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Estimating the distance by Stick Method in a 12th century Arabic manuscript

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Abstract

The stick method is a common and easy way to estimate the object size or its distance to an observer. Today, painters, scouts, artillery observers, woodchoppers, etc. use this method when they have no special rangefinder instrument. We don't know the exact origin of this method but Euclid has described a similar way in proposition 22 of his *optics*. Great astronomer of the 12th century, cAbd al-Raḥmān al-Khāzinī, has explained the stick method in his treatise named *about the marvelous instruments* (*fī Ālāt al-cAjība*). He dedicated his treatise to explain observatory instruments and amplified how to use these instruments and the information gathered by them. al-Khāzinī described 7 instruments used for observing the stars in 7 independent books and added their application as the surveying tools. He presented the geometrical arguments to show us why and how we can use his formulas to gain mathematical information about subjects from data gathered by these instruments. al-Khāzinī talked about the stick method in an appendix without the geometrical arguments. His main goal was to describe a simple way for soldiers and horsemen to calculate their distance from their enemies or estimate the strength of foes armies. Furthermore, he explained estimating the distance from an object by knowing its diameter or vice versa and finally estimating the distance and diameter of the object when they were both unknown. His method was easy and practical for anyone in the military campaign who didn't know complicated mathematical relations. He used a wooden stick about 80 c.m long to do this job. In this paper, I briefly presented al-Khāzinī's original treatise and four manuscripts available in the libraries and talked about how amended al-Khāzinī's treatise in the first step. Secondly, I introduced the English translation of al-Khāzinī's appendix about the stick method with its main Arabic text. Finally, I described his method by modern mathematics symbols and notations and then I tried to rebuild the geometrical arguments according to his previous book about the triquetrum.

Keywords: *About the Marvelous Instruments* treatise, Euclid’s *optics*, estimating the distance, Geometry, al-Khāzinī, stick method.

Introduction

Most of us have seen painters who hold a pencil and look at an object while one eye is shut. Not only it isn't a mysterious way for gathering the object information but also it's a simple geometrical way to estimate the distance to the object or to measure its size. In addition to the painters, some other people like the scouts, artillery observers, woodchoppers also use this method when they have no special tools. I have no clue about the exact origin of this method but I have the historical evidence to prove that it is an ancient method.

If we study Euclid's *optics*, we'll find a similar idea in the 22nd proposition (Kheirandish, 66-69). Assume that point G is the painter's eye and AB is the unknown length of the object (Fig.1). The painter can draw the line segment DE near his eyes and parallel to AB. He would able to estimate the length of AB by knowing the length of DE, AG, and GD because of the similarity of the triangles GDE and GAB:

$$AB = \frac{AG \times GE}{GD} \qquad (1)$$

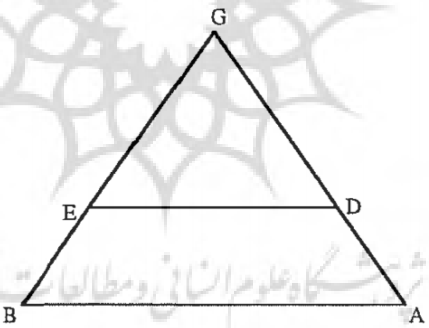

Fig.1. Geometric principle of the painters' method to estimate the object's length (Kheirandish, 66).

ED is the painter's pencil length and GE is his arm length. Both ED and GE lengths are known. So if the painter estimates AG, he'll calculate AB or vice versa. Practically, the painters have to spend many years learning how to estimate AG.

But do we have more pieces of evidence to demonstrate that the stick method is an ancient method? Great astronomer of the 12th century, cAbd al-Raḥmān al-Khāzinī, has a treatise named *about the* *marvelous instruments* (*fīKĀlāts al-ᶜAjība*). Al-Khāzinī described 7 instruments used for observing the stars or surveying in 7 books. These instruments are:

1. Dhāt al-Shuᶜbatayn (the triquetrum);
2. Dhātaal-Thuqbatayn (the diopter);
3. Dhāt al-Muthalath (another kind of the diopter probably invented by al-Khāzinīyhimself);
4. Rubᶜ (the quadrant);
5. The mirror;
6. The astrolabe;
7. Some simple tools like gnomon to estimate the distance (Kiani Movahed, 189).

