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Evaluating the Factors Affecting on Credit Ratings Accepted Corporates in Tehran Securities Exchange by Using Factor Analysis and AHP

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ABSTRACT

Implementation credit rating for Corporates is influenced by Different circumstances, systems, processes, and cultures in each country. In this study, we proposed a Factor analysis modified approach for determine important factors on real data set of 123 accepted corporate in Tehran Securities Exchange for the years 2009-2017 of diverse range of 52 variables. We estimated the priority score for 49 factors. The three factors, Debt to Equity Ratio, Current debt-to-equity ratio and proprietary ratio exclude due to high correlation with others. The results indicated that three macroeconomic factors: Price Index of Consumer Goods and Services, exchange rate and Interest rate determinants were more effective on the credit ratings. In addition, Financial Ratios and non-Financial Ratios Financial Ratios such as Return on equity (ROE), Long-term debt-to-equity ratio, Benefit of the loan, ratio of commodity to working capital, Current capital turnover, return on Working Capital, Quick Ratio, Current Ratio, Net Profit margin, Gross profit margin, had effect on credit rating accepted corporate in Tehran Securities Exchange. The Nonparametric statistical test to validate the consistency between AHP ranking and Factor analysis revealed, the new approach has a moderated consistency with AHP. In conclusion, the Factor analysis modified approach could be applied significantly to evaluate efficiency and ranking factors with minimum loss of information.

1 Introduction

In recent decades, the relevance of ratings has grown and demand for ratings is increasing every day. Numerous researches were undertaken over the years, in order to find out which economic variable influences rating, as the understanding of the relationship between ratings and economic variables could have important policy implications [1, 2, 3, and 4]. The Credit rating is an abstract concept that cannot be measured quantitatively in a direct way; however, this can be determined by the interaction of causal variables. In fact, it can be measured through latent structure which can be identified with

latent variables behind a set of correlated variables. In the estimation of latent variables, the selection of related variables and the estimation of parameters (weights) are more important [5]. In first, issue for the selection of latent variables, we should rely on the standard reduction of information criterion approaches. In the second, as credit rating is unobserved, it is unfeasible estimating the parameters by standard regression techniques. Therefore, to maximizing the information of a dataset included in an index, assignment of the weight to the indicators or sub-indices is critical. In addition, a good composite index should consist of significant information from all the indicators and not be biased toward some indicators. The Factor Analysis (FA) is a statistical method having been developed in 1904 by spearman [6]. Overall, it detects joint variations through unobserved latent variables [6]. In fact, it can reduce the measurable and observable variables to fewer latent variables that share a common variance [7]. One of the most important uses of factor analysis is to sort factors by importance (factor variance) from large to small, and in each factor determine the importance of each variable as an index factor. In general, the rating agencies to allocate a credit rating to a debtor or debt Instrument use a combination of several quantitative and qualitative variables. Therefore, identifying the various factors which affect credit ratings is extremely important [8, 9]. Unlikely, in developing countries, if a financial institution decides to implement credit ratings, there is rare literature with useful experiences to assist the institute. This is because the literature on implementation of credit ratings is limited, and most of available literature is from developed countries which are different in circumstances, systems, processes, and cultures with developing countries. Therefore, in these countries, the factors which affecting credit rating should be determined based on their condition. The major innovations of this study is extracting the factors affecting the credit rating from a wide range of effective variables and determine their priority weight using factor analysis (FA) technique which is seen to be the first attempt to examine these factors in Iran using FA. In this method, regardless of the factors identity, variables are investigated based on volume and frequency distribution. We believe that this method can help decision makers, along with other methods in selecting and prioritizing the factors affecting credit rating. Hence, the purpose of this study was to determine the effective and imperative factors influence on credit ratings in accepted corporate in Tehran Securities Exchange which is denoted in three objectives: First determining of affecting index on credit ratings using the FA approach, second estimating of affecting index on credit ratings through one of most common approach, the Analytic Hierarchy Process (AHP), and finally investigation of consistency between AHP outcomes and FA results. This paper is organized as follows: Section two contains the literature overview of the most important articles in the field, Section three gives the fundamental feature and developing in FA technique and brief description of the AHP model, section four explains the research methodology, section five demonstrate two approach applied to identify and ranking affecting factors on credit ratings, based upon the real data set of an Iranian companies. Finally, Section 6 includes the conclusion of this research.

