$$\cos D = -\tan(\phi)\tan(\varepsilon)$$

in modern notation.

As I have mentioned, this points to the existence of differences between the two treatises of Ibn al-Kammād, *al-Kawr* and *al-Muqtabas*.

4. *Concluding remarks*

As a conclusion, I would like to summarize some of Ibn al-Hā'im's reasons for criticizing Ibn al-Kammād's astronomical work:

- First, the inconsistencies he found in Ibn al-Kammād's text, for instance, in the length of the year, and also, its inappropriate use.

- Second, the lack of accuracy in certain calculations which are not in accordance with the needs of the astronomers.

- Third, that Ibn al-Kammād sought in some instances to derive a model valid for all time, which is erroneous according to Ibn al-Hā'im. From his experience, models and, in consequence, tables, have to be corrected periodically and their validity is for a period of no more than 40 years after compilation.

As for Ibn al-Hā'im's work, from this study there are reasons to think that *al-Zīj alKāmil* in its present form is a reduced version of the original, which probably included some other topics in detail as astrology, timekeeping and eclipses for instance, since Ibn al-Hā'im refers the reader to them. It may also have contained tables although there are no traces in the only copy extant of this *Zīj*[21]

As for Ibn al-Kammād's work, it seems that there were several differences between *al-Kawr* and *al-Muqtabas*, although some tables may be similar in the two works.

In conclusion, Ibn al-Hā'im's criticisms are useful to understand the astronomical work of Ibn al-Kammād who, no doubt, was a major figure in the history of Astronomy in al-Andalus.

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[21]. On the possibility that this zij had tables cf M. Abdulrahman, "Wujud ;adawil fi zij Ibn al-Ha'im". English abstract with the title "Ibn al-Ha'im's zij did have numerical tables", in *From Baghdad to Barcelona*. vol. I pp. 365-381.

body. According to Ibn al-Hā'im this value is not always constant but only rarely and, therefore, it must not be used in a book as a basis[19].

This topic is dealt with in the Latin version of *al-Muqtabas* in canon (porta) 26 (fols. 14r- 15r) releated to the crescent moon and in a more elaborated way than in *al-Kawr* as Ibn al-Hā'im describes it.

3.3 *Timekeeping*

The passage (#9-10), devoted to timekeeping, also gives an idea of the differences that may be found between Ibn al-Kammād's *al-Kawr* and *al-Muqtabas*. Ibn al-Hā'im's criticisms numbered 9 and 10 are related to the determination of the altitude of the sun from the hour and vice-versa (the hour from the altitude of the sun). Ibn al-Hā'im does not give the procedure in Ibn al-Kammād but says that it is only correct at the equinoxes. Therefore it could be the approximate formula of Indian origin.

I. sin t = sin h/sin H

where t is the time elapsed since sunrise in degrees, h is the altitude of the sun at the moment given and H is the meridian altitude of the sun in that day.

In chapter 22 of *al-Muqtabas'* Canons Ibn al-Kammād includes instructions to determine the time of the day from the altitude of the sun using this formula. But afterwards, he explains that this formula is only useful at the equinoxes and, otherwise, another procedure must be used. He then describes the exact formula

II. vers T = vers D - vers D sin h/sin H[20]

where T is the horary angle and D the half length of daylight. Then he explains that the time from sunrise, t, can be obtained making

$$t = D - T$$

In fol. 47v. of the *Muqtabas* we find a table which gives the half length of daylight as a function of a latitude $\phi = 38;30$ and an obliquity of the ecliptic $\varepsilon = 23;33$ which derives from the exact formula:

19. On Islamic visibility tables cf. E.S. Kennedy & M.Agha, "Planetary Visibility Tables in Islamic Astronomy" *Centaurus*, 7 (1960) pp. 134-140.

20. This formula is also of Indian origin and was known in the Islamic East from the 9th century onwards.

17-21 Eclipses

22 Solar year

23-24 Astrology (excess of revolution, *tasyīr* or astrological progressions)

25 Sun

I have analysed elsewhere the criticisms related to the trepidation and the solar year[16]. For instance, Ibn al-Hā'im criticizes that Ibn al-Kammād uses the anomalistic year, instead of the sidereal one, to carry out astrological calculations.

