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Hyper-Rational Choice and Economic Behaviour

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ABSTRACT

In this paper, with help of the concept of hyper-rationality, we model the interaction between two investment companies by an important game as trickery game that has special equilibrium which called hyper-equilibrium. In trickery game, one company can choose cooperation with another company until the last moment and finally changes his action to non-cooperation which incur more loss to an opponent. Indeed, the hyper-equilibrium is the point in which only one player can displace equilibrium to another point by changing his action which causes profit or loss to other players so they cannot change their action. Our findings indicate that the kind of behaviour interactive, environmental conditions, and valuation system are based on hostility causes an equilibrium point to incur the maximum loss to an opponent.

1 Introduction

Game theory aims to help us understand situations in which decision-makers interact. Game theoretic reasoning pervades economic theory and is used widely in other social and behavioral sciences [8,9,10]. Recently Eshaghi and Askari introduced a new concept of rational choice called hyper-rational choice [1]. The theory of hyper-rational choice looking for an explanation the behavior of the person who behaves reasonably and considers the profit or loss of others in addition to the individual profit. Based on the new concept of hyper-rational choice, relations between players (countries, companies, humans, …) can be modeled with help of game theory.

Here, we consider the set of possible choices of rational individual $i \in \{1,2,\dots,n\}$ is shown with $A_i = \{a_1, a_2, \dots, a_n\}$. Given hyper-preferences, how will a hyper-rational individual behave? We assume that given a set of choices $B \subseteq \mathcal{A} = A_1 \times A_2 \times \ldots \times A_n$. We define the weak hyper-preferences of actor i' over the set B as follows:

$$\begin{aligned}(a_1, a_2, \dots, a_n)_i \succcurlyeq' (b_1, b_2, \dots, b_n)_i \Leftrightarrow\ & either\ a_1 \succcurlyeq b_1\ or\ \ a_1 \preccurlyeq b_1\ based\ on\ actor\ i' \\ & preferences\ for\ actor\ 1\ and\ either\ a_2 \succcurlyeq b_2\ \ or\ \ a_2 \preccurlyeq b_2\ based\ on\ actor\ i' \\ & preferences\ for\ actor\ 2\ and\ \ either\ a_i \succcurlyeq b_i\ \ or\ \ a_i \preccurlyeq b_i\ \ based\ on\ actor\ i'\end{aligned}$$

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$$preferences\ and\ either\ a_n \succcurlyeq b_n\ or\ a_n \preccurlyeq b_n\ based\ on\ actor\ i'\ preferences\ for\ actor\ n,$$

where relation $\succcurlyeq$ is complete and transitive. We say that $(a_1, a_2, \dots, a_n)$ is strictly preferred to $(b_1, b_2, \dots, b_n)$, or $(a_1, a_2, \dots, a_n) \succ' (b_1, b_2, \dots, b_n)$, if $(a_1, a_2, \dots, a_n) \succcurlyeq' (b_1, b_2, \dots, b_n)$ but not $(b_1, b_2, \dots, b_n) \succcurlyeq' (a_1, a_2, \dots, a_n)$. We say the actor is indifferent between $(a_1, a_2, \dots, a_n)$ and $(b_1, b_2, \dots, b_n)$, or $(a_1, a_2, \dots, a_n) \sim' (b_1, b_2, \dots, b_n)$, if $(a_1, a_2, \dots, a_n) \succcurlyeq' (b_1, b_2, \dots, b_n)$ and $(b_1,b_2,\dots,b_n) \succcurlyeq' (a_1,a_2,\dots,a_n)$. So, we defined set of hyper-preference over set of preferences.

Definition 1. The relation $\succcurlyeq'$ on B is complete if for all $(a_1, a_2, \dots, a_n), (b_1, b_2, \dots, b_n) \in B$ either $(a_1, a_2, \dots, a_n) \succcurlyeq' (b_1, b_2, \dots, b_n)$ or $(b_1, b_2, \dots, b_n) \succcurlyeq' (a_1, a_2, \dots, a_n)$, or both.

