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ABSTRACT: This paper evaluates and ranks the temporal dimensions, known as fourth dimension of urban design, of a number of places in a city by TOPSIS method. TOPSIS method is technique for order preference by similarity to ideal solution. TOPSIS is one of the renowned methods for classical multi-criteria decision-making (MCDM) problems that defines the positive ideal solution and negative ideal solution to maximize the benefit criteria and minimize the cost criteria. The best solution is a point that has the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. Because of the vagueness of the input data, triangular fuzzy numbers are applied. In addition, Euclidian distance and a new positive and negative ideal solution are used in this paper. This technique is implemented in Marand, Iran to evaluate fifteen important places based on eight criteria of temporal dimensions. Closeness coefficient values verify the ranking order of fifteen important places, which is a vital decision for the urban managers.

Keywords: Urban design, Temporal dimensions, TOPSIS method, Fuzzy numbers.

#### INTRODUCTION

The temporal dimension is one of dimensions of urban design. The time impacts almost every aspect of urban design, such as: 1) on the way the environment is perceived (i.e. over time and on the move); 2) on the way places become imbued with meanings over time; 3) on how places last and adapt; 4) how robust they are (i.e. on how places change over time); 5) their morphological processes; and 6) on the length of time that urban design processes take. Some of the most stimulating discussions of time are found in related fields such as cultural geography, philosophy, anthropology and phenomenology, but a number of theorists have also attempted to relate time factors directly to urban design such as Carmona et al., (2003). Evaluating the temporal dimension of urban design is a vital and complex decision for the urban managers, which several criteria are concerned.

Decision-making is known as a procedure to select the best alternative among a set of feasible alternatives, where decision-making problems considering several criteria are called multi-criteria decision-making (MCDM) problems. It is often required that decision makers should provide qualitative/quantitative assessments for determining the performance of each alternative with respect to each criterion, and the relative importance of evaluation criteria with respect to the overall objective of the problems. Therefore, the MCDM refers to showing, prioritizing, placing, or selecting a set of alternatives under independent or conflicting criteria. These problems will usually result in uncertain and subjective data being present, which makes the decision-making process difficult and tricky. That is why decision-making problems often considered in a

fuzzy environment (FMCDM) where the information available is imprecise/uncertain. The application of fuzzy set theory to multi-criteria evaluation methods has proven to be an effective approach. In this case, positive ideal and negative ideal points to solve decision-making problems with multi-judges are also studied. The general utility of the alternatives with respect to all criteria is often measured by a fuzzy number where the alternatives are ranked based on the comparison of their corresponding fuzzy utilities (Chen and Hwang, 1992). The technique for order preference by similarity to ideal solution called as TOPSIS is one of the renowned methods for classical MCDM problems. The fundamental logic of TOPSIS is to define the positive ideal solution and negative ideal solution in which the ideal solution is the solution that maximizes the benefit criteria and minimizes the cost criteria, whereas the negative ideal solution is the solution that maximizes the cost criteria and minimizes the benefit criteria. In short, the ideal solution is composed of all of best values achievable of criteria, whereas the negative ideal solution consists of all worst values attainable of criteria. The best alternative is a point that has the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. Many researchers have applied TOPSIS method to solve FMCDM problems in the past with different approaches such as Wang and Lee (2007). Because of different observations of different experts for

weighting the criteria, a fuzzy group weight can be considered

necessary. In fuzzy TOPSIS (FTOPSIS), in addition, the

technique of positive and negative ideal solution is easily used to find the best alternative, considering that the chosen

alternative should simultaneously have the shortest distance

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from the positive ideal point and the longest distance from the negative ideal point (Yeh et al., 1999, Chen and Tzeng, 2004). FTOPSIS can also obtain the gap between the ideal alternative and each alternative, as well as the ranking order of alternatives. Wang and Lee (2007) incorporated the fuzzy set theory and the basic concepts of positive and negative ideal to expand multi-criteria decision-making in a fuzzy environment. Wang and Chang (2007) extended fuzzy pair wise comparison and the basic concepts of positive ideal and negative ideal points to expand multi-criteria decision-making in a fuzzy environment. Fuzzy multicriteria decision-making method based on concepts of positive ideal and negative ideal points to evaluate bus companies' performance is researched in Yeh et al., (1999). Chen (2000) extended the TOPSIS for group decision-making in a fuzzy environment and considered fuzzy distance function for evaluation.