After finishing the last book, al-Khāzinī annexed a short chapter to describe how anyone could use a short stick to estimate the distance to an object or to obtain its size. He mentioned that these methods were used by Turkmen soldiers in the military campaigns (Al-Khāzinī, 31-32).

Following, I describe al-Khāzinī's treatise briefly. Then, I submit the annexed chapter of al-Khāzinī's treatise in English, and inithe Mext… step, I introduce the Arabic original text for who is interested in the main source and al-Khāzinī's treatise itself. Finally, I will describe this method by modern mathematics symbols. This chapter in the original treatise has no figure and I drew the necessary figures to complete it.

A brief description of al-Khzznı'ı's rrr ks

Abū al-Fath ᶜAbd al-Raḥmān Mansūr al-Khāzinī (flourished 1115–1130) was one of the most famous astronomers in the 12^{th} century A.D. He was a Greek slave who converted to Islam and worked in the court of Sultān Sanjar (reigned 1118-1157 A.D). His most important book, *mīzān al-ḥekma,* is a complete guide about practical methods to find the specific weight of metals and jewels. The translation of his astronomical tables named *Zīj al-Muᶜtabar al-Sanjarī* to Greek affected the byzantine astronomy renaissance (Abattouy, 480-481). His treatise *about the marvelous instruments* is a small manuscript about some common instruments in his era for observing the stars. He also described the application of them for surveying. Currently, there are four manuscripts of this treatise available (Kiani Movahed, 7):

1. Manuscript No 2/6412 at the Majlis library in Iran. I coded this manuscript by “M” in the Arabic text.
2. Manuscript No 1/681 at the Sepahsālār school library in Iran. I coded this manuscript by “S” in the Arabic text.
3. Manuscript No A.Y-314 at the Istanbul library in Turkey. I coded this manuscript by “I” in the Arabic text.
4. Manuscript No 45H.K 4/6591 at the Manisa library in Turkey. I coded this manuscript by “Q” in the Arabic text.

The third manuscript was published as a facsimile by Prof. Fuat Sezgin in his collection (Sezgin, 114-166) but none of them didn’t emend before finishing my thesis for my master's degree. I emended the Sepahsālār manuscript as the main text because it has the best situation among them and it has no lacuna. If there was a syntax error in the main text, I emended the Arabic text according to the Arabic syntactic and I added the original text as the footnote.

After emending, I checked the emended text with the other three manuscripts. I added the differences to the original text as the footnotes. Also, I added some words to the original text and put them between [] to understand the Arabic text better, as well as, the paragraph numbers. So, [No. ¶] shows the number of the paragraph in the emended Sepahsālār manuscript. / / shows the beginning of the pages in the manuscripts. For example, /Q‹84f/ shows the beginning of sheet No.84 (the front page) in the Manisa manuscript, and /Q‹83b/ shows the beginning of sheet No.83 (the back page) in that manuscript. Sepahsālār manuscript is numbered by stamp (probably in the Majlis library) like modern books and has no front/back page numbering. I used its numbering to reference the pages in my article.

I should say that this annexed chapter in the Majlis manuscript is missed and it is unfinished in the Istanbul manuscript.

Original text in English

A subtle chapter about the Turkmens tricks with which we finish the treatise

This [chapter] consists of two parts: [part] I, Description of the instrument, and [part] II, Its applications.

Part I, Description of the instrument and its terms

[126¶] If they want [to make] it, they have to choose a wooden stick like a swagger stick with a length of almost ten palms[1], and [the length of] that stick should be as long as the distance between their eyes and their fists when they extend their hands toward the object. [2] They use [this] instrument to estimate long distances and because of the vast application of measuring distance and estimating the number of enemy riders during the campaign, they do not decrease [its length] when using it. So, that [stick] with the length between the eyes and the fist replaces the triquetrum.[3] That [instrument] is similar to the leg of the compass[4] when the part between the stick tip and the fist aligned horizontally with the object. So, instead of the opening grading of the triquetrum, its length is measured by the palm and the finger.[5] They estimate the distance between themselves and the object as well as the diameter[6] of the object by that [stick]. Now if we divide the length between the eyes and the fist into 60 equal parts, this [instrument] will be similar and equal to the triquetrum as we described in [its] book[7], and the success belongs to God.

