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Comparison of Public Investment Approaches on Social Welfare Function: A Case Study of Iran

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ABSTRACT

The use of natural resource revenues for achievement of development has been a challenging issue for resource abundant countries. These challenges stem from the fact that incomes from natural resources are non-durable, unpredictable and uncertain. Different countries have pursued various approaches and tools for managing these revenues to avoid economic fluctuations. The international organizations and economic experts propose a diversification in the use of resource revenues through different approaches of public investment. Maximizing the social welfare function has become a common guideline for resource revenue management in resource-rich countries. This article investigates the effect of public investment approaches and their impacts on intergenerational social welfare function of Iran. To this end, the impact of two public investment approaches, namely Permanent Income Hypothesis (PIH) and Bird In Hand (BIH), was examined and compared on a Certainty Equivalent (CE) of social welfare function. Results of the simulation with Iran's economic data indicated substantial positive loss in welfare if switching to the BIH approach. Calculations show that the CE of social welfare function for PIH and BIH is 786.3 and 444.3, respectively. The baseline simulation estimates $gap = 0.77$ confirming the theoretical model's prediction that the approach based on the PIH provides a substantially higher welfare level for economy. In other words, the jump in CE investment necessary for the BIH approach to generate the same level of utility as the PIH approach is estimated in the baseline simulation to be approximately 0.77 times the level expressed by the CE_{BIH}.

1 Introduction

One of the main causes of poverty and income inequality in developing countries is the scarcity of capital and lack of capital accumulation, especially investment in infrastructure. The case for a public investment push is gaining increasing consensus in the policy debate. The IMF [18], for example, has argued that, given current low government borrowing costs, and concerns about the impact on both potential and near-term growth of infrastructure bottlenecks in several countries, this might be a good time for a public infrastructure push. Using both cross country panel estimates and model simulations, IMF [18] concludes that increased public infrastructure investment raises output in both the short and long term, particularly during periods of economic slack and monetary accommodation. A few key characteristics distinguish infrastructure from other types of capital. First, infrastructure investments

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are often large, capital-intensive projects that tend to be “Natural Monopolies”- it is often more cost-effective for services to be provided by a single entity. Second, they tend to have significant up-front costs, but the benefits or returns accrue over very long periods of time, often many decades; this longevity can pose a challenge to private financing and provision. Third, infrastructure investments have the potential to generate positive externalities, so that the social return to a project can exceed the private returns it can generate for the operator. This can lead to under provision of needed investments. For these reasons, these types of investments are provided by the public sector [18]. The development of investment is one of the ways to solve economic problems. Due to resources constraints, the importance of investment efficiency is emphasized. Particularly in a country like Iran under the current circumstances and despite sanctions, the importance of this issue becomes clearer [2]. How to finance public investment depends on the economic conditions of the countries. Some countries with increasing debt, others with increasing taxes and countries that face the abundance of natural resources to finance public investment use from natural resources revenue. In other words, the main source of financing in many economies that face scarcity of capital is revenues from the exploitation of natural resources. Figure (1) illustrates this issue for Iran. The use of natural resource revenues for the achievement of development has been a challenging issue for resource abundant countries. These challenges stem from the fact that incomes from natural resources are non-durable, unpredictable and uncertain. The fluctuation of revenue from natural resources results in fluctuations in government income and has a negative impact on public investment, income distribution and ultimately social welfare.

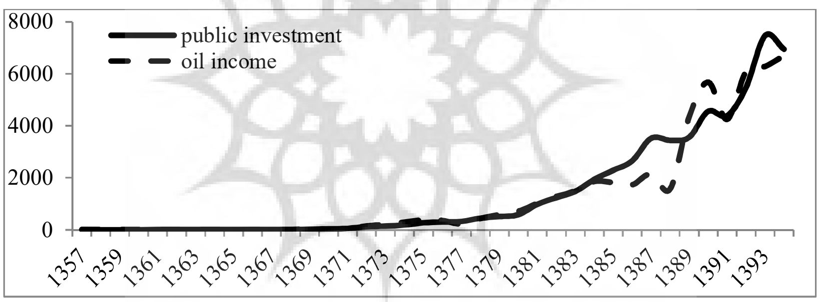

Fig.1: Oil income and public investment in Iran
Source: Central Bank, National Accounts of Iran