Some applicable researches are provided here. Bostenaru (2004) developed a decentralized decision model for retrofitting existing buildings using hierarchical process. Abbasbandy and Asady (2006) presented a modification of the distance-based fuzzy number ranking approach called the sign distance, which produces non-intuitive results in certain cases. Soo and Teodorovic (2006) ranked order transit signal priority strategy alternatives for traffic management in urban planning. They used decision support system (DSS) framework integrating with TOPSIS method. Asady and Zendehnam (2007) defuzzified the fuzzy numbers using minimizer of the distance between the two fuzzy numbers. They also represented new properties for ordering the fuzzy numbers. Under a fuzzy environment, an evaluation on the initial training aircraft and ranking the alternatives based on the fuzzy TOPSIS is done in Wang and Chang (2007). To assign weights and rank expected functions as spatial choices, a conceptual model in AHP is propagated and recommended by Thapa and Murayama (2008). Önüt and Soner (2008) investigate the application of AHP and TOPSIS for the solid waste transshipment site-selection problem in Istanbul, Turkey. Sadi-Nezhad and Khalili (2009) proposed a preference ratio with a moderate modification for negative fuzzy numbers and fuzzy distance measurement for generalized fuzzy numbers. Javadian et al. (2009) presented triangular fuzzy numbers for multiple criteria group decision-making (FMCGDM) problem with TOPSIS based on the new concept of positive and negative ideal solution and compared the efficiency of the algorithm with algorithms in the literature. Ertuðrul and Karakaþoðlu (2009) studied the evaluation of the performance of fifteen Turkish cement firms in the Istanbul Stock Exchange. They applied fuzzy analytic hierarchy process (AHP) to determine the weight of the criteria and then ranked the firms by TOPSIS methods. Caterino et al. (2009) compared analytically two methods (TOPSIS and VIKOR) for seismic structural retrofitting in civil and architectonic management. Wang et al. (2009) used analytical hierarchy process AHP and spatial information technologies for the selection of the appropriate solid waste landfill site in Beijing, China. A geographic information system (GIS) was used to present spatial data. Tansel YC and Yurdakul (2010) proposed the decision support system for the banks to determine a quick credibility scoring of manufacturing firms in Turkey based on the financial ratios and fuzzy TOPSIS approach. They also efficiently applied the FTOPSIS in assessment of traffic police centers. Dursun and Ertugrul-Karsak (2010) developed FTOPSIS for personnel

selection and 2-tuple linguistic representation model. They employed ordered weighted averaging (OWA) operator that encompasses several operators. Evaluation of ecological capability criteria is utilized by means of AHP and Expert Choice software as a case of implementation of indoor recreation in Varjin protected area in Jozi et al. (2010). Erkayman et al. (2011) proposed a fuzzy TOPSIS approach to a logistics center location-selection problem for sustainable development of urban areas. The author applied this method in eastern Anatolia region of Turkey. Hashemi and Amiri-Aref (2011) ranked a number of places in cities with crisp data of TOPSIS method. Amiri-Aref et al. (2012) introduced a fuzzy TOPSIS method using a new distance function for triangular and trapezoidal fuzzy numbers and then compared the results with three references, Chen and Hwang (1992), Chen (2000) and Li (1999) in the literature.

The major purpose of this paper is the application of the fuzzy TOPSIS based on the concept of positive and negative ideal solution in the urban design context while no published paper considered temporal dimensions in urban design with fuzzy logic. Considering the fuzzy data, linguistic variables are applied to determine the weights of all criteria and the rating of each alternative with respect to each criterion. A fuzzy decision matrix and a weighted normalized fuzzy decision matrix are generated. According to the concept of TOPSIS, we applied the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS). Advantages of the new FPIS and FNIS is to present a more reliable and easier way which guarantees that the preferred alternative is closer to the positive ideal solution and farther from the final negative ideal solution. Based on closeness coefficient values, we verify the ranking order of all alternatives and select the best alternative.

## MATERIALS AND METHODS

## Fuzzy numbers and linguistic variables

The representation of multiplication operation on two or more fuzzy numbers is one of useful tools for decision makers in the fuzzy multiple criteria decision-making environment for ranking all the candidate alternatives and selecting the best one. In this section, basic definitions of fuzzy sets, fuzzy numbers, and linguistic variables are reviewed from Zimmermann (1996) and Hwang and Yoon (1981).

**Definition 1.** A fuzzy set  $\tilde{A}$  in a universe of discourse X is characterized by a membership function  $\mu_{\tilde{A}}(x)$  which associates with each element x in a real number in the interval [0, 1]. The function value is termed as the grade of membership of in .

**Definition 2.** A triangular fuzzy number can be defined by a triplet. Its conceptual schema and mathematical form are shown by Eq. (1). A triangular fuzzy number in the universe of discourse that conforms to this definition is shown in Fig. 1.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < \\ \frac{x - a_1}{a_2 - a_1}, & a_1 < x < \\ \frac{a_2 - x}{a_3 - a_2}, & a_2 < x < \\ 0, & a_2 < \end{cases}$$
(1)

**Definition 3.** A trapezoidal fuzzy number can be defined

by a quadruplet  $(a_1, a_2, a_3)$ . Its conceptual schema and mathematical form are shown by Eq. (2). A trapezoidal fuzzy number in the universe of discourse that conforms to this definition is shown in Fig. 2.