A section about its terms

[127¶] [The size of] the stick is equal to [the length of] the swagger stick and its length is [equal to the arm length] from one eye to the fist if a palm equals the four fingers. They call what is between the fist

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1. The palm is an obsolete unit of length, originally based on the width of the human handbreadth.

2. Al- Khāzinī assumes that the stick length is equal to the open arm.

3. The original name in Arabic is Dhāt al-Shucabatayn which means the equipment with to rods/arms.

4. “Compass” here means the triquetrum.

5. The finger (fingerbreadth or finger's breadth) is any of several units of measurement that are approximately the width of an adult human finger.

6. Here, diameter means the subject’s height or width.

7. The first book of al- Khāzinī’s treatise.

and the stick tip “absolute length or “absolute width”.[1] If the stick is held horizontally, the perpendicular on the line that connects the observer and the object, is called “the object width” or if [the stick] is held vertically it is called “[the object] height” and if the stick held down it is called “[the object] depth”.

A section about the method of its application and estimating[٢] the distance and diameter [of the object] with it in general

[128¶] We stand in front of the object and hold the stick horizontally between our eyes and the object, and we look with one eye in such a way that the stick tip is rectilinear to one end of the object and the end of the fist rectilinear to the other end of it while the fist is toward another end [of the object]. Then we recede [from the object] until the stick tip and the end of the fist cover the width of the object. Then we measure between them[3] in terms of fingers and palms, and we call it the “opening number” and we record it. Then, to estimate the distance or the object’s width or height or depth, we say that the ratio of absolute length or absolute width, to the opening number is equal to the ratio of the distance [from the observer to the object] to the object’s diameter. So we calculate one of these two [by knowing others].

Part II estimating the distance and the diameter [of the object]

[This part is divided into] some sections**.**

A section [about] when one of them[4] is unknown and the other is known

[129¶] When the distance [to the object] is unknown and the object’s diameter is known, we multiply the known object’s diameter by the absolute length then we divide the product by the opening number. The quotient is the distance between the observer and the object. But when the object’s diameter is unknown and the distance

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1. Al- Khāzinī’s definition is confusing. The "absolute length" (or "absolute width") is the observer arm length, and there is no fundamental difference between them. What is between the fist and the stick tip is called “opening number” in the next section by al- Khāzinī.

2. In the Arabic original text al-Khāzinī uses “**معرفة**” (macrifa) means “cognition’ in English but in translation, we used “estimating” rather than cognition.

3. That means we measure the length between the stick tip and the end of the fist.

4. Here, the meaning of the phrase "them" is the distance and the diameter of the object.

[to it] is known, we multiply the known distance by the opening number then we divide the product by the absolute length, so the unknown object's diameter will be obtained.[1]

A section about estimating the distance and diameter of the object when they are both unknown

[130¶] We stand in a specified standpoint and obtain the opening number and we call it the first opening number. Then we recede rectilinear to the first standpoint as a known distance and obtain the second opening number, and that is always less than the first [opening number]. Then we subtract the second [opening number] from the first [opening number], and we call the rest "the difference" and we record it. Then we multiply the distance [between the two standpoints] by the second opening number and divide the product by the difference to obtain the distance between the first standpoint to the object. If we multiply the distance [between the two standpoints] by the first opening number and divide the product by the difference, the distance between the second standpoint to the object is derived. If we are going to know its diameter, we multiply the known distance [from the first standpoint to the object] by the [first] opening number and divide the product by absolute length, so the diameter is obtained based on [the unit we use to measure] the distance between the two standpoints.