Here I would like to present some notes on what Ibn al-Ha'im says concerning the solar theory, arcs of visibility and timekeeping in Ibn al-Kammād's works.

3.1 *Solar theory*

The third criticism of the list is related to the Solar Theory. In this passage Ibn al-Hā'im says that Ibn al-Kammād "established the two radices of the apogee for the Hijra and the Persian era from a motion quicker than the one he put on the tables". Ibn al-Hā'im goes on saying that "it is impossible that both questions are correct at the same time. The longitude of the apogee was of 85;49° from the spring equinox according to the observer (i.e. Azarquiel)[17] and according to it based all the questions, for instance, the motion of the fixed stars and trepidation *(iqbal wa idbar)* and the times observed of crossing the equinox *(al-majāzāt al-istiwā'iyya)*[18]. The result is that everything related to the motion of the apogee in the two books *al-Kawr and al-Mu qtabas* is completely erroneus".

3.2 *Visibility of the celestial bodies*

On the visibility *(zuhūrāt wa-ikhtifā'āt)* of the celestial bodies (#6) Ibn al-Hā'im says that Ibn al-Kammād determined the arc of visibility to be equivalent to the difference between the oblique ascension of the sun and the ascension (oblique) of the degree corresponding to the rising of the celestial

16. Cf. E. Calvo, "Ibn al-Kammād's astronomical work in Ibn al-Hā'im's al-Zīj al-Kāmil fi-l-ta'ālīm" in *Science and Technology in the Islamic World*, Turnhout, 2002, pp 109-120.

17. This value was established by Ibn al-Zarqālluh around 1074-75. Cf. Samsó, *Las Ciencias*.... p. 198, n. 131. It is mentioned by Bernardus of Virdun in his Tractatus super totam Astrologiam. Cf. Toomer *The Solar Theory of Az-Zarqal*: An Epilogue, p. 517.

18. Cf. p. 11 (fol.6r) of the ms.

in Cordoba in the year 510 H./ 1116-7 A.D[14]. We also find some information on Ibn al-Kammād in work on *mīqāt* by Abū-l-Hasan 'Alī al Marrākushī (ca. 1262), entitled *Jāmi' al-Mabādī' wa-l-Ghayāt fī 'ilm al-Mīqāt.* There, for instance, the author praises the precision and correctness of the tables included in *al-Amad 'ala al-abad* to determine the solar longitude.

A more wider source of information of Ibn al-Kammād's astronomical work is Ibn al-Hā'im, whose complete name was Abū Muhammad 'Abd al-Haqq al-Gāfiqī al-Ishbilī. He was the author of an astronomical book entitled al-Zīj *al-Kāmil fi'l-Ta'ālīm,* a *zīj* composed in the first years of the thirteenth century and dedicated to the Caliph Abū Abd Allāh Muhammad al-Nāsir (1199-1213). All we know about Ibn al-Hā'im's life is that he appears to have worked in North Africa under the Almohad dynasty[15].

Ibn al-Kammād adopted the modifications introduced by Ibn al-Zarqālluh in the solar model although he intended to modify some of the parameters. These modifications were criticised by Ibn al-Hā'im in the introduction of his *al-Zīj al-Kāmil.* This astronomer can also be inscribed in the zarqāllian tradition.

3. *Ibn al-Hā'im 's criticisms to Ibn al-Kammād's work*

Ibn al-Hā'im includes in the introduction to his *Kitāb* a list of criticisms to the work of Ibn al-Kammād. The 25 topics concerned in the criticisms appear numbered in *abujad* notation in the margin of the manuscript. Here they are in an abridged table:

1 Solar year
2 Trepidation
3-5 Solar theory
6 Arcs of visibility
7 Trigonometry
8 Mediation
9-10 Timekeeping
11 Astrology (Equalization of houses)
12-21 Moon
12 Conjunction and opposition
13-16 Parallax

