International Journal of Business and Development Studies **Vol. 9, No. 1,** (2017) pp 5-26

The Origin and Limitations of Modern Mathematical Economics: A Historical Approach

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Abstract

We have first demonstrated that Debreu's view regarding the publication of *The Theory of Games and Economic Behavior* by von Neumann and Morgenstern in 1944 as the birth of modern mathematical economics is not convincing. In this paper, we have proposed the hypothesis that the *coordinated research programs* in the 1930's, initiated by the *Econometric Society* and the *Cowles Commission for Research in Economics* with the objective of unifying economic theory, mathematics and statistics, can be regarded as the beginning of modern mathematical economics as well as econometrics. We have argued that this unification has failed to satisfactorily bridge the gap between mathematical economics and the real world economic issues. However, contrary to Marshall's view that mathematics is not an engine of inquiry in economics but is only a shorthand language, we have established in this paper that the application of mathematics in modern mathematical economics can, under certain conditions, produce economic results of value.

Keywords: Mathematical Economics, Econometric Society, Cowles Foundation, Marshall, Debreu.

JEL: B15, B16, B23.

1. Introduction

We may broadly categorize the literature on mathematical treatment of economics into classical and modern mathematical economics. We call the contributions of Cournot (1838), Jevons (1871) and Walras (1874) and the further theoretical developments stemming from them as "classical mathematical economics". Derakhshan (2014a, 2014b) critically examine the origin and nature of classical mathematical economics as well as its methodological shortcomings. This paper deals with the origin and limitations of "modern mathematical economics" from a historical perspective.

Marshall's *Principles of Economics* (1890) was a synthesis of classical economics of Smith (1776), Ricardo (1817), Mill (1848) and the classical mathematical economics developed by Cournot, Jevons and Walras. The

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reasons and factors involved in the forty years of sluggish progress in theoretical developments in mathematical economics since Marshall (1890) are explained in Section 2. Debreu (1986, p. 1261) regards the publication of von Neumann and Morgenstern’s *Theory of Games and Economic Behavior* (1944) as the birth of modern mathematical economics. We have critically examined and refuted his view in Section 3.

Section 4 deals with our hypothesis that the creation of the *Econometric Society* in 1930 and the *Cowles Commission* in 1932 together with the *coordinated research programs* in mathematical economics are the epoch-making events laying the foundation for the emergence of modern mathematical economics. The instrumentality of mathematics in discovering economic results of value is the subject matter of Section 5. The role of Nobel Foundation and Nobel Institutions in awarding Nobel prizes to mathematicians and economists working to advance the frontiers of mathematical economics are briefly explained in Section 6. Limitations of modern mathematical economics are discussed in Section 7, and the summary and concluding remarks are presented in Section 8.

2. Forty Years of Sluggish Progress in Theoretical Development in Mathematical Economics since Marshall

Marshall, who was a mathematician before becoming an economist, placed great emphasis on non-mathematical analysis of economics and kept all his mathematical presentations in a long appendix to his book. Even most diagrammatic representations were included in footnotes. Marshall's *Principles of Economics* (1890) dominated economic literature for more than 30 years[1]. In fact, Marshall's book was one of the main sources of economic knowledge in capitalist economies before the Keynesian revolution in 1936.

The contributions made in mathematical economics during the period from Marshall (1890) to the 1930's, can be classified as attempts either to reorganize and consolidate the previous works or to produce new and comprehensive reports on the significance of the already known results. As discussed below, Bowley, Evans and Wicksell are the most influential writers on mathematical economics in this period.

Bowely published his *Mathematical Groundwork of Economics* in 1924. Despite being an acknowledged and well-respected statistician and

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1. The 8th edition of the *Principles of Economics* was published in 1920 in 850 pages. However, the 9th edition, published in 1961, can be regarded only as a reprint of a classical work.

economist at the London School of Economics, his work did not have any impact on the direction of research work in economics. Evans, who studied mathematics at Harvard University and was a professor of mathematics at Berkeley University in 1934, published his *Mathematical Introduction to Economics* in 1930. This, together with his earlier contributions on the applications of calculus of variations to economic analysis (1925), did not play any significant role in advancing the prevailing state of knowledge in economic analysis. Wicksell, who intended to become a professor of mathematics but studied economics upon completing his doctorate in mathematics, published his *Lectures on Political Economy* in 1934, again with no significant impact on the existing trend in economic analysis.