A section in estimating the width of a river (or a ditch) when the other bank cannot be reached

[131¶] First, we find a standpoint on one of the river banks. Then we[2] pull our arm held down the stick toward the other bank. Without moving our hand up or down, we turn until the stick [tip] points to the ground in a specified point, and we know [that point] and then we mark [it] and measure [the distance] between the standpoint and the marked point, so that [distance] is equal to the desired river width.

A section [about the conjecturing a caravan direction]

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1. Considering that the absolute length (or absolute width) and the opening number are both in terms of "palm", the diameter is obtained based on the unit we have chosen to measure the distance.

2. From this sentence al-Khāzinī switches from first-person plural personal pronoun to third-person singular personal pronoun, but I continue the translation by the first-person plural personal pronoun to make the text fluent.

[132¶] When we see a caravan in the low-level [ground] or want to know whether the caravan coming toward us or going away from us or it's stopped, we held the stick horizontally toward that [caravan] until its tip is rectilinear to the head of the caravan, then we deliberate [the situation carefully]. If the caravan is passing above the stick and emerges, then the caravan would go away [from the observer] or if it is covered [by the stick] then it would come toward [the observer]. If the caravan is passing under the stick and emerges, then it would come toward [the observer] and if it is covered [by the stick] then the caravan would go away [from the observer].

A section about estimating the number of enemy riders

[133¶] First, we obtain the width of their cavalry with the stick in terms of the cubit[1] and devote 3 feet for each rider and 4,000 riders for each mile[2]. So, we calculate by this method [the number of enemy riders] and that's what we wanted to remind.

Original text in Arabic

باب لطيف نختم الكتاب من طرف حيل الأتراک

يشتمل على قسمين: [القسم] الأوّل في بيان الآلة و[القسم] الثاني في استعمالها.

فالقسم الأوّل فى بيان ذكر الآلة والاشارة إلى أسمائها

[¶۱۲۶] إذا أرادوا ذلک أخذوا قصباً قدر سوطٍ نحو عشر قبضات ويكون ذلک بقدر ما بين عينهم إلى قبضتهم إذا مدوّا يدهم نحو الشيء المنظور إليه. واعتادوا اعتياداً كيلا يزيد البعد، ولاينقص عند الحاجة إليه لكثرة الاستعمال عند استخراج المسافة وجذر عدد فوارس المخالف في المصاف. فتصير[۳] هي بدل الشعبتين إمّا بين العين والقبضة. فهو كرجل الفرجار وما بين رأسها والقبضة إذا حاذاها معترضاً نحو الشيء المنظور إليه. /**I,۸۰b**/ وقامتها بالقبضات والأصابع بدل الانفتاح ذات الشعبتين. وعرفوا بها المسافة بينهم وبين الشيء المنظور إليه وقطره. وإن قسم[۴] القدر

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1. A cubit is an obsolete unit of length, originally based on the length of the human arm from elbow to tip of the middle finger.

2. The term "mile" in Arabic may be different from European "mile".

3. Q: فيصير

4. Q:- قسم

الذي بين العين والقبضة بستّين قسماً متساويةً شابهَت ذات الشعبتين وطابقها[1] كما مرّ ذكر في بابها وبالله[2] التوفيق.

فصلٌ في أساميها

[¶١٢٧] المقياس هو مقدار السوط وطوله مابين إحدي عينيه إلى القبضة إذا قيس بالقبضات كلّ قبضةٍ أربع أصابع. ويسمّونه[3] الطول المطلق والعرض المطلق منها[4] ما بين القبضة إلى رأسه. إذا قبضتها معترضاً نحو الشيء المنظور إليه والمسافة ما بين الواقف[5] إلى الشيء المنظور إليه والقائم على نهاية المسافة يقال لعرض الشيء المنظور إليه أو سمكه نحو السماء أو عمقه نحو السفل.

