematics, mathematics must be taught to be useful and connected to children's experiences of everyday life (Studies, 2017). Therefore, Freudenthal (1973) believed that the only way to learn math is to reinvent it under appropriate guidance, which should start from the early years of elementary education and naturally be extended throughout the years of study. Therefore, mathematical training in this method is based on reinvented guidance that the student can experience. Three topics that are considered in the realistic mathematics approach are: guided reinvention, didactical phenomenology, and self-developed model (Gravemeijer \& Terwel, 2000). These three components are connected like a lever (Studies, 2017).

Mathematical Literacy: It is competent (what do you mean by competent, please make sure it means what do you want to say) to apply conceptual knowledge and procedural knowledge and mathematical skills (Mariani et al., 2018). In other words, the ability of students to relate and apply their knowledge and math skills derived from the classroom in their daily lives (Kilpatrick, 2005). Thus, the definition of mathematical literacy includes the ability of a person to relate mathematical skills to different fields. In this sense, the underlying process and interpretation of mathematical literacy is a modeling that begins with an issue in realworld situations. Then, the real-world problem is formulated in mathematical language. Then, the math problem is solved and the answer is translated into the real-world language. To evaluate the mathematical literacy, in this research, the pre and post test scores of each student, who underwent CCR Output-Oriented Model, were analyzed with GAMS software (Calasan, et al, 2019). The efficiency or the inefficiency of each student is meant to be an increase in mathematical literacy (Milinković, 2019).

PISA pre-test and post-test: Program for International Student Assessment (PISA) is a test that examines math literacy-reading, literacy and science literacy (Adams \& Wu, 2000). This study was first performed in 2000 and is repeated every year. The study of PISA is being developed and implemented by OECD (Programmed for International Student Assessment \& Organization for Economic Co-operation and Development, 2009), which has 30 members from 30 industrialized nations of the world. It is noted that, non-member countries can also participate (Vázquez-Cano, et al, 2020). For example, in 2000, 11 nonmember countries and 30 member countries of OECD participated in the PISA test (Programmed for International Student Assessment \& Organisation for Economic Co-operation and Development 2009). The number has increased to more than 60 participants in 2006, but Iran has not yet participated in this study (Yeşim Özer Özkan, 2018) \& (Adams \& Wu, 2000).

## 2. Methodology

This quasi-experimental study was performed on the age group of 15 years and the ninth grade of the ninth academic year in Tehran province, region 19 in the academic year of 98-99. The choice of this age group is based on the Pisa exams so that students can answer the standard Pisa questions related to the ninth and eighth grades. In this study, each student is considered a DMU with both training realistic mathematics and problem-solving process. The input of each DMU (Student) includes three components of realistic mathematical instruction (Guided Reinvention, Mathematical Phenomenology and Self-Developed Model)
and five components of the Shewhart problem-solving (Definition, Assessment, Plan, Implementation and Communication). Outputs per DMU efficiency of each student will be measured using the mathematical literacy performance considering the score obtained in the pretest and post-test math. Using simple random sampling from among 22 districts of Tehran province, students of district 3 were randomly selected. Of the 20 female high schools in the area, two were randomly evaluated and surveyed. A total of 122 students were selected as the control group and 122 students as the experimental group (A, B).

Before conducting the experiment, a mathematical pre-test was performed for students who completed the same questions as the post-test with validity and reliability ( $r=0.209-0.743$, and $p=0.243-0.569$ ), and Cronbach's alpha was calculated at 0.754 . After the experiment, the post-test questions were selected based on the PISA 2015 test to evaluate the students' mathematical literacy and to calculate the final efficiency. The questions of this test had realistic mathematical content consistent with the goals of math problemsolving. Initially, mathematical literacy pre-test was given to students. Then, the problem-solving method was taught to the students of the experimental group using the Shewhart cycle and realistic mathematics for 8 weeks. The mathematical literacy post-test was performed in both experimental and control groups. Each test included 10 mathematical questions, each with a score of 10 and the whole test had 100 scores. In each question, eight components of problem-solving and realistic mathematics were selected as inputs and outputs, each component had 1.25 score. The Output-Oriented Model, in this research, 8 inputs and 8 outputs were used for each question which its components are mentioned in the above tables. For each solved problem or given score, in CCR Model, efficiency is considered. Then, the sum of efficiency analysis will be the test efficiency for each student. Ultimately, using CCR Model, the efficiencies of pre- and posttests are determined. After performing the mathematical pre-test among the experimental and control groups, the mathematical problem-solving and realistic mathematics were taught to experimental group students in 8 weeks. Thus,

Score INPUT \& OUTPUT Any Question: An example of the problem based on realistic mathematics education and PISA problem-solving (2015): A pizza seller sells two types of pizza with the same thickness in a small size with a diameter of 30 cm , and a price of $30 \$$, in a large size with a diameter of 40 cm , at a price of $40 \$$, Which pizza is worth buying?

DAPIC Math Problem-solving: 1. Define: (Pizza shape, definition of the circle) 1.25, 2. Assess: (Definition of the area and perimeter of the circle, smaller and larger) 1.25, 3. Plan: (Which approach is appropriate) 1.25, 4. Implement: (Find the perimeter and area and the appropriate ratio) 1.25, 5. Communicate (Review and analysis of the solution) 1.25, Realistic Mathematics Education (RME), 6. Guided reinvention: (innovation and communication with the real world) 1.25, 7. Didactical phenomenology (the real experience of buying pizza) 1.25, 8. Self-developed model: (Communication of Reality with Math) 1.25.