It should be noted, however, that the highly inventive use of calculus of variations in economic analysis in the 1920's has been the most important contribution by mathematical economists, albeit with minimal impact on the attitude of economists towards economic analysis and theorization. Frank Ramsey, a well-known young Cambridge mathematician, successfully applied calculus of variations to study the saving behavior. Ramsey contributed two papers to the literature of mathematical economics. His first paper, which appeared in 1927 in *The Economic Journal,* was on the theory of taxation. However, his second paper published in 1928, again in *The Economic Journal,* is unanimously regarded as a significant contribution to economic optimization. In the *Collected Writings of John Maynard Keynes* (1972, pp. 335-336) he maintains that Ramsey's second paper "is, I think, one of the most remarkable contributions to mathematical economics ever made ..."

As Koopmans (1965) reports, unfortunately, Ramsey's contribution was almost totally ignored by economists until 1960's. From a historical perspective, the earliest attempts to apply variational methods to economic analysis can be traced back to Edgeworth (1881). Besides, the works of Hotelling (1925), Evans (1925) and Roos (1928) are known to be good examples of the efforts made in this direction. von Neumann's paper in game theory in 1928[1] and its potential applications to economic analysis was a significant contribution in the 1920's, which should also be mentioned. This work, which was published in German, again was totally ignored by economists.

[1].“Zur Theorie der Gesellschaftsspiele”, *Mathematische Annalen,* no. l00, pp. 295-3 20.

3. A Critical Analysis of Debreu's View on the Origin of Modern Mathematical Economics

According to Debreu (1986), the symbolic beginning of contemporary (or modern) mathematical economics is the year 1944, when the well-known mathematician John von Neumann and the economist Oskar Morgenstern published their work on *The Theory of Games and Economic Behavior*. According to Debreu (1986, p. 1261), this book "sets a new level of logical rigor for economic reasoning" by presenting for the first time a new mathematical method for economic analysis, i.e. the game theoretic approach. We claim that the following historical evidence may refute Debreu’s hypothesis.

i) The earlier contributions were also quite remarkable. Recall that Keynes (1972, pp. 335-336) regarded Ramsey's second paper (1928) on optimal levels of saving as "one of the most remarkable contributions to mathematical economics ever made". Moreover, contributions of Leonid Kantorovich (1939) on organizing and planning of production and Wassily Leontief (1941) on input-output analysis were so significant that were awarded the Nobel prizes in 1975 and 1973, respectively.

ii) Recall that von Neumann and Morgenstern have made the strongest attack on the mathematical methods used in consumer's utility maximization and producers profit maximization in Walrasian type mathematical economics. They claimed that the exact description of an economic agent's effort to attain the maximum satisfaction can only be obtained by employing a game-theoretic approach: "It is well known that considerable-and in fact unsurmounted-difficulties this task [utility or profit maximization] involves ... It will appear, therefore, that their exact positing and subsequent solution can only be achieved with the aid of mathematical methods which diverge considerably from the techniques applied by older or by contemporary mathematical economics, [i.e.] the mathematical theory of *games of strategy*" (1944, p. 1). This fact can also be seen from the three opening lines of their book: "The purpose of this book is to present a discussion of some fundamental questions of economic theory which requires a treatment different from that which they have found thus far in the literature".

Publication of the *Theory of Games and Economic Behavior* in 1944 did not have any impact on the direction of research work in mathematical economics at the time. The main objective of their book was to establish that "the typical problems of economic behavior become strictly identical with the mathematical notions of suitable games of strategy" (*ibid*, p. 2). From a purely mathematical point of view, von Neumann and Morgenstern's work is a departure from Walrasian tradition. Hence, Debreu arrived wrongly at the conclusion that this book was the beginning of a new era in mathematical economics.

The game-theoretic approach to economic analysis was overlooked by mathematical economists for almost four decades until 1980’s when the game-theoretic approach increasingly was applied to microeconomics and then to macroeconomics. In other words, despite von Neumann and Morgenstern's emphasis on the game-theoretic approach to economic analysis, the actual development of mathematical economics after 1944 was mainly along with the direction of the general equilibrium analysis and optimal properties of growth models developed in the 1950's and 1960's.