فصلٌ في كيفيّة استعمالها ومعرفة المسافة وقطر الشيء بها قولاً كليّاً

[¶١٢٨] وقفنا مستقبلا للشّيء المنظور إليه وجعلنا المقياس معترضاً بين ناظرنا والشيء المنظور إليه وننظر بإحدي العينين حتّى يحاذي طرف المقياس أحد طرفي قطر الشيء المنظور إليه ونهاية القبضة على طرفه الآخر منه وندني القبضة نحو الطرف الثاني. /**Q,٨٣b**/ ونبعد عنها إلى أن يحاذي طرف المقياس ونهاية القبضة الداخلة منها قطر الشيء المنظور إليه معترضاً. ثمّ نقاس[6] بينهما بالقبضات والأصابع ونسمّيها عدد الانفتاح ونحفظها. ثمّ نقول في معرفة المسافة وقطر الشيء المنظور إليه معترضاً وقائماً ومنخفضاً بالقول الكلّ نسبة الطول المطلق أو العرض المطلق إلى عدد الانفتاح بعد النظر، كنسبة المسافة إلى قطر الشيء[7] مثلاً وأمثالاً وجزءً وأجزاءً / **The end of the Istanbul manuscript**/ فنقاس[8] أحدهما بالآخر.

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1. Q: كابقها

2. Q: وإلى اللّه

3. Q: ويسمّوا

4. Q: بها

5. Q: الواحد

6. I,Q: نقيس

7. Al-Khāzinī is making a mistake here. The main text of the treatise is as follows:
«نسبة الطول المطلق إلى العرض المطلق بعد النظر، كنسبة المسافة إلى قطر الشيء»
But the absolute length/width must be divided by the opening number to calculate the distance or diameter of the object.

8. Q: فيقاس

القسم الثاني في معرفة مقدار المسافة وقطر الشيء

[في] فصولٍ.

فصلٌ إذا كان أحدهما مجهولاً والآخر معلوماً

[¶۱۲۹] أمّا إذا كانت المسافة مجهولةً وقطر الشيء معلوماً[1] ضربنا عدد قطر الشيء المعلوم في أصابع الطول المطلق، وقسمنا المبلغ على أصابع الانفتاح. فيخرج من القسمة المسافة بين الواقف والمنظور إليه. وأمّا إذا كان قطر الشيء مجهولاً والمسافة معلومةً، ضربنا عدد المسافة المعلومة في أجزاء الانفتاح، وقسمّنا المبلغ على الطول المطلق. فيخرج عدد قطر /**S,۳۲**/ الشيء المجهول.

فصلٌ في استخراج المسافة وقطر الشيء إذا كانا مجهولين

[¶۱۳۰] وقفنا في موضعٍ محدودٍ وعرفنا[2] أصابع الانفتاح وسمّيناها أجزاء الانفتاح الأوّل. ثمّ نتأخّر عن الموقف المحدود على استقامة الموقف الأوّل تأخّراً[3] معلوماً بالخطّي أو غيرها وعرفنا أصابع الانفتاح الثانية[4] وهي يكون أبداً أقلّ من الأولى.[5] ثمّ نقصنا الثانية من الأولى[6] وسمّينا الباقية[7] فضلةً وحفظناها. ثمّ ضربنا عدد التأخّر في أصابع الانفتاح الثاني وقسمّنا المبلغ على الفضلة، فيخرج المسافة بين الموقف الأوّل والشيء المنظور إليه. وإن ضربنا عدد التأخّر في أصابع الانفتاح الأوّل وقسمّنا المبلغ على الفضلة، يخرج المسافة بين الموقف الثاني إلى الشيء المنظور إليه. وإن أردنا معرفة قطره، ضربنا عدد المسافة المعلومة في أصابع الانفتاح وقسمّنا /**Q,۸۴f**/ المبلغ على الطول المطلق، فيخرج عدد قطره بقدر ما في أجزاء التأخّر.

فصلٌ في معرفة عرض نهرٍ أو هبوطٍ لايمكن الوصول إلى الجانب الآخر منه

[¶۱۳۱] طلبنا أوّلاً موقفاً حاذي جانبا النهر الخطّ الأفقي. ثمّ مدّ يده بالمقياس نحو طرفه الآخر أحاذي طرف على جانبه الأبعد. ثمّ استدار حوله من غير بعسير يده

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1. S: المعلوماً

2. Q: وفرضنا

3. Q: بأجزاء

4 .S,Q:**الثاني**

5. Q: الأول

6. Q: الأول

7. I: الثانية

فوقاً وتحتاً حتّى لاقا أرضاً يمكن وصوله إليها وأشار إلى موضعٍ ويعلم، ثمّ علامة ثمّ يمسح مابين موقفه والعلامة، فهو مثل عرض النهر المطلوب.