2 Literature Review

Over the years, several researches carried out to find which economic variables influence rating and which measure could have important policy implications on the relationship between ratings and the fundamentals. At 1966, for first time, Beaver [10] used Financial Ratios to estimate firm's performance in USA After that, the study of Altman [11] was a milestone in applying multiple variable models in prediction of firms' failure. They used multiple discriminate analyses as the appropriate statistical technique. In 1973, the patterns of financial ratio factors were obtained and stability of this

pattern during the time was exanimated [12]. After a few years Chen and Shimerda illustrated the representative ratio could be adequate to capture most of the information which provides by all ratio [13]. Argent introduced first predicting model based on non-financial indicators. Then, Kasey and Watson represented the significant effect of non-financial factors on prediction corporate failure [14]. In another study a new model was proposed to predict default for borrowers of a Swedish bank over period 1994-2000 which considered the financial and non-Financial Ratios such as accounting ratios, borrowers behavior, macroeconomic conditions such as the output gap, the yield curve and consumers expectation of future economic development [15]. GUL and CHO [16] investigated the effect of capital structure of firms on default risk. They used Moody's KMV option pricing model to obtain the probability of default manufacturing firms during 2005-2016. Also they found the rise in short - term debt to assets leads to increase the risk of default whereas the increase in long-term debt to assets leads to decrease the default risk. In Brazil Market, a Generalized Estimating Equations (GEE) model was used for investigation of effect of ten independent variables: profitability, leverage, size, growth, financial coverage, liquidity, financial market performance, corporate governance, control and internationalization on credit rating explanation. They showed that leverage and internationalization (p-value = 0.01) and financial market performance was significant (p-value=0.05) [17]. In Turkey, the effects of the main macroeconomic determinants such as economic growth rate, inflation rate, external debt, foreign direct investment, savings and current account debt were investigated on the sovereign credit rating. They demonstrated inflation rate and external debt have a significant negative relationship with the sovereign credit rating [18].

Boumparis et al [19] compared the effected macroeconomic and financial determinants on sovereign ratings in a multivariate Panel Vector Autoregressive framework with relationship between nonperforming loans and sovereign ratings. They showed that affects from non-performing loans on sovereign ratings more than economic and financial variables such as; GDP growth, government debt-to-GDP ratio, investment-to-GDP ratio and the fiscal balance-to-GDP ratio. In 2019, Josephson and Shapiro [20] showed that the observed equilibrium outcome and rating inflation depend on rating quality constraints as well as the relative demand by constrained and unconstrained investors. Also, the studies of credit risk effect and macroeconomic factors on profitability of ASEAN banks shown that credit risk and GDP growth negatively affect Return on Equity (ROE) and Return on Assets (ROA) (p-value:0.05). However, ROA and ROE were influenced by an increase in inflation rate [21]. Ali and Charbaji [22] applied factor analysis model for 42 Financial Ratios on international commercial airlines to test the theoretical structure and grouping of financial ratios. They demonstrated there is a significant difference between theoretical and the empirical classification and also detected five significant factors. Other research used the factor analysis on 29 Financial Ratios and found the 8 underlying factors [23]. Karatas [24] for eliminating redundancy among factors before conducting logistic regression and discriminate analysis applied factor analysis on financial ratios. Another researcher identified the Financial Ratios for each factor by using factor analysis model, before conducting discriminate analysis, to develop a model for failure prediction [25]. Xuana et al [26] investigated the factors affecting business performance of small and medium-sized enterprises in Vietnam. They applied a least squares estimation method in the multivariate regression model for estimate Factors affecting. The results of his study have demonstrated to those indicators such as the level of access to government support policies, education of the firm's owner, the size of the firms, the social relations of firms and the rate of revenue growth affect the business performance of small and medium-sized enterprises. Ocal et al [27] determined the financial indicators effect on the financial state of Turkish