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14. Cf. Angel Mestres,"Maghribi Astronomy in the 13th Century: a Description of Manuscript Hyderabad Andra Pradesh State Library 298" *From Baghdad to Barcelona*, pp. 383-443, especially p. 404

15. On Ibn al-Ha'im cf. J. Samsó, *Las Ciencias*, pp. 320-325 and E. Calvo, "Astronomical Theories Related to the Sun in Ibn al-Ha'im's al-Zij al-Kamil fi'l-Ta'alim" *Zeitschrift für Geschichte der ArabischIslamischen Wissenschaflen* 12 (1998) 51-111

These modifications, trepidation, solar model with variable eccentricity, appear in a number of Andalusian and Maghribian *zijes* in what is designated as "the zarqāllian tradition".

As an example of the influence of his work in later astronomers we can see some aspects of the work of an andalusian astronomer of this period, namely Ibn al-Kammād who was inscribed in this Zarqāllian tradition but who made some new modifications to the parameters and elements in the models of this tradition and who was critizised afterwards by later astronomers in the same tradition due to some of these modifications.

2. *Ibn al-Kammād and his modifications to the Zarqāllian astronomical theories*

Although nothing is known of his life, Ibn al-Kammād was probably a disciple of Ibn al-Zarqāfluh. His complete name was Abū Ja'far Ahmad ibn Yūsuf Ibn al-Kammād (although in later sources, Ibn Ishāq and Ibn al-Ha'im for instance, is called Abū-l'Abbās) He was the author of several *zijes* or astronomical handbooks. We know the title of three of them:

-*al-Amad 'alā al-abad ("The* Valid for the Eternity").

-*al-Kawr 'alā al-dawr* ("The Periodic Rotations")

-*al-Muqtabas* ("The Compilation" -of the other two-)

None of them has survived apparently in its original Arabic version with the exception of some passages of the second, *al-Kawr* (in ms. 939, 4 Escorial), and chapter 28 of the last one. *Al-Muqtabas* was also translated into Latin in 1262 by Johannes of Dumpno. This translation, which is preserved in the Biblioteca Nacional in Madrid (in ms. 10023), played an important role in the transmission of Ibn al-Zarqālluh's astronomical theories into the Latin World and its reception by the astronomers of the Renaissance.[13]

Some elements of Ibn al-Kammād's astronomical work are preserved in later sources. One of them is the *zīj* of Ibn Ishāq al-Tūnisī, an astronomer who worked in Marrākush in the first half of the XIIIth century (fl. 1222 A.D.). There we are told, for instance, that Ibn al-Kammād cast a horoscope

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13. On Ibn al-Kammād cf. J. Samsó, *Las Ciencias de los antiguos en al-Andalus* (Madrid, 1992) pp. 320-324; J. Vernet, "Un tractat d'obstetricia astrologica" in *Estudios de Historia de la Ciencia Medieval*. Barcelona, 1979, pp. 273-300 and E. Calvo, "Ibn al-Kammād's astronomical work in Ibn al-Hā'im's al-Zīj al-Kāmil fi-l-ta'ālīm" in *Science and Technology in the Islamic World*, Turnhout, 2002, pp 109-120. On his tables cf. J. Chabás & B.R. Goldstein, "Andalusian Astronomy: al-zīj al-Muqtabis of Ibn al-Kanmād" *Archive for the History of Exact Sciences* 48 (1994) pp. 1-41 and BR. Goldstein & J. Chabas, "Ibn al-Kammād's Star List" *Centaurus*, 38 (1996) pp. 317-334.

The Toledan Tables represent the first original development of Andalusī Astronomy, which was extremely influential in Europe up to the Scientific Revolution, namely the theory of trepidation which tries to justify two facts attested by observation: the obliquity of the ecliptic and the precession are not constant values. The theory of trepidation established that the equinoctial points had a very slow motion forwards and backwards along a limited arc of the ecliptic. A geometrical model justifying such a motion is designed by Ibrahim b. Sinān (908-946), the grandson of Thābit b. Qurra[10] and introduced in al-Andalus. Related to this there is a book entitled *Liber de Motu Octave Sphere* ("The book on the motion of the eighth sphere") traditionally attributed to Thābit b. Qurra although it can be probaly related to the work of Toledan astronomers.