Two further points should be noted. Firstly, von Neumann's generalization of Brouwer's fixed point theorem to prove the existence of an optimal growth path has been a remarkable contribution, but this has no relation with game-theoretic approach to economic behavior. Secondly, a number of contributions, which von Neumann and Morgenstern made in their game-theoretic applications to economic analysis, provided a new level of mathematical rigor in economic reasoning. For example, the introduction of *convex analysis* into economic theory (1944, chapter III, section 16) has made the concept of *convexity* an integral component in topical issues in mathematical economics.[1] However, these contributions did not change the direction of research work in mathematical economics for nearly three decades, hence contrary to Debreu’s claim, cannot be valued as the beginning of a new era in mathematical economics.

4. Coordinated Research Programs: The Origin of Modern Mathematical Economics and Econometrics

I now propose the hypothesis that the coordinated research programs in the 1930’s can be considered as the origin of modern mathematical economics and econometrics. The establishment of new academic research institutions aimed at coordinating and encouraging advances in mathematical economics was a new phenomenon in the beginning of the 1930's. Recall that there has been no efforts in coordinating the research

[1].For example, topics like consumption theory, production theory, welfare economics, efficiency analysis and more importantly, theory of general equilibrium.

works in classical mathematical economics, i.e. the literature developed during the period from Cournot (1838) to Jevons (1871) and Walras (1874) as well as during the period characterized by their extensions to Marshall (1890) and beyond until the 1930's.

To elaborate this point, we note that despite the independent contributions made by Walras and Jevons, Walras initially respected Jevons's work and acknowledged his priority in formulating the *Equation of Exchange*, which was identical to Walras's *Condition of Maximum Satisfaction.* However, soon Walras took an unfriendly position against Jevons and accused him as a plagiarist of his work. This fact demonstrates that there was no research coordination between Jevons and Walras. It is also known that Pareto, the successor to Walras in the chair of political economy at Lausanne, departed from Walras's tradition and gave up economics and concentrated exclusively on sociology[1].

The starting point in the formation of modern mathematical economics and econometrics has been the recognition of the fact that any further advances in classical mathematical economics necessarily depended on the integration of statistical techniques into mathematical economic analysis. The *Econometric Society* was established on December 29, 1930, in Cleveland, Ohio, with the objective of unifying *economic theory, mathematical analysis* and *statistics.* In fact, the sub-title of the Econometric Society was "An International Society for the Advancement of Economic Theory in its Relation to Statistics and Mathematics." The early attempts by Ragnar Frisch, Professor of Economics, University of Norway, Oslo; Charles Roos, Permanent Secretary of the American Association for the Advancement of Science, Washington; and Irving

[1]. To provide further evidence on the absence of coordination in the classical mathematical economics, we refer to Walras (1874). In the preface to the first edition (pages 35 and 36) he maintained that "This work was completely written and almost completely printed...when...my attention was drawn to a work on the same subject, entitled: *The Theory of Political Economy,* in 1871, by W. Stanley Jevons, Professor of Political Economy at Manchester...I acknowledge Mr. Jevons's priority so far as his formula is concerned, without relinquishing my right to claim originality for certain important deductions of my own. I should not enumerate these points which competent readers will readily discover. I need only add that, as I see it, Mr. Jevons's work and my own, far from being mutually competitive in any harmful sense, really support, complete, and reinforce each other to a singular degree".

The above passage might indicate a harmony between the work of Jevons and Walras; but the lack of coordinated research program soon put an end to this friendly attitude. Donald Walker (1987, pp. 861-862) reports that "[Walras's] initial cordiality towards Jevons, as a fellow pioneer in mathematical economics, was dissipated by Jevons's failure to recognize Walras's [main] contributions ... and eventually Walras ... came to regard Jevons as a plagiarist of his work (Walras to M. Pantaleoni, 17 August 1889) ... Walras felt neglected by Alfred Marshall [too] ... Walras wrote in 1904 that "I have not the least doubt about the future of my method and even my doctrine; but I know that success of this sort does not become clearly apparent until after the death of the author" (Walras to G. and L. Renard, 4 June 1904)".

Fisher, Professor of Economics, Yale University, had a profound impact on the formation of the Econometric Society. Irving Fisher became the President and Chairman of the Council, while the other two, together with Arthur Bowley, Professor of Statistics at LSE, Joseph Schumpeter, Professor of Economics at Harvard University and Alfred Cowles[1], Director of the *Cowles Commission for Research in Economics*[2], (as treasurer), among others, formed the Council of 10 members. Section *I* of the Constitution reads as follows: "The Econometric Society is an international society for the advancement of economic theory in its relation to statistics and mathematics."

