



Research Paper

Portfolio Optimization Based on Semi Variance and Another Perspective of Value at Risk Using NSGA II, MOACO, and MOABC Algorithms

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ARTICLE INFO

Article history:

Received 2020-08-22

Accepted 2020-12-28

Keywords:

SemiVariance

Value at Risk

NSGA II Algorithm

MOACO Algorithm

MOABC Algorithm

ABSTRACT

This study examines the criterion of value at risk from another perspective and presents a new type of mean-value at Risk model. To solve the portfolio optimization problem in Tehran Stock Exchange, we use NSGA II, MOACO, and MOABC algorithms and then compare the mean-pVaR model with the mean-SV model. Given that, finding the best answer is very important in meta-heuristic methods, we use the concept of dominance in the discussion of multi-objective optimization to find the best answers and show that, at low iterations, the performance of the NSGA II algorithm is better than the MOABC and MOACO algorithms in solving the portfolio optimization problem. As the iteration increases, the performance of the algorithms improves, but the rate of improvement is not the same, in a way, the performance of the MOABC algorithm is better than that of the NSGA II and MOACO algorithms. Then, to compare the performance of the “mean-percentage of Value at Risk” model and the “mean-semi variance” model, we examine both models in the standard mean-variance model and show that the mean-pVaR model, compared to the mean-SV model, Has better performance in stock portfolio optimization.

1 Introduction

One of the most significant concerns of investors in the capital market is electing the optimal portfolio in terms of profitability. To this end, the variety of portfolio selection methods in investment has widened [14]. The most important model for optimizing the portfolio is introduced in 1952 by Markowitz. After that, in this regard, various models and methods are presented for solving these models. To this purpose, lots of researchers in the world and Iran have done various researches. The notion of portfolio optimization and diversification is instrumental in developing and understanding financial markets and

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financial decision-making. In this regard, the publication of Harry Markowitz's portfolio theory gained many successes [15]. Since its launch, Markowitz has made many changes to people's attitudes to investing and portfolio and has been used as an efficient tool for portfolio optimization [21]. The first demonstration of portfolio selection by Markowitz in 1952 considering variance as a risk measure was presented. His method is known as the mean-variance method. Through this model, the risk was measured quantitatively for the first time. The Markowitz model considers the highest return and the lowest variance Simultaneously [26]. While variance captures only the risks associated with achieving the average return, PMPT recognizes that investment risk should be based on each of the investor's specific goals and that any outcomes above this goal do not represent economic or financial risk. PMPT's downside risk' measure makes a clear distinction between downside and upside volatility. In PMPT only volatility below the investor's target return incurs risk; all returns above this target cause "uncertainty", which is nothing more than a riskless opportunity for unexpectedly high return [36].

Firstly, the Markowitz model illustrates the potentiality to offer a combination of return maximization and risk minimization. Hence, Markowitz's initial model does not have the potentiality to resolve the problem of portfolio optimization in dealing with some real-world constraints such as the number of assets in the portfolio or the minimum amount of investment in designated assets [25]. In addition, an efficient boundary search is possible at the right time with the help of precise mathematical methods with a small number of assets. However, by the increase in the number of portfolio assets and in terms of real-world constraints, model-solving using deterministic methods such as quadratic planning, in solving the limited portfolio optimization problem based on the constraints, are not very efficient. The weaknesses of the Markowitz model lead to the fact that this model does not have the necessary efficiency and therefore Meta-heuristic algorithms have been considered to solve this problem. These approaches, unlike exact optimization methods, seek points that are as close as possible to the global optimization, so as to satisfy the decision-maker to an acceptable level [29]. Meta-heuristic methods are also called "inexact" methods cause of stochastic mechanisms that play an essential role in creating their structure. In general, these approaches are based on the order or rules of one natural organism. These methods in preparing solutions to complex computational problems are important and efficient, On the other hand, some meta-heuristic research is devoted to the study of finance. Therefore, in this study, we solve the portfolio optimization problem using The NSGA II algorithm, the MOACO algorithm, and the MOABC algorithm, by the mean-variance model and mean- pVaR model, which is derived from the mean-VaR model.

2 Theoretical Basics

In recent years, the problem of portfolio optimization is a standard problem in the financial world and has received a lot of attention. Selecting an optimal weighting of assets is a critical issue for which the decision-maker considers several aspects [28]. Some metaheuristic-based research has been devoted to the study of Finance [31]. One specific area of interest is portfolio selection, in which different types of metaheuristics have been employed in the optimization process [27], both under single objective and multi-objective perspectives. The latter perspective gathered popularity in this particular area since portfolios focus on two major objectives: return and risk.

2.1 Return

The return is the reward that the investor receives in the investment process, which is achieved from two options: change of asset price (difference in asset price at the end of the period compared to the

begging of the period) and earnings received during the phase [39].

2.2 Risk

For the first time, Harry Markowitz [34] presented a numerical index for risk based on quantitative definitions. He specified risk as to the standard multi-period deviation of a variable. There is a further aspect of risk definition that addresses only the negative side of fluctuations [34]. In this study, we have utilized variance and value at risk, as risk measures, which are shortly described below.

2.2.1 Semi Variance

Traditionally, portfolio optimization problems use variance (or standard deviation) as a measure of risk. Although commonly accepted, this measure is not the most appropriate for assessing risk, since it considers equally adverse deviations (below average) as well as favourable ones (above average). However, as Markowitz admitted, an investor is only concerned with adverse variations. In this context, Markowitz proposed an alternative measure of risk, the "Semi variance", which considers only adverse deviations. Semi variance is mathematically defined as (1).

$$S = E \left(\min(0, R_p - C)^2 \right) \quad (1)$$

where E is the expected value, R_p is the portfolio return and C is a benchmark [24].

2.2.2 Value at Risk (VaR)

Value at Risk is defined as the maximum amount of investment that may be lost in a specified time interval. Calculation of VaR can be done through two methods: parametric method and nonparametric method [34]. In this paper, we apply the parametric method for calculating VaR as described in (2).

$$VaR_p = M(Z_\alpha * \sigma - r_p) \quad (2)$$

Where VaR_p is Value at Risk, M is the amount of asset, σ is Standard deviation, r_p is the return of the portfolio and α is the confidence level [19].