INPUT

- $I_{1}=$ define
- $I_{2}=$ assess
- $I_{3}=p l a n$
- $\mathrm{I}_{4}=$ implement
- $I_{5}=$ communicate
- $\mathrm{I}_{6}=$ Guided reinvention
- $\mathrm{I}_{7}=$ Didactical phenomenology


Fig2. Input Output Chart

- The first week- Definition of the problem: Students must first understand the problem well to solve the problem. Therefore, the first task to solve a problem is to understand the correct and complete problem. To solve a problem, the following should be clearly explained: what should be found? What is unknown? What is supposed? What is known? Students examine the content of the problem and selectively select the mathematical knowledge appropriate to each topic; the teacher guides the students upon their request. Students learn how to use and apply the learned mathematical definitions.
- The second week- problem assessment: At this stage, the student must learn how to relate the unknown and the known components. Students should evaluate the situations so that they can come up with mathematical models or simple methods to solve each problem. They should learn what the relationship is between data and the unknown in the problem. Students should learn to summarize data and unknowns to help them solve problems.
- The third week-Plan: Students are asked to submit a map to solve the problem and select the best applicable map after adding up the proposed maps. Each map should include a figure, table, and diagram of the data in the problem and students should be able to objectively understand the conditions of the problem so that they can guess and find the correct way to solve it.
- The fourth week- implementation: Once the map has been prepared, it should be implemented. The basic point is that the students have complete oversight over the progress of the project so that if they feel s/he might not solve the problem, they would be able to prepare a new plan.
- The fifth week- communication: Upon the completion of the implementation phase, the problem solver must have an overview of all the steps and try the solutions and arguments, and analyses their solution.
- The sixth week- Guided reinvention: Students must discover informal strategies for their mathematical knowledge under the guidance of a teacher. Be in a real problem-solving position to try to improve your math using the real world. Like buying and paying money, choosing a bigger pizza at a reasonable price, calculating the price of discounted goods and etc. At this stage, the real world can inspire mathematical thinking (Gravemeijer \& Terwel, 2000).
- The Seventh week- Didactical phenomenology: At this stage, the student learns how to generalize real life to the world of mathematics. The student tries to reinvent the learned mathematics with the real world (Gravemeijer \& Terwel, 2000). The use of real-life problems in similarity, communication, and modelling helps students to better understand the issues. Students learn meaningful learning (confusing word choice) and develop math ideas and problem-solving abilities.
- The Eighth week- self-developed model: Students should use the simple methods and models they have learned to solve math problems. They should build their own math model and generalize an informal model to a formal math model. This aim of this step is to connect the informal knowledge of mathematics to the formal knowledge of mathematics that the model itself has developed. The models developed by the students are a familiar point for students that become a part of him after the generalization and formalization of the model.


## 3. Findings

In each table, the DMU represents each ninth-grade student in district 19 of Tehran Province. The number in front of it is the student's performance score in the final exam in December compared to the October exam. How to calculate efficiency with data envelopment analysis method has been that students without receiving new test methods at the beginning of the test year based on Pisa questions 2012-2015, then after 8 weeks and training in problem solving and Realistic math will also be tested based on the Pisa 20122015 questions. The score calculated in each table by data envelopment analysis is the efficiency of each student. If this score is more than 1 and more than one, the student has performed better than his/her own score in the initial test; score below 1 means that the student is inefficient and that the student's performance was not appropriate compared to his/her initial test. Table 2 shows the performance results of experimental
and control groups with different teaching methods. Table 3 shows the results of experimental and control groups with the Only Problem-Solving teaching methods. Table 4 shows the results of experimental and control groups with the Only RME teaching methods. SE in the following table's abbreviation is Super Efficiency the research data in the output-oriented CCR model are as follow:

| CLASS | MEAN | STAND.DEV. | MAX | MIN | CLASS | MEAN | STAND.DEV. | MAX | MIN | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| POINT | POINT |  |  |  |  |  |  |  |  |  |
| $\boldsymbol{A}$ | 48.422 | 2.800 | 62 | 27 | $\boldsymbol{A}$ | 91.185 | 2.486 | 100 | 82 | 61 |
| $\boldsymbol{B}$ | 60.137 | 4.246 | 85.5 | 30 | $\boldsymbol{B}$ | 93.658 | 1.030 | 100 | 84 | 61 |
| $\boldsymbol{C}$ | 55.856 | 7.735 | 72 | 30.5 | $\boldsymbol{C}$ | 45.097 | 2.800 | 65 | 30 | 61 |
| $\boldsymbol{D}$ | 37.527 | 6.214 | 57.5 | 21 | $\boldsymbol{D}$ | 34.772 | 3.458 | 57 | 20 | 61 |

Table1. Statistical description of the 4 Class
According to the tables above in the post-test math, $91.185 \%$ of class A students and $93.6 \%$ of class B students from the experimental group were efficient. $45.097 \%$ class C students and $34.772 \%$ class D students from the control group were efficient in the test.

Table2. Super Efficiency of the experimental group CLASS a \& CLASS B \& Control group CLASS C \& CLASS D (TWO Process

