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The Fuzziness of Fuzzy Ideals

Amer Himza Almyaly

Department of Mathematics and Computer Applications, College of Science/AL-Muthanna University, Iraq. ORCID CODE: 0000-0003-2448-5121, E-mail: amerhimzi@mu.edu.iq

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Abstract

The backing of the fuzzy ideal is normal ideal in some ring and in same time there fuzzy set whose is not fuzzy ideal and it backing set is ideal, i.e., it crisp is normal ideal. Consequently, in this paper we constructing a fuzziness function which defined on fuzzy sets and assigns membership grade for every fuzzy set whose it backing set are crisp ideal. Now, Let J^R be collection of all fuzzy subsets of ring R and $J = [0,1]$, and the function τ defines from J^R to J, such that the value of the function is greater than zero if the crisp set of fuzzy set is ideal, and the value of the function is equal to one when the crisp set is maximal ideal or is the ring R itself. But if the support of the fuzzy set did not ideal then the value may be equal to or large than 0. Therefore, we add another condition to the fuzziness function to be more determined with respect to the fuzzy set. From above we try to find relation between the fuzzy ideal and its crisp set. This concept is derives from the open grade for all fuzzy set in fuzzy topological space which called smooth topology.

Keywords: Fuzzy ideal; Subring; Maximal ideal; Fuzziness of the fuzzy set.

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1. Introduction

In mathematics, the concept of the fuzzy sets (as the uncertain sets) are somewhat like sets whose elements have degrees of membership and those degrees may be numbers, objects or any other things such that we can give a degree for every element and this degree illustrates the importance of this element. This is similar as in probability theory, we give probability for every event between 0 and 1, if the event is unattainable then the probability will be 0 and if it sure then the probability is 1, other than the probability is between 0 and 1. The concept of the fuzzy sets was introduced by (Zadeh 1965). It has multi of the applications in mathematics, physics, engineering, medical, computer, Information systems, Expert systems and control systems sciences (Taghavifard & his colleagues 2014; Shahbazi & Kianifar 2016; Salman and Khalaf 2019; Ghatari et al., 2018). For example, the Fuzzy logic is very significant when it apply in medical decision making. Since medical and healthcare data can be unstable or fuzzy, applications in this domain have a great potential to benefit a lot by using fuzzy logic based approaches.

Here, we work in field of the Algebra theory. Liu introduced some concepts in Algebra as fuzzy subgroup, fuzzy invariant subgroup, fuzzy subgroupid, fuzzy subring and fuzzy ideal. Also, he gave the fuzzy left ideal and the fuzzy right ideal and discussed the homomorphism between two rings. He proved every fuzzy left (right) ideal is fuzzy ideal in skew field (Liu 1982). Actually, Azriel is the first whose define the fuzzy ideal but he defined on just group (Rosenfeld 1971). Those researchers used one fuzzy function which named fuzzy set $A: \Omega \to J$ when Ω is universal set and $J = [0,1]$. But here, we used two fuzzy functions which will be explaining it later. Sostak remarked the fuzziness of the concept of openness of the fuzzy set so that he give for every fuzzy set some degree to been fuzzy open, and this degree may start from 0 (completely is not open sets) to 1 (completely open sets) (Sostak 1985), therefore he defined a fuzzy topological space as pair (Ω, F) where Ω is a nonempty set and $F \in J^{J^{\Omega}}$, that is a fuzzy topology is a function $F: J^{\Omega} \to J$ which assigns to every fuzzy subset of Ω the real number in $[0,1]$. Ramadan studied a similar concept for Sostak's concept, which namely "smooth topological space", using the lattices L and L' instead of $J = [0,1]$ (Ramadan 1992). This concept (Smooth Topology) gives the grade of open of any fuzzy set in fuzzy topological space. Thus, we employed this concept but in the ring theory and by various conditions to give the membership grade of the fuzzy ideal for any fuzzy set, i.e., we allow to fuzzy set to be fuzzy ideal to some degree. In fact, we can exchange J by the lattice L but we need results to be more explained. During this paper, $J = [0,1]$,R be a ring and let J^R is the set of all fuzzy subsets of ring R.

2. The aim research

To find function which explain the relationship between the normal ideal and fuzzy ideal such that we can expect what the properties of fuzzy ideal while we do in normal ideal and vice versa.