The first issue of *Econometrica,* the Journal of the Econometric Society, was published in January 1932. Ragnar Frisch was appointed as the editor. The academic background of the associate editors clearly demonstrates the emphasis of the Society on the unification of economics, statistics and mathematics: Alvin Hansen was Professor of *Economics* from University of Minnesota, Frederick Mills was Professor of *Statistics* from Columbia University and Harold Davis was Associate Professor of *Mathematics* from Indiana University.

To provide further evidence on the importance of the Econometric Society in the formation of modern mathematical economics it is opportune to refer to some points which Ragnar Frisch has made in his first editorial to *Econometrica* (1932, vol. 1, p. 2): "Econometrics is by no means the same as economic statistics. Nor it is identical to what we call general economic theory, although a considerable portion of this theory has a definitely quantitative character. Nor should econometrics be taken as synonymous with the application of mathematics to economics. Experience has shown that each of these three view points, that of statistics, economic theory and mathematics, is a necessary, but not by itself a sufficient condition for a real understanding of the quantitative

1. Alfred Cowles was not among the first Council of 10 members in the *Econometric Society.* When L. V. Bortkiewicz from University of Berlin, who was a member of the Council, died in August 1931, Alfred Cowles was appointed as a member and the treasurer of the Council.

2. The Cowles Commission for Research in Economics founded in 1932 by Alfred Cowles and a group of economists and mathematicians concerned with the applications of quantitative techniques to economics and the related social sciences at Colorado Springs. The Commission moved to Chicago in 1939 and was affiliated with the University of Chicago until 1955 when it moved to Yale. The research staff of the Commission along with other members of the Yale Department of Economics established the *Cowles Foundation for Research in Economics* in order to sponsor and encourage the development and application of quantitative methods in economics.

relations in modern economic life. It is the *unification* of all three that is powerful. And it is this unification that constitutes econometrics."

According to Ragnar Frisch in his first editorial to *Econometrica,* the mutual penetration of quantitative economic theory and statistical observations is, in fact, the essence of econometrics. This view profoundly changed the tradition of economic theorization within the framework of classical mathematical economics. "Theory, in formulating its abstract quantitative notions must be inspired to a large extent by the technique of observation. And fresh statistical or other factual studies must be healthy elements of disturbance that constantly threatens and disquiet the theorist and prevents him from coming to rest on some inherited, obsolete set of assumptions." (*ibid*, p. 2)

The emphasis given by Ragnar Frisch on the role of mathematics in quantitative economics is interestingly confusing. On the one hand, he maintained that "Mathematics is certainly not a magic procedure which in itself can solve the riddles of modern economic life, as is believed by some enthusiasts. But when combined with a thorough understanding of the economic significance of the phenomena, it is an extremely helpful *tool*.'' On the other hand, he held the view that "Indeed, it will be an editorial principle of *Econometrica* that no paper shall be rejected solely on the ground of being too mathematical. This applies no matter how highly involved the mathematical apparatus may be." (*ibid,* pp. 2-3)

The objective of the coordinated program of research work mapped out by the Econometric Society[1] was not to compete in any harmful sense with traditional classical mathematical economics of Jevons and Walras; on the contrary, it claimed to possess the capability of completing and enriching it. To respect the founder of classical mathematical economics, the first volume of *Econometrica* was honored by a portrait of Cournot accompanied by a paper in French entitled "Cournot et L'Ecole Mathematique" by Rene Roy.

Applications of statistical methods to economic analysis, strongly supported by the Econometric Society, produced a more positive attitude

[1]. Contributions of the "Cowles Commission" towards further developments in coordinated research program should not be overlooked. As mentioned earlier, this Commission was established in Colorado in 1932, moved to Chicago in 1939 and to Yale (as Cowles Foundation) in 1955. Its main contributions in advancing quantitative methods in economics are summarized in the *Report of Research Activities* as follows: "The activity analysis formulation of production and its relationship to the expanding body of techniques in linear programming became a major focus of research at Chicago period. The Walrasian model of competitive behavior was examined with a new generality and precision in the context of a modern reformulation of welfare theory" [see Cowles Foundation (1983), p. 1].

towards *measurement* in economic analysis. Recall that utility maximization was the corner-stone of the contributions of Jevons, Walras and Pareto. It is not therefore surprising that a debate concerning the *determinateness of utility* was given higher priority in the first round of coordinated research works. In this debate, a number of questions regarding utility functions and their measurements were discussed in detail. The published works of Lange (1934, 1935), Phelps-Brown (1934), Allen (1935) and Bernandelli (1935) in the newly published journal of the *Review of Economic Studies,* (first published in 1933), were among the most significant contributions resulting from this debate. *Econometrica* and the *Review of Economic Studies* were the two journals acting as continuous sources of encouragement for further research work in mathematical economics and econometrics.