| $\begin{gathered} \hline \text { CLAS } \\ \text { SA } \end{gathered}$ | SE | $\begin{gathered} \hline \text { CLAS } \\ \text { SA } \end{gathered}$ | SE | $\begin{gathered} \hline \text { CLAS } \\ \text { SA } \end{gathered}$ | SE | $\begin{gathered} \hline \text { CLAS } \\ \text { SA } \end{gathered}$ | SE | $\begin{gathered} \hline \text { CLAS } \\ \text { SA } \end{gathered}$ | SE | $\begin{gathered} \hline \text { CLAS } \\ \text { SA } \end{gathered}$ | SE | $\begin{gathered} \hline \text { CLAS } \\ \text { SA } \end{gathered}$ | SE | $\begin{gathered} \hline \text { CLAS } \\ \text { SA } \end{gathered}$ | SE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DMU | 1.3 | DMU | 1.2 | DMU | 0.7 | DMU | 1.0 | DMU | 0.9 | DMU | 0.3 | DMU | 0.7 | DMU | 1.0 |
| 1 | 2 | 31 | 7 | 1 | 4 | 31 | 0 | 1 | 1 | 31 | 6 | 1 | 4 | 31 | 0 |
| DMU | 0.9 | DMU | 1.6 | DMU | 0.6 | DMU | 1.3 | DMU | 0.7 | DMU | 0.1 | DMU | 0.6 | DMU | 0.7 |
| 2 | 1 | 32 | 8 | 2 | 7 | 32 | 1 | 2 | 8 | 32 | 8 | 2 | 7 | 32 | 6 |
| DMU | 0.8 | DMU | 1.4 | DMU | 0.6 | DMU | 1.2 | DMU | 0.8 | DMU | 0.6 | DMU | 0.6 | DMU | 0.9 |
| 3 | 4 | 33 | 6 | 3 | 9 | 33 | 5 | 3 | 8 | 33 | 5 | 3 | 9 | 33 | 8 |
| DMU | 0.9 | DMU | 1.0 | DMU | 0.7 | DMU | 1.2 | DMU | 0.7 | DMU | 0.6 | DMU | 0.7 | DMU | 0.8 |
| 4 | 3 | 34 | 9 | 4 | 4 | 34 | 0 | 4 | 8 | 34 | 4 | 4 | 4 | 34 | 9 |
| DMU | 0.8 | DMU | 1.7 | DMU | 0.7 | DMU | 0.9 | DMU | 0.8 | DMU | 0.4 | DMU | 0.7 | DMU | 0.9 |
| 5 | 7 | 35 | 0 | 5 | 3 | 35 | 7 | 5 | 7 | 35 | 4 | 5 | 3 | 35 | 7 |
| DMU | 1.7 | DMU | 1.1 | DMU | 0.7 | DMU | 1.3 | DMU | 0.9 | DMU | 0.7 | DMU | 0.7 | DMU | 0.7 |
| 6 | 1 | 36 | 4 | 6 | 3 | 36 | 4 | 6 | 2 | 36 | 7 | 6 | 3 | 36 | 6 |
| DMU | 0.8 | DMU | 1.0 | DMU | 0.8 | DMU | 1.0 | DMU | 0.1 | DMU | 0.8 | DMU | 0.8 | DMU | 0.4 |
| 7 | 9 | 37 | 1 | 7 | 5 | 37 | 8 | 7 | 8 | 37 | 8 | 7 | 5 | 37 | 4 |
| DMU | 1.9 | DMU | 1.3 | DMU | 0.8 | DMU | 1.3 | DMU | 0.8 | DMU | 0.2 | DMU | 0.8 | DMU | 0.6 |
| 8 | 8 | 38 | 3 |  | 4 | 38 | 1 | 8 | 7 | 38 | 1 | 8 | 4 | 38 | 7 |
| DMU | 0.9 | DMU | 1.1 | DMU | 0.9 | DMU | 1.1 | DMU | 1.0 | DMU | 1.3 | DMU | 0.9 | DMU | 0.7 |
| 9 | 0 | 39 | 6 | 9 | 2 | 39 | 4 | 9 | 0 | 39 | 3 | 9 | 2 | 39 | 7 |
| DMU | 2.3 | DMU | 1.2 | DMU | 0.8 | DMU | 1.1 | DMU | 0.0 | DMU | 1.0 | DMU | 0.8 | DMU | 0.7 |
| 10 | 0 | 40 | 9 | 10 | 4 | 40 | 9 | 10 | 3 | 40 | 3 | 10 | 4 | 40 | 6 |
| DMU | 0.9 | DMU | 1.2 | DMU | 1.0 | DMU | 0.9 | DMU | 0.1 | DMU | 1.1 | DMU | 0.9 | DMU | 0.9 |
| 11 | 6 | 41 | 7 | 11 | 0 | 41 | 6 | 11 |  | 41 | 3 | 11 | 9 | 41 | 6 |
| DMU | 2.3 | DMU | 1.6 | DMU | 0.8 | DMU | 1.3 | DMU | 0.8 | DMU | 0.4 | DMU | 0.8 | DMU | 0.9 |
| 12 | 5 | 42 | 8 | 12 | 8 | 42 | 0 | 12 | 4 | 42 | 4 | 12 | 8 | 42 | 7 |
| DMU | 1.5 | DMU | 1.4 | DMU | 1.0 | DMU | 1.3 | DMU | 0.5 | DMU | 0.8 | DMU | 1.0 | DMU | 0.5 |
| 13 | 3 | 43 | 6 | 13 | 3 | 43 | 1 | 13 | 4 | 43 | 9 | 13 | 3 | 43 | 6 |
| DMU | 1.1 | DMU | 1.3 | DMU | 0.9 | DMU | 0.9 | DMU | 0.8 | DMU | 0.5 | DMU | 0.9 | DMU | 0.9 |
| 14 | 8 | 44 | 2 | 14 | 4 | 44 | 8 | 14 | 7 | 44 | 8 | 14 | 4 | 44 | 8 |
| DMU | 1.7 | DMU | 1.4 | DMU | 1.1 | DMU | 0.9 | DMU | 0.2 | DMU | 1.2 | DMU | 0.9 | DMU | 0.9 |
| 15 | 2 | 45 | 4 | 15 | 9 | 45 | 6 | 15 | 9 | 45 | 2 | 15 | 6 | 45 | 6 |
| DMU | 1.2 | DMU | 1.6 | DMU | 1.3 | DMU | 1.0 | DMU | 0.2 | DMU | 0.9 | DMU | 0.9 | DMU | 0.5 |
| 16 | 2 | 46 | 3 | 16 | 9 | 46 | 2 | 16 | 6 | 46 | 9 | 16 | 7 | 46 | 8 |
| DMU | 1.2 | DMU | 1.0 | DMU | 1.5 | DMU | 0.9 | DMU | 0.8 | DMU | 0.9 | DMU | 0.5 | DMU | 0.9 |
| 17 | 7 | 47 | 2 | 17 | 5 | 47 | 2 | 17 | 8 | 47 | 8 | 17 | 5 | 47 | 2 |