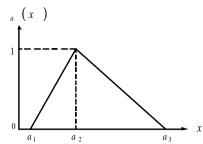


Fig. 1: A triangular fuzzy number

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < \\ \frac{x - a_1}{a_2 - a_1}, & a_1 < x < \\ 1, & a_2 < x < \\ \frac{a_1 - x}{a_2 - a_4}, & a_2 < x < \\ 0, & a_n < \end{cases}$$
(2)

**Definition 4.** The  $\alpha_{\underline{cut}} \tilde{A}_{\alpha}$ , and strong  $\alpha_{\underline{cut}} \tilde{A}_{\alpha^{+}}$  of the fuzzy set  $\tilde{A}$  in the universe of discourse X is defined by

$$\tilde{A}_{\alpha} = \{x | \mu_{\tilde{A}}(x) \ge \alpha, x \in, \text{ where } \alpha \in [0, 1]\}$$

$$\tilde{A}_{\alpha^{+}} = \{x | \mu_{\tilde{A}}(x) > \alpha, x \in, \text{ where } \alpha \in [0] \}$$

The lower and upper points of any are represented by and , respectively, and we suppose that both are finite. For convenience, we denote by and by (Fig. 3).

**Definition 5.** Assuming that both  $\tilde{A}$  and  $\tilde{B}$  are fuzzy numbers and  $\lambda \in \mathbb{R}$ , the notions of fuzzy sum,  $\oplus$ , fuzzy product by a real number, -, and fuzzy product,  $\otimes$ , are defined as follows (Wang and Chang, 2000):

$$\mu_{(\tilde{\alpha} \oplus \tilde{b})}(z) = \sup \{ \min(\mu_{\tilde{\alpha}}(x), \mu_{\tilde{b}}(y)) : (x, y) \in \mathbb{R}^2 \text{ and } x + y =_{\text{weight of the } \tilde{f}\text{th criterion, } \tilde{\mathcal{C}}_j \text{ and } \tilde{x}_{ij} \text{ is the performance } \tilde{\mathcal{C}}_j \text{ and } \tilde{x}_{ij} \text{ is the performance } \tilde{\mathcal{C}}_j \text{ and } \tilde{x}_{ij} \text{ is the performance } \tilde{\mathcal{C}}_j \text{ and } \tilde{x}_{ij} \text{ is the performance } \tilde{\mathcal{C}}_j \text{ and } \tilde{x}_{ij} \text{ is the performance } \tilde{\mathcal{C}}_j \text{ and } \tilde{x}_{ij} \text{ is the performance } \tilde{\mathcal{C}}_j \text{ and } \tilde{x}_{ij} \text{ is the performance } \tilde{\mathcal{C}}_j \text{ and } \tilde{x}_{ij} \text{ is the performance } \tilde{\mathcal{C}}_j \text{ and } \tilde{x}_{ij} \text{ is the performance } \tilde{\mathcal{C}}_j \text{ and } \tilde{\mathcal{C}}_j \text$$

$$(\lambda \cdot \tilde{a})(z) = \begin{cases} \tilde{a} \left(\frac{z}{\lambda}\right), & \lambda \neq 0 \\ I_{\{0\}}(z), & \lambda = 0, \end{cases}$$

where  $I_{\{0\}}(z)$  is the indicator function of ordinary set  $\{0\}$ , and  $\mu_{(\tilde{a}\otimes \tilde{b})}(z) = \sup\{\min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)) : (x, y) \in \mathbb{R}^2 \text{ and } x \times y = \tilde{y} = \tilde{y} = 0\}$ 

Let  $\tilde{A}$  and  $\tilde{B}$  be two positive fuzzy numbers and  $\alpha \in [0,1]$ . The basic operations on positive fuzzy numbers with  $\alpha_{cut}$  operator are as follows:  $(\tilde{a} \oplus \tilde{b})_{\alpha} = [a_{\alpha}^{-} + b_{\alpha}^{-}, a_{\alpha}^{+} +$ 

$$(\tilde{a} \otimes \tilde{b})_{\alpha} = [a_{\alpha}^{-} \times b_{\alpha}^{-}, a_{\alpha}^{+} \times$$

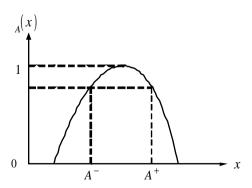


Fig. 3: An example of an  $\alpha_{cut}$ .