construction companies by factor analysis on 25 Financial Ratios and detected five underlying factors. Also Chong et al [28] showed using a lot of ratios it is not necessary for assessing financial performances and the financial ratios. Generally, it has not normal distributional pattern and only after remedial actions for certain ratios their normality can be improved. In new research, the impacts of Industrial Production, GDP, Crude Oil Price, Inflation and Exchange Rates, on credit rating of Indian Corporates were investigated. It was represented that for selected economic factor, the credit rating responds in linear, as well as nonlinear manner and, the Economic factors such as Industrial Production, GDP, and Exchange Rates have a linear relationship to credit rating, whereas Inflation and Crude Oil price have a non-linear impact upon the credit rating [4]. In Gulf Cooperation Council (GCC) Countries with high-income economy, foreign ownership and state ownership are negatively and significantly related to specific loan loss provisions. In fact, consumer price index and the gross domestic product have a negligible relationship with loan loss provisions. Because of outer factors is not affected of the behavior of the borrowers and low fluctuations in the economy, the poor effect macroeconomic factors result that loan loss provisions are not responsive to the changes in consumer price index and gross domestic product [29].

3 Preliminaries

3.1 Factor Analysis

Factor analysis is a statistical method, was used in several sciences such as finance, operations research, product management, psychometries and personality theories. It was developed by Spearman in 1904. The FA characterized correlated variables in terms of a potentially lower number of unobserved variables which called factors. It also describes variability among observations [4, 30, 31, 32, 34, 35, and 36]. This method, like the empirical method, finds joint variations through unobserved latent variables. The observed variables are modeled as linear combinations of the potential factors, plus "error" terms [33]. Let we are given a set $\{x_i\}_{i=1}^n$, of n observations from a variable vector X in \mathbb{R}^p , that each observation x_i has p dimensions:

$$x_i = (x_{i1}, x_{i2}, ... x_{ip})$$
 (1)

And it is an observed value of a variable vector $X \in \mathbb{R}^p$. Therefore, X is composed of p random variable:

$$X = (X_1, X_2, \dots X_p) \tag{2}$$

Where X_j for j=1... p is a one – dimensional random variable.

The factor analysis model used is a generalization of

$$X = \Phi * F + E + \mu \tag{3}$$

where Φ is a $(p \times k)$ matrix of the $[q_{jl}]$ (= loading of the j-th variable on the l-th factor), F is a $(k \times 1)$ matrix of the $[F_i]$ (=l-th common factor), E is a $(p \times 1)$ matrix of the $[e_i]$ (= unobserved stochastic error terms with zero mean and finite variance, which may not be the same for all i) and μ is a $(p \times 1)$ matrix of the $[\mu_i]$ (= mean of variable j). The factor analysis model can be written algebraically as follows. If you have p variables $X_1, X_2, ..., X_p$ measured on a sample of

n subjects, then variable i can be written as a linear combination of *m* factors F_1 , F_2 , ..., F_m where, as explained above m < p. Thus,

$$X_i = a_{i1}F_1 + a_{i2}F_2 + \dots + a_{im}F_m + e_i \tag{4}$$

Where the a_i 's are the factor loadings (or scores) for variable i and e_i is the part of variable X_i which cannot be explained by the factors [33].

3.2 Analytic Hierarchy Process

The Analytic Hierarchy Process is a mathematical method to define priorities and support complex decision which developed by Saaty [37]. In fact, that is a measurement method via pair -wise comparisons and relies on the judgments of experts to derive priority scales. It has been used by several researchers and decision makers due to mathematical properties of the method. In fact, the hierarchical structure of AHP methodology is able to provide a comprehensive framework for making multicriteria decisions by organizing problems into a hierarchical structure [38]. AHP methodology involves four main steps; create structure the decision hierarchy, construct matrices, calculate weight of the elements to each level and finally, check and balance of decisions which explain below:

Create Structure the Decision Hierarchy

You can define of a structure hierarchy as a pyramid that the goal of the decision is on the top the criteria is graded in several successive steps. Therefore, higher levels control lower levels of the hierarchy and alternatives are seen at the lowest level. The decision hierarchy must be extensive enough to include the main concerns of the decision makers and small enough to allow timely changes.

Construct Matrices for Calculate a Set of Pairwise Comparison

To compare the elements, for each criterion in the upper level, one matrix must be built which is calculated by comparing the relative importance of the criterion y_i to versus the criteria y_j , $j = 1 \dots n$ with respect to the criterion or property of them. For the measurement and compare of quantitative as well as qualitative criteria a verbal scale is used. In other words, the ratio a_{ij} is inferred of the weights w_i and w_j

$$a_{ij} = \frac{w_i}{w_i}$$
 (5)

The values of a_{ij} (or a_{ji}) are allowed to be integers in the range of 1 to 9, corresponding to the intuitive judgement scale given in Table 1.