| DMU | 1.7 | DMU | 1.0 | DMU | 1.2 | DMU | 0.8 | DMU | 0.2 | DMU | 0.5 | DMU | 0.7 | DMU | 0.8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 18 | 7 | 48 | 4 | 18 | 3 | 48 | 8 | 18 | 7 | 48 | 6 | 18 | 6 | 48 | 8 |
| DMU | 1.5 | DMU | 1.2 | DMU | 1.1 | DMU | 1.2 | DMU | 0.3 | DMU | 1.4 | DMU | 0.5 | DMU | 0.2 |
| 19 | 0 | 49 | 0 | 19 | 9 | 49 | 7 | 19 | 3 | 49 |  | 19 | 6 | 49 | 7 |
| DMU | 1.0 | DMU | 1.3 | DMU | 0.9 | DMU | 1.1 | DMU | 0.1 | DMU | 1.2 | DMU | 0.9 | DMU | 0.4 |
| 20 | 3 | 50 | 2 | 20 | 4 | 50 | 6 | 20 | 5 | 50 | 2 | 20 | 4 | 50 | 4 |
| DMU | 1.3 | DMU | 1.2 | DMU | 1.1 | DMU | 1.0 | DMU | 0.1 | DMU | 1.0 | DMU | 0.8 | DMU | 1.0 |
| 21 | 1 | 51 | 7 | 21 | 4 | 51 | 0 | 21 | 0.1 | 51 | 9 | 21 | 9 | 51 | 0 |
| DMU | 1.4 | DMU | 1.0 | DMU | 1.0 | DMU | 1.1 | DMU | 0.3 | DMU | 0.2 | DMU | 1.0 | DMU | 0.6 |
| 22 | 5 | 52 | 4 | 22 | 4 | 52 | 9 | 22 | 0.3 | 52 | 2 | 22 | 4 | 52 | 7 |
| DMU | 2.1 | DMU | 1.4 | DMU | 1.7 | DMU | 1.1 | DMU | 0.4 | DMU | 1.1 | DMU | 0.7 | DMU | 0.6 |
| 23 | 0 | 53 | 6 | 23 | 9 | 53 | 7 | 23 | 9 | 53 | 5 | 23 | 9 | 53 | 7 |
| DMU | 1.2 | DMU | 1.3 | DMU | 1.1 | DMU | 1.0 | DMU | 0.3 | DMU | 0.2 | DMU | 0.7 | DMU | 0.6 |
| 24 | 5 | 54 | 7 | 24 | 2 | 54 | 6 | 24 | 3 | 54 | 8 | 24 | 7 | 54 |  |
| DMU | 1.6 | DMU | 1.3 | DMU | 0.8 | DMU | 1.2 | DMU | 0.8 | DMU | 0.3 | DMU | 0.8 | DMU | 0.3 |
| 25 | 5 | 55 | 5 | 25 | 5 | 55 | 5 | 25 | 8 | 55 | 2 | 25 | 5 | 55 | 3 |
| DMU | 1.0 | DMU | 1.8 | DMU | 1.3 | DMU | 1.2 | DMU | 0.3 | DMU | 0.4 | DMU | 0.7 | DMU | 0.6 |
| 26 | 9 | 56 | 6 | 26 | 3 | 56 | 5 | 26 | 1 | 56 | 2 | 26 | 7 | 56 | 7 |
| DMU | 1.0 | DMU | 2.2 | DMU | 1.0 | DMU | 2.4 | DMU | 1.0 | DMU | 0.1 | DMU | 1.0 | DMU | 0.4 |
| 27 | 8 | 57 | 5 | 27 | 9 | 57 | 2 | 27 | 3 | 57 | 7 | 27 | 9 | 57 | 5 |
| DMU | 1.4 | DMU | 1.5 | DMU | 1.5 | DMU | 1.8 | DMU | 0.3 | DMU | 0.9 | DMU | 0.5 | DMU | 0.8 |
| 28 | 0 | 58 | 3 | 28 | 6 | 58 | 2 | 28 | 1 | 58 | 7 | 28 | 6 | 58 | 2 |
| DMU | 1.0 | DMU | 1.4 | DMU | 0.9 | DMU | 2.3 | DMU | 0.0 | DMU | 0.4 | DMU | 0.9 | DMU | 0.5 |
| 29 | 0 | 59 | 6 | 29 | 6 | 59 | 5 | 29 | 8 | 59 | 4 | 29 | 6 | 59 | 6 |
| DMU | 2.0 | DMU | 1.9 | DMU | 1.0 | DMU | 4.1 | DMU | 0.6 | DMU | 0.2 | DMU | 1.0 | DMU | 0.1 |
| 30 | 1 | 60 | 5 | 30 | 0 | 60 | 3 | 30 | 8 | 60 | 4 | 30 | 0 | 60 | 3 |
|  |  | DMU | 1.2 |  |  | DMU | 1.5 |  |  | DMU | 0.8 |  |  | DMU | 0.9 |
|  | 61 | 7 |  |  | 61 | 0 |  |  | 61 | 9 |  |  | 61 | 8 |  |

Table3. Super Efficiency of the experimental group CLASS a \& CLASS B \& Control group CLASS C \& CLASS D (Only