This process is very important during the operation of the software as it enables us to easily design an algorithm for the algebra of rings where we can find all fuzzy ideals by the software thus we can design a program that gives fuzzy ideals in any ring.

3. Materials and Methods

3.1 Fuzzy set concept: Let$\Omega \neq \emptyset$ be a set, the function $\lambda: \Omega \to J$ whose assigned real number $\lambda(x)$ in $J\ \forall x \in \Omega$, then λ be named fuzzy subset in Ω. The value $\lambda(x)$ we will name (grade of membership of x inλ), (Klir & Yuan 1995).

3.2 Fuzzy point concept: any fuzzy subset in nonempty set Ω as $\lambda(\sigma) = \begin{cases} s \neq 0 & \text{if } \sigma = \gamma \\ 0 & \text{if } \sigma \neq \gamma \end{cases}$ we called fuzzy point with support γ and value s, which denote byγ_s, (Klir & Yuan 1995).

3.3 Inclusion concept: The fuzzy point γ_t belong to fuzzy set λ, and written $\gamma_s \in \lambda$, if $\lambda(\gamma) \geq s$, (Klir & Yuan 1995).

3.4 α-level concept: Let$\Omega \neq \emptyset$ be a set and λ is fuzzy subset in setΩ, theα-level set of fuzzy set λ is normal set in Ω and denoted itby λ_α and defined as: $\lambda_\alpha = \{\rho \in \Omega: \lambda(\rho) \geq \alpha, \alpha \in (0,1]\}$, (Klir & Yuan 1995).

3.5 The backing set concept: Let $\Omega \neq \emptyset$ be a set and λ is fuzzy set in setΩ, the backing set of λ, symbolize by suppλ, is the normal set of Ω which it elements all have nonzero membership degree in λ. Such that., $\mathrm{supp}\lambda = \{\rho \in \Omega: \lambda(\rho) > 0\}$, (Klir & Yuan 1995).

3.6 Fuzzy Subring concept: Let R is ring and λ be fuzzy subset in Rthen λ is called fuzzy subring ofR, if $\forall\ \gamma, \rho \in R$, then:

1- $\lambda(\gamma - \rho) \geq \min\{\lambda(\gamma), \lambda(\rho)\}$,

2- $\lambda(\gamma\rho) \geq \min\{\lambda(\gamma), \lambda(\rho)\}$. (Liu 1982)

3.7 Fuzzy ideal concept: In a ringR, we called the fuzzy subset λ ofR fuzzy ideal in R, if $\forall \gamma, \rho \in R$, then:

1- $\lambda(\gamma - \rho) \geq \min\{\lambda(\gamma), \lambda(\rho)\}$,

2- $\lambda(\gamma\rho) \geq \max\{\lambda(\gamma), \lambda(\rho)\}$. (Liu 1982)

3.1 Example about Fuzzy ideal concept: LetR be a ring and λ be fuzzy subset define as following:$\lambda(\gamma) = r, \forall \gamma \in R$ and $r \in [0,1]$, then λ is fuzzy ideal in R.

Note, each fuzzy ideal is fuzzy subring but the reverse is not true in general as the next example:

3.2 Example: In ring of real numbers set $\mathbb{R}$, let the fuzzy set $\lambda: \mathbb{R} \to J$ defines as:$\lambda(\gamma) = \begin{cases} 1/2 & \text{if } \gamma \in Z \\ 0 & \text{otherwise} \end{cases}$, when Z is integer number set,then λ is fuzzy subring but is not fuzzy ideal.

Note, In examples 1.1 and 1.2, we notice can be having infinite of a fuzzy subring and ideal sets in any ring.

3.8 Fuzzy maximal ideal concept: The fuzzy ideal λ of ringR is said by fuzzy maximal ideal, if the level ideal$\{ \gamma \in R: \lambda(\gamma) = 1 \}$ is a maximal ideal within^{R}, (Kambhojkar & Bapat 1993).

Note, the ring^{R} may be have fuzzy maximal ideal or it have not as following example.