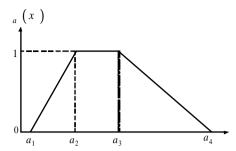


Fig. 2: A trapezoidal fuzzy number  $\tilde{A}$ .

and 
$$\lambda \in \mathbb{R} \setminus \{0\}$$
 if, then we have:  $(\lambda \cdot \tilde{a})_{\alpha} = \lambda a_{\alpha}$ , namely,  $\lambda < 0$  if,  $(\lambda \cdot \tilde{a})_{\alpha} = [\lambda a_{\alpha}^{-}, \lambda a_{\alpha}^{+}]$   $\lambda > 0$  if,  $(\lambda \cdot \tilde{a})_{\alpha} = [\lambda a_{\alpha}^{+}, \lambda a_{\alpha}^{-}]$ 

**Definition 6.** A linguistic variable is a variable the values of which are linguistic terms. Linguistic terms have been found intuitively easy to use in expressing the subjectiveness and/or qualitative imprecision of a decision maker's assessments (L.A. Zadeh, 1975).

**Definition 7.** A fuzzy MCDM problem with alternatives and criteria can be concisely expressed in a fuzzy decision matrix format as:

$$\widetilde{D} = \begin{bmatrix} X_{11} & \widetilde{X}_{12} & \widetilde{X}_{13} & \dots & \widetilde{X}_{1n} \\ A_{2} & \widetilde{X}_{21} & \widetilde{X}_{22} & \widetilde{X}_{23} & \dots & \widetilde{X}_{2n} \\ \widetilde{X}_{21} & \widetilde{X}_{32} & \widetilde{X}_{23} & \dots & \widetilde{X}_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{m} & \widetilde{X}_{m1} & \widetilde{X}_{m2} & \widetilde{X}_{m3} & \dots & \widetilde{X}_{mn} \end{bmatrix},$$
(5)
$$\widetilde{W} = [\widetilde{W}_{1}, \widetilde{W}_{2}, \dots, \widetilde{W}_{n}],$$

where  $\tilde{x}_{ij}$  (i = 1, ..., m, j = 1, ..., n), and  $\tilde{w}_{j}$  (j = 1, ..., n), are linguistic fuzzy numbers. Note that  $\tilde{w}_{j}$  represents the weight of the *j*th criterion,  $\tilde{C}_{j}$  and  $\tilde{x}_{ij}$  is the performance rating of the *j*th alternative,  $A_{j}$  with respect to the *j*th criterion,  $C_{j}$ . The weighted fuzzy decision matrix is:

$$\tilde{V} = \begin{bmatrix} \tilde{v}_{11} & \tilde{v}_{12} & \tilde{v}_{13} & \dots & \tilde{v}_{1n} \\ \tilde{v}_{21} & \tilde{v}_{22} & \tilde{v}_{23} & \dots & \tilde{v}_{2n} \\ \tilde{v}_{31} & \tilde{v}_{32} & \tilde{v}_{23} & \dots & \tilde{v}_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{v}_{m1} & \tilde{v}_{m2} & \tilde{v}_{m3} & \dots & \tilde{v}_{mn} \end{bmatrix} = \begin{bmatrix} \tilde{w}_1 \otimes \tilde{x}_{11} & \tilde{w}_2 \otimes \tilde{x}_{12} & \dots & \tilde{w}_j \otimes \tilde{x}_{1j} & \dots & \tilde{w}_n \otimes \tilde{x} \\ \tilde{w}_1 \otimes \tilde{x}_{21} & \tilde{w}_2 \otimes \tilde{x}_{22} & \dots & \tilde{w}_j \otimes \tilde{x}_{2j} & \dots & \tilde{w}_n \otimes \tilde{x} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{w}_1 \otimes \tilde{x}_{i1} & \tilde{w}_2 \otimes \tilde{x}_{i2} & \dots & \tilde{w}_j \otimes \tilde{x}_{ij} & \dots & \tilde{w}_n \otimes \tilde{x} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \tilde{w}_1 \otimes \tilde{x}_{m1} & \tilde{w}_2 \otimes \tilde{x}_{m2} & \dots & \tilde{w}_j \otimes \tilde{x}_{mj} & \dots & \tilde{w}_n \otimes \tilde{x} \end{bmatrix}$$

**Definition 8.** The Euclidian distance between two triangular fuzzy numbers and is calculated as follows.

$$D^{2}(A,B) = \sqrt{\frac{1}{3}((a_{1}-b_{1})^{2} + (a_{2}-b_{2})^{2} + (a_{3}-b_{3})^{2})}^{(7)}$$

**Definition 9.** Since we use the qualitative criteria, the linguistic variables are used. A linguistic variable is a variable the values of which are linguistic terms. Linguistic terms have been found intuitively easy to use in expressing the subjectiveness and/or qualitative imprecision of a decision maker's assessments (L.A. Zadeh, 1975).

**Definition 10.** Fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS) for two triangular fuzzy numbers  $\tilde{A} = (a_1, a_2, a_3)$  and  $\tilde{B} = (b_1, b_2, b_3)$  are defined in the following. Suppose  $\tilde{A}_k = (a_1^k, a_2^k, a_3^k)$ , k = 1, 2, ..., n be TFN. For determining FNIS, follow the below procedure: 1: List all  $a_1^k, k = 1, 2, ..., n$ ; l = 1, 2, 3.