Table 1: Corresponding to the Intuitive Judgement Scale in AHP

a_{ij}	Definition
1	The criteria y_i and y_j are equally important
3	The criterion y_i is slightly favored over y_j in importance
5	The criterion y_i is strongly favored over y_j in importance
7	The dominance of the criterion y_i over y_j is affirmative
9	The importance of the criterion y_i is an order of magnitude higher than that of y_j
2, 4, 6, 8	Intermediate values used when compromises are needed between the aforementioned values

Calculate Weight (Priority) of the Elements to Each Level

To calculate the relative contribution of each element into hierarchical structure relative to the upper goal or criterion and in relation to the main goal, first, the calculation regarding the weight of each element in relation to its immediately upper element (criterion) is made, and the local mean weight of the element is established. Then we calculate the global weight (regarding the main goal) of the respective element, multiplying its local average priority by the local average priorities of the hierarchically superior criterion.

Check and Balance of Decision.

To check the compatibility of the AHP results with the expectations and if flaws are identified, the previous process should be reviewed. It is highly recommended to avoid gaps between the model and expectations. Whenever necessary, the elements or criteria which not considered should be included to complete model [38].

4 Methodology

The present study is an applied study which its purpose is the development of applied knowledge in credit rating context. It divided into total of two sections; in first Section, we look for importance determinants of credit ratings by use a Factors analysis (FA) modified approach. Total 52 variables (potential factors) were extracted from Iranian researchers' studies by library method [39-42] and financial statements and explanatory notes. The statistical population was composed of all Corporates listed on the Tehran Stock Exchange.

The Corporates needs to meet the following conditions:

- 1. They should be listed on the Tehran Stock Exchange prior to 2009 and continue to 2017.
- 2. For increasing in comparability, the fiscal year in all corporate should be ended in March
- 3. During this period, no changes in their fiscal year or activities have been happened.
- 4. They are not included in financial intermediaries and investment companies.
- 5. To calculate the research variables, the necessary data are available.

A total of 123 Corporates which applying the above limitations are selected. The research data are elicited from the financial statements and explanatory notes of the listed firms via central bank of Iran and Stock Exchange websites and Rahavard Novin software. Variables were shown in Table 3.

In Section two, among range of 52 considered variable in first section, the most effective factors would have selected by using the AHP method which is a survey-descriptive method to estimate relative magnitudes of factors base on individual experts' experiences. A pairwise questionnaire was distributed among experts of Credit Ratings based on snowball sampling technique which sampling based on the highest capacity for reasoning and explaining the subject matter. Base on snowball sampling technique, we selected 17 experts with Credit Ratings experience in Financial Organizations. The experts' information is listed in Table 2.

Table 2: The Experts' Information with Credit Ratings Experience in Financial Organizations

Financial Organ-	Quantity	Educ	ation	Work ex	perience
ization		Master's degree	Ph.D.	Below 15 years	Above 15 years
Sepah Bank,	7	4	3	3	4

Table 2: Continue

Financial Organ-	Quantity	Educ	Education		Work experience	
ization		Master's degree	Ph.D.	Below 15 years	Above 15 years	
Mellat Bank	5	3	2	2	3	
Day bank	5	5	1	5	0	
Total	17	12	5	10	7	

Finally, at the time of decision making, inconsistency would have minimized by used the least squares method.

5 Data and Experimental Results

In order to identify affecting factors on credit ratings of Iranian Corporates, the 52 variables (potential factors) were adapted from studies carried out on Iranian Corporates [39,40 and 41] was taken into consideration for the time period of 2009- 2017. During this period, the relevant data were available for 123 Iranian companies included 1305 observations. The quantitative and qualitative variables extracted from annual financial statements of these companies though Rahavardnovin software. The macroeconomic variables extracted from central banks website of Iran. Variables were shown in Table 3.