The completeness condition ensures that all action profiles can be compared with each other. Considering situations which the individual faces, we define hyper-rationality as follows:

Definition 2. (Hyper-rational) An individual will be called hyper-rational under certainty if is a rational (see Definition 1 in [1]) and their hyper-preferences (individual or for others) satisfy at least one of the following conditions:

- The actor chooses from the set of available alternatives (actions) based on individual preferences;
- The actor chooses from the set of available alternatives (actions) based on preferences for other actors.

It can be concluded that each hyper-rational actor is a rational actor, but each rational actor is not a hyper-rational actor. In order to describe a game, the set of possible choices of rational individual i $\in \{1,2,\dots,n\}$ is shown with $A_i = \{a_1, a_2, \dots, a_n\}$ [3,4,5]. So, each individual player i has a set of actions A_i available to him and a particular element in the set of actions is denoted by $a_i \in A_i$. A profile of actions for the players is given by

$$(a) = (a_1, a_2, \dots, a_n) \in \prod_{i=1}^{n} A_i,$$

or alternatively by separating the action of player i from all other players, denoted by $-i$:

$$(a) = (a_i, a_{-i}) \in (A_i, A_{-i}).$$

Finally there are payoff functions for each player i:

$$U_i: A_1 \times A_2 \times \ldots \times A_n \to \mathbb{R}$$

$$U_i(a) = U_i(a_1, a_2, \dots, a_n) = b \in \mathbb{R}.$$

Now, we apply hyper-rational choice theory as a basis and main element of modeling in game theory. With help of hyper-rationality, we analyze conditions of a strategic game. In the concept of hyper-rationality, the player thinks about profit or loss of other players in addition to his personal profit or loss and then will choose an action which is desirable to him. In order to describe a game based on concept of hyper-rational choice, the payoff functions for each player i is given by:

$$U_i^j: A_1 \times A_2 \times \ldots \times A_n \to \mathbb{R}$$

$$U_i^j(a_1, a_2, \dots, a_n) = \begin{cases} U_i(a_1, a_2, \dots, a_n) & if \quad i = j \\ U_j(a_1, a_2, \dots, a_n) & if \quad i \neq j, \end{cases} \tag{1}$$

where U_i^j shows that if player i considers profit (loss) of player j, he will choose an action from a set of available actions which will benefit (lose) player j, for every $i, j \in \{1,2,\dots,n\}$. In other words, based on player i' preferences for player j, he thinks about profit or loss of another player in addition to his personal profit or loss and then will choose an action from a set of available actions which is desirable to him. Below, we show the best response functions based on hyper-preferences of players with B, P and L. Precisely, we define the best response function B_i based on individual benefit by

$$B_i(a_{-i}) := \{a_i \in A_i : u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i})\ for\ all\ a_i' \in A_i\}, \tag{2}$$

any action in $B_i(a_{-i})$ is at least as good based on individual benefit for player i as every other action of player i when the other players' actions are given by a_{-i}. We call B_i the best response function of player i based on individual benefit. Precisely, we define the best response function P_i based on profit for other players by

$$P_i(a_{-i}) := \{a_i \in A_i : u_{-i}(a_i, a_{-i}) \geq u_{-i}(a_i', a_{-i})\ for\ all\ a_i' \in A_i\}, \tag{3}$$

any action in $P_i(a_{-i})$ for player i relative to every other action of player i is at least the best based on profit for other players when the other players' actions are given by a_{-i}. We call P_i the best response function of player i based on profit for other players. Precisely, we define the best response function L_i based on the loss of other players by

$$L_i(a_{-i}) := \{a_i \in A_i : u_{-i}(a_i, a_{-i}) \leq u_{-i}(a_i', a_{-i})\ for\ all\ a_i' \in A_i\}, \tag{4}$$

any action in $L_i(a_{-i})$ for player i relative to every other action of player i is at least as good based on the loss for other players when the other players' actions are given by a_{-i}. We call L_i the best response function of player i based on the loss of other players. From relations (2), (3) and (4) we have,

$$B_i(a_{-i}) \cup P_i(a_{-i}) \cup L_i(a_{-i}) = A_i.$$

In the following, we define strictly dominant action and weakly dominant action based on the loss of other players.