2.2.3 Percentage of Value at Risk (pVaR)

If we analyze the Value at Risk, it is possible to say that the Value at Risk deals with two components: 1) The amount of investment (M), 2) the maximum percentage of investment that may be lost in a specified time interval, as described in (3).

$$pVaR = Z_\alpha * \sigma - r_p \quad (3)$$

Thus, the above concept shows the maximum percentage of the lost investment in a certain term and with a certain level of confidence that the investor will bear in the investment process. Hence, pVaR is the percentage of assets at risk. Choosing the value or price of an asset in the mean-VaR model is very thought-provoking, as M can be considered the last price of the study phase, the average of the last month, the average of the last year, or even the average of the period. Obviously, the choice of M can be effective in the analysis and selection of the portfolio. Notwithstanding, the mean-pVaR model does not consider the value of the asset and merely deals with the portion of value at risk. In this respect, it can be an advantage of the studied model. Considerably, this type of portfolio optimization model in Iran has been done no research, So far.

2.3 Portfolio Optimization Models

After the introduction of the Markowitz mean-variance model in 1952, several types of research have been done by financial researchers in order to develop and complete portfolio optimization models and provide different solutions to solve such ones. For instance, models that include other criteria to measure portfolio risk, such as semi variance, VaR, and CVaR.

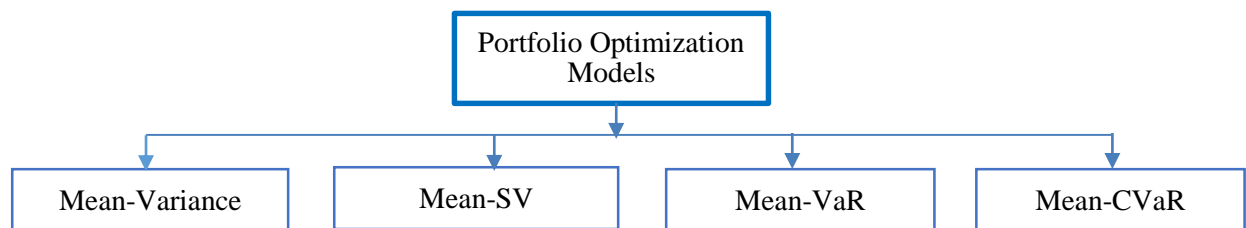


Fig 1: Portfolio Optimization Models

The algorithms that exist to solve the optimization problems can be divided into two categories: precision algorithms and approximate algorithms. Exact algorithms are able to find optimal solutions accurately, but approximate algorithms are able to find near optimal solutions for difficult optimization problems and are divided into three categories of heuristic, meta-heuristic, and hyper-heuristic. The two main problems of the heuristic algorithms are their local optimality, and their inability to apply them to various problems. The meta-heuristic algorithms presented to solve the heuristic algorithms are a variety of approximate optimization algorithms that have local optimization solutions that are applicable to a wide range of problems [41].

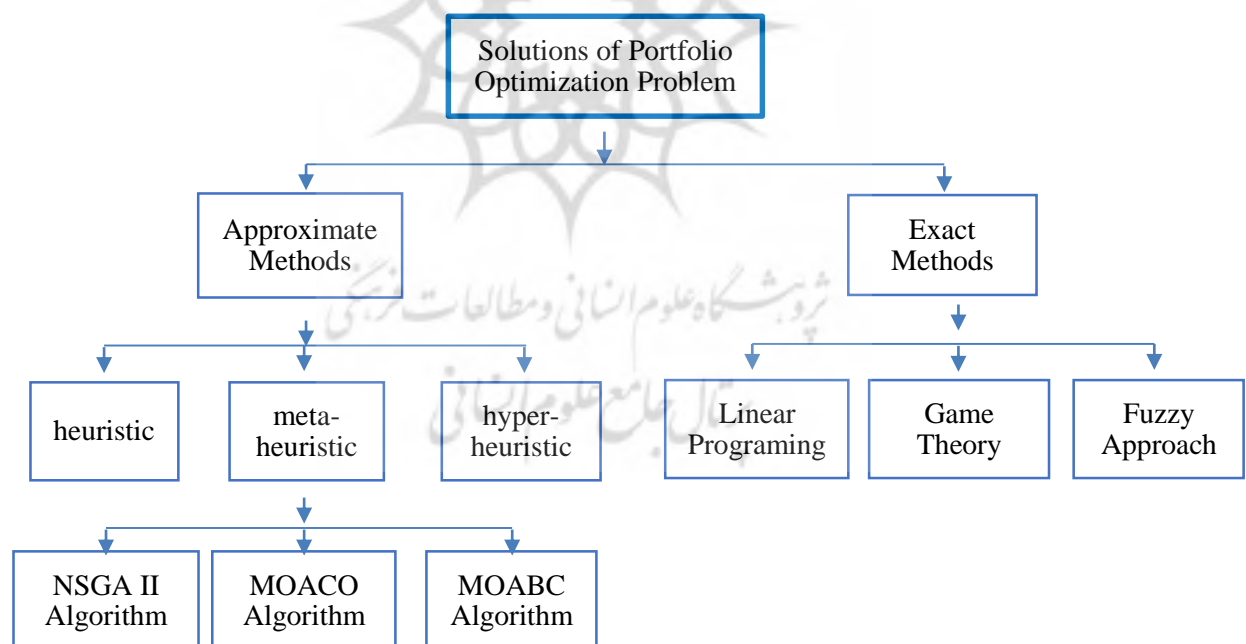


Fig 2: Solutions of Portfolio Optimization Problem

Given the fact that meta-heuristic algorithms are regarded as optimal algorithms for solving optimization problems and have a random nature, solving a problem through different methods may lead to

different solutions. Therefore, the evaluation of algorithms and the selection of suitable algorithms with the help of various indices have attracted the attention of the researchers [16].