Table 3: List of the 52 Variables of Affecting Credit Ratings

Item num-	Item description	Item	Item description
ber	-	number	
V1	operating profit margin	V27	Long-term debt-to-equity ratio
V2	Return on equity (ROE)	V28	Current debt-to-equity ratio
V3	Return on Asset (ROA)	V29	proprietary ratio
V4	Net Profit margin	V30	Debt Coverage Ratio
V5	Gross profit margin	V31	interest Coverage Ratio
V6	profit margin	V32	The financial burden of the loan
V7	Profit for Profit	V33	Financial costs for operating profit
V8	Return on Working Capital	V34	Financial costs to net profit
V9	Fixed asset returns	V35	Percentage of institutional ownership
V10	Benefit of the loan	V36	Percentage of major shareholders
V11	Current Ratio	V37	Amount of ownership of the largest shareholder
V12	Instant ratio	V38	Concentration of ownership
V13	Cash ratio	V39	Separation of CEO from Board of Directors
V14	The ratio of current assets	V40	Board size
V15	Cash adequacy ratio	V41	Ratio of non-commissioned board members
V16	Cash Flow Ratio	V42	Reliance power
V17	Working capital net	V43	Annual adjustments
V18	Period of inventory of materials and goods	V44	size of the company

Table 3: Continue

Item num-	Item description	Item	Item description
ber		number	
V19	Periodicals Collection	V45	Company history
V20	The ratio of commodity to working capital	V46	GDP
V21	Current capital turnover	V47	GDP growth rate
V22	Turnover of fixed assets	V48	Price Index of Consumer Goods and Services
V23	Total asset turnover	V49	Inflation
V24	Debt Ratio	V50	Interest rate
V25	Debt to Equity Ratio	V51	Unemployment
V26	The ratio of fixed assets to equity	V52	exchange rate

5.1 Preliminary Analysis

Factor analysis was initially performed on all 52 variables related to indexes of affecting credit ratings. We tried to do a Factor Analysis in SPSS. To avoid undertaken a non-positive define matrix, high correlated variables (proprietary ratio, debt-to-equity ratio, and current-to-equity ratio) were eliminated, then to ensure out data was suitable to conducting factor analysis, the Kaiser–Mayer–Olkin (KMO) test and Bartlett's test of sphericity were applied. The KMO test measure the adequacy of sample in terms of distribution of values for factor analysis execution and the acceptable values should be greater than 0.5. In addition, if we have an identity matrix, factor analysis is meaningless. Therefore, Bartlett's test of sphericity was done to determine if the correlation matrix is an identity matrix. Our results indicate that there is no identity matrix and our variables are appropriate for factor analysis (see Table 4).

Table 4: Results of KMO and Bartlett's Test

	KMO and Bartlett's Test	
Kaiser-Meyer-Olkin Measure of Sampli	ng Adequacy.	.606
	Approx. Chi-Square	46973.928
Bartlett's Test of Sphericity	Df	1176
-/-	Sig.	.000

5.2 Factor Extraction

The factors were extracted based on the fundamental theory of factor analysis which believes observed value can be composed as a linear combination of hypothetical factors. The base factors is chosen such as the base vector is an element with most responsible for occurring variances. The variables are sorted according to their contribution to the variance, several extraction methods such as Unweight least squares, Maximum Likelihood, Principal components, generalized least squares, Principal Axis, Image factoring, Alpha factoring is available for determined the factors. In this study, principal component analysis was used because we had a lot of variables which are highly associated. Generally, the principal component analysis reduces the number of observed variables to a smaller number of principal components which account most of the variance of the observed variables. However, as extracting few factors might eliminate valuable common variance, we decided all factors with eigenvalues greater than zero be retained and account for total variance.

5.3 Factor Rotation

Since non-rotated factors are ambiguous for better interpretation, factors should be rotated. Indeed, the goal of the rotation is to achieve optimal simple structure, which attempts each variable have load on as few factors as possible. Two rotation techniques are orthogonal and oblique. In orthogonal rotation which has two types; Quartimax and Varimax rotation, it is assumed that the factors are uncorrelated. Quartimax minimizes the number of factors needed to explain each variable, while, Varimax involve minimization the number of variables that have high loadings on each factor. In contrast, Oblique rotation is more complex than orthogonal rotation and used when factors are correlated. It produces a pattern matrix which contains the factor loadings and factor correlation matrix includes the correlations between the factors. In this study, Orthogonal (Varimax) rotation was applied because there was a negligible correlation among extracted factors. After factor rotation, variables were loaded maximally to only one factor and minimally to the remaining factors. Finally, for simple interpretation, in SPSS we sorted the loadings by size in a descending order and Suppress small coefficients below Absolute value 0.3.