فصلٌ [في معرفة اتّجاه السيارة]

[١٣٢¶] وإذا رأى في المغارة سايراً أو أراد أن نعلم هو يدنوا منه أو يبعد عنه أو هو واقفٌ، أشار[1] بالمقياس نحوه معترضاً حتّى حاذي نهاية طوله رأس الساير، ثمّ تأمّل. فإن كان الاعتبار من فوق المقياس وقد برز فهو يبعد عنه سايراً أو أن استتر فهو يدنو إليه. وأمّا[2] إذا كان الاعتبار من تحت المقياس وقد برز من تحت فهو يدنوا منه وإن استتر فهو يبعد عنه.

فصلٌ في معرفة عدد فرسان العدوّ[3]

[١٣٣¶] وعرفنا أولاً عرض فرسانهم بالخطّي والذرعان وجذرنا بقدرها لكّل فارسٍ ثلاث خطواتٍ وفي كلّ فرسخ[4] طولاً أربعة آلافٍ فارسٍ. وعلى هذه نقاس[5] وذلك ما أردنا أن نذكر. /

The end of the Sepahsalar & Manisa manuscripts/

1. Q: اشارة

2. Q: وبها

3. Q:- العدوّ

4. Q: فرس

5. Q: **يقاس**

The description of the original text

The instrument

As you see, the annexed chapter of al-Khāzinī's treatise includes two main parts. In the first part, he describes the instrument and some terms related to it. In the second part, he describes the methods to estimate the distance and size of an object. He never mentioned Euclid's *optics* 22nd proposition, but these methods were based on that proposition. We don't know al-Khāzinī had the Arabic version of *optics* or, as he mentioned, these methods were common in his era.

In the first part, al-Khāzinī says that the observer must take a wooden stick in ten palms length. The palm is an ancient length measurement unit that equals the width of the human handbreadth (Dilke, 26). On the other hand, each palm equals 4 fingers. The finger is another ancient length measurement unit equals to the width of an adult man finger (Dilke, 26). Both of them are anthropic measurement units (Fig.2). The observer must hold the stick by his hand so the maximum available length of it is 9 palms. Al-Khāzinī assumes that the length of the human hand, from wrist to shoulder is 10 palms approximately.

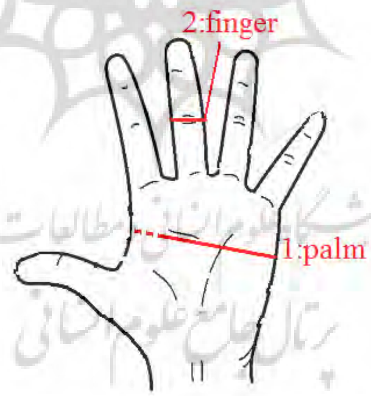

Fig.2 Palm (1) and finger (2) ancient length units.

How to use the stick

Al-Khāzinī describes that the observer must hold the right arm with the stick in front of his eyes and shut one of them to estimate the distances (Fig.3). The observer can estimate the object length or the height if he holds the stick horizontal or vertical, respectively. If the observer holds the stick vertically al-Khāzinī calls the arm length "absolute length" and if he holds it horizontally al-Khāzinī calls the arm length "absolute width". Both "absolute length" and "absolute width" are 10 palms and there is no difference between them. Then, the observer must go back or forward to see the entire length or width of the subject covered by the stick. The observer must measure the length of the stick from its tip to his fist. Al-Khāzinī calls this length "the opening number".

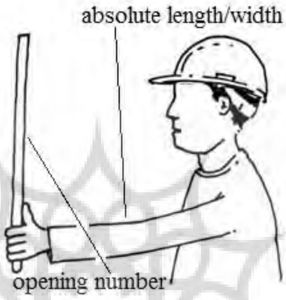

Fig.3 using the stick.