2.4 Multi-Objective Evolutionary Algorithms

Over the past decade, a number of multi-objective evolutionary algorithms (MOEAs) have been suggested. The primary reason for this is their ability to find multiple Pareto-optimal solutions in one single run. Since the principal reason why a problem has a multi-objective formulation is because it is not possible to have a single solution which simultaneously optimizes all objectives, an algorithm that gives a large number of alternative solutions lying on or near the Pareto-optimal front is of great practical value [8]. In this study, we use the NSGA II algorithm, the MOACO algorithm, and the MOABC algorithm, which all of them are multi-objective algorithms, to solve the portfolio optimization problem.

2.5 Multi-Objective Ant Colony Optimization Algorithm (MOACO)

Ant Algorithm Optimization (ACO) is a population-based meta-heuristic technique developed by Marco Dorigo in 1992. This algorithm combines a random search technique and learning mechanism [20]. This algorithm is based on ants' living systems and imitating their behavior for searching food [9]. The initial algorithm aimed to find an optimal path in a graph based on the ant colony behavior of searching for a path between the nest and the food source [33]. Ants exchange food information with pheromones that they sour along the way. An ant returns to the nest by finding a food source. When the ants return to the nest on a shorter path, more pheromones and a shorter path will remain. [9].

Dorigo et al., Dorigo and Stützle [10, 11, 12], Ant colony optimization (ACO) is a swarm intelligence technique that was initially conceived for tackling single-objective combinatorial optimization problems. Given its success on these problems, ACO algorithms were soon extended to tackle multi-objective combinatorial optimization problems (MOCOPs), resulting in the introduction of multi-objective ant colony optimization (MOACO) algorithms. In other words, these MOACO algorithms tackle multi-objective problems in terms of Pareto optimality [22]. The ant colony optimization algorithm MOACO is derived from the ACO algorithm, which is presented to solve multi-objective optimization problems, which uses the optimal Pareto principle and its output becomes a set of Pareto optimal instead of one solution [23].

2.6 Non-dominated Sorting Genetic Algorithm II (NSGA II)

The genetic algorithm first proposed by John Holland is a kind of search algorithm based on the mechanism of natural selection and genetic science. Genetic algorithm is a comprehensive probabilistic search method that follows the process of natural biological evolution [18]. The work of the genetic algorithm is deceptively simple, very easy to understand and, in simple terms, the simplest way that humans believe that animals have evolved accordingly [17]. This algorithm combines the robustness of survival of the best string structure with the random information exchange operation and forms a very powerful search algorithm. To solve the problem with this algorithm, at first, the response must be encoded so that the algorithm can be evaluated and implemented by various operators [29]. The non-dominated Sorting Genetic Algorithm (NSGA) proposed in Srinivas and Deb, was one of the first such evolutionary algorithms [8]. NSGA II Algorithm is one of the fastest and most powerful optimization algorithms that have less operational complexity than other methods that with using A fast non-dominated sorting approach, Density estimation and

Crowded comparison operator, makes the optimal points of Pareto and gives the designer the freedom to choose the design he wants from among the optimized designs [8].

2.7 Multi-Objective Artificial Bee Colony Algorithm (MOABC)

Bee Colony, or ABC, is a meta-heuristic algorithm based on bee social life introduced by Karaboga in 2005 to optimize numerical problems [32]. This algorithm performs better than other mass intelligence algorithms [20]. The artificial bee colony (ABC) algorithm was designed for numerical optimization problems, based on the foraging behavior of honey bees. Since the performance of metaheuristic algorithms depends on the number and the choice of parameters, the main advantages of the ABC algorithm are derived from the fact that the algorithm uses only 3 control parameters: colony size, maximum cycle number, and limit [2].

ABC algorithm employs three classes of artificial bees: employed bees, onlookers, and scouts. Employed bee stays on a food source (candidate solution) and examines the neighborhood. Onlookers are allocated to a food source based on the information which they gain from employed bees. If a food source does not improve for a certain number of cycles, scouts replace that food source with a new, random one [2]. The MOABC algorithm uses the concept of Pareto dominance to determine the flight direction of a bee, and it maintains non-dominated solution vectors that have been found in an external archive. This algorithm is validated using the standard test problems, and simulation results show that the proposed approach is highly competitive and can be considered a viable alternative to solve multi-objective optimization problems [42].

3 Review of some Related Research

Chen [4], in his survey, states that The experimental outcomes demonstrate that real-world constraints have a great impact on the optimal investment schemes, and the MABC algorithm has a better performance than the standard ABC algorithm and other heuristic algorithms, such as GA, SA, PSO, DE. Chen et al [5] have applied a modified ABC algorithm to solve the optimization problem; their results show that the portfolio proportions changes with different types of transaction costs. Finally, they compared the results of the modified ABC algorithm with the ABC algorithm and GA, which showed that the modified ABC algorithm is better than the ABC algorithm and GA in solving the fuzzy portfolio selection problem.

Milan Tuba et al [40], used ABC for portfolio optimization problems and showed, according to experimental tests, it can be concluded that the ABC algorithm has the potential in solving portfolio optimization problems. With minor adaptations, the ABC algorithm can be adjusted for solving constrained portfolio optimization. Chang et al [3], established a heuristic approach to portfolio optimization problems in different risk measures by employing a genetic algorithm (GA) and they compared its performance to the mean-variance model in cardinality constrained efficiency frontier. To attain this object, they collected three different risk measures based upon mean-variance by Markowitz; semi-variance, mean absolute deviation, and variance with skewness. It is also shown that the portfolio optimization problems are solvable by a genetic algorithm if mean-variance; semi-variance, mean absolute deviation, and variance with skewness are applied as the measures of risk.