5.4 Prioritization of Variables Using FA

Although the whole set of causal variables is replaced by a few principal components, which account considerable percent of the whole variation in all sample variables, we are considered as many components as the number of explanatory variables. This is due to fact, truncating the data in factor analysis caused discarding information which could affect accurate estimation of factors. Therefore, we account for all 100% of variation in our data. Thus, the factor analysis model can be rewritten algebraically as follows:

$$X_i = a_{i1}F_1 + a_{i2}F_2 + \dots + a_{im}F_m \tag{6}$$

Where, the a_i 's are the factor loadings (or scores) for variable *i*. The Prioritization of variables was determined as follows:

At first, to obtain eigenvalues and eigenvectors, the correlation matrix was calculated. Let's $\lambda_1 \ge \lambda_2$ $\ge \cdots \ge \lambda_m$ as the sorted eigenvalues, compute the following weightings, which determine share of each factor *i* in the model:

$$w_i = \frac{\lambda_i}{\sum_{j=1}^m \lambda_j}; \quad i = 1, \dots, m$$
(7)

Each weighting actually determines the share of each eigenvalue out of a whole. Then the factor components were selected by determination of the dominant eigenvalues and compute:

$$I_i = \sum_{i=1}^m w_i a_i \tag{8}$$

The value of I_i gives a combined measure to evaluate prioritization of variables and rank of X_i . Therefore, the variables were prioritized based on upper formula. Each rank in Table 5 represents the effect of variables on credit ratings which extracted from factor analysis modified approach.

5.5 Prioritization of Factors Using AHP

For the purposes of this paper, we limited the hierarchy to four levels which the top level was the objective of selecting the best factor; the second level consisted of Financial Ratios, non-Financial Ratios and Macroeconomics factors; 3th level formed sub-criteria for upper level criteria. For instance, Financial Ratio divided to below sub-factors: Profitability ratios, Liquidity ratios, Coating ratios, Activity ratios and Leverage ratios. The bottom of hierarchy was all alternative candidates (factors) that affecting on credit rating (Fig. 1).

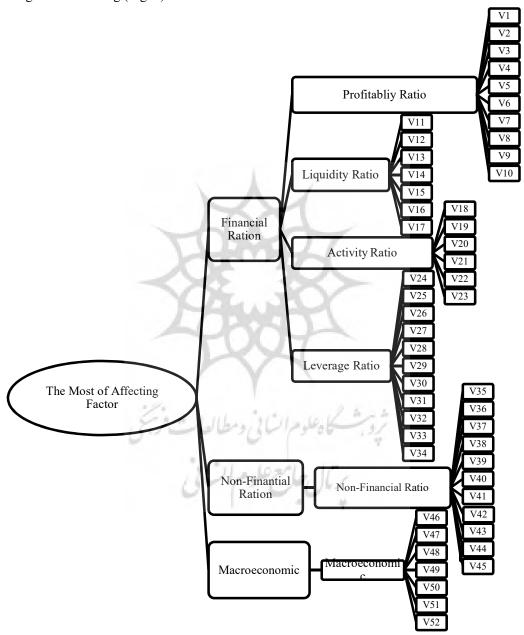


Fig. 1: Hierarchical Pyramid for 52 Variables Affecting on Credit Rating

The factors are pairwise compared under each selection criterion to see which one is relatively better. In this study a pairwise questionnaire was filled by 17 experts that had experience in Credit Ratings in Financial Organization. After the pairwise comparison, the last step of synthesizing results throughout the hierarchy is to compute the overall ranking or weights of decision alternatives using the standard AHP weighting. Thus, the decision maker obtains the best decision for his/her problem: the alternative having the largest synthesized final priority. The decision maker needs to be consistent in his/her pairwise judgements, to ensure that the decisions made are acceptable. Thus the consistency ratio (C.R.) is set to be less than 0.10. Otherwise the decision maker should re-judge or re-evaluate his/her preference judgments in a pairwise comparison matrix. Details of the formulation are given in [37]. When a serious inconsistent comparison was existing in pairwise comparison matrix, the priority weights calculated are not reliable; on the other hand, achieving true and fair results is at the desired level of inconsistent. Therefore, Researchers proposed methods for identifying the comparison which causes inconsistency and improving its value [43-46]. In this study, the least-square method used for calculating the priority of alternative in a comparative matrix as fallow [44]:

Let $A = [a_{ij}]$ that $a_{ij} = \frac{1}{a_{ji}}$ and $a_{ij} \in \left\{\frac{1}{9}, \frac{1}{8}, \dots, 1, 2, \dots, 9\right\}$, assumew_i's are priorities of alternative then we have $\forall i, j \stackrel{W_i}{/_{W_j}} = a_{ij} \leftrightarrow w_i - w_j a_{ij} = 0$ due to inconsistency and error in the phrase $w_i - w_j a_{ij} = 0$, define the least-squares problem to find optimal w_i values by solving the following homogeneous equation:

$$\frac{\partial e(w, a_{ij})}{\partial w_k} = 0 \quad k = 1, 2, ..., n$$
(9)

Subject to:

$$e(w, a_{ij}) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} (w_i - w_j a_{ij})^2$$
(10)

Then we have:

$$-\sum_{i=1}^{k-1} a_{ik}(w_i - w_k a_{ik}) + \sum_{j=k+1}^{n} (w_k - w_j a_{kj})$$

$$= -\sum_{i=1}^{k-1} a_{ik} w_i + \left(\sum_{i=1}^{k-1} a_{ik}^2 + n - k\right) w_k - \sum_{j=k+1}^{n} a_{kj} w_j = 0$$

$$= 0$$
(11)

To avoid the obvious answer, add the condition $\sum_{i=1}^{n} w_i = 1$ to the above equations. Then we have $B_{(n+1)n}w = b$ that in this case:

$$b_{kj} = \begin{cases} -a_{jk} & 1 \le j < k \\ \sum_{k=1}^{k-1} a_{ik}^2 + n - k & j = k \\ -a_{kj} & k < j \le n \end{cases}$$

$$j, k = 1, 2, \dots, n$$

$$b = [0, 0, \dots, 1]^T$$
(12)

The data of the pairwise questionnaire analysed and the factors related priority extracted using MATLAB software. The output results of the software, which shows the prioritization of the effective factors on Credit Ratings based on AHP approach, are presented in Table 5.

Table 5: Results of Priority Scores Calculated by AHP and FA Modified Approach.

Variables	AHP	FA modified		
variables	Priority scores	Rank	Priority scores	Rank
V1	31216.896	11	3.876819	16
V2	93029.913	1	6.408957	2
V3	1543.6926	50	1.626905	42
V4	29044.286	12	4.370383	12
V5	22829.657	16	4.365059	13
V6	4402.3825	41	1.890957	37
V7	6682.6448	37	1.549162	44
V8	35570.261	8	5.89135	8
V9	8849.8747	32	2.607696	19
V10	68858.349	2	6.353515	3
V11	50736.24	6	5.054316	11
V12	51766.42	5	5.145308	10
V13	36828.843	7	0.794292	49
V14	26269.52	14	1.64926	41
V15	5544.7265	38	2.046085	25
V16	9014.0526	30	1.756842	40
V17	10044.23	27	2.01171	33
V18	18800.737	17	1.59628	43
V19	4345.2087	42	3.421261	18
V20	11331.954	25	5.945279	6
V21	24591.375	15	5.934818	7
V22	1225.8272	51	1.840235	39
V23	4802.5991	39	2.075254	23
V24	15151.062	19	1.386559	47
V25	32894.338	9		
V26	2152.6722	48	1.413899	46
V27	14115.109	22	6.426919	1
V28	7265.2687	36		
V29	32403.38	10		
V30	13800.932	23	2.039315	27
V31	2092.8758	49	2.032949	28
V32	8960.026	31	2.014247	32
V33	11537.566	24	2.336008	21
V34	8518.8961	33	2.050771	24
V35	9174.002	29	1.867894	38

Table 5: Continue

X7	AHP		FA modified	
Variables	Priority scores	Rank	Priority scores	Rank
V36	15967.746	18	4.321938	14
V37	3125.2095	45	1.498471	45
V38	9627.6615	28	4.319858	15
V39	3528.4623	44	1.983769	35
V40	2520.3302	46	2.026124	29
V41	2469.9236	47	1.097964	48
V42	10887.827	26	2.039377	26
V43	4015.7262	43	2.002505	34
V44	7463.5702	34	1.973375	36
V45	15116.898	20	2.015344	31
V46	4516.7301	40	3.721834	17
V47	26784.616	13	2.563624	20
V48	64446.842	3	6.308726	4
V49	14775.339	21	2.093063	22
V50	58280.504	4	5.789449	9
V51	7387.6695	35	2.024205	30
V52	58280.504	4	6.206739	5

Overall Inconsistency in AHP = 0.04

Results of Nonparametric Correlations (Spearman test) obtained by numerical experiments represented moderate correlation between AHP results and FA modified methods. See Table 6.