3.3 Example: The ring Z_6, we suppose $\lambda: Z_6 \to J$ define as: $\lambda(\gamma) = \begin{cases} 1 & \text{if } \gamma \text{ is even} \\ 1/2 & \text{otherwise} \end{cases}$ then λ is fuzzy ideal and since $\{\gamma \in Z_6: \lambda(\gamma) = 1\} = \{0,2,4\}$ is the maximal ideal in the ringZ_6, thenλ is fuzzy maximal ideal in the ringZ_6

3.4 Example: In ring$\mathbb{R}$, Let the fuzzy set $\lambda: \mathbb{R} \to I$ be a fuzzy maximal ideal, then the level$\{\gamma \in R: \lambda(\gamma) = 1\}$is normal maximal ideal in$\mathbb{R}$, but this is unacceptable since the ring of real numbers set do not contain any maximal ideal. Thus, the ring $\mathbb{R}$ has not any fuzzy maximal ideal.

4. Results & Discussion

4.1 Fuzziness of fuzzy ideal concept: LetR be a ring and Let$\mathcal{F}: J^R \to J$ be a fuzzy function satisfies the next conditions:

$\mathcal{F}(\emptyset) = 0.$

$\mathcal{F}(\lambda) \leq \mathcal{F}(\mu), \ \forall \lambda \leq \mu.$

$\mathcal{F}(\lambda) \geq \min(\lambda(\gamma - \rho), \lambda(\sigma\delta)), \ \forall \lambda(\gamma), \lambda(\rho) > 0 \text{ and } \lambda(\sigma) \text{or} \lambda(\delta) > 0.$

Then we name $\mathcal{F}$ the fuzziness of fuzzy ideals on ringR.

4.1 Example: LetR be any ring and let$\mathcal{F}: J^R \to J$ is define as$\tau(\lambda) = \lambda(\tilde{0})$ ($\tilde{0}$ is the identity element in ringR), then:

I- $\mathcal{F}(\emptyset) = \emptyset(\tilde{0}) = 0.$

II- If$\lambda \leq \mu \Longrightarrow \lambda(\tilde{0}) \leq \mu(\tilde{0})$, therefore$\tau(\lambda) \leq \tau(\mu)$ is satisfied.

III- $\mathcal{F}(\lambda) = \lambda(\tilde{0}) = \begin{cases} \lambda(\tilde{0} - \tilde{0}) \geq \min \lambda(\gamma - \rho), (\forall \lambda(\gamma), \lambda(\rho) > 0), & \text{if} \lambda(\tilde{0}) > 0 \\ \lambda(\tilde{0}w) = \lambda(\tilde{0}) = 0 = \min \lambda(\sigma\delta)(\text{s.t.} \lambda(\delta) > 0), & \text{if} \lambda(\tilde{0}) = 0 \end{cases} \geq \min \left(\lambda(\gamma - \rho), \lambda(\sigma\delta)\right), \forall \lambda(\gamma), \lambda(\rho) > 0$ and, $\lambda(\sigma)$or $\lambda(\delta) > 0$.

Therefore during above,$\mathcal{F}$ is the fuzziness of the fuzzy ideals in ringR.

4.1 Proposition: LetR be a ring,

For each fuzzy subset $\lambda \neq \emptyset$ inR, if suppλ is ideal then$\mathcal{F}(\lambda) > 0$.

$\mathcal{F}(\mathrm{R}) = 1$.

$\mathcal{F}(\lambda) = 1$, where λ is fuzzy maximal ideal.

Proof:

[1] If $\lambda \neq \emptyset$ andsuppλ is ideal, then this follows frommin$\left(\lambda(\gamma - \rho), \lambda(\sigma\delta)\right) > 0$,

$\forall \lambda(\gamma), \lambda(\rho) > 0$ and, $\lambda(\sigma)$or $\lambda(\delta) > 0$. Therefore, by the condition (III)$\tau(\lambda) > 0$.

[2] Since for each element belong to R has membership grade1, and by the condition (III), then$\tau(\mathrm{R}) = 1$.

[3] Let λ be the fuzzy maximal ideal in the ringR, then thesupp$\lambda = \{\gamma \in \mathrm{R}: \lambda(\gamma) = 1\}$ be maximal ideal in R. Now, we define the fuzzy set $\nu: \mathrm{R} \to \mathrm{J}$ such that:

$\nu(\gamma) = \begin{cases} 1 & \text{if } \gamma \in \lambda \\ 0 & \text{if } \gamma \notin \lambda \end{cases}$. Then$\nu$ is fuzzy ideal and from the condition (III) then$\tau(\nu) = 1$. But$\nu \leq \lambda$ therefore by the condition (II)$\tau(\lambda) = 1$■■

From proposition 2.1,$\mathcal{F}(\lambda) > 0$ for each fuzzy ideal$\lambda \neq \emptyset$, but if the fuzzy setλ is not fuzzy ideal then$\tau(\lambda) = 0$ or not as example 4.1. We effort to limitation the fuzziness function τ on the fuzzy ideals, but this is not effective because there exists a fuzzy subset which it is not fuzzy ideal but the support is ideal as following below:

4.2 Example: In ringZ_6, let $\lambda = \{(0,0.3), (2,0.5), (4,0.5)\}$, then λ is not fuzzy ideal since $\lambda(4.0) = \lambda(0) = 0.3 < max(\lambda(4), \lambda(0))$, although thesupp$\lambda$ is ideal inZ_6.