Conducting appropriate fiscal policy may be particularly difficult when a large of government revenue comes selling a government owned exhaustible natural resources such as oil. Large and unpredictable fluctuations in international oil prices may make the determination of appropriate expenditure levels particularly difficult [1]. In addition, since oil wealth is exhaustible, intergenerational equity considerations must also be taken into account. This is the case of the oil exporting countries where most of the government's revenue comes from oil and gas production [7]. Therefore, one of the main issues in the economies that face the abundance of natural resources is how to manage the income from the exploitation of natural resources. If the economy is heavily dependent on exhaustible resources and these resources are depleted over time, how can be assured that the social welfare of future generations would not be weakened? Is it possible to pursue a policy that will ensure sustainability over time?

Different countries have pursued various approaches and tools for managing these revenues to avoid economic fluctuations. The international organizations and economic experts propose a diversification in the use of resource revenues through different approaches of public investment. Hartwick [19] in his rule showed to maintain consumption level at a constant level between different generations, all revenues acquired from exhaustible resources should be invested. On the other hand, maximizing the social welfare function has become a common guideline for creating resource revenue management in resource-rich countries. But, selection of an appropriate social welfare function for the inter-temporal distribution of oil wealth is rarely discussed, despite the importance of this choice for empirical results and allocation decisions. This paper tries to assess the social welfare gained from the public investment approaches of natural resources revenue, and compare impacts of public investment approaches namely, permanent income hypothesis (PIH) and Bird-In-Hand (BIH) approach on social welfare with using Iran's economic data. The rest of the paper is organized as follows. Section 2 discusses literature review. Section 3 describes the model. Section 4 presents the results derived from the model, for Iran's economic data. Finally, Section 5 concludes.

2 Literature Review

Economics is usually defined as “the study of how societies use scarce resources to produce valuable commodities and distribute them among different people.” Such a broad definition should include social welfare. Social welfare involves government spending on many aspects of the public infrastructure. The government not only spends money to provide individuals with education and health care, it also helps to build the schools and hospitals that make these services possible. Expanding this conception of the government’s role, spending on the public infrastructure can include the construction of piers, bridges, and highways to facilitate commercial activity, the development of industrial zones to subsidize business, and even the maintenance of parks to provide workers with suitable forms of recreation. When social welfare reduces economic insecurity, regulates the environment and the workplace, and spends money on the public infrastructure, it both protects people against the market and contributes to the market’s profitability. Separate from the market, social welfare gives people money and a better quality of life [4]. The discussion of the impact of wealth on society goes back to, at least, the 14th century. The specific effects of an influx of natural resource wealth on an economy have been explored in the academic literature since the 1950s and 60s. In the 1950s natural resources were viewed simply as abundant, easily exploitable and a major means of capital accumulation. The impact of natural resources wealth on economic welfare can be seen from two points of view [3]. The first point of view explains the positive impact of the income of natural resources on social welfare. Some development economists believe that the main factor limiting economic growth is the lack of capital, and natural resource revenues can compensate this deficiency. Rosenstein-Rodan in 1943 [22] and 1961 [23], as well as Murphy et al. in 1989 [21], using the big push theory explained poor countries needed a great shock to get out of the poverty circle and natural resources revenues can provide the foreign currency and the capital to these countries. Despite the theories, there is another point of view. Resource-rich countries often face lower growth rates than those of non-resource-rich counterparts —