| $\begin{gathered} \hline \text { CLA } \\ \text { SSA } \end{gathered}$ | SE | $\begin{aligned} & \hline \text { CLA } \\ & \text { SSA } \end{aligned}$ | SE | $\begin{aligned} & \hline \text { CLA } \\ & \text { SSA } \end{aligned}$ | SE | $\begin{aligned} & \hline \text { CLA } \\ & \text { SSA } \end{aligned}$ | SE | $\begin{aligned} & \hline \text { CLA } \\ & \text { SSA } \end{aligned}$ | SE | $\begin{aligned} & \hline \text { CLA } \\ & \text { SSA } \end{aligned}$ | SE | $\begin{gathered} \hline \text { CLA } \\ \text { SSA } \end{gathered}$ | SE | $\begin{gathered} \hline \text { CLA } \\ \text { SSA } \end{gathered}$ | SE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \hline \text { DM } \\ & \text { U1 } \end{aligned}$ | 0.63 | $\begin{aligned} & \hline \text { DM } \\ & \text { U31 } \end{aligned}$ | $\begin{aligned} & 0 . \\ & 45 \end{aligned}$ | DM <br> U1 | 1.00 | DM U31 | 0.64 | DM U1 | 0.03 | $\begin{aligned} & \text { DM } \\ & \text { U31 } \end{aligned}$ | 0.83 | $\begin{gathered} \hline \mathrm{DM} \\ \mathrm{U} 1 \end{gathered}$ | Infea sible | $\begin{aligned} & \hline \text { DM } \\ & \text { U31 } \end{aligned}$ | 1.00 |
| $\begin{aligned} & \hline \text { DM } \\ & \text { U2 } \end{aligned}$ | 0.37 | $\begin{aligned} & \hline \text { DM } \\ & \text { U32 } \end{aligned}$ | $\begin{aligned} & \hline 0 . \\ & 56 \end{aligned}$ | $\begin{gathered} \hline \mathrm{DM} \\ \mathrm{U} 2 \end{gathered}$ | 1.25 | $\begin{array}{r} \mathrm{DM} \\ =\quad \mathrm{U} 32 \end{array}$ | 0.77 | $\begin{gathered} \text { DM } \\ \text { U2 } \end{gathered}$ | 0.04 | $\begin{aligned} & \hline \text { DM } \\ & \text { U32 } \end{aligned}$ | 0.22 | $\begin{gathered} \text { DM } \\ \text { U2 } \end{gathered}$ | Infea sible | $\begin{aligned} & \hline \text { DM } \\ & \text { U32 } \end{aligned}$ | 0.90 |
| $\begin{gathered} \hline \text { DM } \\ \text { U3 } \end{gathered}$ | 0.40 | $\begin{aligned} & \hline \text { DM } \\ & \text { U33 } \end{aligned}$ | $\begin{gathered} \hline 1 . \\ 00 \end{gathered}$ | $\begin{aligned} & \hline \text { DM } \\ & \text { U3 } \end{aligned}$ | 1.00 | $\begin{aligned} & \hline \text { DM } \\ & \text { U33 } \end{aligned}$ | 1.00 | $\begin{gathered} \hline \text { DM } \\ \text { U3 } \end{gathered}$ | 0.02 | $\begin{aligned} & \hline \text { DM } \\ & \text { U33 } \end{aligned}$ | 0.97 | $\begin{gathered} \hline \text { DM } \\ \text { U3 } \end{gathered}$ | 0.33 | $\begin{aligned} & \hline \mathrm{DM} \\ & \mathrm{U} 33 \end{aligned}$ | 0.87 |
| $\begin{gathered} \hline \mathrm{DM} \\ \mathrm{U} 4 \end{gathered}$ | 1.00 | $\begin{aligned} & \hline \text { DM } \\ & \text { U34 } \end{aligned}$ | $\begin{gathered} \hline 1 . \\ 00 \end{gathered}$ | $\begin{gathered} \hline \mathrm{DM} \\ \mathrm{U} 4 \end{gathered}$ | 1.95 | $\begin{aligned} & \hline \text { DM } \\ & \text { U34 } \end{aligned}$ | 0.63 | $\begin{aligned} & \hline \mathrm{DM} \\ & \mathrm{U} 4 \end{aligned}$ | 0.30 | $\begin{aligned} & \hline \text { DM } \\ & \text { U34 } \end{aligned}$ | 0.41 | $\begin{gathered} \hline \mathrm{DM} \\ \mathrm{U} 4 \end{gathered}$ | 0.20 | $\begin{aligned} & \hline \text { DM } \\ & \text { U34 } \end{aligned}$ | 0.94 |
| $\begin{gathered} \text { DM } \\ \text { U5 } \end{gathered}$ | 1.25 | $\begin{aligned} & \text { DM } \\ & \text { U35 } \end{aligned}$ | $\begin{gathered} 1 . \\ 00 \end{gathered}$ | $\begin{aligned} & \text { DM } \\ & \text { U5 } \end{aligned}$ | 5.00 | $\begin{aligned} & \text { DM } \\ & \text { U35 } \end{aligned}$ | 1.67 | $\begin{gathered} \text { DM } \\ \text { U5 } \end{gathered}$ | 0.44 | $\begin{aligned} & \text { DM } \\ & \text { U35 } \end{aligned}$ | infea <br> sible | $\begin{gathered} \text { DM } \\ \text { U5 } \end{gathered}$ | 0.00 | $\begin{aligned} & \text { DM } \\ & \text { U35 } \end{aligned}$ | 1.00 |
| $\begin{gathered} \hline \text { DM } \\ \text { U6 } \end{gathered}$ | 1.00 | $\begin{aligned} & \hline \text { DM } \\ & \text { U36 } \end{aligned}$ | $\begin{aligned} & \hline 0 . \\ & 34 \end{aligned}$ | $\begin{gathered} \hline \text { DM } \\ \text { U6 } \end{gathered}$ | 1.00 | $\begin{aligned} & \hline \text { DM } \\ & \text { U36 } \end{aligned}$ | 2.36 | $\begin{gathered} \hline \text { DM } \\ \text { U6 } \end{gathered}$ | 1.00 | $\begin{aligned} & \hline \text { DM } \\ & \text { U36 } \end{aligned}$ | 0.53 | $\begin{gathered} \hline \text { DM } \\ \text { U6 } \end{gathered}$ | 0.33 | $\begin{aligned} & \hline \text { DM } \\ & \text { U36 } \end{aligned}$ | 0.53 |
| $\begin{gathered} \text { DM } \\ \text { U7 } \end{gathered}$ | 1.38 | $\begin{aligned} & \text { DM } \\ & \text { U37 } \end{aligned}$ | $\begin{aligned} & 0 . \\ & 53 \\ & 53 \end{aligned}$ | $\begin{aligned} & \hline \mathrm{DM} \\ & \hline \end{aligned}$ | 0.56 | $\begin{aligned} & \text { DM } \\ & \text { U37 } \end{aligned}$ | 1.00 | $\begin{gathered} \text { DM } \\ \text { U7 } \end{gathered}$ | 0.99 | $\begin{aligned} & \text { DM } \\ & \text { U37 } \end{aligned}$ | 0.31 | $\begin{gathered} \text { DM } \\ \text { U7 } \end{gathered}$ | 0.45 | $\begin{aligned} & \text { DM } \\ & \text { U37 } \end{aligned}$ | 1.00 |
| $\begin{gathered} \hline \text { DM } \\ \text { U8 } \end{gathered}$ | infea <br> sible | $\begin{aligned} & \hline \text { DM } \\ & \text { U38 } \end{aligned}$ | $\begin{gathered} \hline 2 . \\ 00 \\ \hline \end{gathered}$ | $\begin{aligned} & \hline \text { DM } \\ & \text { U8 } \end{aligned}$ | infea <br> sible | $\begin{aligned} & \hline \mathrm{DM} \\ & \text { U38 } \end{aligned}$ | 1.01 | $\begin{gathered} \hline \mathrm{DM} \\ \mathrm{U} 8 \end{gathered}$ | infea <br> sible | $\begin{aligned} & \hline \text { DM } \\ & \text { U38 } \end{aligned}$ | 0.93 | $\begin{gathered} \hline \mathrm{DM} \\ \mathrm{U} 8 \end{gathered}$ | Infea <br> sible | $\begin{aligned} & \hline \text { DM } \\ & \text { U38 } \end{aligned}$ | 0.70 |
| $\begin{gathered} \hline \mathrm{DM} \\ \text { U9 } \end{gathered}$ | 0.44 | $\begin{aligned} & \hline \text { DM } \\ & \text { U39 } \end{aligned}$ | $\begin{aligned} & \hline 0 . \\ & 60 \end{aligned}$ | $\begin{gathered} \hline \text { DM } \\ \text { U9 } \end{gathered}$ | 1.00 | $\begin{aligned} & \hline \text { DM } \\ & \text { U39 } \end{aligned}$ | infea <br> sible | $\begin{gathered} \hline \mathrm{DM} \\ \text { U9 } \end{gathered}$ | 0.88 | $\begin{aligned} & \hline \text { DM } \\ & \text { U39 } \end{aligned}$ | Infea <br> sible | $\begin{gathered} \hline \text { DM } \\ \text { U9 } \end{gathered}$ | 0.54 | $\begin{aligned} & \hline \text { DM } \\ & \text { U39 } \end{aligned}$ | Infea <br> sible |
| $\begin{aligned} & \text { DM } \\ & \text { U10 } \end{aligned}$ | 1.00 | $\begin{aligned} & \text { DM } \\ & \text { U40 } \end{aligned}$ | $\begin{gathered} 1 . \\ 00 \end{gathered}$ | $\begin{aligned} & \mathrm{DM} \\ & \mathrm{U} 10 \end{aligned}$ | 1.00 | $\begin{aligned} & \hline \text { DM } \\ & \text { U40 } \end{aligned}$ | 0.51 | $\begin{aligned} & \text { DM } \\ & \text { U10 } \end{aligned}$ | 1.00 | $\begin{aligned} & \text { DM } \\ & \text { U40 } \end{aligned}$ | 0.49 | $\begin{aligned} & \text { DM } \\ & \text { U10 } \end{aligned}$ | 1.00 | $\begin{aligned} & \text { DM } \\ & \text { U40 } \end{aligned}$ | 0.09 |
| $\begin{aligned} & \hline \text { DM } \\ & \text { U11 } \end{aligned}$ | 0.37 | $\begin{aligned} & \hline \text { DM } \\ & \text { U41 } \end{aligned}$ | $\begin{aligned} & \hline 0 . \\ & 73 \end{aligned}$ | $\begin{aligned} & \text { DM } \\ & \text { U11 } \end{aligned}$ | 0.63 | $\begin{aligned} & \mathrm{DM} \\ & \mathrm{U} 41 \end{aligned}$ | 0.61 | $\begin{aligned} & \mathrm{DM} \\ & \mathrm{U} 11 \end{aligned}$ | 0.92 | $\begin{aligned} & \mathrm{DM} \\ & \mathrm{U} 41 \end{aligned}$ | 0.73 | $\begin{aligned} & \text { DM } \\ & \text { U11 } \end{aligned}$ | 0.37 | $\begin{aligned} & \hline \mathrm{DM} \\ & \mathrm{U} 41 \end{aligned}$ | 0.80 |