Definition 3. (Strict domination of loss) In a strategic game for player i, action a_i'' is strictly dominant on her action a_i' based on loss for player $-i$, if we have

$$u_{-i}(a_i'', a_{-i}) < u_{-i}(a_i', a_{-i})\ for\ every\ \ a_{-i} \in A_{-i},$$

where u_{-i} is a payoff function that represents player $-i$' preferences. It is defined as strictly dominant action based on benefit for other players similar, but the difference is that direction of the relation $<$ is changed.

Definition 4. (Weak domination of loss) In a strategic game for player i, action a_i'' is weakly dominant on her action a_i' based on loss for player $-i$, if we have :

$$u_{-i}(a_i'', a_{-i}) \leq u_{-i}(a_i', a_{-i})\ \ for\ every\ \ a_{-i} \in A_{-i},$$

and

$$u_{-i}(a_i'', a_{-i}) < u_{-i}(a_i', a_{-i})\ \ for\ some\ \ a_{-i} \in A_{-i},$$

where u_{-i} is a payoff function that represents player $-i$' preferences. It is defined as weakly dominant action based on benefit for other players similar, but the difference is that direction of relations $\leq$ and $<$ is changed.

The actions which are chosen based on the concept of hyper-rationality (hyper-preferences) and rationality of the players may be similar or different. To prevent ambiguity in interactions, we divide actions of players into three classes as strictly dominant action and weakly dominant action based on individual benefit, strictly dominant action and weakly dominant action based on profit for other players and strictly dominant action and weakly dominant action based on the loss for others. The following proposition shows a method for finding equilibrium in the game.

Proposition 5. The action profile a^* is a equilibrium point of strategic game if and only if hold true in at least one of the following conditions:

- Each action of the player is the best response to actions of other players based on personal benefit:

$$a^* \ is \ \ in \ \ B_i(a^*_{-i}) \ \ for \ every \ player \ i,$$

- Each action of the player is the best response to actions of other players based on the benefit of other players:

$$a^* \ is \ in \ \ P_i(a^*_{-i}) \ \ for \ every \ player \ i,$$

- Each action of the player is the best response to actions of other players based on loss of other players:

$$a^* \ is \ in \ \ L_i(a^*_{-i}) \ \ for \ every \ player \ i.$$

We consider equilibrium based on concept of Nash [6,7]. Based on the concept of hyper-rationality, equilibriums can be divided into three classes. The first class is the equilibria which are considered based on personal benefit. The second class is the equilibria which are selected based on profit or loss of other players. The third class is the equilibria which are considered based on individual benefit and loss or profit of other players at the same time. An action profile may be selected which is the equilibrium point of the game based on hyper-preference of the maximum loss for other players and also has the maximum loss for all players, but it is not Nash equilibrium based on the classic concept of rationality. In addition, based on this concept, games may have one (Second Best game), two (Prisoner's Dilemma), three (Trickery game) and four (Chicken game) equilibrium or may has not equilibrium. For example, the Matching pennies is not in equilibrium and Missile crisis game has two equilibriums. Hyper-rationality helps the analyst to interpret every cell of the game table and have more accurate analysis.

2 Hyper-Rational Choice in Economic Behavior

In this section, with help of concept of hyper-rationality, we introduce a new definition of equilibrium in the game as hyper-equilibrium. A hyper-equilibrium is an action profile $b^* = (a'_i, a^*_{-i})$ with the property that only player i can increase or decrease outcome of other players by choosing an action different from a^*_i in equilibrium point $a^* = (a^*_i, a^*_{-i})$, given that every other player $-i$ adheres to a^*_{-i}.

Definition 6. (Hyper-equilibrium) The action profile $b^* = (a'_i, a^*_{-i})$ in a strategic game is a hyper-equilibrium if, only for player i and every action $\mathrm{a}^*_{\mathrm{i}}$ of player i, b^* is at least as good according to player hyper-preferences as the equilibrium point $a^* = (a^*_i, a^*_{-i})$ in which player i chooses a^*_i while every other players chooses a^*_{-i}. Equivalently, for every player i, based on benefit of other players

$$u_{-i}(a_i', a_{-i}^*) \geq u_{-i}(a_i^*, a_{-i}^*) \ for \ all \ \ a_i^* \in A_i,$$

or for every player i, based on loss of other players

$$u_{-i}(a_i', a_{-i}^*) \leq u_{-i}(a_i^*, a_{-i}^*) \ for \ all \ \ a_i^* \in A_i,$$

where u_i is a payoff function that represents player i' preferences. Hyper-equilibrium is a point in which only one player can increase or decrease outcome of other players by changing his action without other players having motivation for the change of action.