the so-called natural resource curse [24]. One of the reasons may explain this curse 'Dutch disease'. The Dutch disease occurs when a rapid influx of resource rent comes from the export of natural resources, and results in high domestic absorption and appreciation of the domestic currency6, which then affects the non-mining sectors detrimentally. Dutch disease can lead to a state becoming petroleum-dependent and a lack of industrial diversification can impede sustainable development. Research has shown that resource-poor countries tend to be more efficient and pursue policies that maximize social welfare [12]. The theoretical literature of natural resource management proposes several public investment approaches. This literature is dominated by the PIH approach, which recommends that governments smooth primary investment expenditure over time, setting it equal to their permanent income. Operationally, the PIH implies that resource revenue is saved in the early years of extraction, and such reserves will be gradually run down as resource depletion takes place. In this sense, the policy prescription of the PIH involves both stabilization, through the avoidance of procyclical policies because of resource price volatility, and intergenerational equity, through the saving of resource wealth for future generations. Uncertainty over the determination of the permanent income remains nevertheless a key obstacle in the implementation of expenditure smoothing according to the PIH. Some theoretical work has advocated the BIH approach which argues for countries that experience resource windfalls to accumulate these revenues in a sovereign wealth fund and investment only the interest from this fund. As the natural resources deplete, the government uses the sovereign wealth fund to smooth its investment spending. Compared to the PIH, the BIH approach is more conservative, because it implies a higher rate of savings from resource revenue and is based on financial returns on liquidated resource wealth not identified natural resource reserves. Specifically, a BIH approach requires countries to use all resource-related revenue to accumulate financial assets and use only the yield from the accumulated financial assets to finance expenditure [24].

The theoretical foundations for analyzing public investment in dynamic macro models go back to Barro [3] and Glomm and Ravikumar [14], among others. Using an ''AK" endogenous growth model, Barro [3] demonstrated that economic welfare is maximized when the share of infrastructure investment in GDP is set equal to its output elasticity in the aggregate production function. Bom and Ligthart [5] study the welfare effects of public infrastructure investment in a small-open economy. They find that that a permanent increase in public investment raises welfare in the long term if the output elasticity of public capital exceeds the public investment-to-GDP ratio which averages 3 percent in OECD countries. Dabla-Norris et al. [11], find a negative correlation between their index of public investment quality and natural resource dependence. IMF [18] argues that governments should strengthen institutions that are responsible for the planning, allocation, and implementation of public investments to enhance the productivity of public investment. Good public investment management would secure a positive welfare multiplier of public investment. While several papers have documented the specific nature of oil price limited works have been done about the welfare consequence of these movements. However, this question appears to be of primary importance from both the economic and the political point of view. For instance, the recent financialization of oil price and the potential welfare social consequences raise the economic question of the trade-off between private and public interest, since financialization is often defined as being beneficial from the private perspective without any beneficial considerations from a social planner's point of view [6]. Chatterjee et al. [9] and Gibson and Rioja [13] introduce public infrastructure into the incomplete-markets framework in other contexts, the present paper is the first to characterize the welfare-maximizing rate of infrastructure investment, both in the short-run as well as the long run. Motamen [20] examined the optimal allocation of oil revenues on Iranian economy in the form of optimal control problem. Valde´s and Engel [7] introduce optimizing rules to maximize the social welfare function (SWF), considering the adjustment cost of government expenditure and the uncertainty of future oil prices. Maliszewski [19] applied this idea by introducing a few ad hoc rules such as ‘‘Bird-in-Hand’’ (BIH), and he found the poor performance of ad hoc rules compared to optimizing rules.

3 Proposed Model

3.1 Social welfare function and governments budget constraint

The optimal policy of spending on government investment is to maximize the social welfare function according to the government's budget constraint. As mentioned above, maximizing the social welfare function has become a common approach to creating resource revenue management guidelines in resource-rich countries. The social welfare function introduced in this section is based on public investment. In other words, the welfare of society is influenced by public investment. The utility specification of the model is the following:

$$W = E\{\sum_{t=0}^{\infty} \beta^t [u(g_t^I)]\} \tag{1}$$

$$u(g_t^I) = -\left(\frac{1}{\alpha}\right) \exp(-\alpha g_t^I) \tag{2}$$

where W is the social welfare function to be maximized, β represents the intertemporal discount rate parameter, and g_t^I is the public investment level at date t. u is the CARA instantaneous utility function, $\alpha > 0$ is the coefficient of absolute risk aversion. The planner’s infinite horizon constrained optimization problem is:

$$\text{Max}\, E\{\sum_{t=0}^{\infty} \beta^t [-\left(\frac{1}{\alpha}\right) exp(-\alpha g_t^I)]\} \tag{3}$$