Contributions of Leonid Kantorovich (1939) in organizing and planning of production, Wassily Leontief (1941) in input-output analysis, Paul Samuelson (1947) in foundations of economic analysis, Tjalling Koopmans (1951) in activity analysis of production and George Dantzig (1951) in simplex algorithm were the main early results in modern mathematical economics. Further expansion in the literature paved the way for the publication of more specialized journals in mathematical economics, i.e., *International Economic Review* in 1960, *Journal of Economic Theory* in 1969, *Journal of Mathematical Economics* in 1974 and *Journal of Economic Dynamics and Control* in 1979 are the well-known journals, which stimulated further research interests in mathematical economics and econometrics.

Having established the importance of coordinated research programs and the significance of the appropriate academic institutions in the establishment and development of modern mathematical economics, we now briefly examine the role of mathematical machinery in discovering economic results, which could not have been known otherwise.

5. The Instrumentality of Mathematics in Discovering Economic Results of Value

It is generally agreed that making use of mathematical symbols, operations, methods, theorems and geometrical representations in economic reasoning would not only facilitate the exposition and generalization of problems but also render them to greater precision of statements by avoiding vague argumentation.

Conflicting views arise when economists face the question that whether the application of mathematics to economic analysis could produce *economic results of value,* which were previously *unknown.* For example, Jevons (1879), in appraising Cournot (1838) wrote that "... this investigation, presents a beautiful example of mathematical reasoning, in which knowledge is apparently evolved out of ignorance." (p. *xxxi,* preface to the 2nd edition)[1]

Alfred Marshall gives the best exposition on the limitations of mathematical economics. According to Marshall, who graduated in mathematics at Cambridge University in 1865 and is known as the founder of the Cambridge School of Economics, there is a tendency in mathematical economics to give emphasize on those issues, which will fit easier into mathematical methods. He strongly warned economists on this unbalanced treatment of economic issues. In his *Principles of Political Economy* [(1890), pp. 850-1] he wrote as follows: "And hence arises a tendency towards assigning wrong proportions to economic forces; those elements being most emphasized which led themselves most easily to analytical methods ... It is a danger which more than any other the economist must have in mind at every turn. But to avoid it altogether, would be to abandon the chief means of scientific progress."

For Marshall, explicit and clear economic meanings together with potentiality of explaining economic observations were the two conditions for successful applications of mathematical methods to economics. Being very conservative in using mathematical symbols in economic analysis, he wrote in 1906 that "I never read mathematics now; in fact I have forgotten even how to integrate a good many things. But I know I had a growing feeling in the later years of my work at the subject that a good mathematical theorem dealing with an economic hypothesis was very unlikely to be good economics; and I went more and more on the rule - (1) Use mathematics as a short-hand language, rather than as an engine of inquiry, (2) Keep to them till you have done, (3) Translate into English, (4) Then illustrate by examples that are important in real life, (5) Burn the mathematics, (6) If you can't succeed in (4), burn (3). This last I did often." (*ibid,* p. 776, vol. 2, *Notes*)

As mentioned before, the most controversial question in mathematical treatment of economics is whether economic truths are discoverable through the instrumentality of mathematics. Recall that Marshall held the

[1].See Derakhshan (2014a) for a critical evaluation of Jevons's judgment.

view that mathematics was only a shorthand language and not an engine of inquiry. As I will discuss in the following sections, contrary to Marshall's view, mathematics can be an engine of inquiry in modern mathematical economics if the nature and scope of our economic problem calls for the application of certain mathematical methods. In other words, there are economic questions, which can only be attended by certain mathematical methods. This is where mathematical methods can really contribute to economic analysis.

6. The Role of Nobel Foundation and Nobel Institutions in the Promotion of Mathematical Treatment of Economics

A number of Nobel prizes in economics are awarded to the mathematicians who demonstrated successful applications of mathematics to economic analysis. It should be mentioned that in accordance with Nobel's will[1], the first Nobel prizes were awarded in 1901 to Physics, Chemistry, Medicine, Literature and Peace. However, the Sweden's Central Bank or Sveriges Riksbank, donated large sums to the Nobel Foundation in 1968 for awarding the Nobel Prize in Economic Sciences. The Nobel Foundation[2] decided subsequently, that no prizes in any other disciplines in sciences and humanities will be awarded.