Calculation

The observer knows his arm length. In the first case (Fig.4), he wants to gain the distance between himself to the object (c). If he read the opening number (b), he can calculate the object's width (or height) (h) by using Equ.2:

$$h = \frac{b \times c}{a} \qquad (2)$$

Vice versa, if the observer estimates the object width/height (h), he can calculate the distance between himself to the object (c) by using Equ.3:

$$c = \frac{a \times h}{b} \qquad (3)$$

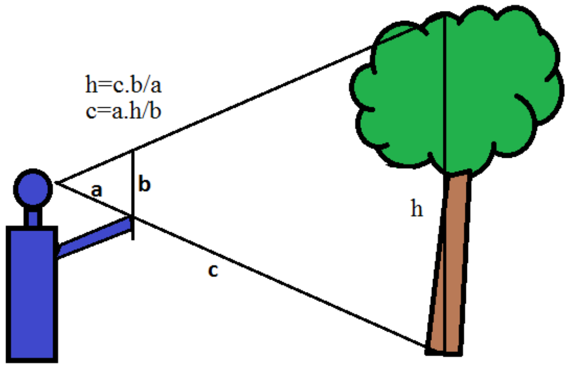

Fig.4. Estimating the object’s height/width when the distance between the observer and the object is known.

Al-Khāzinī uses *optics* 22nd proposition in the first case although he doesn’t mention *optics*.

In the second case, the observer can estimate neither the object's width/height nor its distance. So, he must read the opening number from his standpoint and then recede to another standpoint rectilinear to the first one and read the opening number again (Fig.5). He must subtract the second opening number (b’) from the first one (b). Al-Khāzinī calls b-b’ “the difference”. The observer can calculate the distance between the first standpoints to the object by using Equ.4:

$$c = \frac{d \times b'}{b - b'} \qquad (4)$$

And he also can calculate the width/height of the object by Equ.2.

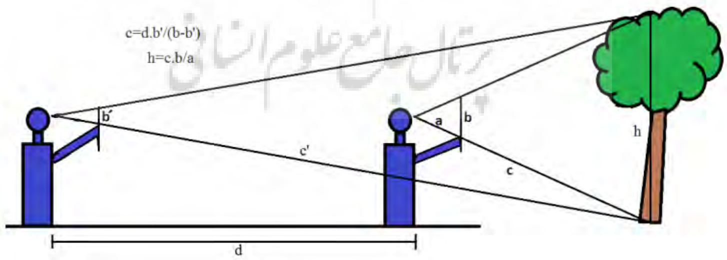

Fig.5 Estimating the object’s height/width when the distance between the observer and the object is unknown.

The proof of this case is more complicated and al-Khāzinī didn't present the argument for it but we can use his argument for the triquetrum from the first book of his treatise (Kiani Movahed, 204-205).

In Fig.6, assume that point E is the first standpoint and AB is the subject's height (h). ZC is the first opening number (b), and EZ is the observer's arm (a). Point T is the second standpoint and ET is the distance between two standpoints (d). SO is the second opening number (b').

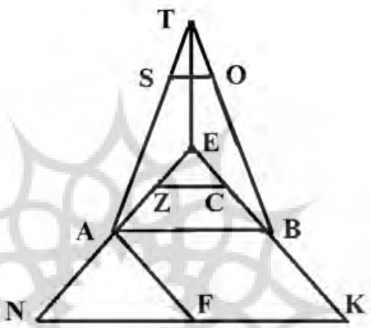

Fig.6 The geometric argument of the second case (Kiani Movahed 2019, 205).