Subject to constraint:

$$A_{t+1} = (A_t + OR_t - g_t^I)R, \quad t = 0,1,2, \dots.., \quad A_0 = 0 \tag{4}$$

$$\lim_{t\to\infty} R^{-t} A_{t+1} = 0 \tag{5}$$

Equation (4) represents the government's flow budget constraint. The initial financial wealth endowment of the government is $A_0 = 0$. OR_t is the exhaustible resource income $R = (1 + r)$ is the constant gross interest rate. In addition, I assume that $\beta R = 1$. In conclusion, the transversality condition (5) guarantees that the government is neither borrowing nor lending in the long run. In order to achieve the government's intertemporal budget constraint, equation (4) can be rewritten as follows:

$$A_{t+1} = (A_t + OR_t - g_t^I)(1 + r), \quad t = 0,1,2, \dots.. \tag{6}$$

$$A_t = g_t^I - OR_t + \frac{A_{t+1}}{1+r} \tag{7}$$

By dividing the sides of equation (7) by (1 + r) the following equation is obtained:

$$\frac{A_{t+1}}{1+r} = \frac{g^I_{t+1} - OR_{t+1}}{1+r} + \frac{A_{t+2}}{(1+r)^2} \quad (8)$$

By substituting equation (8) in equation (7) and repeating the exercise forward gives:

$$A_t = g^I_t - OR_t + \frac{g^I_{t+1} - OR_{t+1}}{1+r} + \frac{A_{t+2}}{(1+r)^2} \quad (9)$$

$$\frac{A_{t+2}}{(1+r)^2} = \frac{g^I_{t+2} - OR_{t+2}}{(1+r)^2} + \frac{A_{t+2}}{(1+r)^3} \quad (10)$$

$$A_t = g^I_t - OR_t + \frac{g^I_{t+1} - OR_{t+1}}{1+r} + \frac{g^I_{t+2} - OR_{t+2}}{(1+r)^2} + \frac{A_{t+2}}{(1+r)^3} \quad (11)$$

$$A_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (g^I_s) - \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (OR_s) + \lim_{t\to\infty} \left(\frac{1}{1+r}\right)^{t} A_{t+1} \quad (12)$$

With applying the transversality condition (5):

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (g^I_s) = A_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (OR_s) \quad (13)$$

This budget constraint should hold exactly for all future dates. Thus, it should hold in expectation terms at time t:

$$E_t[\sum_{s=t}^{\infty} R^{t-s}(g^I_s)] = A_t + E_t[\sum_{s=t}^{\infty} R^{t-s}(OR_s)] \quad (14)$$

equation (14) represents the government's intertemporal lifetime budget constraint.

3.2 Prices and income of natural resources

The resource income in each period is obtained from the resource price in amount of resources. Therefore, resource income is expressed as follows:

$$OR_t = P_t X_t \quad (15)$$

In equation (15) OR_t represents resource income, P_t price, and X_t amount of resources. It is assumed that amount of resources extracted from an exogenous process follows the following:

$$X_t = (1-\delta)X_{t-1}\ ,\ \delta > 0 \quad (16)$$

δ represents depletion rate. Hamilton [16] states that resource prices follow a random walk without drift of the following kind:

$$P_t = P_{t-1} + \varepsilon_t, \quad \varepsilon_t \approx N(0, \sigma^2_\varepsilon) \quad (17)$$