In Iran, Davoodi and Sadri [6] presented a model in which the constraints of the Short-selling prohibition and despite the transaction costs were used. They applied VaR as a risk measure and for portfolio optimization used GA-continuous and PSO algorithms and concluded that in equal conditions in terms of the number of iterations and population numbers, the efficiency of the PSO algorithm is more than

the GA-continuous algorithm in solving the portfolio optimization problem. Eslami Bidgoli and Tayebi Sani [13], presented a model in which minimizing VaR as a goal and its limitation as the minimum expected return and Short-selling prohibition. They worked out the problem of portfolio optimization by the GA algorithm, ACO algorithm, and the hybrid algorithm of GA and ACO. They concluded that the hybrid algorithm performed better than the GA algorithm, And VaR performance is superior to variance. Sinai and Zamani [38], solved the portfolio optimization problem by using GA and ABC algorithms and concluded that the portfolios formed by the bee algorithm are closer to the optimal portfolios resulting from the Markowitz model solution.

Fallah Shams et al. [14] optimized the portfolio using the ant colony algorithm and compared four models with distinct risk criteria (variance, semivariance, VaR, and CVaR). they concluded that the mean-variance model provides the worst efficient boundary and the mean- CVaR of the best efficient boundary and the mean-variance model takes the least run time and the mean-CVaR about the most run time. Although CVaR has a better efficient boundary, it is not a good measure in the run time, especially in large portfolio sizes. Rahmani et al [35] used genetic algorithm, ant colony algorithm, and artificial bee colony algorithm to optimally select the portfolio. They used the Markowitz model for this purpose and came to the conclusion that; in case the investors intend to apply a low-risk strategy for investment, the findings suggest that using the artificial bee colony algorithm can help them to achieve the optimum results as this algorithm detects the portfolio with lower risk compared to other algorithms.

4 Research Models

This study utilizes the Markowitz basic model to optimize the portfolio, and compares two models of the mean - SV and mean- pVaR to each other. The objective function in both models is a two-objective function for minimizing risk and maximizing return. In these models, Capital budget constraint, number of stocks constraint (Portfolio consisting of 30 stocks), the lower band (In this study, the least acceptable shares in the portfolio are equal to 1%), minimum liquidity (in this study, 25% is considered) And Short-selling prohibition is taken into account.

4.1 The mean – SV Model

This model seeks to minimize semi variance and maximize return on the portfolio. The mean-SV model presents in the following description:

$$\text{Min } \delta_p^2 = \sum_{i=1}^N \sum_{j=1}^M x_i x_j \delta_i \delta_j \rho_{ij} = \sum_{i=1}^N \sum_{j=1}^M x_i x_j \delta_{ij} \quad (4)$$

$$\text{Max } r_p = \sum x_j r_j \quad (5)$$

$$\text{st: } \sum x_i = 1 \quad (6)$$

$$\sum z_i = 30 \quad (7)$$

$$1\% \leq x_i \leq 99\% \quad (8)$$

$$L_i \geq 25\% \quad (9)$$

$$x_i \geq 0 \quad i=1.2....n \quad (10)$$

4.2 The mean – pVaR Model

This study uses pVaR as a risk measure. The mean-pVaR model seeks to minimize pVaR and maximize portfolio returns. This model presents below:

$$pVaR = Z_\alpha * \sigma - r_p \quad (11)$$

$$pVaR_p = Z_\alpha \sqrt{\sum_{i=1}^n x_i^2 \sigma_i^2 + 2 * \sum_{i=1}^n \sum_{j<i}^m x_i x_j \rho_{ij} \sigma_i \sigma_j} - \sum x_j r_j \tag{12}$$

$$pVaR_p = \sqrt{\sum_{i=1}^n (x_i \sigma_i Z_\alpha)^2 + 2 * \sum_{i=1}^n \sum_{j<i}^m x_i x_j \rho_{ij} \sigma_i \sigma_j (Z_\alpha)^2} - \sum x_j r_j \tag{13}$$

therefore, the mean-pVaR model is presented in The following description:

$$Min pVaR_p = \sqrt{\sum_{i=1}^n x_i^2 (pVaR_i + r_i)^2 + 2 * \sum_{i=1}^n \sum_{j<i}^m x_i x_j (pVaR_i + r_i) * (pVaR_j + r_j) \rho_{ij}} - \sum_{i=1}^n x_i r_i \tag{14}$$

$$Max r_p = \sum x_j r_j \tag{15}$$

$$st: \sum x_i = 1 \tag{16}$$

$$\sum z_i = 30 \tag{17}$$

$$1\% \leq x_i \leq 99\% \tag{18}$$

$$L_i \geq 25\% \tag{19}$$

$$x_i \geq 0 \quad i=1.2....n \tag{20}$$

Where δ_p^2 is the variance of portfolio, r_p is the return of portfolio, r_j is the return of asset j, $pVaR_p$ is the percentage of assets at risk of portfolio, x_i is asset share i, x_j is asset share j, $pVaR_i$ is percentage of assets at risk of asset i, $pVaR_j$ is percentage of assets at risk of asset j, δ_{ij} is semi covariance asset i and asset j, ρ_{ij} is correlation coefficient asset i and asset j, L_i is Liquidity ratio and α is confidence level (In this study $\alpha=95\%$).

5 Methodology and Hypotheses

This survey extracts the monthly data of all firms in the first market of the Tehran Stock Exchange between 2012 and 2017 from the Novin Rah-e-Avard software of the Tehran Stock Exchange. Then, amongst the firms that are listed on the Tehran Stock Exchange are considering active between 2012 and 2017, 50 firms are selected in the following description:

- First, according to the number of active firms in each industry, we settle the weight of that industry. The weight of each industry is equal to the number of active firms in that industry divided by the whole number of active firms in the Tehran Stock Exchange.
- Then, according to the weight of each industry, we determine the number of firms surveyed. The number of firms surveyed in each industry is equal to the number 50 (the number of samples considered) multiplied by the settled weight of that industry.