A number of Nobel prizes in economics are awarded to mathematicians mainly for demonstrating how economic applications of certain mathematical methods can provide useful economic results, which

[1]. Alfred Nobel (1833-1896), was a Swedish chemist, engineer, inventor and businessman, and best known for his invention of dynamite in 1867. In 1897, the Norwegian Parliament approved his will in which Alfred, who had no wife and children; left most of his huge wealth (over 1.6 million GBP at the time) in trust to fund what is known today as the Nobel Prize. The Nobel Foundation was then established in 1900 acting as an investment company, to financially manage what Alfred Nobel bequeathed in order to fund the awards. Nobel's brothers, Ludvig and Robert, founded the "Petroleum Production Company of Nobel Brothers" in Baku, Azerbaijan in 1876 and then in St. Petersburg in 1879 for production and distillation of oil in the Caspian region. Alfred Nobel, who was a major producer of cannon and other armaments, invested in this oil company too. The Royal Swedish Academy of Sciences admitted Alfred Nobel as a member in 1884 and Uppsala University awarded him an honorary doctorate in 1893. An interesting question has always been that why Nobel, who owned more than 90 armaments factories at his death, became the benefactor of awards for science, medicine, literature, and peace. It is said that in 1888 Ludvig died and a French newspaper mistakenly announced Alfred's death saying that "the merchant of death is dead…Alfred Nobel, who became rich by finding ways to kill people faster than ever before, died yesterday." (Frederic Golden, "The Worst and the Brightest", *Time,* 16 October 2000). Apparently, Alfred Nobel read this announcement and became disturbed on how the history may judge him by his invention of dynamite.

[2]. The final decisions on selecting the Nobel Prize winners who are usually called the Laureates are taken by the Nobel Institutions and not by the Nobel Foundation. These institutions are currently as follows: Royal Swedish Academy of Sciences with 350 members (Physics, Chemistry and Economics), Nobel Assembly at Karolinka Institutet with 50 members (Physiology and Medicine), Swedish Academy with 18 members (Literature) and Norwegian Nobel Committee with 5 members (Peace).

otherwise could not have been obtained. The first Sveriges Riksbank Prize in Economics in the memory of Alfred Nobel was awarded to Ragnar Frisch and Jan Tinbergen in 1969 for “having developed and applied dynamic models for the analysis of economic processes.” Ragnar Frisch obtained his Ph.D. in mathematical sciences and Jan Tinbergen’s Ph.D. dissertation was entitled “The Minimization Problems in Physics and Economics”. Leonid Kantorovich, a Russian mathematician, was awarded the Nobel Prize in Economics in 1975 for developing the mathematical theory of linear programming and applying it to economic problems of optimum allocation of resources. Gerard Debreu, a French mathematician, is another example who won the Nobel Prize in Economics in 1983 for his contributions to the general equilibrium analysis.

For more recent examples, we may refer to Alvin E. Roth and Lloyd Shapley who won the Nobel prizes in 2012 for their contributions on the theory of stable allocations and the practice of market design. Alvin Roth received his B.A. degree from Columbia University, School of Engineering and Applied Sciences and M.A. and Ph.D. degrees from Stanford University all in Operation Research. Lloyd Shapley received B.A. degree in Mathematics from Harvard University and Ph.D. and Post Ph.D. degrees in Game Theory from Princeton University. Shapley defined game theory as “a mathematical study of conflict and cooperation.” Jean Tirole, who won the Nobel Prize in 2014 for his contribution to market power and regulation, received degrees in Engineering and Mathematics in France before obtaining Ph.D. degree in Economics from MIT. Bengt Holmstrom and Oliver Hart, who received the Nobel prizes in 2016 for their contribution to Contract Theory, were mathematicians. Holmstrom studied mathematics and physics at the University of Helsinki before receiving his Ph.D. in Operations Research from Stanford University. Oliver Hart received his B.A. degree in Mathematics from Cambridge University and then M.A. and Ph.D. degrees in Economics from Princeton University.