By substituting equations (16) and (17) in (15):

$$OR_t = (P_{t-1} + \varepsilon_t)[(1-\delta)X_{t-1}] \quad (18)$$

3.3 Public investment approaches

Considering the social welfare function and the government's budget constraint in Section 3.1, the value equation (Bellman equation) for the problem is given by:

$$V(A_t) = \max_{g_t^I}\{u(g_t^I) + \beta E_t V(A_{t+1})\} \tag{19}$$

The first order condition of the Bellman Equation (19) is given by:

$$u'(g_t^I) - \beta E_t V'(A_{t+1}) \frac{\partial A_{t+1}}{\partial g_t^I} = 0 \tag{20}$$

$$u'(g_t^I) = R\beta E_t V'(A_{t+1}) \tag{21}$$

Differentiating the Bellman equation and using the Envelope Theorem gives:

$$V'(A_t) = \beta E_t V'(A_{t+1}) \frac{\partial A_{t+1}}{\partial A_t} \tag{22}$$

$$V'(A_t) = \beta R E_t V'(A_{t+1}) \tag{23}$$

By equating the equations (21) and (23) can be obtained:

$$u'(g_t^I) = V'(A_t) \tag{24}$$

Substituting again (23) at $t + 1$ in (21) give the Euler equation, describing optimization behavior over time:

$$u'(g_t^I) = R\beta E_t[u'(g_{t+1}^I)] \tag{25}$$

Equation (25) shows the marginal utility of public investment.

Now, let's introduce uncertainty about the income of resources in investment approaches. The CARA utility specification implies that (Euler equation) becomes:

$$\exp(-\alpha g_t^I) = \beta R E_t \exp[-\alpha g_{t+1}^I] \tag{26}$$

Since income generates natural distribution resources, thus;

$$g_{t+1}^I = g_t^I + \log(\beta R)^{\frac{1}{\alpha}} + \frac{\alpha}{2}\sigma_\varepsilon^2 + \varepsilon_{t+1} \tag{27}$$

Based on this dynamics, the level of public investment is projected in the future:

$$E_t(g_{t+1}^I) = g_t^I + log(\beta R)^{\frac{1}{\alpha}} + \frac{\alpha}{2}\sigma_\varepsilon^2 \tag{28}$$

$$E_t(g_{t+2}^I) = E_t\left(E_{t+1}(g_{t+2}^I)\right) = E_t\left(g_{t+1}^I + log(\beta R)^{\frac{1}{\alpha}} + \frac{\alpha}{2}\sigma_\varepsilon^2\right) \tag{29}$$

$$= g_t^I + 2log(\beta R)^{\frac{1}{\alpha}} + \alpha\sigma_\varepsilon^2$$

$$E_t(g_s^I) = g_t^I + (s-t)[log(\beta R)^{\frac{1}{\alpha}} + \frac{\alpha}{2}\sigma_\varepsilon^2] \quad (30)$$

Now define $\kappa = log(\beta R)^{\frac{1}{\alpha}} + \frac{\alpha}{2}\sigma_\varepsilon^2$ so that (30) becomes$E_t(g_s^I) = g_t^I + (s-t)\kappa$. The expected present discounted value of public consumption is obtained as follows:

$$E_t[\sum_{s=t}^{\infty} R^{t-s}(g_s^I)] = g_t^I \sum_{s=t}^{\infty}(1+r)^{t-s} + \boldsymbol{\kappa}\sum_{s=t}^{\infty}(1+r)^{t-s}(s-t) \quad (31)$$

$$E_t[\sum_{s=t}^{\infty} R^{t-s}(g_s^I)] = g_t^I\left(\frac{1+r}{r}\right) + \boldsymbol{\kappa}\left(\frac{1+r}{r^2}\right) \quad (32)$$