Table4. Super Efficiency of the experimental group CLASS a \& CLASS B \& Control group CLASS C \& CLASS D (Only RME

| CLA | SE | CLA | SE | CLA | SE | CLA | SE | CLA | SE | CLA | SE | CLA | SE | CLA | SE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SSA |  | SSA |  |  |  |  |  |  |  |  |  |  |  |  |  |
| DM | 0.75 | DM | 1. | DM | Infeas | DM | 0. | DM | 0.20 | DM | 0. | DM | 0.25 | DM | 0. |
| U1 |  | U31 | 00 | U1 | ible | U31 | 57 | U1 |  | U31 | 36 | U1 |  | U31 | 40 |
| DM | Infeas | DM | 1. | DM | Infeas | DM | 0. | DM | Infeas | DM | 0. | DM | Infeas | DM | 0. |
| U2 | ible | U32 | 00 | U2 | ible | U32 | 45 | U2 | ible | U32 | 50 | U2 | ible | U32 | 48 |
| DM | 0.80 | DM | 1. | DM | 1.00 | DM | 1. | DM | 0.67 | DM | 1. | DM | 0.67 | DM | 1. |
| U3 |  | U33 | 00 | U3 |  | U33 | 17 | U3 |  | U33 | 00 | U3 |  | U33 | 25 |
| DM | 1.00 | DM | 0. | DM | 1.00 | DM | 0. | DM | 0.19 | DM | 0. | DM | 0.23 | DM | 0. |
| U4 |  | U34 | 45 | U4 |  | U34 | 56 | U4 |  | U34 | 57 | U4 |  | U34 | 26 |
| DM | 2.00 | DM | 1. | DM | 0.50 | DM | 0. | DM | 0.80 | DM | 1. | DM | 1.00 | DM | 1. |
| U5 |  | U35 | 00 | U5 |  | U35 | 83 | U5 |  | U35 | 00 | U5 |  | U35 | 00 |