Due to the fact that the risk and return information of all firms operating in the Tehran Stock Exchange is not complete (so that the lack of information is not more than three consecutive months), in order to generate more diversity, we select 50 active firms in Different industries that contain complete information, as the number of samples. In such a situation, it will be practicable to opt for at least one firm from any significant industry in the stock portfolio. Therefore, according to the diversity that is gained, the selected sample represents the whole market. This study utilizes the NSGA II, MOACO, and MOABC algorithms to solve the portfolio optimization problem in the mean - SV model and mean - pVaR model. These algorithms have been developed using MATLAB codes. It also uses the Excel software for data analysis and the SPSS software for statistical analysis. Because the answers obtained from the run of meta-heuristic algorithms are not the exact ones, therefore several solutions are achieved in different runs. That is why it is important to determine the best run to find the best answer. In this regard, to find the optimal results in each model, we run each algorithm 10 times by the iteration of 1000 and

once by the iteration of 5000. In the next step, to get the best Pareto in the iteration of 1000, we must notice the concept of dominance in the discussion of multi-objective optimization. This means that x_1 dominates to x_2 , if and only if x_2 would not be worse than x_1 in any of the targets [7]. To this purpose, among all of the solutions obtained from 10 performances of each algorithm in each model, we tend to find out the solutions that dominate the other solutions and draw the efficient Pareto that includes the best solutions in each model and each algorithm. It should be noted that no research has been done by this type of method to have the best Pareto in Iran, So far. Based on the above, in this study, we select the best Pareto from all of the solutions, which are resulted from 10 runs in each algorithm in each model. Then, using the concept of the Sharp ratio that indicates the return of each portfolio, the following hypotheses are also proposed.

- There is a significant difference between the investment Sharp ratio of portfolios resulting from the performance of the MOABC algorithm and portfolios resulting from the performance of the NSGA II algorithm.
- There is a significant difference between the investment Sharp ratio of portfolios resulting from the performance of the MOABC algorithm and portfolios resulting from the performance of the MOACO algorithm.

To obtain the Sharp ratio for each of the proposing portfolios on the Pareto- front, we employ (21), in which the risk-free return is considered to be 18% per year.

$$Sharp\ Ratio = \frac{r_p - r_f}{\sigma_p} \tag{21}$$

Where, r_p is the return of portfolio, r_f is risk-free return and σ_p is standard deviation[37].

We should notice that the variance of each portfolio is not calculated in the software output, Therefore the calculation of the Sharp ratio for the proposed portfolios in the mean-SV and mean-pVaR models is not easily Calculable. To reach the purpose, according to the specified percentage of investment per stock in each portfolio " x_i " that is obtained from the run of each algorithm and (22), we calculate the variance of the portfolio and then proceed to calculate The portfolio sharp ratio.

$$\sigma_p^2 = \sum_{i=1}^N \sum_{j=1}^M x_i x_j \sigma_{ij} \tag{22}$$

Due to the lots calculations of the portfolio variance with 30 stocks and its computational complexity, we utilize MATLAB software to calculate the variance and after calculating the variance of each portfolio, the Sharp ratio of that portfolio is calculated, too.

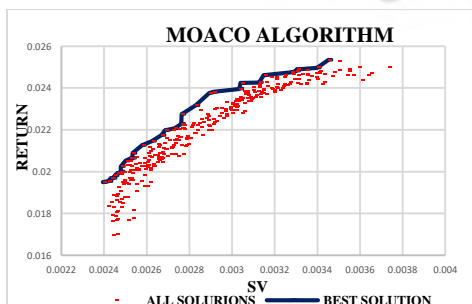


Fig 3: Selecting the Best Pareto among all Solution Resulted from 10 Runs of MOACO in mean-SV Model

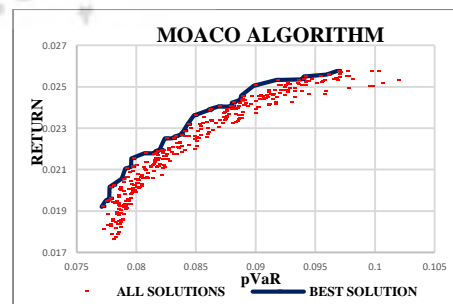


Fig 4: Selecting the Best Pareto among all Solution Resulted from 10 Runs of MOACO in mean-pVaR Model

6 Findings and Data Analysis

6.1 Finding the Best Pareto of Each Algorithm in Each Model

As mentioned, at first we run 10 times, each of the NSGA II, MOACO, and MOABC algorithms in two mean-pVaR and mean-SV models, in 1000 iterations. Then, by the concept of dominance, we determine the best solutions among all of the ones that result in the runs of each algorithm. These solutions, dominant to other solutions, and are the same best Pareto- front resulting from the run of each algorithm in each model as is shown in Figures 3-8.

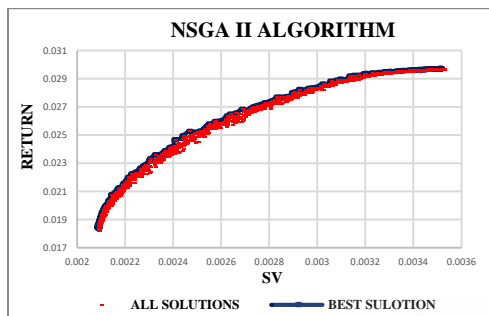


Fig 5: Selecting the Best Pareto among all Solution Resulted from 10 Runs of NSGA II in mean-SV Model

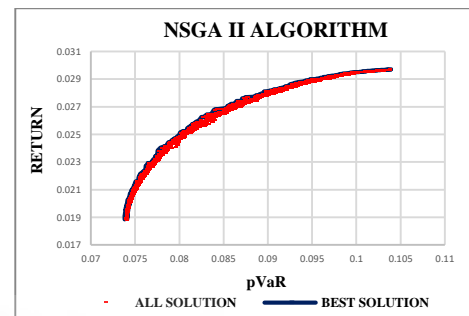


Fig 6: Selecting the Best Pareto among all Solution Resulted from 10 Runs of NSGA II in mean-pVaR Model

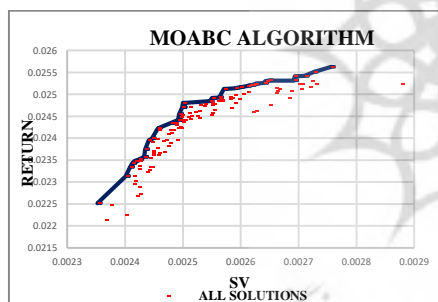


Fig 7: Selecting the Best Pareto among all Solution Resulted from 10 Runs of MOABC in mean-SV Model