Many Nobel laureates in Economics, who did not have formal education in mathematics, were great enthusiasts for mathematical treatment of economics. The best example is Paul Samuelson, the first American who won the Nobel Prize in Economics in 1970 for “having done more than any other contemporary economists to raise the general analytical and methodological level in economic science.” He entered the University of Chicago in January 1932 at the age of 17 and received his

Ph.D. degree in Economics in 1941under the supervision of Joseph Schumpeter and Wassily Leontief. He became full Professor of Economics at M.I.T at age 32. He believed that "mathematics is the natural language for economists." Samuelson completely transformed the structure of economic analysis from the verbal and diagrammatic approach, prevailing in the pre-1930's, to mathematical treatment of economic issues and quantitative methods of reasoning in economic theorization. Recall from Section 4 that the prime objective of the Econometric Society, after its establishment in 1932, was the determination and measurement of utility function. It is not therefore surprising that the early research work of Samuelson in 1936, at age 21, while being a Ph.D. student at Chicago, was on the measurement of utility functions. In this regard, Samuelson can be regarded as an outstanding economist amongst the first generation of economists flourishing from the coordinated research programs of the Cowles Commission. Samuelson played a unique and incomparable role in the advancement of modern mathematical economics during his 60 years of academic work, until his death in 2009.

An examination of the background and the contributions of the Nobel Prize winners from 1969 reveal the fact that the Nobel Foundation and the Nobel Institutes have had a pivotal role towards the success of the Cowles Commission coordinated research work in modern mathematical economics.

7. Limitations of Modern Mathematical Economics in Discovering Economic Results of Value

It has always been an easy choice for economists as well as mathematicians to select a well-defined mathematical method and then apply it to a set of economic problems with predetermined assumptions and constraints, in order to facilitate the application of the assumed mathematical method. This approach is unlikely to produce any significant economic results. On the contrary, the starting point in mathematical treatment of economics should be to explore the complexities of the economic problem in order to identify the suitable mathematical methods, rendering those complexities for rigorous explanation, mathematical modeling and logical method of deriving conclusions based on the appropriate and realistic assumptions. Hence, economic results of value in mathematical treatment of economic problems may be expected if and only if the complexities of real economic issues can mathematically be modeled and then analyzed by the powerful machinery of mathematical reasoning.

The analysis of the nature of an economic problem and the identification of its complexities in order to treat them mathematically necessitate the following conditions: *i*) familiarity of economists to appropriate advanced mathematical techniques, *ii*) further advancement in certain existing mathematical methods and *iii*) discovery of new mathematical theorems and methods appropriate to manage the complexities of the economic problem. If the above-mentioned conditions are satisfied, then mathematics can be the engine of inquiry and its applications may lead to *real economic contributions.*

From a historical standpoint, a number of early economic examples for the three conditions listed above are the followings. Applications of mathematical optimal control to optimum allocation of economic resources over time [see for example Chow (1972, 1975) or Aoki (1976)] or the application of Ito's lemma in deriving Black-Scholes-Merton differential equation in option pricing model (1973) in finance are good examples for the first condition. von Neumann's *minimax theorem* (1944), which is a fundamental result in the theory of zero-sum games, or the Kakutani's generalization of *Brouwer's fixed-point theorem* (1941), which were both motivated by problems in economic game theory, provide examples for the second condition. Finally, Kantorovich's contribution to economics, mentioned earlier, is a clear example of the last condition, in which Kantorovich (1939) invented the linear programming, the general mathematics of finite systems of linear inequalities, in order to solve problems in optimum allocation of resources. Since then, linear programming has been added to the literature of applied mathematics. Another early example for the last condition is the simplex method in quadratic programming, invented by mathematician Philip Wolfe (1959) in order to solve problems in optimum investment decisions. Strategic game theory developed by von Neumann and Oskar Morgenstern (1944) provides another significant example regarding the last condition.

We may now refer to a problem of prime theoretical importance, which may exert adverse effects on the efficiency of economic applications of mathematical methods in producing economic results of value. Further development in certain mathematical methods induced by the advancement in mathematical economics may generate new round of intellectual interests among mathematical economists and mathematicians to define new economic problems with new assumptions and constraints, in order to treat them mathematically. This growth generating cycle has certainly influenced the direction of economic theorization by widening the existing gap between theoretical mathematical economics and the real world economic life. The present situation in economic literature, which is characterized by *too much* mathematics and an endless fascinating journey in abstract economic applications of mathematical methods support this argument.