2.1 Trickery Game

Recently Eshaghi and Askari introduced Trickery game [2]. This game shows a good reason for the trickery of some players in everyday life, which one of the players can with cunning change his action that reduces the payoff, other players. Consider two international investment companies. These two companies can cooperate (C) or not cooperate (D) in the field of investment. We consider row company as player 1 and column company as player 2. The trickery game G' table is given in Table 1.

Table 1: Trickery Game G'

Player 1	*Player 2*	
	C	*D*
C	*3, 4*	*2, 4*
D	*3, 1*	*2, 1*

NOTE: C= cooperate; D= non-cooperate

In the following, we investigate depends on the claim that each player in the Trickery game is hyper-rational. Based on definitions of hyper-rationality we have;

- **Each player is thinking of making a profit to another player**.

In game G', for player 1 we have: based on concept of hyper-rationality, given fixed C for player 2, we can see that if player 1, seeks to incur profit to his opponent, he will choose C (player 2, earns a reward 3), it can conclude that pair of action (C, C) is chosen. With choosing D by player 2, we can see that if player 1, seeks to incur profit to his opponent, he will choose C (player 2, earns a reward 4), it can conclude that pair of action (C, D) is chosen. Therefore, for the player 1, based on the profit of another player, C is a strictly dominant action.

In game G', for player 2 we have: based on concept of hyper-rationality, given fixed C for player 1, we can see that if player 2, seeks to incur profit to his opponent, he will choose C (player 1, earns a reward 4), it can conclude that pair of action (C, C) is chosen. With choosing D by player 1, we can see that if player 2, seeks to incur profit to his opponent, he will choose C (player 1, earns a reward 3), it can conclude that pair of action (D, C) is chosen. So, for the player 2, based on the profit of another player, C is a strictly dominant action.

- **Player 1 is looking to profit for player 2 and player 2 seeks to lose of player 1.**

In game G', for player 1 we have: based on concept of hyper-rationality, given fixed C for player 2, we can see that if player 1, seeks to incur profit to his opponent, he will choose C (player 2, earns a reward 3), it can conclude that pair of action (C, C) is chosen. With choosing D by player 2, we can see that if player 1, seeks to incur profit to his opponent, he will choose C (player 2, earns a reward 4), it can conclude that pair of action (C, D) is chosen. Therefore, for the player 1, based on the profit of another player, C is a strictly dominant action.

In game G', for player 2 we have: based on concept of hyper-rationality, given fixed C for player 1, we can see that if player 2, seeks to incur loss to his opponent, he will choose D (player 1, earns a reward 2), it can conclude that pair of action (C, D) is chosen. With choosing D by player 1, we can see that if player 2, seeks to incur loss to his opponent, he will choose D (player 1, earns a reward 2), it can conclude that pair of action (D, D) is chosen. So, for players 2, based on the loss of another player, D is a strictly dominant action.

- **Each player is thinking of making a loss to another player.**

In game G', for player 1 we have: based on concept of hyper-rationality, given fixed C for player 2, we can see that if player 1, seeks to incur loss to his opponent, he will choose D (player 2, earns a reward 1), it can conclude that pair of action (D, C) is chosen. With choosing D by player 2, we can see that if player 1, seeks to incur loss to his opponent, he will choose D (player 2, earns a reward 1), it can conclude that pair of action (D, D) is chosen. Therefore, for the player 1, based on the loss of another player, D is a strictly dominant action.

In game G', for player 2 we have: based on concept of hyper-rationality, given fixed C for player 1, we can see that if player 2, seeks to incur loss to his opponent, he will choose D (player 1, earns a reward 2), it can conclude that pair of action (C, D) is chosen. With choosing D by player 1, we can see that if player 2, seeks to incur loss to his opponent, he will choose D (player 1, earns a reward 2), it can conclude that pair of action (D, D) is chosen. So, for players 2, based on the loss of another player, D is a strictly dominant action.