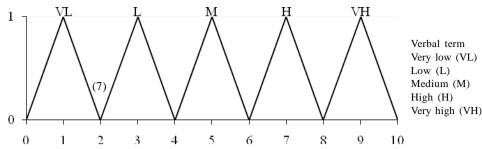


Fig. 4: Linguistic variables for the ratings.

2: Sort increasingly a...

3: Select the first three  $a_i^k$  as minimum TFN of  $\tilde{A}_k$ , i = 1, 2, ..., n.

4: Record this as  $\tilde{A}_{min}$  where:

$$\tilde{A}_{min} = \bigwedge_{k=1,2,\dots,n} \tilde{A}_k. \tag{8}$$

For determining FPIS, follow the below procedure:

- 1: List all
- 2: Sort increasingly.
- 3: Select the last three as maximum TFN of

4: Record this as where: 
$$\tilde{A}_{max} = V_{k=1,2,\dots,n} \tilde{A}_k$$
. (9)

The proposed Fuzzy TOPSIS algorithm

Step 1: The linguistic ratings or fuzzy values,

$$\tilde{x}_{ij}$$
,  $(i=1,\ldots,m,j=1,\ldots,n)$ 

for alternatives with respect to criteria and then, the appropriate linguistic variables  $\tilde{w}_{j}$ , (j = 1, ..., n) as weights of the criteria must be chosen.

**Step 2**: The raw data are normalized to eliminate anomalies with different measurement units and scales in several MCDM problems. However, the purpose of linear scales transform normalization function used in this study is to preserve the property that the ranges of normalized triangular fuzzy numbers to be included in [0.1]. Suppose  $\tilde{p}$  denotes normalized fuzzy decision matrix, then

$$\begin{split} \tilde{R} &= \left[ \tilde{r}_{ij} \right], i = 1, 2, \dots, m, \ j = 1, 2, \dots, n, \\ \tilde{r}_{ij} &= \left( \frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+} \right), \quad j \in B, \qquad c_j^+ = \max_i c_{ij} \quad \text{if} \ j \in B, \\ \tilde{r}_{ij} &= \left( \frac{a_{ij}^-}{c_{ij}}, \frac{a_{ij}^-}{b_{ij}}, \frac{a_{ij}^-}{a_{ij}} \right), \quad j \in C, \qquad a_j^- = \min_i a_{ij} \quad \text{if} \ j \in C, \end{split}$$

where B is the benefit criteria set and C is the cost criteria set. **Step 3**: by using Eq. (6), the weighted normalized fuzzy decision matrix  $V = \begin{bmatrix} \tilde{v}_{ij} \end{bmatrix}_{m \ge 0}$  will be generated. **Step 4**: Fuzzy positive ideal solution (FPIS) and the fuzzy

negative ideal solution (FNIS) for two triangular fuzzy numbers  $\tilde{A} = (a_1, a_2, a_3)$  and  $\tilde{B} = (b_1, b_2, b_3)$ should be

obtained. So and for each criterion are obtained as follows. 
$$\tilde{v}_{j}^{*} = \bigvee_{i} \tilde{r}_{ij}, \ j = 1, 2, ..., n \tilde{V}^{*} = (\tilde{v}_{1}^{*}, \tilde{v}_{2}^{*}, ..., \tilde{v}_{n}^{*})^{(11)}$$
$$\tilde{V}^{-} = (\tilde{v}_{1}^{-}, \tilde{v}_{2}^{-}, ..., \tilde{v}_{n}^{-}) \ \tilde{v}_{j}^{-} = \bigwedge_{i} \tilde{r}_{ij}, \ j = 1, 2, ..., n$$

**Step 5:** Distance between the possible alternative  $\tilde{v}_{ij}$  and the positive ideal solution  $\tilde{v}_i^*$  and the negative ideal solution  $\tilde{v}_i^*$  can be calculated respectively by using:

$$L_i^+ = \sum_{j=1}^n D^2(\tilde{v}_{ij}, \tilde{v}_j^*), \quad i = 1, 2, ..., m,$$

$$L_i^- = \sum_{j=1}^n D(\tilde{v}_{ij}, \tilde{v}_j^-), \quad i = 1, 2, \dots, m.$$

Step 6: The closeness coefficient represents the distances to FPIS and FNIS simultaneously by taking the relative closeness to the FPIS. The closeness coefficient (CC) of each alternative is calculated as:

$$CC_i = \frac{L_i^-}{L_i^- + L_i^+}, \qquad i = 1, 2, ..., m.$$

Fuzzy value

(0.1.2)(2,3,4)

(4,5,6)

(6,7,8)

(8,9,10)

While  $L_i^- \ge 0$  and  $L_i^+ \ge 0$ , then,  $CC_i \in [0,1]$ , clearly. Step 7: According to the descending order of  $CC_1$ , we can determine the ranking order of all alternatives and select the best one from among a set of feasible alternatives.