This work was continued by Ibn al-Zarqālluh who is the greatest exponent of the theoretical astronomical activities of this period in which he showed his innovating ingenuity[11]. He is the author of a work on the motion of the fixed stars preserved in hebrew translation in which he studied three different models of trepidation. He devoted 25 years to solar observation and wrote a treatise on the solar theory which is not preserved although many references can be found in later works both Arabic and Latin[12]. From these sources we know that he is the first astronomer to state the proper motion of the apogee in about one degree (1°) in 279 Julian years. He also devised a model to justify the variable solar eccentricity which was well known in Europe up to the time of Copernicus.

← Andalusr Science", in Saliba and King eds. *From Deferent to Equant .~ A Volume of Studies in the History of Science in the Ancient and Medieval Near East in Honour of Prof. E.S. Kennedy* (New York, 1987) pp. 373-401.

10. In his Kitāb fī'harakāt al-shams. Cf. A. S. Sa'idan ed., Rasā'il Ibn Sinān (Kuwait 1983) pp. 274-304.

11. On Ibn al-Zarqālluh cf J. Samsó, *Las Ciencias de los Antiguos en al-Andalus*. Madrid, 1992, pp. 147-152, 166-240; J. Samsó, "Azarquiel e Ibn al-Banna" *Relaciones de Ja Peninsula Ibérica con el Magreb* (Siglos XIII-XVI) C.S.I.C. Madrid, 1989, pp. 36 1-372; J. Vemet "Al-Zarqālī" Dictionary of *Scientific Biography*. vol. XIV. New York, 1976, pp. 592-595 and E. Calvo, "Ibn al-Zarqālluh" *Encyclopaedia of the History of Science, Technology and Medicine in Non-Western Cultures*. Kluwer Academic Publishers. Dordrecht, 1997 , pp. 415-416.

12. On his solar theory cf Samsó, Las Ciencias... pp. 207-2 18; G.J. Toomer, "The Solar Theory of az-Zarqal. A History of Errors", Centaurus, 14 (1969) pp. 306-336., and G..J. Toomer, "The Solar Theory of Az-Zarqal: An Epilogue" in Saliba and King eds. *From Deferent to Equant*, pp. 513-519.

Although the use of these instruments was more difficult and entailed a greater effort of imagination than that of the standard astrolabe, they were greately influential in later astronomers and instrument makers in al-Andalus as well as in the Islamic Orient and also in the Latin Europe[6].

1.2.2 Astronomical Theories and astronomical tables or *zijes*

Although Andalusian astronomers knew Ptolemy's works at least from the 10[th] Century, they never fully abandoned the Indian tradition. There is a good example of this fact in the *Tabulae Jahen,* or "The *zij* of Jayyān", which is the city of Jaen in the south of Spain. Only the canons of this *zij* are extant, preserved in a Latin translation by Gerard of Cremona. This *zij* was an adaptation of Khwārizmī's *Sindhind* to the coordinates of Jaen. But the author, Ibn Mu'ādh al-Jayyānī (d. 1093), seems to be using a different version of the work from the one adapted to Cordoba by Maslama al-Majrītī a century earlier, and introduces more materials which are either new or derived from Ptolemaic sources[7]. The author demonstrat es awareness of developments in the Islamic Orient associated with al-Bīrūnī and his pre decessors[8].

The *Tabulae Jahen* were not as successful as the Toledan Tables which we only know trough a Latin translation (The *Tabulae Toletanae)* extant in a great number of manuscripts. These tables seem to be an adaptation of all the available astronomical materials (Indian, Helenistic) to the coordinates of Toledo. They were compiled in a short period of time around 1069 by a group of Toledan astronomers led by the *qādī* Sā'id (d. 1070), including Ibn al-Zarqālluh and 'Alī ibn Jalaf[9].

← posteriores" in Ochava Espera y Astrofísica (Barcelona, 1990) pp. 22 1-238 and E. Calvo "La labor de difusión de la cultura árabe por parte de Alfonso X y su contribución a la formación del lenguaje científico. Los Libros del Saber de Astronomía" In *La civilización islámica en al-Andalus y los aspectos de tolerancia* (Casablanca, 2003) pp. 27-42.