The resource income process is expressed in Equation (18), now have:

$$E_t(OR_{t+1}|OR_t) = E_t[(P_t + \varepsilon_{t+1})(1-\delta)X_t] = E_t[(P_t(1-\delta)].E_t(X_t) = P_t[(1-\delta)X_t] \quad (33)$$

$$E_t(OR_{t+2}|OR_t) = E_t[E_{t+1}(OR_{t+2})] \quad (34)$$

$$= E_t[P_{t+1}(1-\delta)^2X_t] = (1-\delta)^2P_tX_t$$

$$E_t(OR_{t+2}|OR_t) = (1-\delta)^{s-t}P_tX_t = (1-\delta)^{s-t}OR_t \quad (35)$$

$$E_t(OR_s) = (1-\delta)^{s-t}OR_t \quad (36)$$

$$E[\sum_{s=t}^{\infty} R^{t-s}(OR_s)] = OR_t\sum_{s=t}^{\infty}(1+r)^{t-s}(1-\delta)^{s-t} = \left(\frac{1+r}{r+\delta}\right)OR_t \quad (37)$$

Computing the present discounted value of income gives:

$$E[\sum_{s=t}^{\infty} R^{t-s}(OR_s)] = OR_t\sum_{s=t}^{\infty}(1+r)^{t-s}(1-\delta)^{s-t} = \left(\frac{1+r}{r+\delta}\right)OR_t \quad (38)$$

We can now substitute the equations (32) and (38) in (14):

$$g_{t,PIH}^I\left(\frac{1+r}{r}\right) = \left(\frac{1+r}{r+\delta}\right)OR_t + A_t^{PIH} - \boldsymbol{\kappa}\left(\frac{1+r}{r^2}\right) \quad (39)$$

By sorting out the above equation, the PIH approach is achieved:

$$g_{t,PIH}^I = \left(\frac{r}{r+\delta}\right)OR_t + \left(\frac{r}{1+r}\right)A_t^{PIH} - (\frac{\alpha}{2r})\sigma_\varepsilon^2 \quad (40)$$

Considering$\phi = (\frac{\alpha}{2r})\sigma_\varepsilon^2$, the PIH approach in Equation (40) shows the dynamics of the budget constraint obtained from (4):

$$A_{t+1}^{PIH} = (1+r)[A_t^{PIH} + OR_t - \left(\frac{r}{r+\delta}\right)OR_t - \left(\frac{r}{1+r}\right)A_t^{PIH} + \phi] \quad (41)$$

$$A_{t+1}^{PIH} = A_t^{PIH} + \left(\frac{\delta(1+r)}{r+\delta}\right)OR_t + (1+r)\phi] \quad (42)$$

Solving this difference equation gives:

$$A_t^{PIH} = A_0^{PIH} + \left(\frac{\delta(1+r)}{r+\delta}\right)\sum_{s=0}^{t-1}OR_s + t(1+r)\phi \quad (43)$$

By substituting equation (43) in (40), obtain the PIH approach as a function of only exogenous terms:

$$g_{t,PIH}^{I} = \left(\frac{r}{r+\delta}\right)(OR_t + \delta \sum_{s=0}^{t-1} OR_s) + \left(\frac{r}{1+r}\right) A_0^{pih} + (rt-1)\phi \tag{44}$$

The BIH approach is supposed to resource windfalls to accumulate in a sovereign wealth fund and investment only the interest from this fund. In the BIH approach, only the interest rate is used. So, equation (40) is rewritten as follows:

$$g_{t,BIH}^{I} = \left(\frac{r}{1+r}\right) A_t^{BIH} \tag{45}$$

To consider the dynamics of the government's budget constraint in this approach by returning to equation (4) and placing in the BIH approach:

$$A_{t+1}^{BIH} = (1+r)[A_t^{BIH} + OR_t - \left(\frac{r}{1+r}\right) A_t^{BIH}] \tag{46}$$

$$A_{t+1}^{BIH} = A_t^{BIH} + (1+r)OR_t \tag{47}$$

Solving the difference equation obtained in (48) gives:

$$A_t^{BIH} = A_0^{BIH} + (1+r)\sum_{s=0}^{t-1} OR_s \tag{48}$$

By substituting equation (48) in (45) obtain the BIH approach as a function of only exogenous terms:

$$g_{t,BIH}^{I} = \left(\frac{1}{r+\delta}\right)[(1+r)\sum_{s=0}^{t-1} OR_s) + A_0^{BIH}] \tag{49}$$