#### RESULTS AND DISCUSSION

In this section, first we present a real case of investigating temporal dimensions in urban design study with triangular fuzzy data and introduce the evaluating criteria of temporal dimensions in Marand, Iran, to illustrate this TOPSIS approach. Then important places in this city are recognized. A rank order of the places based on the temporal dimensions criteria is provided by the TOPSIS method.

A temporal dimension is one way to measure physical change. It is perceived differently from the three spatial dimensions. There is only one of it, and that we cannot move freely in time but subjectively move in one direction. The equations used in physics to model reality do not treat time in the same way that humans commonly perceive it. The equations of classical mechanics are symmetric with respect to time, and equations of quantum mechanics are typically symmetric if both time and other quantities (such as charge and parity) are reversed. In these models, the perception of time flowing in one direction is an artifact of the laws of thermodynamics (time is perceived as flowing in the direction of increasing entropy). The best-known treatment of time as a dimension is Poincaré and Einstein's special relativity (and extended to general relativity), which treats perceived space and time as components of a four dimensional manifold, known as space-time. Eight temporal dimensions, as the qualitative criteria, are recognized by experts and evaluated in the Marand City in the following.

Identity-Oriented (IO): Presence of religious elements and the Shrine, existence of well-known poets and scholars in different historical periods and ancient fortress dating back thousands of years caused Marand identity richness, but the increasing erosion of ancient castle and historical elements because fading over time this feeling will.

Memorably (Me): Historical memory of a city means the city has special places and defines what had happened in the places. One of the most memorable times in the context of activities that occur in the city of Marand, the ceremony of Shabihkhani¹ based on the mourning of Imam Hussain in certain places have been done. In addition, in the past the city celebration in the downtown Square was held. Although holding the celebrations going on for a long time, it has lasting memories of the city.

Sense of place (SP): Sense of place in the parts of Marand, due to historical and ancient elements, is highlighted. However, the new buildings and structures of city, is very weak sense of place. Security (Se): In Emam Khomeini Square of Marand city, due to the active users, a high security in daylight is domain. However, at the night, this place is not currently active and it has been reduced security. In addition, in the ruined buildings fabric of over time undermines security and gathers criminal people.

Variability (Va): Marand faces in different seasons are different. These changes in their faces and street trees, seasonal fruits and people show activity. Elements without time restriction, like the Mishoo Mountain and elements with lowest time restriction, like ancient castle, the large mosque, the Marand Market mosque and Imam Khomeini Square have remained stable and variable over time.

Sense of richness (SR): Marand city due to color changes caused by seasonal changes in the sense of time is completely evident. In addition, sales of seasonal products enhance the sense of richness of visual, auditory, and olfactory. Texture, especially approximately the historical mosque and market, and during some of the richness of ancient tissue pathways are feeling a sense of time.

**Survival** (Su): Mishoo Mountains in Marand are two lasting elements. Buildings and structures due to housing, from 1956 to, researchers have witnessed the destruction of the city gardens and survival of this valuable element in the physical city and in the minds of people.

Sense of Belonging (SB): People of Marand city a have high sense of historical elements, especially to the city mosque and market, and ancient castle This feeling has grown over

time and has made Marandi. However, in recent years, sense of historical context Marand has been reduced. Moreover, in the modernized context, the sense of belonging is very pale. Fifteen important places that have temporal dimension in Marand city are recognized by overlapping cognitive map of a group of Marand's citizens. These are as follow. Holy Ahmad (HA), Holy Ibrahim (HI), Marand mosque (MM), Marand market mosque (MMM), Imam Square (IS), Imam1 St. (I1), Imam2 St. (I2), Imam3 St. (I3), Old texture (OT), middle texture (MT), new texture (NT), Bagmazar cemetery (BC), ancient mount (AM), wheat-saler square (WS), oratory (O). Eight criteria of temporal dimension (IO, M, SP, Se, V, SR, Su, SB) are evaluated in those places. All of the criteria are benefit index. It means that the more score, the more suitable place. A group of experts in urban design obtains weights of criteria. Table 1 represents the initial decision making matrix of fuzzy ratings of possible alternatives with respect to criteria and the weights of criteria. After computing the normalized and weighted normalized decision matrix, FPIS and FNIS are also shown in two last rows of Table 1. The values L; and L; are then calculated and the closeness coefficient of each place is illustrated in column CC, of Table 1. Finally, According to the closeness coefficient, ranking the preference order of these alternatives is obtained.