| $\begin{gathered} \hline \text { DM } \\ \text { U6 } \end{gathered}$ | 0.50 | $\begin{aligned} & \hline \text { DM } \\ & \text { U36 } \end{aligned}$ | $\begin{aligned} & \hline 1 . \\ & 57 \end{aligned}$ | $\begin{gathered} \hline \text { DM } \\ \text { U6 } \end{gathered}$ | 1.00 | $\begin{aligned} & \hline \text { DM } \\ & \text { U36 } \end{aligned}$ | $\begin{gathered} 1 . \\ 42 \end{gathered}$ | $\begin{gathered} \hline \text { DM } \\ \text { U6 } \end{gathered}$ | 0.24 | $\begin{aligned} & \hline \text { DM } \\ & \text { U36 } \end{aligned}$ | $\begin{aligned} & \hline 0 . \\ & 50 \end{aligned}$ | $\begin{gathered} \hline \text { DM } \\ \text { U6 } \end{gathered}$ | 0.20 | $\begin{aligned} & \hline \text { DM } \\ & \text { U36 } \end{aligned}$ | $\begin{aligned} & \hline 0 . \\ & 21 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DM | 0.57 | DM | 0. | DM | 0.42 | DM | 0. | DM | 0.48 | DM | 0. | DM | 0.38 | DM | 0. |
| U7 |  | U37 | 30 | U7 |  | U37 | 60 | U7 |  | U37 | 40 | U7 |  | U37 | 29 |
| DM | 3.50 | DM | 1. | DM | 2.00 | DM | 0. | DM | 2.50 | DM | 0. | DM | 2.00 | DM | 0. |
| U8 |  | U38 | 00 | U8 |  | U38 | 60 | U8 |  | U38 | 25 | U8 |  | U38 | 25 |
| DM | 0.57 | DM | 0. | DM | 1.00 | DM | 0. | DM | 0.55 | DM | 0. | DM | 0.43 | DM | 0. |
| U9 |  | U39 | 29 | U9 |  | U39 | 79 | U9 |  | U39 | 50 | U9 |  | U39 | 25 |
| DM | 3.50 | DM | 0. | DM | 2.67 | DM | 0. | DM | 0.21 | DM | 0. | DM | 0.19 | DM | 0. |
| U10 |  | U40 | 15 | U10 |  | U40 | 60 | U10 |  | U40 | 50 | U10 |  | U40 | 21 |
| DM | 1.00 | DM | 0. | DM | 0.69 | DM | 1. | DM | 0.43 | DM | 0. | DM | 0.43 | DM | 0. |
| U11 |  | U41 | 39 | U11 |  | U41 | 03 | U11 |  | U41 | 45 | U11 |  | U41 | 25 |
| DM | 0.21 | DM | 1. | DM | 0.67 | DM | 0. | DM | 0.46 | DM | 0. | DM | 0.46 | DM | 0. |
| U12 |  | U42 | 00 | U12 |  | U42 | 83 | U12 |  | U42 | 29 | U12 |  | U42 | 17 |
| DM | 0.50 | DM | 1. | DM | 2.08 | DM | 1. | DM | 0.83 | DM | 0. | DM | 0.83 | DM | 0. |
| U13 |  | U43 | 00 | U13 |  | U43 | 06 | U13 |  | U43 | 72 | U13 |  | U43 | 58 |
| DM | 0.40 | DM | 1. | DM | 1.00 | DM | 3. | DM | 0.52 | DM | 1. | DM | 0.46 | DM | 0. |
| U14 |  | U44 | 00 | U14 |  | U44 | 00 | U14 |  | U44 | 00 | U14 |  | U44 | 80 |
| DM | 1.00 | DM | 2. | DM | 0.33 | DM | 1. | DM | 1.25 | DM | 0. | DM | 1.00 | DM | 0. |
| U15 |  | U45 | 00 | U15 |  | U45 | 33 | U15 |  | U45 | 60 | U15 |  | U45 | 63 |
| DM | 0.25 | DM | 0. | DM | 0.50 | DM | 0. | DM | 0.46 | DM | 0. | DM | 0.38 | DM | 0. |
| U16 |  | U46 | 50 | U16 |  | U46 | 65 | U16 |  | U46 | 31 | U16 |  | U46 | 20 |
| DM | 1.00 | DM | 1. | DM | 4.00 | DM | 0. | DM | 0.24 | DM | 0. | DM | 0.21 | DM | 0. |
| U17 |  | U47 | 00 | U17 |  | U47 | 15 | U17 |  | U47 | 22 | U17 |  | U47 | 31 |
| DM | 1.00 | DM | 1. | DM | $0.37$ | DM | 0. | DM | 0.49 | DM | 0. | DM | 0.39 | DM | 0. |
| U18 |  | U48 | 00 | U18 |  | U48 | 57 | U18 |  | U48 | 28 | U18 |  | U48 | 29 |
| DM | 0.50 | DM | 1. | DM | $0.68$ | DM | 0. | DM | 0.50 | DM | 0. | DM | 0.50 | DM | 0. |
| U19 |  | U49 | 00 | U19 |  | U49 | 50 | U19 |  | U49 | 16 | U19 |  | U49 | 25 |
| DM | 1.00 | DM | 1. | DM | 0.43 | DM | 0. | DM | 0.33 | DM | 0. | DM | 0.33 | DM | 0. |
| U20 |  | U50 | 00 | U20 |  | U50 | 23 | U20 |  | U50 | 56 | U20 |  | U50 | 31 |
| DM | 1.00 | DM | 1. | DM | $0.51$ | DM | 0. | DM | 0.16 | DM | 0. | DM | 0.14 | DM | 0. |
| U21 |  | U51 | 00 | U21 |  | U51 | 58 | U21 |  | U51 | 28 | U21 |  | U51 | 16 |
| DM | 1.00 | DM | 1. | DM | 0.31 | DM | 0. | DM | 0.33 | DM | 0. | DM |  | DM | 0. |
| U22 |  | U52 | 00 | U22 |  | U52 | 52 | U22 |  | U52 | 38 | U22 | . 30 | U52 | 58 |
| DM | 1.00 | DM | 1. | DM | 1.33 | DM | 1. | DM | 0.50 | DM | 1. | DM | 0.47 | DM | 0. |
| U23 |  | U53 | 45 | U23 |  | U53 | 60 | U23 |  | U53 | 25 | U23 |  | U53 | 80 |
| DM | 2.00 | DM | 0. | DM |  | DM | 2. | DM |  | DM | 2. | DM | 2.00 | DM | 2. |
| U24 |  | U54 | 50 | U24 |  | U54 | 00 | U24 |  | U54 | 00 | U24 | 2.00 | U54 | 50 |
| DM | 1.00 | DM | 1. | DM | 0.68 | DM | 0. | DM | 0.35 | DM | 0. | DM | 0.32 | DM | 0. |
| U25 |  | U55 | 00 | U25 |  | U55 | 95 | U25 |  | U55 | 50 | U25 | 0.32 | U55 | 39 |
| DM | 0.50 | DM | 1. | DM | $0.60$ | DM | 0. | DM |  | DM | 0. | DM |  | DM | 0. |
| U26 |  | U56 | 00 | U26 |  | U56 | 44 | U26 |  | U56 | 16 | U26 | 3 | U56 | 28 |
| DM | 1.00 | DM | 1. | DM | 0.59 | DM | 0. | DM | 0.20 | DM | 0. | DM |  | DM | 0. |
| U27 |  | U57 | 00 | U27 |  | U57 | 54 | U27 |  | U57 | 36 | U27 | 0.16 | U57 | 14 |
| DM | 1.00 | DM | 1. | DM | 0.57 | DM | 0. | DM | 0.24 | DM | 0. | DM | 0.25 | DM | 0. |
| U28 |  | U58 | 00 | U28 |  | U58 | 52 | U28 |  | U58 | 54 | U28 |  | U58 | 61 |
| DM | 1.00 | DM | 1. | DM | 0.46 | DM | 0. | DM | 0.22 | DM | 0. | DM | 0.23 | DM | 0. |
| U29 |  | U59 | 00 | U29 |  | U59 | 95 | U29 |  | U59 | 40 | U29 |  | U59 | 39 |
| DM | 1.00 | DM | 1. | DM | 0.33 | DM | 0. | DM | 0.36 | DM | 1. | DM | 0.36 | DM | 1. |
| U30 |  | U60 | 00 | U30 |  | U60 | 71 | U30 |  | U60 | 25 | U30 |  | U60 | 60 |
|  |  | DM | 1. |  |  | DM | 0. |  |  | DM | 1. |  |  | DM | 0. |
|  |  | U61 | 33 |  |  | U61 | 44 |  |  | U61 | 00 |  |  | U61 | 89 |