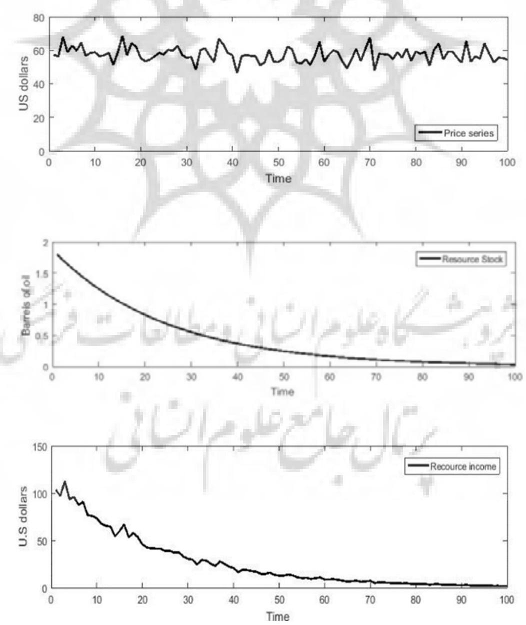

Fig.2: Price series, resource stock and income dynamics

Source: Researcher Findings

4 Simulation and Comparison of Investment Approaches

This subsection calibrates and simulates a finite horizon version of the model using a few parameters and initial values for Iran. In the baseline simulation, the subjective discount rate is assumed to be $\beta = 0.96$, therefore, the real rate of return is $r = 0.041$. The coefficient of absolute risk aversion is set at $\alpha = 0.05$ and the initial foreign assets level is set at $A_0 = 0$.
The amount of proven exhaustible oil reserves for Iran was estimated to be 1.8 billion barrels at the end of 2017 .The depletion rate is set at $\delta = 0.04$. The rest of the oil price series is simulated according to (17). The average and variance for the price series has been initially set at $\mu = 57$ and $\sigma^2 = 31$. Figure 2 shows a random realization of the price, stock and resource income dynamics.

To assess the impact of public investment approaches on social welfare, we define a Certainty Equivalent (CE) for both approaches. Define CE_{BIH} andCE_{PIH}, respectively, such that:

$$\sum_{t=0}^{T} \beta^t [u(CE_{BIH})] = E\{\sum_{t=0}^{T} \beta^t [u(g_{t,BIH}^I)]\} \tag{50}$$

$$\sum_{t=0}^{T} \beta^t [u(CE_{PIH})] = E\{\sum_{t=0}^{T} \beta^t [u(g_{t,PIH}^I)]\} \tag{51}$$

By definition $u^{-1} = -\left(\frac{1}{\alpha}\right) \ln(-\alpha(.))$, the CE for PIH and BIH approach is as follows:

$$CE_{BIH} = u^{-1}\{\frac{E[\sum_{t=0}^{T} \beta^t [u(g_{t,BIH}^I)]]}{\sum_{t=0}^{T} \beta^t}\} \tag{52}$$

$$CE_{PIH} = u^{-1}\{\frac{E[\sum_{t=0}^{T} \beta^t [u(g_{t,PIH}^I)]]}{\sum_{t=0}^{T} \beta^t}\} \tag{53}$$

Figure 3, shows the investment and asset accumulation prospects under the two approaches.

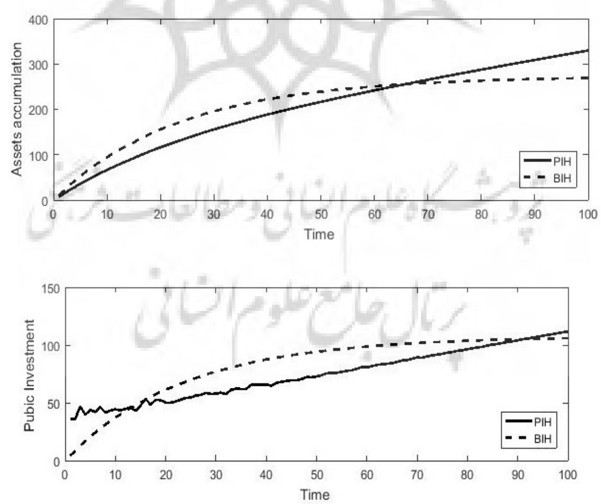

Fig.3: Asset accumulation and public investment in PIH and BIH approach

Source: Researcher Findings

In Table 1, the CE value for both approaches is calculated.