Let us show the computational results in details. Consider the first column (IO), for example. According to equation (10) we normalize the fuzzy data of IO respect to HI and then calculate the weighted normalized fuzzy decision variable, where  $c_1^+ = 10$  and  $\tilde{w}_1 = (0.8, 0.9, 1)$ . So we have:  $\tilde{r}_{12} = (0.2, 0.3, 0.4) \Rightarrow \tilde{v}_{12} = (0.16, 0.27, 0.4)$ . Then, for column IO, the value of  $\tilde{v}_{11}$ ,  $\tilde{v}_{12}$ , ...,  $\tilde{v}_{1,15}$  are computed. FNIS and FPIS in step 4 of fuzzy TOPSIS algorithm related are obtained according to equations (8) and (9), respectively, i.e.,  $\tilde{v}_1^* = (0.8, 0.81, 1)$  and  $\tilde{v}_1^- = (0.0, 0.9, 0.16)$ . The distance from the value of  $\tilde{v}_{12}$  to  $\tilde{v}_1^*$  as well as  $\tilde{v}_1^-$  is respectively computed:

$$D^{2}(\tilde{v}_{12}, \tilde{v}_{1}^{*}) = \sqrt{\frac{1}{3}((0.16 - 0.8)^{2} + (0.27 - 0.81)^{2} + (0.4 - 1)^{2})} = 0.594755$$

$$D^{2}(\tilde{v}_{12}, \tilde{v}_{1}^{-}) = \sqrt{\frac{1}{3}((0.16 - 0)^{2} + (0.27 - 0.09)^{2} + (0.4 - 0.16)^{2})} = 0.196299$$

Table 1: Decision matrix

Temporal dimensions		Ю	M e	SP	Se	V a	SR	Su	SB	$CC_o$	Rank
	$W_j$	VH	M	Н	Н	L	L	M	M		
Important Places	ΗA	VH	Н	VH	L	L	M	M	VH	0.65005	6
	ΗI	L	L	M	L	L	L	M	M	0.346785	12
	M M	VH	VH	Н	Н	Н	Н	VH	Н	0.790587	1
	MMM	VH	VH	Н	Н	Н	Н	Н	Н	0.770067	2
	IS	Н	Н	Н	Н	Н	Н	Н	Н	0.711276	3
	I1	Н	Н	Н	Н	Н	Н	Н	M	0.687556	5
	12	M	M	M	M	M	M	M	M	0.492771	8
	13	M	L	M	M	M	M	M	M	0.468034	9
	OT	Н	M	Н	VL	L	Н	VL	Н	0.457854	10
	ΜT	M	M	M	L	M	L	L	M	0.410021	11
	NT	VL	VL	V L	L	Н	VL	L	VL	0.157631	14
	ВC	Н	Н	Н	VL	VL	Н	Н	Н	0.536805	7
	A M	VH	VH	VH	VL	M	VH	Н	VH	0.697708	4
	W S	Н	Н	Н	Н	Н	Н	Н	Н	0.711276	3
	0	VL	VL	V L	M	L	VL	L	VL	0.1636	13
FPIS	V-	(0.8,0.81,1)	(0.8,0.81,1)	(0.63, 0.64, 0.8)	(0.6,0.61,0.8)	(0.26,0.3,0.4)	(0.27, 0.32, 0.4)	(0.45, 0.48, 0.6)	(0.45, 0.48, 0.6)		
FPIS	<i>V</i> -	(0,0.09,0.16)	(0,0.09,0.16)	(0,0.07,0.16)	(0,0.09,0.15)	(0,0.04,0.05)	(0,0.03,0.04)	(0,0.05,0.08)	(0,0.05,0.12)		

Finally, the total distance is  $L_2^+ = 2.629873$  and  $L_2^- = 1.396171$  and the closeness coefficient of HI will be

## **CONCLUSION**

In this paper, a decision method based on the concepts of fuzzy numbers in a multi-criteria decision-making problem has been developed. A fuzzy TOPSIS method is used in order to rank fifteen important places in Marand, Iran based on temporal dimension of urban design. Eight criteria in urban design that have temporal dimension are recognized. Euclidian distance function and new simple method to find maximum or minimum fuzzy numbers are used. Results show that where should Marand be improved the temporal dimension. It is found out that Marand mosque, Marand market mosque, and Imam Square have the highest value in ranking and Holy Ibrahim, oratory and new texture have the lowest value in ranking and need special attention for development.

For future extension, considering other fuzzy numbers and fuzzy distance functions and comparing the results can be a major work that may influence on managers viewpoints. Group decision-making methods can be another extension of with work.