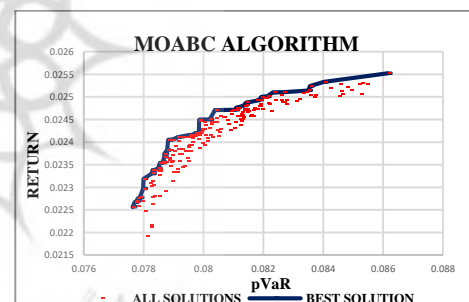


Fig 8: Selecting the Best Pareto among all Solution Resulted from 10 Runs of MOABC in mean-pVaR Model

According to the above, we select the best Pareto obtained from the run of each of the NSGA II, MOACO, and MOABC algorithms in the two models mean-pVaR and mean-SV, and in the iteration of 1000. This study considers this Pareto the basis to compare the performance of studied algorithms and models.

6.2 Comparison of Performance of NAGA II, MOACO and MOABC Algorithms

To this end, we analyze the portfolios formed by the run of the NSGA II, MOACO, and MOABC algorithms in the mean – SV model and mean - pVaR model in the iteration of 1000 and also in the iteration of 5000. then using the T-test in the SPSS software, we test the study hypotheses. the results are shown in Figures 9, 10, and Table 1.

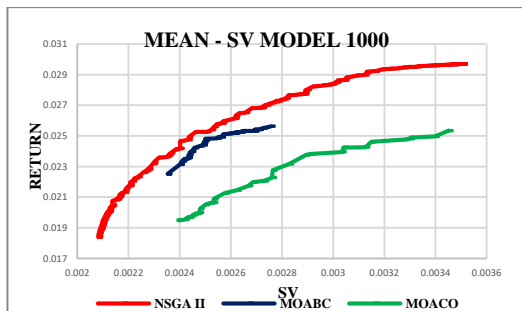


Fig 9: Compare of Best Pareto of MOACO, NSGA II, and MOABC Algorithms in the mean- SV Model and Iteration of 1000

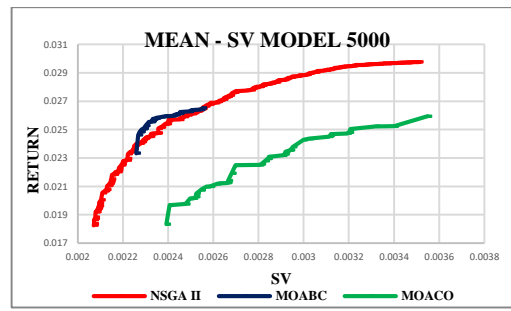


Fig 10: Compare of Best Pareto of MOACO, NSGA II, and MOABC Algorithms in the mean- SV Model and Iteration of 5000

According to the demonstration in Figure 9, in the iteration of 1000, in the mean-SV model, the Pareto-front resulting from the NSGA II algorithm dominates the Pareto- fronts resulting from the MOABC and the MOACO algorithms. In other words, for an identified amount of risk, the portfolio return resulting from the MOABC algorithm is slightly lower than the portfolio returns resulting from the NSGA II algorithm and is relatively higher than the return resulting from the MOACO algorithm. However, by looking at Fig. 10, it is clear that in the iteration of 5000, the Pareto- front resulting from the MOABC algorithm dominates the Pareto- fronts resulting from NSGA II and MOACO algorithms. In other words, in the iteration of 5000 for an identified amount of risk, in most points, the portfolio return resulting from the MOABC algorithm is higher than the portfolio returns resulting from the NSGA II algorithm and MOACO algorithm.

Table 1: Results of T-Test in the mean- SV Model

	MOACO		NSGA II		MOABC		Sig of Levene’s Test for Equality	Sig of T-test for Equality
	No.	Mean sharp ratio	No.	Mean sharp ratio	No.	Mean sharp ratio		
portfolio resulting from MOABC and NSGA II algorithms (iteration of 1000)	-	-	221	0.153	71	0.156	0.000	0.322
portfolio resulting from MOABC and MOACO algorithms (iteration of 1000)	32	0.12	-	-	71	0.156	0.000	0.000
portfolio resulting from MOABC and NSGA II algorithms (iteration of 5000)	-	-	197	0.166	36	0.167	0.000	0.912
portfolio resulting from MOABC and MOACO algorithms (iteration of 5000)	40	0.125	-	-	36	0.167	0.000	0.000

In this regard, by examining the results of the T-test of SPSS software output, which is shown in Table 1, It is clear that the Sig of Levene’s Test for Equality of variances is less than 5%, therefore, the assumption of the equality of variances between the two communities is rejected. On the other hand, because the Sig of T-test for Equality of means is less than 5%, therefore there is a significant difference between the mean of the two communities, otherwise, there is no significant difference between the mean of the two communities [1,30]. Therefore, it can be stated that in the mean-SV model, in the iterations of 1000 and 5000 and at a 5% error level; there is a significant difference between the investment Sharp ratio of portfolios based on the MOABC algorithm and portfolios based on the MOACO

algorithm. However, there is no significant difference between the investment Sharp ratio of portfolios based on the MOABC algorithm and portfolios based on the NSGA II algorithm. On the other hand, since the average Sharp ratio of portfolios based on the MOABC algorithm, is higher than the average Sharp ratio of portfolios based on the MOACO algorithm and NSGA II algorithm then, we can get the result, in this sense the MOABC algorithm has a Preference over the two other algorithms.

To conduct a more comprehensive review, we perform the above operations for the mean-pVaR model. The results are similar to the results of the performance of the studied algorithm in the mean-SV model in the iterations of 1000 and 5000. According to Fig. 11 and Fig. 12, in the iteration of 1000, for an identified amount of risk, the portfolio return resulting from the MOABC algorithm is slightly lower than the portfolio returns resulting from the NSGA II algorithm, and also relatively is higher than the portfolio returns resulting from the MOACO algorithm. In addition, with increasing iteration and reaching the iteration of 5,000, the portfolio return resulting from the MOABC algorithm is higher than the portfolio returns resulting from the MOACO algorithm and the NSGA II algorithm.