Table 1: CE value for PIH and BIH approach

Parameter	value	CE_{BIH}	CE_{PIH}
β	0.96	444.3	786.3
δ	0.04		
α	0.05		
σ^2	31		
r	0.041		

Source: Researcher Findings

The CE calculation for PIH and BIH approaches shows that PIH in sum provides a greater level of higher welfare level for the Iranian economy. Finally, to estimate the gap between two approaches, we use the following equation:

$$gap = \frac{CE_{PIH} - CE_{BIH}}{CE_{BIH}} \tag{54}$$

The baseline simulation estimates $gap = 0.77$ confirming the theoretical model's prediction that the approach based on the PIH provides a substantially higher welfare level for economy. In other words, the jump in CE investment necessary for the BIH approach to generate the same level of utility as the PIH approach is estimated in the baseline simulation to be approximately 0.77 times the level expressed by the CE_{BIH} .

5 Conclusions

Fiscal policy is very challenging in oil-producing countries, where a large portion of revenue is generated from sales of oil. This is caused by two factors: volatility and unpredictability of oil prices and exhaustibility of oil reserves. The first factor causes difficulties in planning government expenditure, the adjustment of which is costly. Due to the second factor, it is important to save for future generations if society believes that oil wealth belongs not only to the current but also to future generations of oil producing countries. To limit the impact of revenue volatility and provide intergenerational distribution of oil wealth, many oil-producing countries have established oil revenue funds, while others are discussing the establishment of such funds and their design. Oil revenue funds are those sovereign wealth funds that are accumulated from oil-related revenue. These funds are funded by taxes paid by oil producers, fiscal surpluses, privatization of oil-related property, and investment profits of the fund. The objectives of each country determine the optimal design of a fund: accumulation rules, withdraw rules, investments, and the like. The background motivation of this research was to focus on a particular aspect of the wealth management problem for governments of resource-rich economies. Resource wealth gives rise to uncertain income paths, creating the need to design approaches that minimize welfare losses. In order to expand the literature on the economics of natural resource management, this paper constructed a simple model of an economy endowed with a stochastic income from exhaustible natural resources. The stylized features of the specified model do not allow us to provide straightforward policy recommendations, because country-specific parameters often play a crucial role in determining the design of spending policies. However, the results provide a clear understanding of the mechanisms and properties of the two alternative public investment approaches under observation. Specifically, we assumed that the planner of the economy decides to spend present and future resource income according to two public investments. One of the important factors in increasing social welfare is attention to infrastructure and public investment. Public investment financing differs from country to country. Countries that own natural resources (such as Iran) use the revenue from these resources to finance public investment, but how to manage natural wealth has brought challenges for these countries. The proposal of international organizations and experts focuses on the investment approach and maximization of social welfare functions. Illustrating how to manage the income of resources with respect to social welfare and public investment, this study first introduced a simple model using maximization of social welfare function with respect to budget constraints. Subsequently, using the maximization of the social welfare function and the Bellman equation, the function of public investment approaches was obtained. In Section 4, the model was evaluated with Iranian economic data according to two approaches. In the last step, in order to evaluate social welfare, the CE for both approaches was introduced. The CE calculation for PIH and BIH approaches showed that PIH in sum provides a greater level of higher welfare level for the Iranian economy. In other words, the jump in CE investment necessary for the BIH approach to generate the same level of utility as the PIH approach is estimated in the baseline simulation to be approximately 0.77 times the level expressed by the CE_{BIH}.

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