6. On this diffusion cf. for instance D. King, *Islamic Astronomical Instruments* (London, 1987) n. VII-X. E. Poulle, "Un instrument astronomique dans l'Occident Latin. La Saphea", in A Giuseppe Ermini (Spoleto, 1970) pp. 491-510; E. Calvo "*La lámina Universal de Ali b. Jalaf....* pp. 221-238.

7. For instance lunar and planetary models and the numerical parameters quoted in the canons derive from al-Khwārizmī but the table of solar and lunar equations was calculated according to a Ptolemaic exact method. Here we find a description of a table of constant precession, a topic not found in al-Khwārizmī's zīj

8. For instance, the division of the astrological houses and the so-called "method of the zījes" to determine the azimuth of the qibla.

9. On the Toledan Tables cf. G.J. Toomer, "A Survey of the Toledan Tables", *Osiris,* 15 (1968) pp. 5-174 and L. Richter Bernburg, "Sāþid, the Toledan Tables and →

Another mathematician was the *qādī* of Jaen Abū 'Abdallāh Muhammad ibn Muādh al-Jayyanī (d. 486/1093) from whom two works have been studied and published, namely his *Maqāla fi'Sharh al-Nisba* ("Commentary on the concept of ratio") which deals with a question that interested many other mathematicians in the islamic world as Ibn al-Haytham and Omar Khayyām. In this book we find a defence of Euclid's definition of ratio.

The second one is the *Kitāb majhulāt qisī al-kura* ("Unknown arcs of the Sphere"), the first treatise on spherical trigonometry compiled in al-Andalus, and the first devoted to this discipline independently from Astronomy. It is a complete treatise on spherical trigonometry in which he studies the solution of all possible cases of spherical triangles.

1.2 *Astronomy in the lifetithe of Khayyam*

As for astronomy, Andalusian astronomers were quite interested in the development of astronomical instruments, most of which were analog computers; only two of them were meant for use in observations, namely the armillary sphere described by Ibn al-Zarqālluh and the instrument designed by Jābir ibn Aflah which is regarded as the precedent of the *torquetum.*

1.2.1 Instruments

In this period, some new instruments, which seem of andalusian origin, were devised. The first of them is the equatorium described by Ibn al-Samh Ibn al-Zarqālluh and Abū-l-Salt (460/1067-529/11 34)[4] which consist in a set of Ptolemaic planetary models made to scale and are intended to make easier the calculations of planetary longitudes.

Another type instruments are the universal astrolabes devised by Ibn al-Zarqālluh and 'Alī ibn Jalaf which are intended to obviate the main shortcoming of the standard astrolabe in which a special plate is required for each latitude. The instruments devised by Ibn al-Zarqālluh and 'Alī ibn Jalaf are, as the astrolabe, the result of a stereographic projection but the centre of projection is the equinoctial points (Aries 0 and Libra 0) instead of the pole, and the plane of projection is the solsticial colure, instead of the equator. The horizon can be represented by a diameter of the instrument, which for instance is represented by a rotating ruler in Ibn al-Zarqālluh's instrument[5].

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4. On these instruments cf. M. Comes, Ecuatorios andalusies (Barcelona, 1990).

5. On the Ibn al-Zarqālluh' safīhas cf. R. Puig Al-sakkāziyya de Ibn al-Naqqāš al-Zarqālluh. Edición, traducción y estudio, Barcelona, 1986 and R. Puig, Los Tratados de Construcción y Uso de la Azafea de Azarquiel. I.H.A.C. Madrid, 1987. On 'Alī ibn Jalaf's universal instrument cf. E. Calvo "La lámina Universal de 'Alī b. Jalaf (s. XI) en la versión alfonsí y su evolución en instrumentos →

Ibn Tufayl, al-Bitrūjī and Ibn Rushd intended to modify the astronomical and cosmological theories of Ptolemy to make them accord to Aristotelian principles.