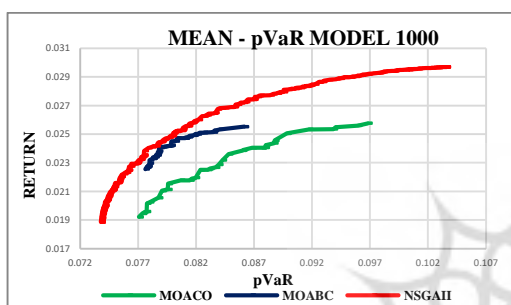


Fig 11: Compare of Best Pareto of MOACO, NSCA II, and MOABC Algorithms in the mean- pVaR Model and Iteration of 1000

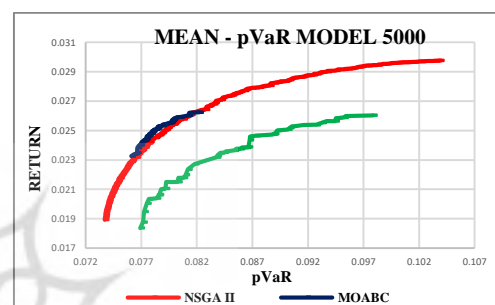


Fig 12: Compare of Best Pareto of MOACO, NSCA II, and MOABC Algorithms in the mean- pVaR Model and Iteration of 5000

Table 2: Results of T-Test in the mean- pVaR Model

	MOACO		NSGA II		MOABC		Sig of Levene's Test for Equality	Sig of T-test for Equality
	No.	Mean sharp ratio	•No.	Mean sharp ratio	No.	Mean sharp ratio		
portfolio resulting from MOABC and NSGA II algorithms (iteration of 1000)	-	-	254	0.166	48	0.162	0.000	0.084
portfolio resulting from MOABC and MOACO algorithms (iteration of 1000)	35	0.135	-	-	48	0.162	0.000	0.000
portfolio resulting from MOABC and NSGA II algorithms (iteration of 5000)	-	-	197	0.173	63	0.177	0.000	0.174
portfolio resulting from MOABC and MOACO algorithms (iteration of 5000)	52	0.14	-	-	63	0.177	0.000	0.000

According to Table 2, in the mean-pVaR model, in the iterations of 1000 and 5000 and at a 5% error level; there is a significant difference between the investment Sharp ratio of portfolios based on the MOABC algorithm and portfolios based on the MOACO algorithm. However, there is no significant difference between the investment Sharp ratio of portfolios based on the MOABC algorithm and portfolios based on the NSGA II algorithm. Therefore, according to the above-mentioned cases, with increasing the number of iteration of each algorithm, the average portfolio return resulting from the MOABC algorithm is higher than the average portfolio returns resulting from the NSGA II algorithm

and MOACO algorithm. On the other hand, according to the structure of each algorithm, the Pareto-front of the MOABC algorithm covers a more limited range of risk than the MOACO algorithm and the NSGA II algorithm. In other words, the conclusions show that selecting and optimizing the stock portfolio using the MOABC algorithm has acceptable performance and by increasing the iteration in this algorithm, its performance will increase. Moreover, in a similar range of risk, it will have better results than NSGA II and MOACO algorithms. However, in low iteration, the performance of the NSGA II algorithm is more desirable and more acceptable. On the other hand, according to the structure of each algorithm, the Pareto-front of the MOABC algorithm covers a more limited range of risk than the MOACO algorithm and the NSGA II algorithm. In other words, it can be concluded that selecting and optimizing the stock portfolio using the MOABC algorithm has acceptable performance and by increasing the iteration in this algorithm, its performance will increase. By the way, in a similar range of risk, it will have better results than NSGA II and MOACO algorithms. However, in low iteration, the performance of the NSGA II algorithm is more desirable and acceptable.

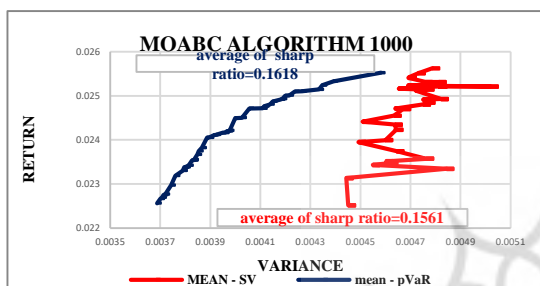


Fig 13: Compare of Performance of the mean- SV & the mean- pVaR Models in MOABC Algorithm in the Iteration of 1000

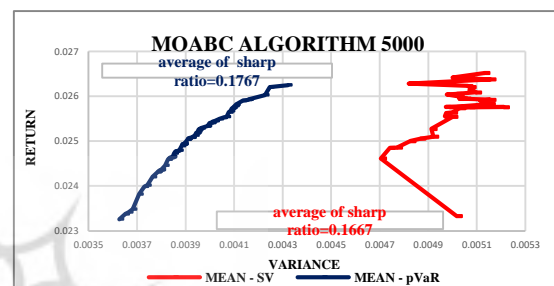


Fig 14: Compare of Performance of the mean- SV & the mean- pVaR Models in MOABC Algorithm in the Iteration of 5000

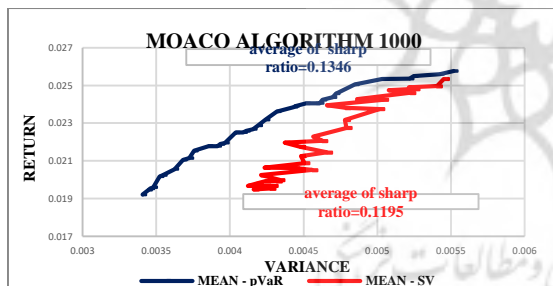


Fig 15: Compare of Performance of the mean- SV & the mean- pVaR Models in MOACO Algorithm in the Iteration of 1000