To elaborate further the mechanism of the above-mentioned growth generating cycle in modern mathematical economics, the institutional arrangements should also be taken into consideration. Historically, the rapid pace of industrialization and economic growth in the 1950's and 1960's and the increasing role and potentiality of computer programming in modeling and planning in industry and trade, produced a greater optimism in modern mathematical economics. The concomitant increasing involvements of mathematical economist in the governing bodies of academic research and educational institutions as well as in Governmental research departments have been conducive in the publications of mathematically oriented papers in economic journals, which in turn provide more job opportunities for young economic graduates. This new institutional setting has produced higher incentives for economic students to study mathematical economics with minimum direct reference to the real world economic problems. This growth generating cycle will continue to exist in the foreseeable future.

The complexities inherent in real world economic problems result from the multi-dimensionality of economic issues, which include social, cultural, political and historical factors. Mathematical machinery is not yet developed enough to capture the interaction of these dimensions for a better understanding of the real economic issues. No matter how advanced is a mathematical method, its application to explore the inherent complexities of economic issues necessarily requires abstracting a one-dimensional economic model from a multi-dimensional real economic problem. This is the most serious limitation of mathematical economics. The objective of modern mathematical economics was to bridge the gap between mathematical treatment of economics and the real world economic analysis by unifying economic theory, mathematics and statistics. However, the methodological shortcomings in the unification of these three categories in order to capture the complexities of the real multi-dimensional economic issues have remained as the most serious limitation of modern mathematical economics. Hence, the key to overcome this limitation is further advancement in economic methodology and the logic of abstraction in deriving pure mathematical models from the real world economic problems, which is beyond the scope of this paper.

8. Summary and Concluding Remarks

The starting point in the examination of the origin of modern mathematical economics is the realization of the fact that there has not been a continual course of development in classical mathematical economics from Pareto (1897) and Marshall (1890) to the works done until the 1930's. Different individual contributions during this period can be categorized as attempts either to reorganize and consolidate previous results or to provide new and comprehensive reports on the significance of the already known contributions. Bowley (1924), Evans (1930) and Wicksell (1934) are the renowned publications in this connection. During this period, the community of economists had remained faithful to non-mathematical Marshallian-type economic analysis.

According to Debreu (1986), the publication of *The Theory of Games and Economic Behavior* by von Neumann and Morgenstern in 1944 marks the beginning of modern mathematical economics on the ground that this book introduced a new level of logical rigor in economic reasoning as well as formulating for the first time a new mathematical method for economic analysis, i.e. the mathematical theory of games of strategy. Using historical evidence, we have demonstrated that Debreu’s analysis is not convincing.

We have proposed the hypothesis that the *coordinated research programs* designed and implemented by new institutional settings in the 1930’s are the origin of modern mathematical economics. To clarify this point, we have demonstrated the absence of any coordinated research work done in classical mathematical economics prior to the 1930's. We have shown that the establishment of the *Econometric Society* on December 29, 1930 and the *Cowles Commission for Research in Economics* in 1932 with the prime objective of unifying economic theory, mathematical analysis, and statistics as well as promoting and encouraging research work in this direction, have marked the beginning of modern mathematical economics. *Econometric Society* explicitly admitted that the mutual penetration of quantitative economic theory and statistical observations could profoundly change the tradition of economic theorization in classical mathematical economics. Moreover, the role of the Nobel Foundation and the Nobel Institutions in awarding the Nobel prizes to mathematicians and economists working to advance the frontiers of mathematical economics cannot be ignored.

To examine the nature of modern mathematical economics we have briefly referred to the limitations of mathematical treatment of economic issues. Marshall’s conditions on the successful applications of mathematics to economic analysis are discussed. Contrary to Marshall's view that mathematics is not an engine of inquiry but is only a shorthand language, we have established that mathematics can, under certain conditions, become the engine of inquiry in modern mathematical economics. Whenever the complexities inherent in an economic problem facing an economist necessarily requires *i*) familiarity with certain relevant mathematical methods, *ii*) further advancement of the existing mathematical methods and *iii*) the discovery of new methods or even new disciplines in mathematics, then one may expect economic results of value by using mathematics in economic analysis.

A question of theoretical significance is how to identify truly the complexities of an economic problem in order to apply the appropriate mathematical method for arriving at a better understanding of those complexities. Economic problems and their complexities cannot truly be understood in isolation of social, political, cultural and historical dimensions. Hence, the identification of the complexities of economic issues falls within the domain of the *logic of abstraction* in the sense of abstracting an economic behavior mathematically from the real-life economic performance while preserving the properties of relations between the economic and non-economic dimensions. Further elaboration on this point is not the subject matter of this paper.

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