Table 6: Results of Spearman's Rho Correlations between AHP and FA

			AHP	FA
Spearman's rho	AHP	Correlation Coefficient	1.000	.593**
		Sig. (2-tailed)		.000
		N	52	49
	FA	Correlation Coefficient	.593**	1.000
		Sig. (2-tailed)	.000	
	- 22	N	49	49

6 Conclusions

This study carried out by applying Factors analysis (FA) modified approach to look for importance determinants of credit ratings, for diverse range of 52 variables by sampling of 123 accepted Corporates in Tehran Securities Exchange during 2009-2017. We determined priority score for 49 variables. The variables Debt to Equity Ratio (V25), Current debt-to-equity ratio (V28) and proprietary ratio (V29) eliminated due to high correlation with other variables. On based our evidence and experimental results in Tehran Securities Exchange, looked that the most effective factors on credit rating Corporate was in order Long-term debt-to-equity ratio(V27), Return on equity (ROE) (V2), Benefit of the loan (V10), Price Index of Consumer Goods and Services(V48), exchange rate(V52), The ratio of commodity to working capital (V20), Current capital turnover (V21), Return on Working Capital (V8), Interest rate(V50), Instant ratio (V12), Current Ratio (V11), Net Profit margin (V4),Gross profit margin (V5), Percentage of major shareholders (V36),Concentration of ownership (V38) and etc, see

Table 5. The most effective factors were selected by using the AHP method, as individual experts' experiences are utilized to estimate relative magnitudes of factors through pair-wise comparisons based on mathematics and psychology, used in several studies [46-50]. Lastly, at the time of decision making, inconsistency between the results of AHP experiment was minimized by used the least squares method which overall inconsistency was 0.04. The questionnaire outcomes was listed that the most effective factors in order: operating profit margin (V2), Benefit of the loan (V10), Price Index of Consumer Goods and Services (V48), exchange rate (V52), Interest rate (V50), Instant ratio (V12), Current Ratio (V11), Cash ratio (V13), Return on Working Capital (V8), Debt to Equity Ratio (V25), Ownership ratio (V29), operating profit margin (V1), Net Profit margin (V4), GDP growth rate (V47), The ratio of current assets (V14), Current capital turnover (V21) and etc, see Table 5.

Results obtained by numerical experiments employed as well as the case study, show that there is a moderate correlation between results of AHP and FA modified methods. Thus, we may be able to use FA modified approach to evaluate efficiency and ranking variables with enough significance and minimum loss of information. Clearly we concluded, Financial Ratios mentioned in preeminent researches on credit risk and default prediction [10, 11, 13, 14] which were combination of Liquidity ratios, Activity ratios, Solvency ratios and Profitability ratios, have high Priority in our results. It ordered as Long-term debt-to-equity ratio(V27), Return on equity (ROE) (V2), Benefit of the loan (V10), The ratio of commodity to working capital (V20), Current capital turnover (V21), Return on Working Capital (V8), Instant ratio (V12), Current Ratio (V11), Net Profit margin (V4), Gross profit margin (V5). Also, the non-Financial Ratios and macroeconomic factors such as Price Index of Consumer Goods and Services (V48), exchange rate (V52), Return on Working Capital (V8), Interest rate (V50), Instant ratio (V12), Current Ratio (V11), Net Profit margin (V4), Percentage of major shareholders (V36), Concentration of ownership (V38), which in recent studies [4, 15, 16] have been considered as an effective indicator, have a high priority in result of our research Since the basic data of this study derived from the perspective of Iran with different economic and policy conditions, comparison of our results with overseas countries is difficult. However, we recommended comparing different between effecting variable on credit rating of Iranian companies with effecting variable on credit rating of prominent foreign companies in other countries moreover we proposed investigating the credit rating with chosen variables by using mathematical models in future researches.

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ثروش كاوعلومران في ومطالعات

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