Table5. Efficiency of the Control group class C, D \& experimental group class A, B for 3 processes

|  | PRETEST |  |  |  |  | POST TET |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CLASS | EFFICENCY | EFFICENCY |  |  |  |  |  |  |  |
| BOTH | RME | EFFICENCY | CLASS | EFFICENCY | EFFICENCY | EFFICENCY | TOT |  |  |
|  | 0.67 | 0.63 | 0.61 |  | A | 1 | 1 | 1 | 61 |
| A | 0.88 | 0.72 | 0.67 | BOTH | RME | PS | AL |  |  |
| B | 0.88 | 1 | 1 | 1 | 61 |  |  |  |  |
| C | 0.75 | 0.57 | 0.75 | C | 0.71 | 0.63 | 0.68 | 61 |  |
| D | 0.54 | 0.71 | 0.67 | D | 0.59 | 0.72 | 0.65 | 61 |  |

## 4. Discussion

According to the first objective of the research, we can use new educational methods to increase students' mathematical literacy. New and active methods of teaching mathematics play an essential role in learning, including solving problem mathematics and realistic teaching methods. Problem solving and its challenges are one of the most constructive ways of learning. Using realistic mathematics will help you understand the problem correctly and think correctly to get the answer using the real world. The task of teaching methods is to create a mental exercise for the student, so that the student learns reasoning and seeks a logical reason for everything in nature; thus, the new teaching of mathematics can lead to the student's growth. Regarding the use of the new educational methods, this research is in line with Alizadeh(2017),Yaftian\& Maleki (2020),Ebrahimi(2016),she, et al(2018), Yeşim Özer Özkan(2018), Mariani, et al(2018)and Putri, et al(2019).

According to the second objective of the research, one of the most important issues in education is to assess and evaluate students in appropriate ways. As shown in the research, the use of mathematical models and data envelopment analysis methods can be one of the tools of measurement and educational evaluation. In terms of using the data envelopment analysis methods, this research is in line with Balf, et al(2010), Chen(2007), Emrouznejad\& Yang (2018)and $\operatorname{Res}(2016)$.

The first question of this research was about the prediction of possible performance of students that have been taught by RME and PSM and their mathematical literacy has been challenged. The findings showed that the students of the experimental group were efficient after the realistic math training and mathematical problem solving compared to the control group in the output-oriented CCR model. They were taught to recognize and develop formal and informal knowledge of mathematics to use in real-life situations to solve various problems (Prakash, 2019). Students were encouraged to learn formal methods of solving math problems and symbols through in the math class discussion. Students participated in the classroom by discussing their solution methods and inspired each other with their informal math knowledge. The discussion in the math class taught them how to interact for developing mathematical knowledge (Hussain Malik \& Abbas Rizvi, 2018). As the efficiency results of the experiments and control groups in the pre-test and post-test of mathematics indicated, the problem-solving education methods make students more motivated and turn them into active learners who can solve the problem ("Using Realistic Mathematics Education and the DAPIC Problem-Solving Process to Enhance Secondary School Students' Mathematical Literacy," 2017).

The second question was a research about the efficiency of the educational methods (PSM, RME, BOTH) has a better result on the increase of students' mathematical literacy. According to Table 5, the comparison of the proposed methods showed that the mathematical literacy enhancement through the realistic mathematical education method had a better performance than the problem-solving method. In addition, the integration of the two realistic and problem-solving mathematical methods to increase mathematical literacy yields a more efficient performance than either method alone. The efficient performance obtained by the realistic mathematics education is a representative of the students' interests in math learning. The speed of and the focus on problem solving, the proper use of definitions and formulas, were the result of students' engagement in solving the problems and their connection to real world integration of two methods, which
are more efficient together than applied individually ("Problem Posing with Realistic Mathematics Education Approach in Geometry Learning," 2017).

In the third discussed question, it can be mentioned that, whether Data Envelopment Analysis, can be proper for the evaluation of students' efficiency and (PSM, RME, BOTH) teaching methods. DEA concerning efficiency has been precisely calculated and the comparison of efficiencies was feasible. There was a ranking possibility, for efficient students, regarding their efficiency calculation. Given that, population's Mean was not important and the hypothesis is not used and instead of T-test DEA was used for the evaluation of efficiency (Res., 2016). Findings of the research revealed that, DEA of CCR Model has been replaced by time-consuming statistical mythologies that the advantages can be referred to as follows: The efficiency of the unit in a relative manner can be measured, $\bullet$ We should not be sensitive to units of measurement and the multiplicity of inputs and outputs of different units, Flexibility and high versatility must be taken into account for application on a variety of issues, An estimate of the necessary changes must be provided in inputs and outputs for the transfer of inefficient units to the efficient boundary, Useful information on different inputs and outputs must be provided to make appropriate resource allocation decisions, Performance of all units should be assessed with the best performance possible in that system, We should be able to use these methodologies in systems with one or more input/output, Calculations can be carried out easily, Fairly good results are provided when using small samples,

Another finding concluded that the realistic mathematics education for the development of student experience and thinking leads them to discover a solution and eventually implement it in the real world to answer the desired problem (Zakaria \& Syamaun., 2017). Hence, better education of the learning process and math learning will be easier with the new methods. Regarding the inputs and outputs that affect the efficiency and information obtained from the evaluation, inefficient students can lead the control group toward efficiency. It also used these new educational methods to increase students' mathematical literacy.

Therefore, changes in the type and number of inputs or outputs allow for changes in evaluation results. In addition, due to the relative efficiency calculation, the change in the number of DMUs will change the efficiency score of all units. In evaluating each system, the number of decision-maker units, the mathematical planning model, must be made and solved, which will result in a high volume of computing. According to this study, using the GAMS software will solve this problem. Choosing the right modelling for ease in mathematical teaching can be one of the most effective methods in mathematical education. Choosing new education and learning methods of math teaching, such as learning how to use the realistic mathematics education method and integrating it with real-world problem solving will enhance mathematics learning.

Suggestions and Restrictions: Since DEA is purely a mathematical and numerical technique, measurement errors may lead to major changes in the results, which should be re-checked after identifying the efficient unit of inputs and outputs to ensure its accuracy. Data envelopment analysis method is only a mathematical method based on linear programming and does not have the ability to compare the qualitative variables of decision units, but it can measure the efficiency of units relatively, and it is a management method. It is suggested that data envelopment analysis method be used to calculate students' regression and progress during the school year. Since in the envelopment analysis method, the performance of each student during the academic year is studied, it can be suggested as a suitable tool for evaluating students individually and in the classroom. In addition, teaching mathematics in a realistic way will give students a better visualization of mathematical concepts, and it is recommended to better understand the various concepts of mathematics.

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