- **Player 1 is looking to loss to player 2 and player 2 seeks to profit for player 1.**

In game G', for player 1 we have: based on concept of hyper-rationality, given fixed C for player 2, we can see that if player 1, seeks to incur loss to his opponent, he will choose D (player 2, earns a reward 1), it can conclude that pair of action (D, C) is chosen. With choosing D by player 2, we can see that if player 1, seeks to incur loss to his opponent, he will choose D (player 2, earns a reward 1), it can conclude that pair of action (D, D) is chosen. Therefore, for the player 1, based on the loss of another player, D is a strictly dominant action.

In game G', for player 2 we have: based on concept of hyper-rationality, given fixed C for player 1, we can see that if player 2, seeks to incur profit to his opponent, he will choose C (player 1, earns a reward 4), it can conclude that pair of action (C, C) is chosen. With choosing D by player 1, we can see that if player 2, seeks to incur profit to his opponent, he will choose C (player 1, earns a reward 3), it can conclude that pair of action (D, C) is chosen. So, for the player 2, based on the profit of another player, C is a strictly dominant action.

The above game is an asymmetric game. This game has three Nash equilibriums (C, D), (D, D) and (C, C). In the trickery game, player 1 has weakly dominant action C and weakly dominated action D based on individual benefit. Player 2 has weakly dominant action D and weakly dominated action C based on individual benefit. Also, in this game, player 1 has strictly dominant action D and strictly dominated action C based on loss of player 2. Player 2 has strictly dominant action D and strictly dominated action C based on loss of player 1. According to the concept of classic rationality, two Nash equilibria (C,D) and (D,D) have equal payoff for player 1, but considering the concept of hyper-rationality, based on loss for player 2 given, $(D,C)\sim'(D,D)\pm'(C,C)\pm'(C,D)$, player 1 prefer (D,D) to (C,D). With assumption choosing strictly dominant action D for player 2 and hyper-rationality, concept of hyper-rationality for player 1 rules that if he seeks to benefit player 2, he will choose C, so the equilibrium of game is (C,D), if he seeks to cause loss to player 2, he will choose D because the opponent will sustain more loss and his outcome will remain fixed, therefore the equilibrium game is (D,D). Therefore, the concept of hyper-rationality helps to understand a common research language between psychologists and economists.

In this game, there are two Nash equilibrium points and based on the definition of Nash equilibrium, no player will be motivated to change his action in equilibrium point. Based on concept of hyper-rationality, given fixed defect of player 2, we can see that if layer 1, seeks to incur loss to his opponent, he will be motivated to change his action in equilibrium point (C,D) and can transfer game equilibrium to point (D,D). We want to see what reaction player 2, will show if player 1, chooses D. With choosing D by player 1, choosing C and D will give equal outcome for player 2, but if player 2, prefer loss of opponent, we will find that he doesn't choose strictly dominated action C and by choosing strictly dominant action D will incur more loss to player 1. So, we conclude that pair of action (D,C) is not chosen and player 2, is not motivated to change his action in equilibrium point (D,D). Therefore, action profile (D,D) is a hyper-equilibrium. In Trickery game, player 1, can use this hyper-equilibrium as a credible threat, to force player 2 to choose cooperation. So, action profile (C,C) is an equilibrium which chooses based on the collective benefit and pressure of the player 1.

3 Conclusion

In this paper, using the concept of hyper-rationality we examined interaction between two investment companies. This concept can describe some of the human behaviors as well. Based on the concept of hyper-rationality, a player may not recognize that what action is the most beneficial to him but can choose an action which causes the maximum loss or benefit for other players. Moreover, in trickery games, we achieved special equilibrium which are not considered as game equilibrium based on the classic concept of rationality, and we call hyper-equilibrium. The hyper-equilibrium is the point in which only one player can displace equilibrium to another point by changing his action which causes profit or loss to other players so they cannot change their action. The hyper-preferences indicate that the kind of behavior interactive, environmental conditions, and valuation system are based on hostility and players at this point have considered the maximum loss to other.

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