1.1 *Mathematics in al-Andalus in the 11th Century*[2]

As for Mathematics, the most prominent mathematicians of this period were al-Mu'taman, Ibn Sayyid and Ibn Mu'ādh.

King Abū 'Āmir Yūsuf ibn Ahmad al-Mu'taman of Saragossa was the author of an important treatise entitled *Kitāb al-Istikmāl* which has been studied by A. Djebbar and J. P. Hogendijk[3]. The latter has dicovered several incomplete manuscripts in which the remaining fragments of the treatise deal with number theory, plane geometry, conic sections and the geometry of the sphere. The extant parts of this work bear witness of the high mathematical level reached by the author who probably had at his disposition the best mathematical books available at that time including Euclid's *Elements*, Archimedes' *On the Sphere and Cilinder*, Theodosius and Menelaos' *Spherics*, Appolonius' *Conics*, Ptolemy's *Almagest*, Thābit ibn Qurra's treatise on the amicable numbers and on Menelaos' theorem, the Banū Mūsā on the measurement of plane and spherical figures, Ibn al-Haytham's *Optics* and some more. Al-Mu'taman does not limit himself to reproduce his sources since he very often offers original solutions which in some cases are more elegant than the precedent ones and he also gives some theorems that would be attributed later to European mathematicians.

Abū Zayd 'Abd al-Rahmāan ibn Sayyid flourished in Valencia between 456/1063 and 490/1096 and was the master of Muhammad ibn Yahyā ibn al-Sā'igh, known as Ibn Bajja (463/ 1070?-573/ 1138) who was a great philosopher and physicist. None of the writtings of Ibn Sayyid is extant but we know that he worked on arithmetical series, following the tradition of the *Arithmetic* of Nichomacus of Gerasa. His disciple, Ibn Bajja gives some information about his research in Geometry, in which he studied conic sections and such classical problems as the trisection of the angle.

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2. On this topic cf. A. Djebbar, "Las matemáticas en al-Andalus a través de las actividades de tres sabios del siglo XI" in *El Legado Cientifico andalusí* (Madrid, 1992) pp. 23-35; id., Deux mathématiciens peu connus de l 'Espagne du XI siècle: al-Mu'taman et Ibn Sayyid (Paris, 1984).

3. Cf. J.P. Hogendijk, "Al-Mu'taman ibn Hud, 11th Century King of Saragossa and Brilliant Mathematician" *Historia Mathematica*, 22 (1995) pp. 1-18; id. "The Geometrical parts of the Istikmāl of Yūsuf al-Mu'taman ibn Hud (11th Century). An analytical Table of Contents "*Archives Internationales d'Histoire des Sciences* and id., "Al-Mu'taman's Simplified Lemmas for Solving Alhacen's Problem" in *From Baghdad to Barcelona*, vol. I, pp. 59-101.

Science in al-Andalus in the lifetime of Khayam

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Introduction

The aim of this paper is to describe the scientific activities in al-Andalus in the lifetime of Khayām, i.e., the end of the 11th Century and the beginning of the 12th, with special attention to the so called Zarqāllian tradition in astronomical theory. The lifetime of Khayyām coincides with the most fruitful period in the scientific activities of al-Andalus. This period corresponds to the end of the petty kingdoms and the beginning of the first of the African invasions of the peninsula, namely the Almoravid.

1. *The Exact Sciences in al-Andalus in the lifetime of Khayyām*[1]

The 11th century is considered the golden period in the history of Exact Sciences in al-Andalus. This is the result of the scientific activities of the precedent times: The middle of the 9th century is the beginning of a stream of orientalization of the Andalusian culture which made that every single innovation coming from Orient was introduced, adopted and regarded as the expression of the highest refinement. The 10th is the first of three centuries in which Andalusian science flourished. The 12th Century is the age of the philosophers in al-Andalus, since a school with members such as Ibn Bajja,

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1. For a general view of the topic cf. J. Samsó, «The Exact Sciences in al-Andalus» in *The Legacy of Muslim Spain* Salma Khadra Jayyusi ed. E. J. Brill (Leiden, 1992) pp. 952-973.