Therefore, we will add condition for the definition 2.1:

$\tau(\lambda) = 0$ if $\min\left(\lambda(\gamma - \rho), \lambda(\sigma\delta)\right) = 0$ when$\lambda(\gamma), \lambda(\rho) > 0$ and, $\lambda(\sigma)$or $\lambda(\delta) > 0$.

Now, if the function $\mathcal{F}: J^R \to J$ satisfy the definition 2.1 with condition (IV), we call it τ-fuzziness of fuzzy ideals on ring Rvv

Note, it is that obvious all $\mathcal{F}$-fuzziness functions are fuzziness functions but the inverse is not satisfied in general as following example. Really, the $\mathcal{F}$-fuzziness function check the backing of any fuzzy set is normal ideal or no.

4.1 Main Theorem: In a ringR. Let $\mathcal{F}: J^R \to J$ is a τ-fuzziness of fuzzy ideals in ringR then $\mathcal{F}(\lambda) = 0$ where$\lambda = \emptyset$ orsuppλ is not ideal and $\mathcal{F}(\lambda) > 0$ ifλ is ideal.

Proof: Letλ is fuzzy set in ringR, $\mathcal{F}: J^R \to J$ be a $\mathcal{F}$-fuzziness of fuzzy ideals onR. If $\lambda = \emptyset$ then by the condition (I)$\mathcal{F}(\lambda) = 0$. Let$\lambda \neq \emptyset$ andsuppλ is not ideal then one or both from following are not satisfied:

1- $\exists \gamma, \rho \in \text{supp}\lambda$ such that $\gamma - \rho \notin \text{supp}\lambda$,

2- $\exists \sigma \in \text{supp}\lambda$ and $\delta \in R$ such that $\sigma\delta \notin \text{supp}\lambda$.

And this meansmin$\big(\lambda(\gamma - \rho), \lambda(\gamma\rho)\big) = 0$ where$\lambda(\gamma), \lambda(\rho) > 0$ and, $\lambda(\sigma) > 0$. Therefore, from the condition (IV)$\mathcal{F}(\lambda) = 0$. Now, if thesuppλ is ideal then by proposition 4.1, $\mathcal{F}(\lambda) > 0$.

5. Conclusion

During this paper, we determine the conditions of a function to be the fuzziness function which gives a degree (between 0 and 1) for every fuzzy set in a ring. The fuzziness function gives for every fuzzy set some degree to been fuzzy ideal. Also, we added another condition to the fuzziness function (which named $\mathcal{F}$-fuzziness function later) to be more determined with respect to the fuzzy set and proved every $\mathcal{F}$-fuzziness function will be fuzziness function but the converse is not true in general. Actually, we can proved if the backing of fuzzy set is normal ideal then the value of fuzziness function with respect to a fuzzy set is large than 0 but if the support of the fuzzy set is not normal ideal then the value may be equal to or large than 0. This problem is solved by give the $\mathcal{F}$-fuzziness function such that if the support of a fuzzy set is not ideal then the value of the $\mathcal{F}$-fuzziness function with respect this fuzzy set is equal to0. Finally, in the fact, we can exchange $J = [0,1]$ by the lattice L or poset S and may have more results but we need results to be more explain.

We try to make a comparison between the crisp sets and fuzzy sets so that we can predict behavior the crisp set when work in fuzzy set and vice versa. This feature enables us to solve some problems that we encounter with fuzzy logic by relying on real logic or knowing the properties of Fuzzy sets by relying on real logic. Therefore, so here we dealt with the theory of rings, because the conversion between the two logics is faster than others and we hope in the future that the process of conversion and forecasting between systems which will be more complicated. Here the conversion process needs many concepts in fuzzy topological space and fuzzy ring space, therefore, any conversion process needs to call more than one concept in different subjects.

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