We extend AE and BE in such a way that EN=EK=TA=TB. Then, we connect the point N to K and draw line segment AF parallel to BK. So:

$$EN = EK = AT = BT \wedge AB \parallel OS \rightarrow_{\triangle} SOT \sim_{\triangle} ABT \qquad (5)$$

$$(5) \rightarrow \frac{AT}{AB} = \frac{TS}{OS} \qquad (6)$$

$$ZC \parallel KN \rightarrow_{\triangle} NKE \sim_{\triangle} ZCE \qquad (7)$$

$$(7) \rightarrow \frac{EN}{KN} = \frac{EZ}{ZC} \qquad (8)$$

If we assume that $AT = TS$ then $AB = OS$ and if we assume that $EN = EZ$ then $KN = ZC$. So we have:

$$NF = KN - KF = KN - AB = ZC - OS \qquad (9)$$

$$AF \parallel BK \rightarrow \triangle AFN \sim \triangle EKN \qquad (10)$$

$$(10) \rightarrow \frac{EK}{KN} = \frac{AF}{NF} \rightarrow KN = \frac{NF.EK}{AF} \qquad (11)$$

$$AF = BK \rightarrow KN = \frac{NF.EK}{BK} \qquad (12)$$

If the distance between the observer and the object is too long and the object not too tall then $TE \approx BK$ and we have:

$$KN = \frac{NF.EK}{TE} \qquad (13)$$

$$AB \parallel NK \rightarrow \triangle ABE \sim \triangle ENK \qquad (14)$$

$$(14) \rightarrow \frac{AE}{AB} = \frac{EN}{KN} \qquad (15)$$

$$(13) \wedge (15) \rightarrow \frac{AE}{AB} = \frac{EN.TE}{NF.EK} \qquad (16)$$

$$EN = EK \rightarrow \frac{AE}{AB} = \frac{TE}{NF} \qquad (17)$$

$$(17) \rightarrow AE = \frac{TE.AB}{NF} = \frac{TE.OS}{ZC - OS} \qquad (18)$$

And again we can calculate the width/height of the object by Equ.2.

Other applications

Al-Khāzinī continues the annexed chapter with 3 other cases. In the first one, he describes a method to estimate the river width. The observer must stand on the bank of the river while his face is toward another bank (Point A in Fig.7). He holds the stick vertically while his arm is parallel to the ground surface. He turns until he sees the ground without changing his arm angle and the opening number. After that, he must mark a point on the ground that is rectilinear to the stick tip (point B) and measure the distance between his standpoint to the marked point (w’). Considering w equals w‘, the observer obtains w indirectly.

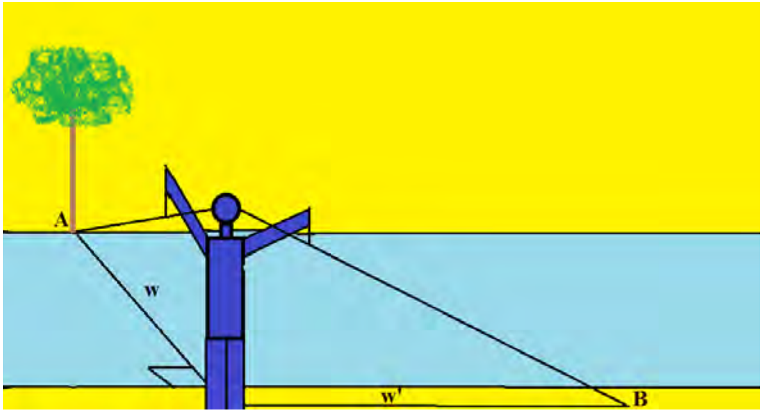

Fig7. Estimating the river width by the stick.

The second case is about conjecturing the vector of a caravan. The observer holds the stick horizontally while its tip is placed at the head of the caravan. If the caravan passed above the stick, it means the caravan is going away from the observer. If the caravan was covered by the stick then the caravan is coming toward the observer.

The last case is about estimating the number of enemy riders. The observer must estimate the width of the enemy horsemen by the stick in terms of the mile and consider 4000 riders per mile.

Conclusion

The stick method is an old method to estimate the object size or to find the distance between the observer and the object. There is a similar way to estimate the distance and size of the object in Euclid's *optics*. Al-Khāzinī has described this method in his treatise *about the marvelous instruments* (*fī Ālāt al-ᶜAjība*) in detail but he hasn't presented any argument. Furthermore, he has described some other methods to estimate the river or trench width or the number of the enemy's cavalry. In this paper, you can read al-Khāzinī's original text with its English translation. Furthermore, I described his method by modern mathematics symbols and notations.

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