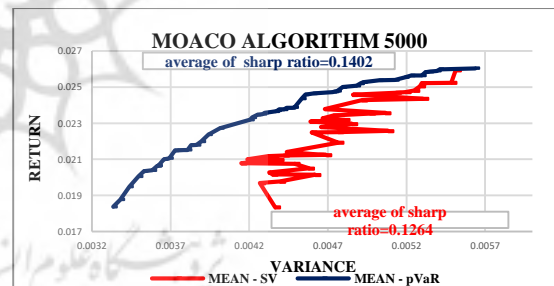


Fig 16: Compare of Performance of the mean- SV & the mean- pVaR Models in MOACO Algorithm in the Iteration of 5000

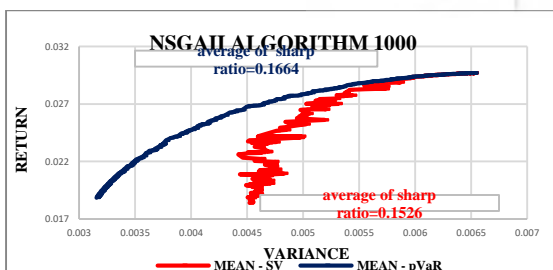


Fig 17: Compare of Performance of the mean- SV & the mean- pVaR Models in NSGA II Algorithm in the Iteration of 1000

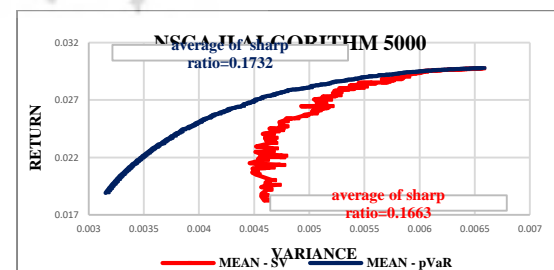


Fig 18: Compare of Performance of the mean- SV & the mean- pVaR Models in NSGA II Algorithm in the Iteration of 5000

6.3 Comparison of Performance of SV and pVaR

To reach the purpose, this study compares the average of Sharp ratio of portfolios resulting from the MOACO, NSGA II, and MOABC algorithms in the mean-pVaR and mean-SV models in the iteration of 1000 and 5000. The risk criterion in the models of the mean-SV and the mean-pVaR is not the same. therefore, we compute the variance of each of the portfolios that are proposed by applying the percentage of assets in the portfolios resulting from the run of the under investigating algorithms. then we compare two of the models studied in the standard mean-variance model, in such a way that the performance of both models in the mean-variance model is measured. Then, according to the calculated variance of each SV and pVaR obtaining from the performance of each algorithm in the iteration of 1000 and 5000, we draw the Pareto- front in the NSGA II, MOACO, and MOABC algorithms. According to Figures 13 to 18. By examining Figures 13 to 18, the conclusion is that, in all three algorithms and both iterations, the Pareto- front of the mean-pVaR model dominates the Pareto- front of the mean-SV model. In other words, in the portfolio optimization problem, using of pVaR comparing to SV as a risk measure, due to having a higher return for an identified amount of risk and the higher average of Sharp ratio, it has more performance that is desirable.

7 Discussion and Conclusion

This study surveys the Value at Risk criterion from another perspective and presents the model of the “mean- percentage of Value at Risk”, and applies the NSGA II, MOACO, and MOABC algorithms for solving the portfolio optimization problem. We apply the concept of dominance in multi-objective optimization discussion to reach the best Pareto-front resulting from all runs of studied algorithms. Then, the comparison of performances of the NSGA II and MOACO algorithms to the MOABC algorithm in solving the portfolio optimization problem is drawn. In the following, we compare the performance of SV and pVaR as risk measures to each other in the mean-variance standard pattern. The results are as follows:

According to our illustrations, in the iteration of 1000, the performance of the NSGA II algorithm is better than the MOABC algorithm for solving the portfolio optimization problem in both mean-SV and mean-pVaR models, and in this regard, the performance of the MOABC algorithm is better than the MOACO algorithm. We show, by increasing the number of iteration of each algorithm and reaching the iteration of 5000, the performance of all algorithms improves, but the rate of increase of their improvement is not equal. In other words, by increasing iteration and reaching the iteration of 5000, the performance of the MOABC algorithm is better than the NSGA II and MOACO algorithms, in portfolio optimization in both mean-SV and mean-pVaR models. It is worth mentioning that most studies have shown that the artificial bee colony algorithm is more efficient than the genetic algorithm and ant colony optimization algorithm in solving the portfolio optimization problem. However, in this study, we show that, in low replications, the performance of the NSGA II algorithm is better than the MOABC algorithm and MOACO algorithm in solving the portfolio optimization problem. And by increasing the repetition, the performance of the MOABC algorithm has been significantly improved, so that its performance is better than the performance of the NSGA II algorithm and MOACO algorithm in solving the portfolio optimization problem. We show the comparison of the two models: in the portfolio optimization problem, the mean-pVaR model is more efficient than the mean-SV model, due to having a higher average return for an identified amount of risk and a higher average of Sharp ratio. In other words, using the pVaR comparing to the SV as a risk measure has a higher desirable for solving the portfolio optimization problem. As we said, the pVaR is derived from the VaR, Hence, it can be expressed; our results

are in accordance with the results of other studies and indicate the higher desirability of the VaR than the SV as a risk criterion. According to the above, we suggest researchers utilize the following in future studies:

- Due to the structure of meta-heuristic algorithms and the existence of some limitations in each of them in solving the stock portfolio optimization problem, future studies can be done to increase the efficiency and eliminate the limitations of these algorithms. For example, hybridize efficient matching heuristic algorithms to solve the portfolio optimization problem.
- using other evaluation criteria such as Sortino, Treynor, etc. for comparing and evaluating the performance of the meta-heuristic algorithm in portfolio optimization.
- applying some special models, in which, the Predicted return resulting from time-series is replaced average return. For instance, using the "Predicted return- variance" model instead of the "mean-variance" model and comparing them to one another.

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