#### **ENDNOTES**

1. A kind of theater in Iran

#### REFERENCES

- Abbasbandy, S., & Asady, B. (2006). Ranking of fuzzy numbers by sign distance. *Inform Sci.*, 176 (1), 2405–2416.
- Amiri-Aref, M., Javadian, N., & Kazemi, M. (2012). A New Fuzzy Positive and Negative Ideal Solution for Fuzzy TOPSIS. *Wseas T Circuits Syst.*, 3(11), 92-103.
- Asady, B., & Zendehnam, A. (2007). Ranking fuzzy numbers by distance minimization. Appl Math Model., 31 (1), 2589– 2598.
- Bostenaru, D. (2004). Multi-criteria decision model for retrofitting existing buildings. Nat Hazards Earth Syst Sci., 4, 485–499.
- Carmona, M., Heath, T., Oc,T.& Tiesdell, S. (2003). Public Places Urban Spaces. Architectural Press, Kent,UK.
- Caterino, N., Iervolino, I., Manfredi, G.& Cosenza, E., (2009).
  Comparative Analysis of Multi-Criteria Decision-Making Methods for Seismic Structural Retrofitting. Comput Civil Infrast. Eng., 24, 432–445.
- Chen, C.T. (2000). Extensions to the TOPSIS for group decision-making under fuzzy environment. *Fuzzy Sets Syst.* 114 1–9
- Chen, M.F. & Tzeng, G.H. (2004). Combining grey relation and TOPSIS concepts for selecting an expatriate host country. *Math Comput Model.*, 40 (13), 1473–1490.
- Chen, S.J., & Hwang, C.L. 1992. Fuzzy Multiple Attribute Decision Making Methods and Applications. Berlin: Springer.
- Dursun, M., & Ertugrul-Karsak, E. (2010). A fuzzy MCDM approach for personnel selection, *Expert Syst Appl.*, 37 (1), 4324–4330.
- Erkayman, B., Gundogar, E., Akkaya, G, & Ýpe, M. (2011). A Fuzzy Topsis Approach For Logistics Center Location

- Selection. in. 2011 New Orleans International Academic Conference, New Orleans, Louisiana USA, 14-16 March.
- Ertuðrul, Ý.,& Karakaþoðlu, N. (2009). Performance evaluation of Turkish cement firms with fuzzy analytic hierarchy process and TOPSIS methods. *Expert Syst. Appl.*, 36 (1), 702–715.
- Hashemi, M.& Amiri-Aref, M. (2011). Application of TOPSIS method for evaluating the temporal dimensions of Marand d *Engineering and Engineering Management*, Singapore, 7-9 Dec.
- Hwang, C.L., & Yoon, K. (1981). Multiple Attribute Decision Making: Methods and Application. New York: Springer.
- Javadian, N., Kazemi, M., Khaksar-Haghani, F., Amiri-Aref, M., & Kia, R. (2009). A General Fuzzy TOPSIS Based on New Fuzzy Positive and Negative Ideal Solution. International Conference on Industrial Engineering and Engineering Management, Hong Kong, 8-11 Dec.
- Jozi, S.A., Zaredar, N., & Rezaeian, S. (2010). Evaluation of Ecological Capability using Spatial Multi Criteria Evaluation Method (SMCE). *Int. J. Envir. Sci. Develp.*, 1(3), 273-277.
- Li, R.J. (1999). Fuzzy method in group decision making. Comp. Math. Appl., 38 (1), 91-101.
- Önüt, S., & Soner, S. (2008). Transshipment site selection using the AHP and TOPSIS approaches under fuzzy environment. *Waste Manag.*, 28 (9) 1552-1559.
- Sadi-Nezhad, S., & Khalili Damghani, K. (2009). Application of a fuzzy TOPSIS method base on modified preference ratio and fuzzy distance measurement in assessment of traffic police centers performance. *Applied Soft Comput.* 10(4), 1028-1039.
- Soo, H.Y., & Teodorovic, D. (2006). A DSS framework for Advanced Traffic Signal Control System Investment Planning. *J Public Trans.*, 9(4), 87-106.
- Tansel ÝÇ, Y. & Yurdakul, M. (2010). Development of a quick credibility scoring decision support system using fuzzy TOPSIS. *Expert Syst. Appl.*, 37 (1), 567–574.
- Thapa, R.B. & Murayama, Y. (2008). Land evaluation for peri-urban agriculture using analytical hierarchy process and geographic information system techniques: a case study of Hanoi. *Land Use Policy*., 25(2), 225-239.
- Wang, G., Qin, L., Li, G., & Chen, L. (2009). Landfill site selection using spatial information technologies and AHP: A case study in Beijing. *J Envir Manag*, 90 (8), 2414-2421.
- Wang, T.C., & Chang, T.H. (2007). Application of TOPSIS in evaluating initial training aircraft under a fuzzy environment. *Expert Syst. Appl.*, 33 (1), 870–880.
- Wang, Y.J.,& Lee. H.S. (2007). Generalizing TOPSIS for fuzzy multiple-criteria group decision-making. *Comput Math Appl.*, 53, 1762–1772.
- Yeh, C.H., Deng, H., & Pan, H. (1999). Multi-criteria analysis for dredger dispatching under uncertainty. *J Oper Res Society*, 50 (1), 35–43.
- Zadeh, L.A. (1975), The concept of a linguistic variable and its application approximate reasoning. *Inform Sci.*, 8 (4), 301–357.
- Zimmermann, H.J. (1996). Fuzzy Set Theory Appl. Boston: Kluwer Academic Publishers.